Joint Bandwidth and Power Allocation for MIMO Two-Way Relays-Assisted Overlay Cognitive Radio Systems

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Abstract—This paper studies the achievable cognitive sum rate of an overlay cognitive radio (CR) system assisted with multiple antennas two-way relays in which primary users (PUs) cooperate with cognitive users (CUs) for mutual benefits. In this context, the problem of both bandwidth and power allocation is investigated. We propose that the CUs are allowed to allocate a part of the PUs spectrum to perform their cognitive transmission. In return, acting as amplify-and-forward two-way relays, they are exploited to support PUs to reach their target data rates over the remaining bandwidth. Power expressions for optimal transmit power allocated per PU and CU antenna are derived under primary quality-of-service constraint in addition to bandwidth and power budget constraints. More specifically, CUs act as relays for the PUs transmission and gain some spectrum as long as they respect these constraints. After deriving the optimal transmit powers, we employ a strong optimization tool based on swarm intelligence to optimize the full and complex relay amplification gain matrices in addition to the bandwidths released to primary and cognitive transmission. Furthermore, three different utility functions are considered in our optimization problems depending on the level of fairness among CUs.

Index Terms—Overlay cognitive radio systems, two-way relaying technique, multiple-input multiple-output antennas, particle swarm optimization algorithm.

I. INTRODUCTION

A. Background

Cognitive radio (CR) is proposed as an intelligent novel approach to solve the spectrum deficit problem [1]. The basic idea of CR is that cognitive users (CUs) are allowed to utilize the spectrum of licensed users which are also known as primary users (PUs) in an opportunistic fashion. CR can be grouped into three main techniques; interweave, underlay, and overlay [2]. The original idea of cognitive radio was based on the interweave technique, where the CUs are allowed to access

the primary spectrum only when the PUs are inactive [3]. In other words, CUs sense primary spectrum holes before utilizing them. In the underlay technique, CUs access the spectrum simultaneously with PUs under some interference limitation constraints to maintain a certain quality-of-service (QoS) of the primary transmission [4]. In the overlay technique, CUs are allowed to use a part of their resources to enhance the primary signal. In counterpart, the PUs may release some bandwidths for CUs data transmission [5]. This technique can be exploited with cooperative relaying techniques to enhance the system performance.

Two-way relaying (TWR), has lately attracted a lot of attention in the literature. In conventional TWR, exchanging different messages between two terminals takes place into two phases only to accomplish the transmission instead of four phases in the traditional one-way relaying. In the first phase, the users transmit their signals simultaneously to the relays. Subsequently, in the second phase, the relays broadcast the signal to the users [6]. The authors in [7] investigated the performance and the spectral efficiency of the TWR transmission and compared them to the traditional one-way relaying transmission. To perform this, several relay strategies are used: decode-and-forward (DF), compress-and-forward (CF), and amplify-and-forward (AF), [8], [9]. In DF protocol, the relay decodes the received signal and removes the noise before transmitting a clean copy of the original signal to the destination, while in CF protocol, the relay compresses the received signal from the source node and forwards it to the destination without decoding the signal. In AF protocol, the relay amplifies the received signal before broadcasting it to the destination. In our framework, we are interested in AF protocol due to its low computational complexity and low delay in the relay node (i.e., AF allows faster transmission without processing delay). Moreover, AF requires much less computing power compared to DF and CF protocols.

The multiple-input multiple-output (MIMO) technique provides more degrees of freedom to the system to enhance the system throughput. Various studies have employed MIMO antennas with TWR [10], [11]. Moreover, many studies employed TWR system in conjunction with interweave and underlay cognitive modes in order to minimize the total power consumption or to enhance the cognitive sum rate while respecting the primary QoS, respectively, [12], [13]. The work in [12] and [13] compared the single and multiple antenna cases in underlay cognitive scenario.

B. Overlay Cognitive Radio: Literature Review

Leasing model, which is also called property rights model, can be categorized into two categories; spectrum leasing and time leasing. For spectrum leasing, PUs who own the spectrum can possibly lease a part of the spectrum to CUs for appropriate remuneration and can exploit the existence of CUs to enhance their performance and QoS. In counterpart, CUs can use the leased spectrum for their own transmission by performing decentralized power control [14]–[16]. On the other hand, for time leasing, PUs can lease a portion of their time for CUs' transmission. In return, in the remaining time, CUs help the PUs to enhance their performance. Indeed, the data transmission takes place in three time slots, in the first time slot, PUs transmit their signals to CUs, while in the second time slot, CUs broadcast the primary signal to the primary destination, finally, the CUs use the remaining time slot for their own transmission [17], [18]. In this paper, the "overlay" model term is used to indicate the spectrum leasing model.

Overlay CR technique has been introduced in literature as a solution to enhance the spectral efficiency of primary transmissions while exploiting the existence of CUs [19]. Most of the studies model CUs as one-way relays that DF the primary signals to the destination in order to improve the system reliability. This operation requires the knowledge of the primary user's data sequence and/or codebook [19]. Furthermore, in overlay CR, CUs need to know the primary channel gains in addition to encoding techniques if they will decode the primary signal. However, compared to interweave and underlay CR, overlay offers to CUs the possibility to freely transmit their signals without any constraints in terms of time and transmitted power. It just requires that PUs know the existence of overlaying cognitive relays in order to coordinate their transmissions. Note here that priority is given to PUs in contrast with the non-cognition case where relays have the unique role of forwarding the primary signals.

Few work have employed TWR systems with overlay CR technique [20]–[24]. These studies assume the absence of direct link between primary terminals and utilize CU relays to forward their signals simultaneously. This scenario overcomes the hidden terminal problem which happens when the PUs are shadowed or are in severe multipath fading. In return, CUs are allowed to share a part of the primary bandwidth to perform their communication. This spectrum sharing scenario might involve some forms of coordination and cooperation between the two types of users (i.e., primary and secondary users), mainly when this cooperation is optimized. For instance, this can be implemented when all users belong to the same network as suggested in [20]. The authors have considered a device-to-device (D2D) communication scenario overlaying a cellular network where D2D users, playing the role of CUs, communicate bi-directionally with each other while assisting the two-way communications between a cellular user and its base station. The work in [21] and [22] proposed a typical model comprising a pair of PUs and a pair of CUs. The objective was to find an optimal power allocation at the single relaying cognitive node that minimizes the outage probability

at the cognitive receiver for given outage constraints on the primary system. A joint relay selection and resource allocation algorithm for TWR overlay CR networks is also proposed in [23], where the best relays (CUs) with higher channel gain are selected to act as relays to help for primary transmission.

C. Contributions

In this paper, we investigate the problem of bandwidth and power allocation for TWR-MIMO overlay cognitive networks using multiple AF relays. The AF scheme is less complex than the DF one as it does not require the decoding of the primary signals in the relay level. Therefore, the knowledge of the primary sequences and codebooks is not required with the AF scheme. The contributions of this paper can be summarized as follows:

- We formulate an optimization problem that maximizes the TWR-MIMO overlay cognitive rate while taking into account all transceiver power budgets in addition to the PU QoS requirements. In our framework, we assume that each CU is allowed to share the primary bandwidth in order to perform its transmission. In return, they are engaged to complete the primary transmission by amplifying and forwarding the PU data over the remaining bandwidth.
- Due to the non-convexity of the problem, we firstly derive expressions of the transmit powers allocated to primary and cognitive users for a fixed user bandwidth and relay amplification gain. Then, we employ a meta-heuristic approach based on particle swarm optimization (PSO) algorithm to find sub-optimal CU bandwidth allocation in addition to the relay amplification matrix gains.
- We consider different cognitive objective functions depending on the level of fairness among CUs.
- Finally, we analyze the performance of our scheme under different system parameters and we compare the algorithm performance in terms of convergence speed and computational complexity with a recently proposed heuristic approach entitled the grey wolf optimizer (GWO) [25].

