Reducing network cost of many-to-many communication in unidirectional WDM rings with network coding

Long Long and Ahmed E. Kamal
Department of Electrical and Computer Engineering, Iowa State University
Email: {longlong, kamal}@iastate.edu

***Abstract*—In this paper we address the problem of traffic grooming in WDM rings with all-to-all and its generalization to many-to-many service by using network coding. We consider minimizing the number of Line Terminating Equipment (LTE) on two types of unidirectional rings, namely, single-hub and un-hubbed rings, as our objective. In single-hub rings, we investigate the minimum cost provisioning of uniform all-to-all traffic in two cases: where network coding is used to linearly combine data, and where it is not used and data is transmitted without coding. We generalize the service mode to many-to-many and evaluate the cost of provisioning. In un-hubbed ring, we propose a multi-hub approach to obtain the minimum cost provisioning in the case of all-to-all and many-to-many traffic. From our numerical results, we find that under many to many traffic, network coding can reduce the network cost by 10-20% in single-hub rings and 1-5% in un-hubbed rings.**

I. Introduction

Wavelength-division multiplexing (WDM) technology allows an aggregate traffic on the order of Tbps to be carried on a single fiber, with each wavelength carrying traffic in the tens of Gbps order. However, the traffic demands of network applications are at much smaller granularities than wavelength bit rates. In order to utilize wavelength capacity more effeciently, a number of flows from multiple network connections with sub-wavelength granularities may be packed onto the same wavelength. This process of allocating low bit rate tributary streams to wavelengths with high bandwidth is referred to as *traffic grooming*. There are two types of traffic grooming problems, static and dynamic. The objective of the static problem is usually to minimize the overall network cost, given the traffic demands, whereas in the dynamic problem, maximizing the throughput or minimizing the blocking probability of connections.

The static traffic grooming problem of unicast traffic has been widely studied in the literature [6], [7]. Among a number of network architectures, ring topologies drew significant attention in the research community due to the availability of legacy SONET equipment. Recently, multipoint traffic has become more important in a number of application environments, and this is why a number of studies addressing multipoint traffic grooming have recently appeared in the literature [9], [10], [11]. In ring networks, both all-to-all [1], [2] and multicast [8] traffic scenarios have been studied. Most of the literature addressing this problem focuses on evaluating and reducing the dominant cost in the optical network, namely, Electronic Add-Drop Multiplexers (ADM), which is required at a node if it either has data to transmit or receive from another counterpart. The number of ADMs required at a node is only a function of the number of lightpaths established and terminated at the node. Another cost function that is similar to the number of ADM is the number of e-Dac grooming ports presented in [4], which can also be referred to as LTE ports.

In this paper, we address the static traffic grooming problem of a class of multipoint traffic in unidirectional ring networks with the number of LTE ports as the network cost. The number of wavelengths and the cost of other optical equipment, such as the optical splitter (which is negligible compared to the electronic LTEs) are not factors to be considered here. We consider uniform all-to-all traffic grooming which can be exemplified by a number of applications including video conferencing. Since in reality it is more likely that there are multiple video conferences going on simultaneously, we also consider a generalized case where there are multiple all-to-all communication groups on the same ring. We consider two types of unidirectional rings, namely, single-hub and un-hubbed rings. In a single-hub ring [5], all the traffic has to be sent to the hub and then forwarded to the destination by the hub. In an un-hubbed ring, there is no such hub.

All-to-all traffic can be implemented using two approaches, unicast and multicast. In unicast mode, traffic duplication can only be implemented in the electronic domain, whereas in multicast mode, traffic duplication can be done in the optical domain by using optical splitters. If a node needs to send a traffic stream on two outgoing links, the node requires two LTE ports in unicast mode but only one in multicast mode. All-to-all traffic that can be divided into multiple multicast sessions will benefit significantly in terms of the network cost. This requires multicast capable nodes to be deployed in the network. The corresponding node architecture is referred to as the Tap-and-Continue node and is introduced in [8]. Since each node on the unidirectional ring has only one incoming link and one outgoing link, and routing of all lightpaths is fixed along the direction of the ring, there is only one possible multicast, which is dropped and forwarded on the same wavelength at each node. We use such node architecture to implement any multicast needed in the network.

Network coding [3], is a promising new technique that has been applied to a variety of network applications in order to improve the performances, such as in multi-hop wireless networks [14], network tomography [12] and content distribution in peer-to-peer networks [13]. To the best of our knowledge, however, it has not been applied to optical networks for the cost saving objective. We choose single-hub rings to apply network coding in order to reduce the network cost, since all the traffic need to be sent to the hub on the ring. Therefore, no additional O-E-O converter are needed at the hub in order to receive and perform algebraic operations on the traffic. Network coding can therefore be used to improve network capacity to reduce the amount of traffic that has to be sent to the nodes, and hence reduce the number of lightpaths that carry traffic. The network cost can, therefore, be reduced. Though there is no hub in un-hubbed rings, we can select a node to act as a hub such that the same network coding operations in single-hub rings can be performed at this node, and hence reduce the network cost in un-hubbed rings.

The paper is structured as follow. In Section II, we introduce network coding and its benefit in saving LTE ports in optical networks. We will explore the network costs with or without applying network coding in single-hub and un-hubbed rings in Section III and IV, respectively. Numerical results of multiple many-to-many communication will be shown in Section V. Finally in Section VI, we will conclude the paper.

II. Network Coding in Optical Network

Network coding is a novel technique which was originally proposed for improving network capacity, particularly in multicast scenarios [3]. Besides the traditional routing functions, network nodes are designed to linearly combine packets arriving at input edges and transmit those combinations on output ports. In an optical network, a node receives information from multiple incoming links and combines the data on electronic domain. The encoded information is then converted into optical signals and sent out through its outgoing links. This approach can result in reducing the number of lightpaths when sub-wavelength channels are groomed on lightpaths and under multicasting service. Since the number of LTE ports at each node is the total number of lightpaths terminated and initiated at this node, then using network coding, the total number of lightpaths can be minimized, and consequently the network cost be minimized. On the contrary, if the network cost is fixed in terms of LTEs, which fixes the number of lightpaths that can be set up, network coding should be able to increase the network capacity, hence achieving higher throughput.

