Blocking Probabilities in Circuit-Switched WDM Networks Under Multicast Service

Ahmed E. Kamal
Department of Electrical and Computer Engineering
Iowa State University
Ames, IA 50011-3060
U.S.A.

Anwar K . Al-Yatama
Department of Electrical and Computer Engineering
Kuwait University
P.O. Box 5969 Safat
Kuwait 13060

Abstract

We evaluate the call blocking probabilities in optical networks which support multicast (or multipoint) service. Due to the difficulty of the problem, we restrict our attention to the case of the completely connected networks, for which the blocking probabilities act as a lower bound for other topologies. Nodes may, or may not be equipped with wavelength converters. In both cases, routing can be done either directly, using one hop, or indirectly, and even in the latter case the number of hops can be either restricted to two hops, or unrestricted. We evaluate the blocking probability by first estimating the load offered to each link, and then use the iterative reduced load approximation method to refine this estimate. Using this estimate, we then obtain the probabilities of blocking and success. Numerical examples are presented, and are compared to simulation.

Keywords: Optical networks; wavelength division multiplexing; multicast service; circuit switching; blocking probability; Markov process; analysis.

I Introduction

The advent of Wavelength Division Multiplexing (WDM) [1, 2] provided a solution to the problem of speed mismatch between the fiber bandwidth and the electronics speed, which is better known as the electronic bottleneck problem [3]. As such, an aggregate bandwidth in the tens of Terabits per second range can be achieved by supporting multiple simultaneous transmissions, each utilizing a different wavelength, or optical frequency. This can provide a means to serve those applications which individually or collectively require a large amount of bandwidth. Such applications include the class of service with multicast, or multipoint traffic which requires the delivery of data from the source to a group of destinations [4]. Such applications include video distribution, network news distribution, multi-party conferences, etc. The nature of the optical networks requires that the support of multicast traffic either creates an independent channel from the source to each destination, or that destinations cooperate in relaying the traffic they receive to other destinations. Although the first approach is easy to implement, its bandwidth requirements are very large due to the creation of multiple channels. The bandwidth requirements of the second approach are modest compared to those of the first one, but they require some special provision to implement. The problem is that in optical networks the optical layer will establish a connection such that it starts at the source, and terminates at one of the destinations. In this paper we assume that there is a provision by which that destination will relay the same traffic, after reading it, to other destinations. This can be implemented easily by converting the optical signal to the electric domain, and then converting it back again to the optical domain, but in this case optical transparency may not be achievable. Alternatively, it can also be implemented in the optical domain directly through the use of active optical switching elements that control the fractions of the optical signals that can be diverted to each of its two outputs [5].

The purpose of this paper is to derive expressions for the call blocking probabilities in optical networks using WDM and carrying multicast circuit-switched traffic. Several studies have derived the call blocking probabilities for unicast circuit-switched applications, but to the best of our knowledge, this is the first study to obtain such blocking probability for the multicast service.

A few studies dealt with multicast routing in optical networks, e.g., [6, 7, 8]. However, those studies were concerned with packet-switched communication, and were concerned with either the establishment of the routing tree, or the derivation of the packet delay. The recent study in [9] considered the performance of unscheduled packet multicasting in broadcast-and-select networks, and found that randomizing the transmissions is better than the persistent approach. More recently, the work in [10] evaluated the distribution of the receiver busy times in a WDM star network under multicast packet switching. References [11, 12, 13] considered WDM support for IP multicasting. The work in [14] (also in [5], pp. 516–527), is probably the only study dealing with multicast services in circuit-switched service over linear lightwave networks, and presents algorithms for the construction of multicast connections. A simulation study of the call blocking probability is also presented.