D. Paper Organization

The remainder of this paper is organized as follows. Section II presents the MIMO-TWR overlay cognitive system model using AF relays. The problem formulation is described in Section III. Section IV gives optimal power expressions for the different considered utilities and describes the PSO algorithm for bandwidth and relay gain optimization. The numerical results are discussed in Section V. Finally, the paper is concluded in Section VI.

We need to make some notes about the notations used in this paper. The superscripts $(.)^T$ and $(.)^H$ correspond to the transpose and the hermitian operator, respectively. $\mathbb{C}$ and $\mathbb{R}$ denote the field of complex and real numbers, respectively. $\mathbb{E}(.)$ and $\text{Tr}(.)$ denote the expectation and the trace operator. $(x)^+$ denotes a maximum between x and zero.

II. System Model

We consider an overlay half duplex CR network with two primary users PU_1 and PU_2 in addition to a cognitive network consisting of L CUs and one cognitive base station. All nodes are equipped with M antennas. A non-line of sight link between PU_1 and PU_2 is considered as illustrated in Fig. 1. The L CUs act as two-way relays for the PUs over the primary bandwidth. In exchange, the PUs may release some of their bandwidths to the CUs to accomplish their own data transmission as long as the PUs maintain their QoS.

Let T denote the time duration that a primary user is allowed to transmit data over the bandwidth B_{tot}. In our overlay MIMO-CR scheme, we assume that the total bandwidth is divided into $L+1$ fractions denoted B_0, B_1,...,B_L, where the primary transmission is held over B_0 while for each lth CU, we allocate the bandwidth fraction B_l such that there is no inter-user interference between all the primary and secondary nodes $\left(\sum_{l=0}^{L} B_l = B_{\text{tot}}\right)$, as shown in Fig. 2.

Assuming independent and identically distributed (i.i.d) complex Gaussian signals, PUs exchange their messages via L CUs as follows: In the first phase, both PU_1 and PU_2 transmit their messages $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ simultaneously to the L CUs with a covariance power matrix of vector $\boldsymbol{x}_t$ denoted $\boldsymbol{P}_t = \mathbb{E}(\boldsymbol{x}_t \boldsymbol{x}_t^H)$ Watt/Hz (power per unit frequency), where $t = \{1, 2\}$. Perfect synchronization between PU_1 and PU_2 is assumed [26]. In the second phase, the CUs play the role of relays by transmitting the amplified signal to the PUs with a covariance power matrix denoted $\boldsymbol{P}_{r_l}$ Watt/Hz, where $l = 1, ..., L$. During PUs transmission and reception, the CUs transmit their data $\boldsymbol{x}_{c_l}$ to the cognitive base station over the remaining bandwidth (i.e., B_1,...,B_L) with a covariance power matrix denoted $\boldsymbol{P}_{c_l} = \mathbb{E}(\boldsymbol{x}_{c_l} \boldsymbol{x}_{c_l}^H)$ Watt/Hz, where $l = 1, ..., L$.
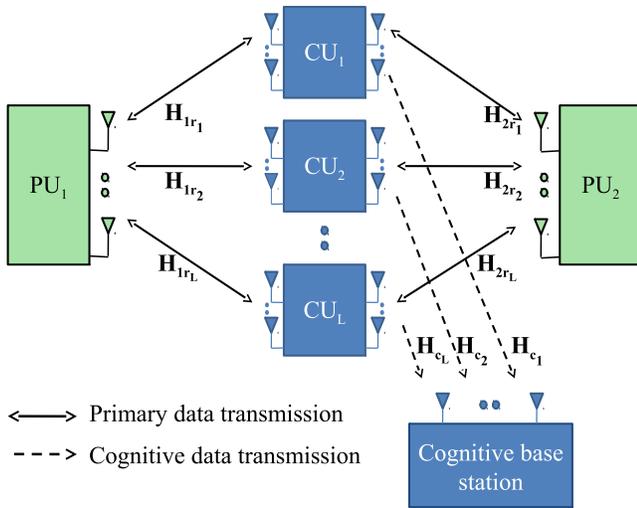
Fig. 1 Overlay TWR-MIMO system model.

Let us define $\bar{E}_p$ and $\bar{E}_c$ as the peak energy at each PU and the peak energy at each CU, respectively. $\boldsymbol{H}_{1r_l} \in \mathbb{C}^{M \times M}$, $\boldsymbol{H}_{2r_l} \in \mathbb{C}^{M \times M}$, and $\boldsymbol{H}_{c_l} \in \mathbb{C}^{M \times M}$ are the MIMO channel gains in the first time slot between PU_1 and the lth
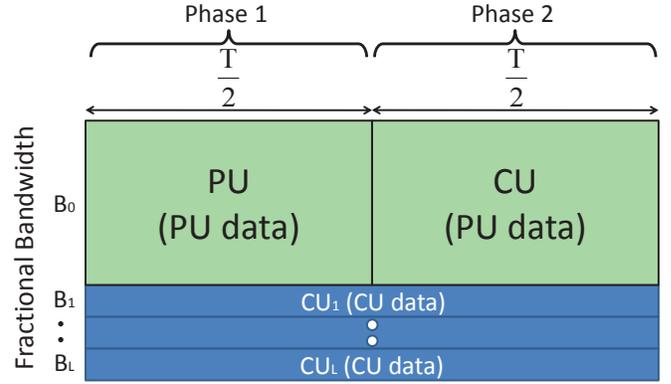
Fig. 2 Time-bandwidth allocation.

CU, PU_2 and the lth CU, and the lth CU and the cognitive base station, respectively. Where in the second time slot the MIMO channel gains are denoted by $\hat{\boldsymbol{H}}_{1r_l} \in \mathbb{C}^{M \times M}$, $\hat{\boldsymbol{H}}_{2r_l} \in \mathbb{C}^{M \times M}$, and $\hat{\boldsymbol{H}}_{c_l} \in \mathbb{C}^{M \times M}$. All the channel gains are assumed to be i.i.d fading channel gains and constant during the coherence time. If the channels are highly correlated during two consecutive time slots, then channel reciprocity is assumed i.e., $\boldsymbol{H}_{1r_l} = \hat{\boldsymbol{H}}_{1r_l}, \boldsymbol{H}_{2r_l} = \hat{\boldsymbol{H}}_{2r_l}, \boldsymbol{H}_{c_l} = \hat{\boldsymbol{H}}_{c_l}$ In addition to that, perfect channel state information at transmitters and receivers are considered. Without loss of generality, all the noise variances are assumed to be equal to $\mathcal{N}_0$.

Let $\boldsymbol{V}_t$ and $\boldsymbol{U}_t$, where $t \in \{1, 2\}$ be two unitary precoder and decoder matrices, respectively, employed by PUs. In the first phase, PU_t employs the precoder matrix $\boldsymbol{V}_t$ such as: $\boldsymbol{x}_t = \boldsymbol{V}_t \tilde{\boldsymbol{x}}_t$ where $\boldsymbol{x}_t$ is the transmitted signal after being precoded by PU_t. Subsequently, during the second phase, PU_t employs the decoder matrix $\boldsymbol{U}_t$ such as: $\boldsymbol{r}_t = \boldsymbol{U}_t^H \boldsymbol{y}_t$, where $\boldsymbol{y}_t$ and $\boldsymbol{r}_t$ are the received signals at PU_t before and after decoding, respectively. The choice of $\boldsymbol{V}_t$ and $\boldsymbol{U}_t$ will be defined later.

The approach of the paper was initially designed to a scenario where PUs and CUs belong to the same legacy system; e.g. LTE with LTE device-to-device (D2D) communications. In the case where it must be extended to deal with different legacy scenario, then the following rules apply:

- Each CU needs to support both standards.
- Each CU needs to have two separate RF chains: one to be used for the secondary transmissions over the secondary system (e.g., 802.11ac standard) and one to be used over the primary system (e.g., LTE standard) to relay PUs' data.
- This requires doubling the number of antennas since transmission over both systems will be simultaneous: If M is the current number of antennas in the paper, each CU would need "$2M$" antennas: M to be used over the primary system (LTE) and another M to be used over the secondary system.
- This will lead to increase the costs of the CU devices, but they would still be able to use the spectrum for free.
- It should be noted that the different legacy scenario assumes that the two standards use overlapping spectrum bands. Otherwise, PUs cannot share a portion of their bandwidth with CUs, since the technology used by CUs

would be operational on different frequency bands.