Applying network coding to optical networks introduces new issues besides the routing and wavelength assignment. For example, where and how should network coding be performed? The answer to this question is straightforward in a single-hub ring network. Since all traffic is collected by the hub, the hub is the perfect node to combine packets. However, it is not as clear in an un-hubbed ring. The traffic that participates in the network coding operation has to go through O-E-O conversion, which requires more LTEs, hence making the problem a trade-off between reducing the number of network coding nodes and the reduction in LTEs due to using fewer lightpaths. How to achieve network coding requires determining the coding scheme and finite field size $GF(q)$ from which we choose coding coefficients. Of course, network coding does not come for free, since more computations are needed. Compared to the cost of LTEs, such resource consumption is almost negligible. Therefore, we only consider the number of LTEs as the network cost in this paper.

III. Cost Analysis in Single-hub Unidirectional Ring

In this section, we address the problem of grooming all-to-all traffic in a single-hub unidirectional ring. Under all-to-all service, each node should receive data from all other nodes on the ring. The problem can be stated as follows: Given a group of nodes n and grooming ratio g, each node i, generates and transmits traffic at rate r_i, and must receive the traffic sent by other nodes such that the network resources, the LTEs in particular, are minimized.

A. Uniform all-to-all traffic

We assume each data unit has to be transmitted to the hub before being relayed to the destination(s). As we mentioned earlier, instead of the unicast method, we implement traffic delivery by using multicast from the hub node and the data is duplicated by using a splitter in the optical domain. The number of LTEs can therefore be reduced. Moreover, we allow traffic bifurcation in order to minimize the total cost.

The all-to-all communication process involves two steps. The first step is to deliver traffic upstream from nodes to the hub. In the second step, the hub grooms the traffic into the minimum number of wavelengths and multicasts the groomed traffic downstream to every node on the ring. In the upstream direction, each node requires $\lceil \frac{r}{g} \rceil$ LTE ports to send the data, and the hub needs $n\lceil \frac{r}{g} \rceil$ LTE ports to receive the traffic. Either grooming the traffic from different nodes before sending it to the hub, or sending data to the hub directly by each node, will not change the total number of LTE ports during the upsteam process. This only effects the number of wavelengths used. However, the number of wavelengths is not a factor of the network cost according to our assumptions. Thus, the total cost in the upstream step includes the LTE ports used by the nodes to transmit traffic and the hub to receive traffic, which is $n\lceil \frac{r}{g} \rceil + n\lceil \frac{r}{g} \rceil = 2n\lceil \frac{r}{g} \rceil$.

Let us consider downstream now. The total amount of traffic units collected at the hub is nr. Since traffic bifurcation is allowed, the minimum number of wavelengths can be used to pack all the traffic, denoted by $\lceil \frac{nr}{g} \rceil$, which is also equal to the number of LTE ports required by the hub to transmit and by each node to receive. Each node employs a tap-and-continue function which splits the optical signal and receives a small portion of power that is just enough to be detected and leave the rest of power to propagate on the ring. Such a power splitting function will reduce the number of LTEs from $2n$ to $n + 1$ in a downstream process if the hub broadcasts a wavelength to every node. Broadcasting each wavelength consumes $n+1$ LTEs such that broadcasting all of the traffic, which requires $\lceil \frac{nr}{g} \rceil$ wavelengths, hence using a minimum cost of $(n+1)\lceil \frac{nr}{g} \rceil$ LTE ports for the downstream traffic delivery.

To sum up, the resources consumed in both upsteam and downstream result in an overall minimum cost of $2n\lceil \frac{r}{g} \rceil + (n+1)\lceil \frac{nr}{g} \rceil$ LTE ports to achieve all-to-all communication.

B. Application of Network Coding

When using network coding, it is obvious that the hub is a perfect place to perform network coding, since all data has to be delivered to the hub first and then converted into electronic signals for grooming, and hence no additional LTEs will be needed for O-E-O conversion to perform network coding. Therefore, the encoding operation is performed at the hub and the decoding is done at each node. We can also consider this problem in the upstream and downstream contexts. Since upstream is unicast and no network coding is needed, the number of LTEs consumed in the upstream process remains the same. In order to save sub-wavelength channels in the downstream data delivery, we use the following coding scheme. Each node has to receive data from the other $n-1$ nodes. Since each node needs data from different $n-1$ nodes, then without implementing network coding but when using splitters, each node has to receive all data units from the hub, which is denoted by nr, in order to achieve minimum network cost. However, if a node receives linear combinations of the traffic instead of the original data, only $n-1$ linearly independent combinations are needed. By including its own data, each node has n linearly independent combinations, from which original data of all other nodes can be decoded, given the coding coefficients are known. This is the basic idea for using network coding to save lightpaths and hence LTEs.

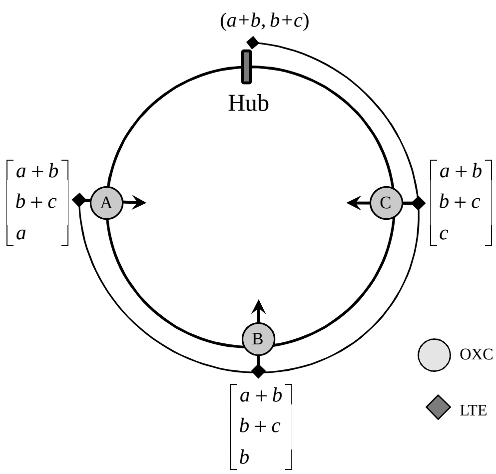

Fig. 1. Application of network coding in downstream in a Single-hub ring

The example shown in Figure 1 illustrates how to use network coding in the downstream process. We suppose that the hub has received the traffic from nodes A, B and C whose data units are denoted by a, b, c, respectively. The grooming factor is 2 and transmission rate at each node is 1. Hence, each wavelength is able to accommodate the traffic transmitted by two nodes. Instead of sending all the traffic a, b and c to each node on the ring, the hub encodes the data and generates code words $a+b$ and $b+c$ using modulo 2 addition. These combinations are independent from each other and also from each of the original data units a, b and c. In this case, each node will receive two code words, in addition to having its own data in its buffer; For example, node A will have combinations $a, a+b$ and $b+c$, where

$$\begin{bmatrix} a \\ a+b \\ b+c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Therefore, the coefficient matrix shown above has a full rank such that a, b and c can be decoded from the combinations. This is a simple coding scheme in which the coefficients are binary numbers, selected from $GF(2)$. Following the same method, node B and C are also able to obtain the original data a, b, c by decoding the code words received from the hub as shown in the Figure 1. Apparently, this coding scheme can be applied to the n-node case where the hub needs to generate $n-1$ linearly independent combinations which are also independent from all raw data units and broadcast them to each node.

Hence, in the case where the number of nodes is n and the traffic rate associated with each node is r, the cost of upstream transmission does not change from the case without network coding, which is denoted by $2n\lceil \frac{r}{g} \rceil$. In the downstream direction, however, the total traffic that the hub has to deliver is reduced to $\lceil (n-1)r \rceil$ which requires $\lceil \frac{(n-1)r}{g} \rceil$ wavelengths. The total cost of LTE ports in the downstream is $(n+1)\lceil \frac{(n-1)r}{g} \rceil$.

Therefore, the overall cost savings by the application of network coding in a single-hub ring with all-to-all traffic demand is $(n+1)(\lceil \frac{nr}{g} \rceil - \lceil \frac{(n-1)r}{g} \rceil) \geq 0$. The savings can be either in LTE ports or network bandwidth, depending on the specific network scenarios.

C. Multiple Many-to-Many Groups

In practice, the most common applications which use the all-to-all service mode are multimedia conferences and working cooperation. Usually, there is more than one multimedia conference group simultaneously in the network. Thus, it is essential to consider multiple groups. In this scenario, the network nodes are divided into multiple groups, and within each group, nodes engage in all-to-all communication, while the traffic can be different in different groups. We assume that each node belongs to one group only.

1) Disjoint Groups: As mentioned above, the definition of disjoint groups is straightforward - it refers to the case in which different groups do not include any common node. The problem of optimizing the network cost in a disjoint group is stated as follows: minimize the number of LTE ports used in a single-hub unidirectional ring, given m disjoint groups where each group i $(1 \leq i \leq m)$ has n_i $(n_i \geq 2)$ nodes and each node has to transmit $r_i \geq 1$ units of traffic in a transmission round to all the other nodes within the same group. As stated in the all-to-all case, traffic bifurcation and splitters are allowed, and we assume that there are enough wavelengths to be used such that the number of wavelengths is not a factor of the network cost to be considered.

Let us consider the case without network coding first. The process is similar with the all-to-all case. In the upstream traffic delivery, nodes send traffic to the hub. In the downstream direction, the hub grooms the traffic from each group and sends them back to the nodes belonging to the same group. Our analysis still begins with upstream process. Every group i is independent from each other such that the total number of LTE ports used in upstream is the sum of LTE ports consumed by each group individually. Following our analysis in Section III.A, for each group i, the upstream consumes $2n_i\lceil\frac{r_i}{g}\rceil$ LTE ports. Hence, the total cost of LTEs ports in the upstream process is $\sum_{i=1}^{m} 2n_i\lceil\frac{r_i}{g}\rceil$.

In the downstream direction, the hub first grooms the traffic from the nodes within the same group together. For group i, the number of wavelengths used to carry the traffic is $\lceil\frac{r_i n_i}{g}\rceil$, and $\lfloor\frac{r_i n_i}{g}\rfloor$ of them are filled up and sent back to the nodes belonging to the same group directly. The number of LTEs used for this process is easy to be obtained, since every lightpath used here is fully loaded and used for a unique group. Thus, we only need to pay attention to the remaining portion of aggregate traffic that cannot fill up a single wavelength if it exists, since it may need to be groomed together with the traffic from other group(s) at the hub in order to save the lightpaths and hence LTEs. Each group has at most one piece of such traffic, since traffic bifurcation is allowed such that the rest of the traffic from a group must have filled up an integral number of wavelengths. This piece of traffic of group i is equal to $n_i r_i - g\lfloor\frac{n_i r_i}{g}\rfloor$. We denote this portion of the traffic by p_i, where $0 \le p_i < g$. Hence, the problem can be stated formally as follow:

Problem GMP: Given m pieces of traffic, each of them with p_i units and has to be received by n_i nodes, groom and multicast this traffic at the hub such that the total LTEs used is minimized.

2) NP-completeness: In the optimal solution, each piece of traffic p_i must not be bifurcated and packed into more than one wavelength. We can prove it by contradiction. Assume that one piece p_i is splitted and assembled into two different wavelengths in the optimal solution, which costs each node of this group 2 LTE ports to receive. This results in $2n_i$ LTE ports in total. If we let the hub use a separate wavelength to send p_i, it only takes n_i LTEs at the nodes and 1 more LTE at the hub. Since $2n_i - (n_i + 1) = n_i - 1 \ge 0$, this means that any optimal solution can be transformed to be the solution without traffic bifurcation for the problem.

Moreover, the minimum number of wavelengths used to allocate all the traffic p_i will result in the minimum number of LTEs consumed in this process. Since without traffic splitting, the number of LTEs used by each group to receive the data is fixed, which is n_i for group i, the fewer the number of LTEs used at hub to transmit the traffic, the fewer the number of LTEs that will be used in total. The number of LTEs used at the hub is equal to the number of wavelengths needed to pack all the traffic p_i. Hence, this problem turns out to be a special case of general traffic grooming in a ring network, which has been proven to be NP-complete by reduction from *Bin Packing problem* in polynomial time in [1]. The minimum number of bins required in the bin packing problem is equal to the minimum number of wavelengths used to accommodate all the p_i.

3) Solutions without Network Coding: We now analyze the minimum network cost of many-to-many groups communication without employing network coding. The network cost is still calculated in two steps: upstream transmission and downstream transmission. The network cost of upstream transmission for n groups is $\sum_{i=1}^{m} 2n_i\lceil\frac{r_i}{g}\rceil$, which is composed of LTEs consumed by each group for transmission and reception at the hub. The cost of downstream transmission consists of two parts. The first part is the number of LTE ports used for broadcasting the lightpaths that are fully loaded for each individual group; the second part is the number of LTEs used for transmitting the remaining traffic portion p_i of each group. In order to obtain the solution of this part, we need to solve the problem GMP. Since it is equivalent to the *Bin Packing problem*, we consider two methods to obtain the minimum wavelengths used at the hub to pack all the traffic p_i. The first method is a heuristic algorithm based on First Fit Decreasing (FFD) [15]; in the second one, we formulate the GMP problem as an Integer Linear Program.