III. PROBLEM FORMULATION

In this section, we formulate an optimization problem that maximizes the cognitive objective function for multiple MIMO TWR-CR networks while satisfying the required QoS of the PUs. Different utility metrics leading to different bandwidth and power allocation are presented and discussed depending on the cognitive objective. Without loss of generality, channel reciprocity is assumed.

A. Primary Data Rate

In the first phase, the baseband received signal at the lth CU over B_0 is given as follows

$$\boldsymbol{y}_{r_l} = \boldsymbol{H}_{1r_l}\boldsymbol{x}_1 + \boldsymbol{H}_{2r_l}\boldsymbol{x}_2 + \boldsymbol{n}_{r_l}, \tag{1}$$

where $\boldsymbol{n}_{r_i}$ is the additive Gaussian noise at the lth relay and $\boldsymbol{x}_t$ is the transmitted signal after precoding by PU_t, where $t \in \{1, 2\}$. During the second phase, each relay CU amplifies $\boldsymbol{y}_{r_l}$ by multiplying it by a full matrix $\boldsymbol{G}_l \in \mathbb{C}^{M \times M}$ and broadcasting it to the PUs. Finally, the received signals at PU_1 and PU_2 are respectively given as

$$\boldsymbol{y}_1 = \underbrace{\hat{\boldsymbol{\Phi}}\boldsymbol{x}_1}_{\text{Self Interference}} + \boldsymbol{\Phi}\boldsymbol{x}_2 + \boldsymbol{z}_1, \tag{2}$$

$$\boldsymbol{y}_2 = \boldsymbol{\Psi}\boldsymbol{x}_1 + \underbrace{\hat{\boldsymbol{\Psi}}\boldsymbol{x}_2}_{\text{Self Interference}} + \boldsymbol{z}_2, \tag{3}$$

where $\boldsymbol{\Phi} = \sum_{l=1}^{L} \boldsymbol{H}_{1r_l}^T \boldsymbol{G}_l \boldsymbol{H}_{2r_l}, \hat{\boldsymbol{\Phi}} = \sum_{l=1}^{L} \boldsymbol{H}_{1r_l}^T \boldsymbol{G}_l \boldsymbol{H}_{1r_l}, \boldsymbol{\Psi} = \sum_{l=1}^{L} \boldsymbol{H}_{2r_l}^T \boldsymbol{G}_l \boldsymbol{H}_{1r_l}$, and $\hat{\boldsymbol{\Psi}} = \sum_{l=1}^{L} \boldsymbol{H}_{2r_l}^T \boldsymbol{G}_l \boldsymbol{H}_{2r_l}$, are the equivalent MIMO channels obtained at PU_1 and PU_2, respectively before decoding. $\boldsymbol{z}_t = \sum_{l=1}^{L} \left(\boldsymbol{H}_{tr_l}^T \boldsymbol{G}_l \boldsymbol{n}_{r_l} \right) + \boldsymbol{n}_t$ and $\boldsymbol{n}_t$ are the equivalent amplified noise at PU_t before decoding and the additive Gaussian noise vectors at PU_t, respectively, where $t \in \{1, 2\}$. Using the knowledge of the channel information and channel reciprocity, the PUs can remove the self interference by eliminating their own signals (i.e., $\boldsymbol{x}_1$ for PU_1 and $\boldsymbol{x}_2$ for PU_2). Thus, the received signal $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ after employing the decoders $\boldsymbol{U}_t$ are given, respectively, by

$$\boldsymbol{r}_1 = \boldsymbol{U}_1^H \boldsymbol{\Phi}\boldsymbol{x}_2 + \tilde{\boldsymbol{z}}_1 = \boldsymbol{U}_1^H \boldsymbol{\Phi}\boldsymbol{V}_2\tilde{\boldsymbol{x}}_2 + \tilde{\boldsymbol{z}}_1 \tag{4}$$

$$\boldsymbol{r}_2 = \boldsymbol{U}_2^H \boldsymbol{\Psi}\boldsymbol{x}_1 + \tilde{\boldsymbol{z}}_2 = \boldsymbol{U}_2^H \boldsymbol{\Psi}\boldsymbol{V}_1\tilde{\boldsymbol{x}}_1 + \tilde{\boldsymbol{z}}_2 \tag{5}$$

where $\tilde{\boldsymbol{z}}_t = \boldsymbol{U}_t^H \boldsymbol{z}_t$ is the equivalent amplified noise at PU_t after decoding. The covariance matrix of the noise $\tilde{\boldsymbol{z}}_t$ can be given as

$$
\begin{aligned}
\boldsymbol{C}_{\tilde{\boldsymbol{z}}_t} &= \mathbb{E}[\tilde{\boldsymbol{z}}_t \tilde{\boldsymbol{z}}_t^H] \\
&= \mathcal{N}_0 \sum_{l=1}^{L} \boldsymbol{U}_t^H \boldsymbol{H}_{tr_l}^T \boldsymbol{G}_l (\boldsymbol{U}_t^H \boldsymbol{H}_{tr_l}^T \boldsymbol{G}_l)^H + \mathcal{N}_0 \mathrm{I}_M,
\end{aligned} \tag{6}
$$

where I_M denotes the identity matrix of size M.

Let us now define the unitary precoding and decoding matrices using the singular value decomposition (SVD) which converts the MIMO channel into parallel channels characterized by their associated eigenmodes. Thus, we perform SVDs for the matrices $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ as follows: $\boldsymbol{\Phi} = \boldsymbol{U}_1 \boldsymbol{S}_{\boldsymbol{\Phi}} \boldsymbol{V}_2^H$ and $\boldsymbol{\Psi} = \boldsymbol{U}_2 \boldsymbol{S}_{\boldsymbol{\Psi}} \boldsymbol{V}_1^H$, where $\boldsymbol{S}_{\boldsymbol{\Phi}}$ and $\boldsymbol{S}_{\boldsymbol{\Psi}}$ are diagonal matrices with square roots of the eigenvalues of matrix $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$, respectively. As such, the primary rates of the PU_1 and PU_2 after SVD can be respectively given as

$$R_{p_1} = \frac{B_0}{2} \sum_{m=1}^{M} \log_2 \left(1 + \frac{\boldsymbol{S}_{\boldsymbol{\Phi}}^2(m, m)\boldsymbol{P}_2(m, m)}{\boldsymbol{C}_{\tilde{\boldsymbol{z}}_1}(m, m)} \right), \tag{7}$$

$$R_{p_2} = \frac{B_0}{2} \sum_{m=1}^{M} \log_2 \left(1 + \frac{\boldsymbol{S}_{\boldsymbol{\Psi}}^2(m, m)\boldsymbol{P}_1(m, m)}{\boldsymbol{C}_{\tilde{\boldsymbol{z}}_2}(m, m)} \right). \tag{8}$$

The factor $\frac{1}{2}$ is added as primary transmission is held over two time slots. In order to meet the target transmission rate for the primary network, R_{p_1} [bits/s] and R_{p_2} [bits/s] should be no less than the primary target transmission rate R_0[bits/s], i.e.,

$$R_{p_1} \geq \alpha R_0 \quad \text{and} \quad R_{p_2} \geq (1 - \alpha)R_0, \tag{9}$$

where $\alpha \in [0, 1]$ is the rate profile of the system to characterize the boundary rates at the PUs. From (9), we can show that the fractional bandwidth needed for the primary network should satisfy

$$B_0 \geq \max \left(\frac{2\alpha R_0}{\sum_{m=1}^{M} \log_2 \left(1 + \frac{\boldsymbol{S}_{\boldsymbol{\Phi}}^2(m,m)\boldsymbol{P}_2(m,m)}{\boldsymbol{C}_{\tilde{\boldsymbol{z}}_1}(m,m)}\right)}, \frac{2(1 - \alpha)R_0}{\sum_{m=1}^{M} \log_2 \left(1 + \frac{\boldsymbol{S}_{\boldsymbol{\Psi}}^2(m,m)\boldsymbol{P}_1(m,m)}{\boldsymbol{C}_{\tilde{\boldsymbol{z}}_2}(m,m)}\right)} \right). \tag{10}$$

Therefore, the PUs may release the remaining fractional bandwidth $(B_{\text{tot}} - B_0)$ to CUs.

B. Secondary Data Rate

On other hand, the received signal at the cognitive base station from the lth CU over bandwidth B_l can be given as

$$\boldsymbol{y}_{c_l} = \boldsymbol{H}_{c_l}\boldsymbol{x}_{c_l} + \boldsymbol{n}_{c_l}, \tag{11}$$

where $\boldsymbol{n}_{c_l}$ is the additive Gaussian noise at the cognitive base station. Thus, the received signal $\boldsymbol{r}_{c_l}$ after decoding is given by

$$\boldsymbol{r}_{c_l} = \boldsymbol{U}_{c_l}^H \boldsymbol{H}_{c_l}\boldsymbol{x}_{c_l} + \boldsymbol{U}_{c_l}^H \boldsymbol{n}_{c_l} = \boldsymbol{U}_{c_l}^H \boldsymbol{H}_{c_l}\boldsymbol{V}_{c_l}\tilde{\boldsymbol{x}}_{c_l} + \boldsymbol{U}_{c_l}^H \boldsymbol{n}_{c_l}. \tag{12}$$

Define $\boldsymbol{H}_{c_l} = \boldsymbol{U}_{c_l}\boldsymbol{S}_{c_l}\boldsymbol{V}_{c_l}^H$, where $\boldsymbol{S}_{c_l}$ is a diagonal matrix with square roots of the eigenvalues of matrix $\boldsymbol{H}_{c_l}$. Therefore, the cognitive rate of lth CU at the cognitive base station can be expressed in [bits/s] as

$$R_{c_l} = B_l \sum_{m=1}^{M} \log_2 \left(1 + \frac{\boldsymbol{S}_{c_l}^2(m, m)\boldsymbol{P}_{c_l}(m, m)}{N_0} \right). \tag{13}$$

C. Optimization Problem

Recall that the PU and CU users have as energy budgets $\bar{E}_p$ and $\bar{E}_c$ expressed in Joules, respectively, and that the power budgets of PU and CU equal to $\bar{P}_p$ and $\bar{P}_c$ Watt, respectively. Thus, the energy budget constraints at the tth PU and l^{th} CU are over time duration T given respectively as

$$\frac{T}{2}B_0\text{Tr}(\boldsymbol{P}_t) \leq \bar{E}_p,$$

$$TB_l\text{Tr}(\boldsymbol{P}_{c_l}) + \frac{T}{2}B_0\text{Tr}\Big(\boldsymbol{\Omega}_{1r_l}\boldsymbol{P}_1\boldsymbol{\Omega}_{1r_l}^H + \boldsymbol{\Omega}_{2r_l}\boldsymbol{P}_2\boldsymbol{\Omega}_{2r_l}^H \quad (14)$$
$$+ N_0\boldsymbol{G}_l\boldsymbol{G}_l^H\Big) \leq \bar{E}_c.$$

or equivalently

$$\frac{B_0}{2}\text{Tr}(\boldsymbol{P}_t) \leq \bar{P}_p,$$

$$B_l\text{Tr}(\boldsymbol{P}_{c_l}) + \frac{B_0}{2}\text{Tr}\Big(\boldsymbol{\Omega}_{1r_l}\boldsymbol{P}_1\boldsymbol{\Omega}_{1r_l}^H + \boldsymbol{\Omega}_{2r_l}\boldsymbol{P}_2\boldsymbol{\Omega}_{2r_l}^H \quad (15)$$
$$+ N_0\boldsymbol{G}_l\boldsymbol{G}_l^H\Big) \leq \bar{P}_c.$$

where $\boldsymbol{\Omega}_{1r_l} = \boldsymbol{G}_l\boldsymbol{H}_{1r_l}$ and $\boldsymbol{\Omega}_{2r_l} = \boldsymbol{G}_l\boldsymbol{H}_{2r_l}$ are the MIMO equivalent channel gains.

Let $U(R_{c_l})$ denote the rate utility of the cognitive system. Thus, the optimization problem of MIMO TWR-CR with multiple relays that maximizes the rate utility while satisfying specific power budgets and target primary rate constraints can be formulated as

$$\underset{\boldsymbol{B},\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_{c_l},\boldsymbol{G}_l \geq 0}{\text{maximize}} \quad U(R_{c_l}) \quad (16)$$

subject to:
(C1: Power budget constraints at the primary users):

$$0 \leq \frac{B_0}{2}\text{Tr}(\boldsymbol{P}_t) \leq \bar{P}_p, \quad \forall t = 1, 2, \quad (17)$$

(C2: Power budget constraints at the cognitive users):

$$B_l\text{Tr}(\boldsymbol{P}_{c_l}) + \frac{B_0}{2}\text{Tr}\Big(\boldsymbol{\Omega}_{1r_l}\boldsymbol{P}_1\boldsymbol{\Omega}_{1r_l}^H + \boldsymbol{\Omega}_{2r_l}\boldsymbol{P}_2\boldsymbol{\Omega}_{2r_l}^H$$
$$+ N_0\boldsymbol{G}_l\boldsymbol{G}_l^H\Big) \leq \bar{P}_c, \quad \forall l = 1, ..., L, \quad (18)$$

(C3: Rate constraint for the first primary user):

$$R_{p_1} \geq \alpha R_0, \quad (19)$$

(C4: Rate constraint for the second primary user):

$$R_{p_2} \geq (1 - \alpha)R_0, \quad (20)$$

(C5: Total bandwidth constraint):

$$\sum_{l=0}^{L} B_l = B_{\text{tot}}, \quad (21)$$

where $\boldsymbol{B} = [B_0, B_1, ..., B_L]$ is the vector that contains the fractions of bandwidth assigned to primary and cognitive transmissions. Constraints (17) and (18) represent the peak energy constraints at PUs and CUs, respectively. The term $\text{Tr}\Big(\boldsymbol{\Omega}_{1r_l}\boldsymbol{P}_1\boldsymbol{\Omega}_{1r_l}^H + \boldsymbol{\Omega}_{2r_l}\boldsymbol{P}_2\boldsymbol{\Omega}_{2r_l}^H + N_0\boldsymbol{G}_l\boldsymbol{G}_l^H\Big)$ in constraint (18) is equivalent to the relay amplified power of the lth CU.

D. Utility Selection

In this section, we characterize three different utility metrics that will be employed in the optimization problem (16).

Max C/I Utility:
The utility of this metric is equivalent to the sum data rate of the cognitive network $U(R_{c_l}) = \sum_{l=1}^{L} R_{c_l}$. This approach is known in the literature as Max C/I [27] as it promotes users with favorable channel and interference conditions by allocating to them most of the resources, whereas users suffering from higher propagation losses and/or interference levels will be deprived from the bandwidth as well as the power and will have very low data rates. Note that, thanks to the employed overlay scheme, and thus, the elimination of user interference, the Max C/I utility promotes users with favorable channels conditions only.

Max-Min Utility:
Due to the unfairness of Max C/I resource allocation, the need for more fair utility metrics arises. Max-Min utilities are a family of utility functions attempting to maximize the minimum data rate in the network $U(R_{c_l}) = \min_l(R_{c_l})$ [28]. By increasing the priority of users having lower rates, Max-Min utilities lead to more fairness in the network. In order to simplify the problem for this approach, we define a new decision variable $R_{\min} = \min_l(R_{c_l})$. Therefore, our optimization problem becomes

$$\underset{\boldsymbol{B},\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_c,\boldsymbol{G}_l,R_{\min} \geq 0}{\text{maximize}} \quad R_{\min} \quad (22)$$

subject to:

$$R_{c_l} \geq R_{\min} \quad \forall l = 1, ..., L, \quad (23)$$
$$(C1), (C2), (C3), (C4), (C5). \quad (24)$$

Proportional Fair Utility:
A tradeoff between the maximization of the sum rate and the maximization of the minimum rate could be the maximization of the geometric mean data rate $U(R_{c_l}) = (\prod_{l=1}^{L} R_{c_l})^{1/L}$ [29]. The proportional fair (PF) metric is fair, since a user with a data rate close to zero will make the whole product go to zero. Hence, any algorithm maximizing the geometric means would avoid having any user with very low data rate. In addition to this, the metric will reasonably promote users with good wireless channels (capable of achieving high data rates), since a high data rate will contribute in increasing the product.