Thus, in the downstream transmission, for each group i, it consumes $\lfloor\frac{n_i r_i}{g}\rfloor$ full wavelengths to pack the data belonging to this group, which takes $(n_i+1)\lfloor\frac{n_i r_i}{g}\rfloor$ LTE ports in total to transmit and receive this part of the traffic. To sum up, for each group i, the total number of LTEs used in the ring to transmit this portion of the traffic is $\sum_{i=1}^{m}(n_i+1)\lfloor\frac{n_i r_i}{g}\rfloor$. To accommodate the remaining traffic p_i of each group, we use W to denote the number of wavelengths used at the hub, which is solved by either the heuristic or ILP described above. Hence, we need W LTEs at the hub to transmit and 1 LTE for each node in group i to receive p_i if it exists, which is determined by a binary number, $\lceil\frac{n_i r_i}{g}\rceil - \lfloor\frac{n_i r_i}{g}\rfloor$. If the result is equal to 1, it indicates that p_i for group i is non-zero; otherwise, it is zero. Therefore, the cost of transmitting this portion of traffic of each group is equal to $W + \sum_{i=1}^{m}(\lceil\frac{n_i r_i}{g}\rceil - \lfloor\frac{n_i r_i}{g}\rfloor) \times n_i$.

Thus, summing up the network cost of upstream and downstream for m groups in terms of LTE ports without network coding gives us:

$$\sum_{i=1}^{m}(2n_i\lceil\frac{r_i}{g}\rceil + (n_i+1)\lfloor\frac{n_i r_i}{g}\rfloor + (\lceil\frac{n_i r_i}{g}\rceil - \lfloor\frac{n_i r_i}{g}\rfloor)n_i + W)$$
$$= \sum_{i=1}^{m} n_i(2\lceil\frac{r_i}{g}\rceil + \lceil\frac{n_i r_i}{g}\rceil) + \lfloor\frac{n_i r_i}{g}\rfloor + W$$

4) Solutions with Network Coding: To apply network coding to the same network scenario in the previous section, the hub performs network coding and generates $n_i - 1$ code words for each group following the same coding scheme with coefficients from $GF(2)$. The combinations of original data has the same rate as the original traffic, which is r_i.

Thus, network coding can reduce the number of traffic units needed to transmit to each group in each unit of time by r_i. The upstream transmission consumes the same amount of resources, but the total amount of traffic turns out to be $(n_i - 1)r_i$ units for each group in the downstream process. Following the same rules of calculating the network cost in downstream proposed in last subsection, the network cost in this case also includes two parts. The first part is denoted by $\sum_{i=1}^{m}(n_i+1)\lfloor\frac{(n_i-1)r_i}{g}\rfloor$. In the second part, let W' denote the minimum number of wavelengths obtained by either FFD or ILP algorithm and used at the hub to pack all the remaining traffic p_i' from each group after network coding.

Combining the cost spent in both upstream and downstream process gives us the total number of LTE ports with the application of network coding in a many-to-many traffic scenario, which is expressed as:

$$\sum_{i=1}^{m} 2n_i\lceil\frac{r_i}{g}\rceil + (n_i + 1)\lfloor\frac{(n_i-1)r_i}{g}\rfloor + (\lceil\frac{(n_i-1)r_i}{g}\rceil - \lfloor\frac{(n_i-1)r_i}{g}\rfloor) \times n_i + W'$$

$$= \sum_{i=1}^{m} n_i\left(2\lceil\frac{r_i}{g}\rceil + \lceil\frac{(n_i-1)r_i}{g}\rceil\right) + \lfloor\frac{(n_i-1)r_i}{g}\rfloor + W'.$$

IV. Cost Analysis in un-hubbed Unidirectional Rings

Following the same sequence of the previous section, we will first investigate the traffic grooming problem in an un-hubbed ring with all-to-all traffic and then generalize it to many-to-many group communication. All the assumptions made in the single-hub ring case remain except that a hub is not used. Traffic bifurcation and splitters are still allowed.

A. Bounds of all-to-all Unitary Traffic

Let us first consider a special all-to-all case. There are n nodes and the traffic rate of each node is $r = 1$. With the benefit of an optical splitter, each node needs at least one LTE port to transmit data. In order to receive the data from every other node, at least $\lceil\frac{n-1}{g}\rceil$ lightpaths will be terminated at each node to deliver the data, which requires $\lceil\frac{n-1}{g}\rceil$ LTE ports at each node. Hence, each node requires a total of at least $(1 + \lceil\frac{n-1}{g}\rceil)$ ports to fulfill all-to-all communication. Therefore, the total LTE ports required in the network should be no less than the sum of the cost at all nodes, denoted by $(1 + \lceil\frac{n-1}{g}\rceil) \times n$, which is the *lower bound* of the network cost.

However, if no traffic is groomed, each node will just multicast its data to all the other nodes with the help of splitters. Each multicast requires n LTE ports, one for transmitting data at the sender and $n-1$ at all other nodes to receive. Therefore, n^2 LTE ports are required in this case, which is considered an *upper bound* of the cost.

B. Uniform all-to-all Traffic

We now extend the problem from unitary traffic to non-unitary traffic demands. Given a grooming factor g and a group of n nodes, each of which has r units of traffic, find the minimum number of LTE ports required to fulfill all-to-all communication.

We propose a *multi-hub* approach to achieve all-to-all demands while minimizing the total number of LTE ports. The basic idea is to groom as many sub-wavelength channels as possible and try to use the minimum number of lightpaths to multicast groomed data. The approach can be done in two steps:

1) a number of nodes groom their traffic together onto a wavelength at a node chosen among them;
2) A lightpath is initialized and it broadcasts the traffic groomed to all other nodes on the ring by using splitters.

Unlike the situation in a single-hub ring, where every node sends traffic to the same hub, in an un-hubbed ring, once enough traffic groomed to fill up a wavelength at a node, then this node will set up a lightpath and broadcast the data to the other $n-1$ nodes on the ring. This node is called a "hub". The number of such hubs needed in a ring is equal to the number of lightpaths required to broadcast all the traffic, which is actually the number of wavelengths that accommodate all the traffic. The minimum number of wavelengths used is denoted by $\lceil\frac{nr}{g}\rceil$. We only consider the case where $r < g$, since if $r \geq g$, the traffic of each node can fill up separate $\lfloor\frac{r}{g}\rfloor$ wavelengths and there is no need to groom with the traffic from other nodes. In this case, we only need to consider the remaining traffic, denoted by $r - g\lfloor\frac{r}{g}\rfloor$, which is less than g. Thus, there is no need to consider the case where $r < g$.