IV. Optimal Power Allocation and Particle Swarm Optimization Algorithm

The formulated optimization problem is a non-convex problem and its optimal solution remains unsolved. For this reason, we propose to solve it in two steps. In the first step, we derive power expressions at each iteration for the optimal transmit primary powers (i.e., $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$) and cognitive transmit powers (i.e., $\boldsymbol{P}_{c_l}$) by assuming fixed bandwidths

of both primary and cognitive users and fixed amplification matrix gains (equivalent to fixed relay power), at all CUs. As a result, we convert our formulated problem to a convex one. The primal-dual method is used due to its simplicity and to the fact that it provides an expression of the power allocation per each antenna for the different utility functions. This can help in interpreting the behavior of each terminal in the network thanks to the water-filling expressions that will be derived next. Then, we propose to employ the subgradient method in order to optimize the Lagrangian multipliers. Note that the interior-point method could be also employed to solve the problem by finding numerically the optimal solution using the Newton method. Although the convergence of the subgradient method is slower than the interior-point method, the subgradient method remains competitive mainly for large scale problems as it requires little storage [30]. In the second step, we employ swarm intelligence to jointly optimize the system bandwidths with the CU amplification gain matrices.

A. Optimal Transmit Power Allocation

We can solve our convex optimization problem for fixed $\boldsymbol{B}$ and $\boldsymbol{G}_l, \forall l = 1, ..., L$, by exploiting its strong duality as follows [30]:

$$\underset{\boldsymbol{\lambda} \geq 0}{\text{minimum}} \quad \underset{\boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_{c_l} \geq 0}{\text{maximum}} \quad \mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_{c_l}), \quad (25)$$

where $\mathcal{L}$ is the Lagrangian function [30] which is given in (26). $\boldsymbol{\lambda}$ is a vector that contains all the Lagrangian multipliers of the system, where $\lambda_{p_t}, \lambda_{c_l}, \lambda_{th_t}$ and λ_B, represent the Lagrangian multipliers related to the peak power budget constraint at the tth PU, peak power budget constraint at the lth CU, the primary target rate constraint for the tth PU, and the bandwidth constraint, respectively. It includes also $\lambda_{R_l}, l = 1, \cdots, L$ related to constraint (23) if the Max-Min utility is used. By taking the derivative of the Lagrangian with respect to the $P_t(m, m)$, and $P_{c_l}(m, m)$ where $t \in \{1, 2\}, m = 1, ..., M$ and $l = 1, ..., L$, we can find the optimal primary power allocated to the mth antenna at PU_t as well as the optimal transmit powers allocated to the mth antenna at CU_l that maximize the Lagrangian function and, consequently, the cognitive utility rate. Since the primary powers are independent of the cognitive utility expression, we can derive the closed-form expressions of $P_1(m, m)$ and $P_2(m, m)$ at each iteration for fixed bandwidth and relay amplification factors as given in (27), where $\ln(2)$ is the natural logarithm of 2. However, the expression of the lth CU transmit power depends on the utility approach as follows:

Max C/I Utility: For max C/I utility, the lth CU transmit power over the mth antenna can be expressed as

$$P_{c_l}(m, m) = \left(\frac{1}{\lambda_{c_l} \ln(2)} - \frac{\mathcal{N}_0}{\boldsymbol{S}_{c_l}^2(m, m)} \right)^+ . \quad (28)$$

We can see from (28) that the value of the $P_{c_l}(m, m)$ depends on λ_{c_l} related to constraint (18) (i.e., corresponding to the primary powers and primary bandwidth). Also, it depends on the channel values between the CUs and cognitive base station. In this approach it is clear that, all resources are allocated to the CUs with favorable channel conditions.

Max-Min Utility: By taking the derivative of the Lagrangian of (22)-(24) with respect to $P_{c_l}(m, m)$ and equating it to zero, the lth CU transmit power over the mth antenna can be derived as

$$P_{c_l}(m, m) = \left(\frac{\lambda_{R_l}}{\lambda_{c_l} \ln(2)} - \frac{\mathcal{N}_0}{\boldsymbol{S}_{c_l}^2(m, m)} \right)^+ . \quad (29)$$

By taking the derivative of the Lagrangian with respect to $R_{\min}$, we can deduce that $\sum_l^L \lambda_{R_l} = 1$, which means that $\lambda_{R_l} \in [0, 1]$. By comparing (29) with (28), we can see that λ_{R_l} values control the priority of the resource allocation. However, enhancing the worst channel condition (i.e., corresponding to the minimum rate achieved) could come at the expense of users with good channel conditions which leads to more fairness between the CUs.

Proportional Fair Utility: For PF utility, the lth CU transmit power over the mth antenna can be derived as

$$P_{c_l}(m, m) = \left(\frac{1}{\lambda_{c_l} \ln(2)} \prod_{\substack{k=1 \\ k \neq l}}^{L} B_k.$$
$$\sum_{m=1}^{M} \log_2 \left(1 + \frac{P_{c_k}(m, m) \boldsymbol{S}_{c_k}^2(m, m)}{\mathcal{N}_0} \right) - \frac{\mathcal{N}_0}{\boldsymbol{S}_{c_l}^2(m, m)} \right)^+ . \quad (30)$$

In this approach, a tradeoff between the maximization of the sum rate and the maximization of the minimum rate can be clearly deduced in (30). The lth CU transmit power over the mth antenna $P_{c_l}(m, m)$ depends directly on the sum rate of other antennas at the same CU and the product of the other CUs rates. This approach tries to avoid having any user with very low data rate and maximize the product of the CUs rates simultaneously.

For all utilities, we can employ the subgradient method to find the optimal Lagrangian multipliers of this problem [31]. Hence, to obtain the solution, we can start with any initial values for the different Lagrangian multipliers and evaluate the optimal powers. We then update the Lagrangian multipliers at the next iteration $(i + 1)$ as follows

$$\lambda_{p_t}^{(i+1)} = \lambda_{p_t}^{(i)} - \delta_{p_t}^{(i)} \left(\bar{P}_p - \frac{B_0}{2} \text{Tr}(\boldsymbol{P}_t) \right), \forall t = 1, 2, \quad (31)$$

$$\lambda_{c_l}^{(i+1)} = \lambda_{c_l}^{(i)} - \delta_{c_l}^{(i)} \left(\bar{P}_c - \left(B_l \text{Tr}(\boldsymbol{P}_{c_l}) + \frac{B_0}{2} \text{Tr} \left(\boldsymbol{\Omega}_{1r_l} \boldsymbol{P}_1 \boldsymbol{\Omega}_{1r_l}^H + \boldsymbol{\Omega}_{2r_l} \boldsymbol{P}_2 \boldsymbol{\Omega}_{2r_l}^H + N_0 \boldsymbol{G}_l \boldsymbol{G}_l^H \right) \right) \right),$$
$$\forall l = 1, ..., L, \quad (32)$$

$$\lambda_{th_1}^{(i+1)} = \lambda_{th_1}^{(i)} - \delta_{th_1}^{(i)} (R_{p_1} - \alpha R_0), \quad (33)$$