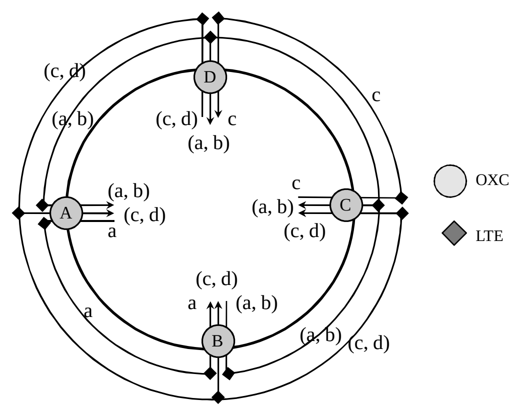

Fig. 2. All-to-all transmission in un-hubbed unidirectional ring using the multi-hub approach

Figure 2 illustrates the all-to-all transmission mechanism proposed in multi-hub approach. In such an un-hub ring network, nodes A, B, C and D need to exchange information represented by a, b, c and d, respectively. In the example, the traffic rate of each stream is 2 and the grooming factor is $g = 4$. Each wavelength can accommodate traffic from two users. Thus, node A sends a to node B and then B packs both a and b into one wavelength and broadcasts the data to every node on the ring except itself. Such a process is called a *broadcast cycle*. Since an optical splitter is used by each node, each broadcast cycle consumes 6 LTE ports, 2 for collecting data and 4 for broadcasting. Nodes C and D execute the similar process with node D being the hub. Thus, two broadcast cycles are needed to accomplish the communication such that the total number of LTE ports required is $2\times 6 = 12$. The following analysis will be based on the transmission mechanism proposed above.

We analyze the network cost in two different cases: when g is a multiple of r or not. In the first case, each wavelength can be filled up without a need for traffic bifurcation. Every $\frac{g}{r}$ nodes groom their traffic together at a node among them into one wavelength. Each such process costs $2(\frac{g}{r}-1)$ LTE ports, since there is a need of $\frac{g}{r}-1$ LTE ports to transmit and another $\frac{g}{r}-1$ LTE ports to receive. Broadcasting the groomed traffic to the other $n-1$ nodes in the ring costs n LTE ports, hence requiring a total of $2(\frac{g}{r}-1)+n$ LTE ports for this broadcast cycle. Therefore, a total of $\lceil\frac{n}{g/r}\rceil$ such broadcast cycles are required to provision the all-to-all traffic demand. In some cases, there might be a cycle that carries a traffic that is equal to $nr-g\lfloor\frac{n}{g/r}\rfloor$ data units, when this traffic does not fill up a wavelength. We let $\frac{g}{r}=k$, which represents the number of nodes whose traffic are groomed together and sent in one broadcast cycle. Then after $\lfloor\frac{n}{k}\rfloor$ cycles, the number of nodes that have not transmitted data is $(\lceil\frac{n}{k}\rceil-\lfloor\frac{n}{k}\rfloor)(n-m\lfloor\frac{n}{k}\rfloor)$. These nodes will transmit in the last cycle. $(\lceil\frac{n}{k}\rceil-\lfloor\frac{n}{k}\rfloor)$ is a binary indicator that there are some nodes left in the last cycle and their number is less than k. Hence, the number of LTE ports needed in the last cycle is $(\lceil\frac{n}{k}\rceil-\lfloor\frac{n}{k}\rfloor)(2(n-k\lfloor\frac{n}{k}\rfloor-1)+n)$ LTE ports.

Thus, the total number of LTE ports needed in all $\lceil\frac{n}{k}\rceil$ broadcast cycles is:

$$\lfloor\frac{n}{k}\rfloor(2k+n-2)+(\lceil\frac{n}{k}\rceil-\lfloor\frac{n}{k}\rfloor)(3n-2k\lfloor\frac{n}{k}\rfloor-2),\qquad \text{where } k=\frac{g}{r}.$$

In this case, we groom the traffic before broadcasting and utilize the minimum number of wavelengths to pack all the traffic such that the minimum number of LTE ports is achieved with the help of nodes deployed with splitters.

However, in the case where g is not a multiple of r, a wavelength cannot be filled up without traffic bifurcation. Minimizing the total number of wavelengths used for broadcasting with traffic bifurcation will result in the minimum number of broadcast cycles. However, each traffic split will increase the number of LTE ports needed. In another way, if we do not split any traffic, we cannot guarantee that the number of broadcast cycles is at a minimum. Each broadcast uses n LTE ports, which means that saving one wavelength will save n LTE ports. However, splitting the traffic from a node will result in two additional LTE ports (one transmitter and one receiver) used at this node. Assume that there are l traffic splits in order to achieve a minimum number of broadcast cycles, then the total number of LTE ports used at the hubs will be increased by $2l$. Therefore, there is a trade-off between the number of traffic splits and the total number of broadcast cycles. In the multi-hub approach proposed here, the minimum network cost is obtained by taking the minimum value of the solutions obtained from the two cases where we split and we do not split the traffic.

Let us consider the two cases separately. First, we consider the case without traffic bifurcation. Without splitting traffic, each wavelength can accommodate traffic from at most $\lfloor\frac{g}{r}\rfloor$ nodes, denoted by k. Then, the total number of broadcast cycles is $\lceil\frac{n}{k}\rceil$ and the number of nodes remaining in the last cycle is $(\lceil\frac{n}{k}\rceil-\lfloor\frac{n}{k}\rfloor)(n-k\lfloor\frac{n}{k}\rfloor)$.

Therefore, the minimum number of LTE ports required in this case is given by:

$$\begin{aligned}&\lfloor\frac{n}{k}\rfloor(2k+n-2)+(\lceil\frac{n}{k}\rceil-\lfloor\frac{n}{k}\rfloor)(3n-2k\lfloor\frac{n}{k}\rfloor-2)\\&=\lceil\frac{n}{k}\rceil(3n-2k\lfloor\frac{n}{k}\rfloor-2)+\lfloor\frac{n}{k}\rfloor(2k\lfloor\frac{n}{k}\rfloor-2n+2k),\\&\qquad\text{where } k=\lfloor\frac{g}{r}\rfloor.\end{aligned}$$

Second, if traffic bifurcation is applied, the minimum number of wavelengths to accommodate all the traffic can be achieved, which is given by $\lceil\frac{nr}{g}\rceil$, which also represents the minimum number of broadcast cycle. Let $\lceil\frac{nr}{g}\rceil = w_{min}$. Hence, the number of LTE ports employed for broadcasting is known, and is given by nw_{min}. However, we know that traffic splitting was used to achieve this. Since each traffic split creates two additional LTE ports, the problem of minimizing the total number LTE ports actually turns out to be a problem of minimizing the number of traffic splits in order to groom the traffic on the minimum number of wavelengths, w_{min}. This problem has been solved by an iterative algorithm proposed in [1]. We use the algorithm here to obtain the minimum number of traffic splits in this situation, given the minimum wavelengths used. We assume that the minimum number of traffic splits obtained by the algorithm in [1] is denoted by sp_{min}. The number of broadcast cycles determines the number of hubs in the ring such that the number of hubs is also equal to w_{min}. If no traffic split happens, collecting traffic at those hub nodes from other nodes before broadcasting consumes $2(n-w_{min})$. However, each traffic split increases the number of LTE ports by 2. Thus, the total number of LTE ports in this collection process is $2(n-w_{min}+sp_{min})$. In addition to the LTE ports used for broadcasting, denoted by nw_{min}, the total number of LTE ports used in the case with traffic bifurcation is $2(n-w_{min}+sp_{min})+nw_{min}$.

Therefore, taking the minimum of the two solutions obtained in the two cases above will give us the overall minimum network cost. Thus, the number of LTE ports of all-to-all traffic without network coding is:

$$\begin{aligned}&min\{\lceil\frac{n}{k}\rceil(3n-2k\lfloor\frac{n}{k}\rfloor-2)+\lfloor\frac{n}{k}\rfloor(2k\lfloor\frac{n}{k}\rfloor-2n\\&\quad+2k),\quad 2(n-w_{min}+sp_{min})+nw_{min}\},\\&\qquad\text{where } k=\lfloor\frac{g}{r}\rfloor,\ w_{min}=\lceil\frac{nr}{g}\rceil,\end{aligned}$$

and sp_{min} is the minimum number of traffic splits obtained by the iterative algorithm proposed in [1] given w_{min}.

C. Application of Network Coding

In order to save wavelength channels, and hence reducing number of LTE ports, the node where network coding is performed at a node which needs to collect all the original data. Thus, we propose a *one-hub* scheme in which only one node acts as a hub. The traffic is gathered and encoded at this node following the same coding scheme proposed in Section III.B, where the network context is a single-hub ring. The hub can be selected from any node in the ring.

Hence, in the upstream process, every node sending traffic to the hub consumes $2(n-1)$ LTE ports. $n-1$ linearly independent code words with traffic rate r are generated and packed into $\lceil r(n-1)/g \rceil$ wavelengths. In the downstream process, we do not need to worry about traffic split - since every node will receive the same traffic broadcast by the hub. Whether the code words are split or not, this will not result in any extra LTEs. Thus, the minimum number of broadcast cycles can be achieved. Each broadcast cycle costs n LTE ports such that the transmission in the downstream direction takes total $n\lceil r(n-1)/g \rceil)$ LTE ports.

Therefore, the total network cost with network coding using one-hub scheme is:

$$2(n-1) + n\lceil r(n-1)/g \rceil$$

Though a one-hub scheme will definitely save LTE ports in downstream, it uses a few more LTE ports than the multi-hub approach described in the previous section. Thus, to sum up, the LTE ports consumed in both upstream and downstream one-hub schemes may not always save LTE ports, depending on the specific network scenario. However, given the traffic demands, we can always use the multi-hub approach to solve the problem without applying network coding. Therefore, by comparing the solutions yielded by the one-hub and multi-hub approaches, we choose the minimum value to be our solution here.

Thus, assuming that LTE_{multi} denote the solution obtained from multi-hub approach, the total network cost in unidirectional rings with n nodes and all-to-all traffic demands r with applying network coding is:

$$min\{2(n-1) + n\lceil r(n-1)/g \rceil, \quad LTE_{multi}\}$$

D. Multiple Many-to-Many Groups

We now extend the all-to-all communication to multiple many-to-many independent groups on an un-hubbed unidirectional ring. Since no node is shared by more than one group, traffic from the same group will only communicate between the nodes within the same group. Hence, traffic from different groups will not be groomed into the same wavelength. Since each group is independent, the total network cost is the sum of the cost of each individual group.

Suppose there are m groups in a unidirectional ring and each group i has n_i nodes with each node in the group sourcing r_i traffic units. The minimum network cost in terms of LTE ports can be represented based on whether network coding is employed or not:

In the case where no network coding is employed, the total network cost in terms of the number of LTE ports is:

$$\sum_{i=1}^{m} min\{\lceil \frac{n_i}{k_i} \rceil (3n_i - 2k_i \lfloor \frac{n_i}{k_i} \rfloor - 2) + \lfloor \frac{n_i}{k_i} \rfloor (2k_i \lfloor \frac{n_i}{k_i} \rfloor - 2n_i + 2k_i), \quad 2(n_i - w^i_{min} + sp^i_{min}) + n_i w^i_{min}\},$$
$$\text{where } k_i = \lfloor \frac{g}{r_i} \rfloor, \; w^i_{min} = \lceil \frac{n_i r_i}{g} \rceil,$$

sp^i_{min} is the minimum number of traffic splits of group i given w^i_{min}.

In the case of using network coding, the total network cost in terms of the number of LTE ports is:

$$\sum_{i=1}^{m} min\{2(n_i - 1) + n_i \lceil r_i(n_i - 1)/g \rceil, \quad LTE^i_{multi}\}.$$

where LTE^i_{multi} is the minimum number of LTE ports of group i obtained by using the multi-hub approach.