$$\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_c) = U(R_{c_l}) - \sum_{t=1}^{2} \lambda_{p_t} \left(\tfrac{B_0}{2} \text{Tr}(\boldsymbol{P}_t) - \bar{P}_p \right) - \sum_{l=1}^{L} \lambda_{c_l} \left(B_l \text{Tr}(\boldsymbol{P}_{c_l}) + \tfrac{B_0}{2} \text{Tr}\left(\boldsymbol{\Omega}_{1r_l} \boldsymbol{P}_1 \boldsymbol{\Omega}_{1r_l}^{H} + \boldsymbol{\Omega}_{2r_l} \boldsymbol{P}_2 \boldsymbol{\Omega}_{2r_l}^{H} \right) \right.$$
$$\left. + N_0 \boldsymbol{G}_l \boldsymbol{G}_l^{H} \right) - \bar{P}_c \right) + \lambda_{th_1} \left(R_{p_1} - \alpha R_0 \right) + \lambda_{th_2} \left(R_{p_2} - (1 - \alpha) R_0 \right) - \lambda_B \left(\sum_{l=0}^{L} B_l - B_{\text{tot}} \right) + \sum_{l=1}^{L} \lambda_{R_l} \left(R_{c_l} - R_{\min} \right). \tag{26}$$

$$P_1(m,m) = \left(\frac{\lambda_{th_2}}{\ln(2) \left(\lambda_{p_1} + \sum\limits_{l=1}^{L} \lambda_{c_l} |\boldsymbol{\Omega}_{1r_l}(m,m)|^2 \right)} - \frac{\boldsymbol{C}_{\tilde{\boldsymbol{z}}_2}(m,m)}{\boldsymbol{S}_{\Psi}^2(m,m)} \right)^{+}, \quad P_2(m,m) = \left(\frac{\lambda_{th_1}}{\ln(2) \left(\lambda_{p_2} + \sum\limits_{l=1}^{L} \lambda_{c_l} |\boldsymbol{\Omega}_{2r_l}(m,m)|^2 \right)} - \frac{\boldsymbol{C}_{\tilde{\boldsymbol{z}}_1}(m,m)}{\boldsymbol{S}_{\Phi}^2(m,m)} \right)^{+}. \tag{27}$$

$$\lambda_{th_2}^{(i+1)} = \lambda_{th_2}^{(i)} - \delta_{th_2}^{(i)} \left(R_{p_2} - (1 - \alpha) R_0 \right), \tag{34}$$

$$\lambda_{B}^{(i+1)} = \lambda_{B}^{(i)} - \delta_{B}^{(i)} \left(B_{\text{tot}} - \left(\sum_{l=0}^{L} B_l \right) \right), \tag{35}$$

$$\lambda_{R_l}^{(i+1)} = \lambda_{R_l}^{(i)} - \delta_{R_l}^{(i)} \left(R_{c_l} - R_{\min} \right), \forall l = 1, ..., L. \tag{36}$$

where $\delta_{p_t}^{(i)}, \delta_{c_l}^{(i)}, \delta_{th_1}^{(i)}, \delta_{th_2}^{(i)}, \delta_{B}^{(i)}$ and $\delta_{R_l}^{(i)}$ are the updated step size according to the nonsummable diminishing step length policy (see [31] for more details). The updated values of the optimal powers and the Lagrangian multipliers are repeated until convergence.

B. Particle Swarm Optimization Algorithm

In the second step, we employ the PSO algorithm to optimize $\boldsymbol{B}$ and $\boldsymbol{G}_l, \forall l = 1, ..., L$. The PSO idea was introduced in 1995 [32] and it is inspired by swarm intelligence, social behavior, and food searching by a flock birds and a school of fish. This approach is widely used in several wireless communication fields due to its simplicity and efficiency [33]–[35]. Due to the following advantages of PSO compared with the other heuristic approaches, we apply it for solving this problem: (i) simple search process and easy to implement by manipulating few numerical parameters (e.g., such as the number of particles and acceleration factors for PSO) (ii) it requires low computational cost attained from small number of agents; and (iii) it provides a good convergence speed [36]. Thus, we propose to implement it to find near-optimal relay amplification gains and bandwidth fractions.

First, the PSO generates N random particles (i.e., a vector contains random $\boldsymbol{B}$ and $\boldsymbol{G}_l, \forall l = 1, ..., L$) $\boldsymbol{W}^{(n)}$, $n = 1, \cdots, N$, of length $1 \times (L(2M)^2 + (L+1))$ to form an initial population set $\mathcal{S}$, where $(2M)$ corresponds to the fact that during the PSO algorithm we optimize complex amplification gain matrices of multiple antenna relays. Note that when $M = 1$, we focus on optimizing a single real entry per relay: the amplification gain. The algorithm computes the achieved utility (16) of all particles by computing the optimal terminal powers derived in Section IV-A for this $\boldsymbol{W}^{(n)}$. Then, it finds the particle that provides the global optimal utility for this iteration, denoted $\boldsymbol{W}^{(\text{global})}$. In addition, for each particle n, it memorizes the position of its previous best performance, denoted $\boldsymbol{W}^{(\text{n,local})}$. After finding these two best values, PSO

Algorithm 1 Proposed Algorithm for Overlay TWR-CR Networks

1: Generate an initial population $\mathcal{S}$ composed of N random particles $\boldsymbol{W}^{(n)}$, $n = 1 \cdots N$.
2: **while** Not converged **do**
3:     **for** $n = 1, \cdots, N$ **do**
4:         Find the optimal primary and cognitive powers by computing (27)-(30) corresponding to the particle $\boldsymbol{W}^{(n)} \in \mathcal{S}$.
5:         Compute the utility U_n depending on the used metric as given in Section III-D.
6:     **end for**
7:     Find $(n_g, q_g) = \arg\max\limits_{n,q} U_n(q)$ (i.e., n_g and q_g indicate the index and the position of the particle that results in the highest utility).
8:     Set $U_{(\text{global})} = U_{n_g}(q_g)$ and $\boldsymbol{W}^{(\text{global})} = \boldsymbol{W}^{n_g}(q_g)$.
9:     Find $q_l = \arg\max\limits_{q} U_n(q)$ for each particle n (i.e., q_l indicates the position of the particle n that results in the highest local utility).
10:    Set $U_{(\text{n,local})} = U_n(q_l)$ and $\boldsymbol{W}^{(\text{n,local})} = \boldsymbol{W}^{n}(q_l)$.
11:    Adjust the velocities and positions of all particles using equations (37) and (38), respectively.
12:    Move to the new iteration $q = q + 1$.
13: **end while**

---

updates its velocity $\boldsymbol{\nu}_j^{(n)}$ and its particle positions $\boldsymbol{W}_j^{(n)}$, respectively at each iteration q as follows:

$$\boldsymbol{\nu}_j^{(n)}(q+1) = \psi \boldsymbol{\nu}_j^{(n)}(q) + c_1 r_1 \left(\boldsymbol{W}_j^{(\text{n,local})}(q) - \boldsymbol{W}_j^{(n)}(q) \right)$$
$$+ c_2 r_2 \left(\boldsymbol{W}_j^{(\text{global})}(q) - \boldsymbol{W}_j^{(n)}(q) \right), \tag{37}$$

$$\boldsymbol{W}_j^{(n)}(q+1) = \left(\boldsymbol{W}_j^{(n)}(q) + \boldsymbol{\nu}_j^{(n)}(q+1) \right)^{+}, \tag{38}$$

where ψ is the inertia weight used to control the convergence speed ($0.8 \leq \psi \leq 1.2$). r_1 and r_2 are two random positive numbers generated for each element j. Finally, c_1 and c_2 are the step size that a particle takes towards the best individual candidate solution $\boldsymbol{W}^{(\text{n,local})}$ and the global best solution $\boldsymbol{W}^{(\text{global})}$. This procedure is repeated until convergence (i.e., the utility remains constant for a certain number of iterations or reaching maximum number of iterations). Details of the proposed algorithm as it is applied to our optimization problem are given in Algorithm 1.

Although PSO's application has been proved to be effective, convergence to its most optimistic solution cannot be guaranteed in theory [37].