V. Numerical Results

We summarize the theoretical solutions analyzed in previous sections in Table I. Within the table, in the all-to-all traffic scenario, n, r and g denote the number of nodes, the traffic rate transmitted at each node and grooming factor, respectively. $k = \lfloor \frac{g}{r} \rfloor$, $w_{min} = \lceil \frac{nr}{g} \rceil$, and sp_{min} is the minimum number of traffic splits given w_{min}. In the many-to-many scenario, m denotes the number of groups on the ring and n_i, r_i denote the number of nodes and traffic rate transmitted by each node in group i; $k_i = \lfloor \frac{g}{r_i} \rfloor$, $w^i_{min} = \lceil \frac{n_i r_i}{g} \rceil$; sp^i_{min} is the minimum number of traffic splits of group i given w^i_{min}; W and W' denote the minimum number of wavelengths obtained by either the FFD algorithm or the ILP and used at the hub to pack all the remaining traffic p'_i from each group with and without applying network coding, respectively.

Based on the theoretical solutions obtained above, we compare the results of two different cases - employing network coding or not employing network coding - in various network scenarios and under different traffic conditions. Since all-to-all communication is a special case of multiple many-to-many group cases where the number of groups is equal to 1, in this section we consider more general cases in which $m \geq 1$ and the traffic rate and the number of nodes of each group is randomly selected.

A. Single-hub Ring

In single-hub rings, we choose different values of traffic rate r_i and the number of nodes n_i for each group i. The total number of nodes is equal to $\sum_i n_i$. Different values of the grooming factor g are used. Since network costs in upstream processes are always the same, both when using network coding and not using network coding under the same network condition, we only consider the cost in the downstream process. The comparison of network costs in terms of the number of LTE ports between the cases with or without network coding is shown in following examples.

Note that the solution of the GMP problem is part of the network cost in the downstream process, and the results obtained

TABLE I

NETWORK COSTS IN SINGLE-HUB AND UN-HUBBED UNIDIRECTIONAL RINGS

single-hub	all-to-all	Without NC	$2n\lceil \frac{r}{g} \rceil + (n+1)\lceil \frac{nr}{g} \rceil$
		With NC	$2n\lceil \frac{r}{g} \rceil + \lceil \frac{(n-1)r}{g} \rceil$
	many-to-many	Without NC	$\sum_{i=1}^{m} n_i(2\lceil \frac{r_i}{g} \rceil + \lceil \frac{n_i r_i}{g} \rceil) + \lfloor \frac{n_i r_i}{g} \rfloor + W$
		With NC	$\sum_{i=1}^{m} n_i \left(2\lceil \frac{r_i}{g} \rceil + \lceil \frac{(n_i-1)r_i}{g} \rceil\right) + \lfloor \frac{(n_i-1)r_i}{g} \rfloor + W'$
un-hubbed	all-to-all	Without NC	$min\{\lceil \frac{n}{k} \rceil (3n - 2k\lfloor \frac{n}{k} \rfloor - 2) + \lfloor \frac{n}{k} \rfloor (2k\lfloor \frac{n}{k} \rfloor - 2n + 2k),$ $2(n - w_{min} + sp_{min}) + nw_{min}\}$, denoted by LTE_{multi}
		With NC	$min\{2(n-1) + n\lceil r(n-1)/g \rceil,\ LTE_{multi}\}$
	many-to-many	Without NC	$\sum_{i=1}^{m} min\{\lceil \frac{n_i}{k_i} \rceil (3n_i - 2k_i\lfloor \frac{n_i}{k_i} \rfloor - 2) + \lfloor \frac{n_i}{k_i} \rfloor (2k_i\lfloor \frac{n_i}{k_i} \rfloor - 2n_i + 2k_i),$ $2(n_i - w^i_{min} + sp^i_{min}) + n_i w^i_{min}\}$
		With NC	$\sum_{i=1}^{m} min\{2(n_i-1) + n_i\lceil r_i(n_i-1)/g \rceil,\ \ LTE^i_{multi}\}$

TABLE II

THE COMPARISON OF NUMBER OF LTE PORTS IN THE SINGLE-HUB RING WITH G=4

	(Traffic,nodes)	(2,18)	(2.5,18)	(2,27)	(2.5,27)	(2,36)	(2.5,36)	(2,45)	(2.5,45)	(2,54)	(2.5,54)	(2,63)	(2.5,63)
FFD	Without NC	33.6	38.5	66	76.2	109.8	129.1	164.8	194.5	245.3	293.5	310.4	368.3
	With NC	23.1	24.2	53.8	58.6	88.7	101.9	141	164.7	212.7	254.3	278	327.9
ILP	Without NC	33.5	38	65.3	75.6	109.5	128.5	163.4	193.3	245	293.1	310.1	367.7
	With NC	23	24	53.2	58.2	88	101.4	140.5	163.4	212.3	253.9	278	327.4

TABLE III

THE COMPARISON OF NUMBER OF LTE PORTS IN THE SINGLE-HUB RING WITH G=8

	(Traffic,nodes)	(2,18)	(2.5,18)	(2,27)	(2.5,27)	(2,36)	(2.5,36)	(2,45)	(2.5,45)	(2,54)	(2.5,54)	(2,63)	(2.5,63)
FFD	Without NC	23.1	24.2	42.7	47	68.3	77.7	91.4	110.3	137.3	160.3	202	237.5
	With NC	20.8	21.3	37.4	39.5	57.1	64.8	83.1	97.3	120.6	141.3	184.1	214.5
ILP	Without NC	23.1	24	42.5	46.6	67.9	76.6	90.9	110	137.3	159.8	201.7	237
	With NC	20.7	21.2	37.1	39.3	56.8	64.5	82.7	97.1	120.4	141.2	183.8	214.1

TABLE IV

THE COMPARISON OF NUMBER OF LTE PORTS IN THE SINGLE-HUB RING WITH G=16

	(Traffic,nodes)	(2,18)	(2.5,18)	(2,27)	(2.5,27)	(2,36)	(2.5,36)	(2,45)	(2.5,45)	(2,54)	(2.5,54)	(2,63)	(2.5,63)
FFD	Without NC	20.7	21.2	31.5	32.4	44	48.4	59.7	65.8	83.6	93.7	104.8	117.3
	With NC	19.7	20	30.2	30.7	39.5	42.3	55.9	60.9	76.8	86.1	95.1	108.4
ILP	Without NC	20.7	21.2	31.4	32.2	44	48.4	59.5	65.8	83.5	93.7	104.8	117.3
	With NC	19.7	20	30.2	30.7	39.5	42.2	55.8	60.8	76.8	86.1	95.1	108.4

by solving the heuristic algorithm, FFD, and formulating the problem as an ILP and then solved by Cplex, may not be equal due to the NP-completeness of problem GMP. Since the solution of problem GMP is denoted by W and W' in the cases without and with network coding, solving the heuristic algorithm FFD and ILP will give us approximate and exact solutions of W and W' respectively. In addition to the other part of the solution, which can be calculated directly, we have the approximate and exact solutions of the network cost in the downstream process. Tables II, III, and IV show the heuristic and exact solutions of the number of LTE ports in different network scenarios in terms of the total number of nodes and the average traffic rate of each group. In each table, a different grooming factor g is used. The numbers in the parentheses represent the average traffic rate transmitted at each node and the average total number of nodes on the ring. Each network cost value is obtained by taking the average value of the solutions of one hundred independent experiments in which the ranges of n_i and r_i are fixed, from which the average traffic rates and the average number of nodes are obtained. The rows starting with "without NC," and "with NC" represent the average number of LTE ports without and with applying network coding.

Under the same network scenarios and grooming factors, the tables shows that the exact network cost is slightly less than its heuristic counterpart in most cases due to the trivial differences between the exact and heuristic solutions of the problem GMP. We can also observe that the number of LTE ports needed in the network increases with the increase of the traffic rates and the number of nodes. Network coding can save network cost in all cases where the grooming factor $g = 4,\ 8$ and 16. The relative savings of the network cost, denoted by the ratio of cost savings to the network cost without applying network coding, are almost the same under different network traffic conditions with the same g. However, the incremental saving of the network cost decreases as g increases. The overall relative cost saving under all the network conditions considered in the examples is between $10\% - 20\%$, which

TABLE V

THE COMPARISON OF NETWORK COST IN THE UN-HUBBED RING WITH G=4

(Traffic,nodes)	(2,20)	(2.5,20)	(2,30)	(2.5,30)	(2,40)	(2.5,40)	(2,50)	(2.5,50)	(2,60)	(2.5,60)	(2,70)	(2.5,70)
Without NC	40	40	81	84.3	145.2	154.1	212.5	233.7	300.3	337.1	405.7	450.8
With NC	40	40	80.1	83.5	138.8	149.6	207.2	229.7	290.6	329	396.6	443.7

TABLE VI

THE COMPARISON OF NETWORK COST IN THE UN-HUBBED RING WITH G=8

(Traffic,nodes)	(2,20)	(3,20)	(2,30)	(3,30)	(2,40)	(3,40)	(2,50)	(3,50)	(2,60)	(3,60)	(2,70)	(3,70)	(2,80)
Without NC	40	40	72.4	77.9	117	131.7	159.3	186.8	218.3	261	285.4	346	363.5
With NC	40	40	71.2	75.5	113	127	156.5	182.1	209.5	251.7	276.8	336	352.5

TABLE VII

THE COMPARISON OF NETWORK COST IN THE UN-HUBBED RING WITH G=16

(Traffic,nodes)	(2,20)	(3,20)	(2,30)	(3,30)	(2,40)	(3,40)	(2,50)	(3,50)	(2,60)	(3,60)	(2,70)	(3,70)	(2,80)
Without NC	40	40	69.1	70.2	103.7	109.6	136.8	147.8	176.9	195.3	225.4	251.7	273.2
With NC	40	40	69.1	69.1	100.8	105.2	134.9	144.2	172.5	189.8	219.7	244.4	268.5

translates to a large CAPEX saving considering the cost of LTEs.

B. *un-hub unidirectional ring*

Unlike single-hub rings, the advantage of network coding in the un-hubbed unidirectional rings is less significant, as shown in Tables V, VI and VII, with $g = 4,\ 8$ and 16, respectively. The first and second element in the parentheses represents the average traffic rate generated at each node and the average total number of nodes, respectively. The rows starting with ”without NC,” and ”with NC” represent the average number of LTEs without and with applying network coding respectively, under different traffic scenarios.

From the tables shown above, we find that network cost saving increases as the total amount of traffic on the ring increases with the same grooming factor. However, the saving realized by employing network coding decreases with the increase of the grooming factor, since the greater the grooming factor is, the fewer lightpaths are used for the communication. Therefore, the fewer lightpaths will be saved by applying network coding, which results in less saving of LTEs. Notice that when the number of nodes in each group is exactly 2 and hence the total number of nodes on the ring is 20, the network cost is a constant regardless of the grooming factor and whether or not network coding is employed. Because we assume that the traffic rate of each node is not greater than g, there is no need of network coding and traffic grooming if only two nodes exchange their data in each group. Each transmission will use one wavelength and 2 LTE ports and hence all-to-all communication in each group consumes 4 LTE ports, which results in 40 LTE ports for 10 groups in any network scenario. Except this case, the overall cost savings under other different network scenarios considered is between 1-5%, which is less significant than the saving obtained by using network coding in single-hub rings.

The reason for the difference between single-hub rings and un-hubbed rings is that in an un-hubbed ring, all the traffic does not need to be transmitted to the same hub when using the *multi-hub* approach. Once a wavelength is filled up at a node, the data is broadcast to all the members belonging to the same group. However, network coding requires a common hub on the ring to collect the data from all the nodes within the same group in the *one-hub* scheme. Even if a wavelength is fully loaded, it has to experience an extra delivery to the common hub, since network coding has to be performed on the data from all the users of the same group. Such extra delivery consumes more LTE ports in un-hubbed rings. Only if the number of nodes and traffic rate of a group satisfy certain conditions can network coding save costs for this group. This means that not every group with an all-to-all traffic demand will benefit from network coding in un-hubbed rings. Therefore, the total saving - the sum of the saving from each group - will not be as high as that in single-hub rings.

VI. CONCLUSIONS

In this paper we provided the first study of the traffic grooming problem of two types of unidirectional rings, single-hub and un-hubbed, with uniform all-to-all, and its extension, many-to-many, traffic scenarios, and with or without network coding. We considered LTE ports as the dominant factor of network cost. Traffic bifurcation and optical splitters are allowed in our analysis of network costs in all the network scenarios. In a single-hub ring, we explored the minimum cost of all-to-all traffic in the cases when network coding was not applied and when it was applied, and from numerical results, we observed 10-20% cost savings in many-to-many group communication scenarios with the deployment of network coding. In the un-hub unidirectional ring, we used the multi-hub approach to derive the minimum cost in both cases. From the observation of numerical results, the savings of LTEs is 1-5%, which is less significant than the saving in the single-hub ring case.

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