V. SIMULATION RESULTS

In this section, we provide selected simulation results for i.i.d Rayleigh fading channels to study the performance of the proposed scheme given in Fig. 1. The total bandwidth and average noise power per unit frequency are assumed to be equal to $B_{\text{tot}} = 5$ MHz and $\mathcal{N}_0 = 1$ Watt/Hz, respectively. The PSO algorithm is executed using these parameters: the initial number of particles is set to $N = 30$ and the maximum number of iterations is equal to 200. It is assumed that the PSO algorithm converges when the utility remains constant for 10 consecutive iterations or reaching maximum number of iterations.

A. System Performance

Fig. 3 plots the achieved cognitive sum rate using PSO algorithm presented in Algorithm 1 versus the rate profile of the system α for different utilities (Max C/I, PF, and Max-Min) with fixed $L = 4$, $R_0 = 10$ Mbits/s, $\bar{P}_p = \bar{P}_c = 20$ dBm and different number of antennas (i.e., $M = \{1, 2\}$). It is shown that the optimal α that maximizes the cognitive sum rate is equal to 0.5 (corresponding to the point where the threshold rate at PU_1 is equal to the threshold rate at PU_2). Also, we notice that the sum rate has a symmetric behvior that reflects the symmetry of the TWR. We compare the three utilities with the uniform bandwidth case using maximum sum rate utility (i.e., we choose B_0 such that it satisfies (10). Then, we consider uniform power distribution among the antennas and distribute the remaining bandwidth to the CUs i.e., $B_l = \frac{B_{\text{tot}} - B_0}{L}, \forall l = 1, ..., L$). This figure also shows the improvement of cognitive sum rate thanks to the use of MIMO antennas (i.e., $M = 2$) instead of the single antenna scheme (i.e., $M = 1$). For instance, for $\alpha = 0.5$, we were able to increase the achievable cognitive sum rate by going from 6.10 Mbits/s to around 9.87 Mbits/s, 4.31 Mbits/s to around 8.05 Mbits/s, and 3.34 Mbits/s to around 7.06 Mbits/s by using two antennas instead of a single one for Max C/I, PF, and Max-Min utilities, respectively.

All simulations show that Max C/I utility leads to the highest sum rate in the network. However, this comes at the expense of fairness as it is shown in Table I. Indeed, the table compares between the different utilities for the same channel realization with fixed $\alpha = 0.5$, $L = 4$, $R_0 = 10$ Mbits/s, and $\bar{P}_p = \bar{P}_c = 20$ dBm. By using one realization, it can be shown that Max C/I enhances the cognitive sum rate, by allocating most of the resources to a unique user having the best channel conditions (i.e., CU_3 for $M = 1$ and CU_2 for $M = 2$). On the other hand, the PF approach maximizes the geometric mean for all the users by allocating almost the same amount of bandwidths to CUs, while Max-Min approach maximizes the minimum cognitive rate and provides the same rate for all cognitive users, hence, leads to fairness performance. The choice of the utility is related to the service used by the CUs. For instance, if the application requires same uplink rates Max-Min utility can be used. However, if it consists in a pure cognitive transmission without priorities, then Max C/I could be employed.
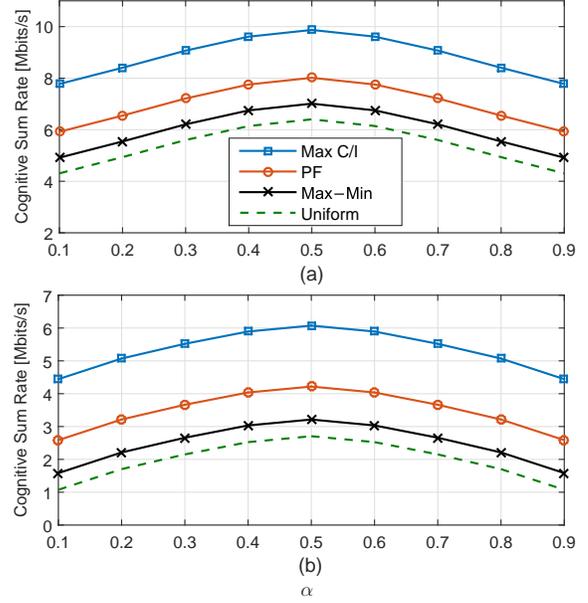
Fig. 3 Cognitive sum rate as a function of α for different utilities with $L = 4$, $R_0 = 10$ Mbits/s, and $\bar{P}_p = \bar{P}_c = 20$ dBm, for different number of antennas (a) $M = 2$, (b) $M = 1$.

TABLE I: Strategy of cognitive users corresponding to each utility with $\alpha = 0.5$, $L = 4$, $R_0 = 10$ Mbits/s, and $\bar{P}_p = \bar{P}_c = 20$ dBm.

	M=1			M=2		
	Max C/I	PF	Max-Min	Max C/I	PF	Max-Min
$\sum_{l=1}^{L} R_{c_l}$ [Mbits/s]	6.51	4.83	3.64	9.87	8.05	7.06
R_{c_1} [Mbits/s]	~ 0	0.87	0.91	~ 0	2.01	1.77
R_{c_2} [Mbits/s]	~ 0	1.39	0.91	9.87	2.35	1.77
R_{c_3} [Mbits/s]	6.51	1.37	0.91	~ 0	1.95	1.77
R_{c_4} [Mbits/s]	~ 0	1.19	0.91	~ 0	1.74	1.77
B_0 [%]	41.90	44.96	44.91	31.85	33.24	33.44
B_1 [%]	~ 0	13.76	17.16	~ 0	16.69	19.56
B_2 [%]	~ 0	13.76	12.51	67.59	16.69	12.78
B_3 [%]	58.10	13.76	10.61	~ 0	16.69	19.01
B_4 [%]	~ 0	13.76	14.81	~ 0	16.69	15.21

In Fig. 4, we aim to investigate the impact of the power budget constraint on the system performance. In this figure, we plot the cognitive sum rate for all the utilities versus the power budget ($\bar{P}_p = \bar{P}_c$) with $\alpha = 0.5$, $L = 4$, $M = 2$, and different values of $R_0 = \{10, 20\}$ Mbits/s. It is shown that increasing the target rate R_0 for the same power budget reduces the cognitive sum rate, since satisfying constraints (19) and (20) requires more bandwidth (i.e., higher B_0) as Fig. 4(b) shows, as well as higher $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ values. As a result, CUs are forced to reduce their transmitted power $\boldsymbol{P}_{c_l}, \forall 1, ..., L$, to support primary transmission and respect constraint (18). For instance, for $\bar{P} = 10$ dBm with Max C/I approach, the total cognitive sum rate is reduced by around 20% by going from around 6.00 Mbits/s to around 4.80 Mbits/s using $R_0 = 20$ Mbits/s instead of $R_0 = 10$ Mbits/s. On the other hand, we can see that increasing the power budget will provide more bandwidth to secondary users to accomplish their transmission. For instance, with 10 dBm, primary transmission needs 60% of the total bandwidth whereas with 30 dBm only 20% of the bandwidth is needed using Max C/I utility. Thus, the secondary sum rate is multiplied by 2.
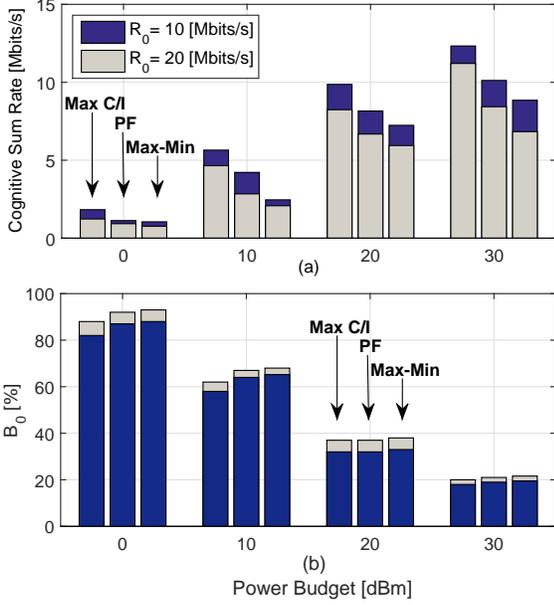
Fig. 4 (a) Cognitive sum rate, (b) primary bandwidth fraction (B_0 [%]) versus power budget constraint ($\bar{P}_p = \bar{P}_c$), for $\alpha = 0.5, L = 4, M = 2$.
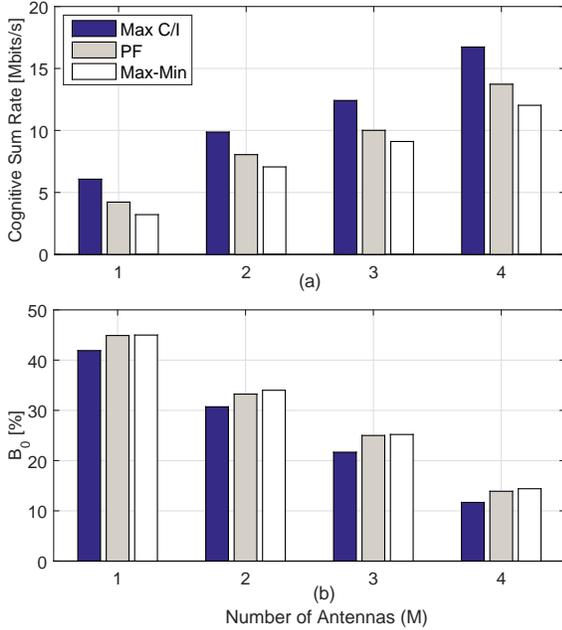
Fig. 5 (a) Cognitive sum rate, (b) primary bandwidth fraction (B_0 [%]), versus number of antennas for $\alpha = 0.5$, $L = 4$, $R_0 = 10$ Mbits/s, and $\bar{P}_p = \bar{P}_c = 20$ dBm.

Fig. 5 illustrates the cognitive sum rate as a function of M for $\alpha = 0.5$, $L = 4$, $R_0 = 10$ Mbits/s, and $\bar{P}_p = \bar{P}_c = 20$ dBm. From this figure, we can deduce that the achievable rate is improving when M increases, in other words, MIMO antennas provide more degrees of freedom to the system which enhances the cognitive sum rate. Similar to the power budget effect, increasing the number of antennas offers more bandwidth to secondary transmissions. Indeed, with MIMO antennas, primary transmission is enhanced as it demands less

relay power and thus more power can be allocated to secondary transmissions.

B. Convergence Speed

In Fig. 6, we compare between the performance of PSO and a recently proposed meta-heuristic approach entitled GWO, which are both employed with the primal-dual method, by investigating their convergence speed defined by the number of iterations needed to reach convergence. Note that an iteration in Fig. 6 corresponds to one iteration of the "while loop" given in Algorithm 1 line 2-13. GWO is inspired by grey wolf hunt. It mimics the leadership hierarchy and hunting mechanism of grey wolves in nature. During an iteration, the algorithm categorizes the candidates (i.e., grey wolves) into four groups for simulating the leadership hierarchy: Group 1 corresponds to the fittest solution, Group 2 and 3 are the second and third best solutions. Finally, Group 4 contains the remaining candidates of the population. Also, the algorithm simulates the hunting, searching for prey, encircling prey, and attacking prey of grey wolves. For example, the hunting corresponds to the position update of each candidate from an iteration to another (see [25] for more details). We plot the cognitive sum rate versus the number of iterations for PSO and GWO. It can be clearly seen that PSO achieves its suboptimal solution faster than GWO. For instance, it requires around 10-20 iterations to converge with the Max C/I utility while GWO needs 30-40 iterations. Moreover, we can notice that PSO reaches a better suboptimal solution than GWO.
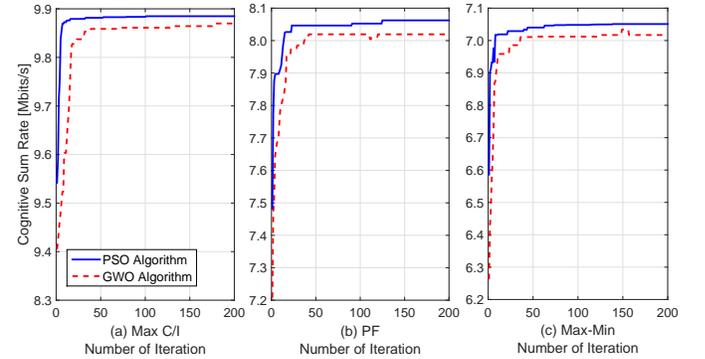
Fig. 6 Algorithms convergence speed for $\alpha = 0.5, L = 4, M = 2, R_0 = 10$ Mbits/s, and $\bar{P}_p = \bar{P}_c = 20$ dBm.

For a given maximum number of iterations of the subgradient method $I_{\max}$, the complexity is given by $\min(I_{\max}, 1/\epsilon^2)$, where ϵ represents the accuracy-guarantee which is defined by the difference between the best value and the iterate value [30]. According to (31)-(36), subgradient needs to calculate $12ML + 8M + 16L + 15$ multiplications and $6ML + 8M + 9L + 12$ additions at each iteration. These operations are computed for each particle of the meta-heuristic approach. On the other hand, PSO and GWO are two meta-heuristic algorithms where the exact number of iterations needed to reach the solution is arbitrary and depends on the studied scenario. However, the computational complexity per iteration can be determined. According to (37) and (38), PSO needs to calculate 5 multiplications and 5 additions for

every element of $\boldsymbol{W}^{(n)}$. Hence, $5(L(2M)^2 + (L+1))N$ multiplications and $5(L(2M)^2 + (L+1))N$ additions are calculated every iteration for the total N particles whereas GWO calculates $13(L(2M)^2 + (L+1))N$ multiplications and $11(L(2M)^2 + (L+1))N$ additions according to [25]. In our simulation results, we set $\epsilon = 0.1$ and $I_{\max} = 200$ iterations for subgradient algorithm. While PSO and GWO algorithms are executed for at most 200 iterations (i.e., the utilities are computed at most $200 \times N$ times), they are stopped if the achieved utility remains constant for a certain number of consecutive iterations.

TABLE II CPU times (sec) and number of iterations for the proposed joint-optimization method. The first row shows the total CPU times, while the second row shows the number of iterations needed when solving the joint power, bandwidth, and amplification factors problem.

	Max C/I		PF		Max-Min	
	PSO	GWO	PSO	GWO	PSO	GWO
Total CPU time	39	59	57	86	76	121
I^*	15	32	22	48	16	39

For 200 realizations, $N = 30$, $L = 4$ and $M = 2$, results show that on average PSO is faster than GWO and requires less time to converge as shown in Table II. In Table II, we compute the CPU times in seconds of both algorithms and record the iteration number (denoted by I^*) needed to reach the near optimal solution of the joint optimization (i.e., optimizing the power, bandwidth, and relay amplification matrices), which exactly marks the instant when the algorithm achieves its steady state utility. Increasing the number of particles N would enhance the convergence speed of the algorithms. In fact, PSO and GWO are able to achieve their solutions with a lower number of iterations but they require more CPU times as they need to perform more additions and multiplications during each iteration. Note that all tests were performed on a laptop machine featuring an Intel(R) Core(TM) i7 CPU and running Windows 8.1. The clock of the machine is set to 2.66 GHz with a 8 GB memory.

VI. Conclusion

In this paper, we investigated multiple-input multiple-output two-way relaying scheme for overlay cognitive radio networks where cognitive users are engaged to support primary transmission over a fraction of the bandwidth. More specifically, we considered multiple amplify-and-forward relays where the primary and cognitive terminal powers were optimized adaptively with the bandwidth and amplification gains. The objective was based on maximizing the cognitive utility while satisfying a certain primary target rate. Starting with expressions of primary and cognitive powers for fixed bandwidths and amplification gains, we employed the heuristic particle swarm optimization algorithm to reach a near-optimal solution. Moreover, in addition to the sum rate objective function, we investigated other utilities that introduce more fairness among cognitive users. In our ongoing task, we are trying to develop an approximation algorithm to solve the formulated optimization problem.

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