A few studies have also dealt with the derivation of the blocking probabilities under unicast service and with circuit switched traffic. Reference [15] considers blocking probabilities in circuit switched networks. However, most of the results in that manuscript are not directly applicable to WDM, especially with multicast traffic. The study in [16] derived the blocking probabilities under circuit switched service in WDM networks, with and without wavelength conversion, and assuming a given wavelength utilization. The effects of path length, switch size, and interference length on the blocking probabilities were evaluated. This study was extended in [17] when multiple fiber links are used. A similar study in [18] used Erlang's fixed point iterative method to arrive at the blocking probabilities. In [19] the blocking probabilities with and without wavelength conversion were also derived, and the solution was obtained iteratively using the reduced load approximation technique [20, 21], which is a generalization of the Erlang's fixed point method. Alternate routing with fixed, and least lightly loaded strategies was considered. The authors in [22] derived the blocking probabilities when only a group of nodes are capable of wavelength conversion. Erlang's fixed point approach was not used on the premise that its use enhances the results only in a minor way when the blocking probabilities are small. The same authors derived the blocking probabilities when the arriving traffic is non-Poisson in [23]. The used the moment matching approach, and the Bernoulli-Poisson-Pascal approximation. In [24] the blocking probabilities in all-optical networks employing limited wavelength translation were derived in terms of the link utilization. It was shown that the benefits of full-range wavelength translation can be almost achieved with a limited-range wavelength translation. Reference [25] considered the same problem in a mesh network. Harai et al. in [26] obtained the blocking probabilities in all optical switching networks when alternate routing is used, and found that alternate routing enhances the system performance. In [27], the same authors introduced a general model using the layered graph approach [28]. The model can be used for any routing or wavelength selection strategies. However, the complexity of the model limits its applicability to networks with a few nodes, and a few wavelength channels. The authors of reference [29] used the same model in [23] in order to study the performance of different routing strategies they proposed. In [30], a model based on circuit-switched alternate routing [15] is used to evaluate the probability of blocking when adaptive, joint route selection and wavelength selection is used, and without wavelength conversion. Reference [31] has provided analysis of the fixed-route, first-fit wavelength selection algorithm under no wavelength conversion. In [32], an exact method to evaluate the blocking probabilities in wavelength routed networks under fixed path routing, and random wavelength selection was presented. However, in order to obtain a closed form expression for the probability distributions, an approximation was presented to convert the process into a time reversible one. In addition, a path decomposition technique was also presented to reduce the complexity of the problem.

In this paper we derive the blocking probabilities under multicast service when the topology forms a completely connected graph. We have chosen this topology for two reasons. First, it is a simple topology that avoids all the routing decisions which must be made by other special topologies which will affect (and in most cases complicate) the derivation of the blocking probabilities. Second, since a link exists between each pair of nodes in the completely connected topology, and since this number of routes from a source to any group of destinations is *at least* equal to the number of routes with any other topology, the blocking probabilities derived in this study serve as lower bounds on the blocking probabilities in other topologies. Moreover, some networks, such as the vBNS [33, 34], implement a virtually completely connected topology using ATM virtual paths. The same modeling techniques presented in this paper can be extended to model those networks, while taking the virtual paths and the type of traffic into account.

We derive the blocking probabilities with and without wavelength conversion. We also consider three cases for the maximum number of hops allowed: a single hop, which corresponds to direct routing; a two-hop indirect routing[1]; and an unrestricted indirect routing strategy. As expected, when the limit on the number of hops increases, the blocking probabilities improve. However, increasing such a limit requires a more complicated control mechanism, and as will be shown in the numerical examples section, this enhances the blocking probabilities only marginally over the two hop case. It should be noted that in the case of no wavelength conversion, as the limit on the number of hops increases, the problem becomes more intractable. Therefore, in that case we only evaluate the blocking probabilities in the cases of direct routing, and indirect routing with a limit

[1]This is also widely known as dynamic routing, in which alternative paths leading to the destination(s) are explored on demand.

of two hops only, similar to the model in [19, 24]. With the unrestricted indirect routing strategy, we rely on simulation to assess the effectiveness of that method.

The approximate approach we follow in this paper is to estimate the successful call arrival rate to a certain link given the number of available channels (wavelengths) on that link and on an adjacent link. This joint availability condition is necessary to capture the strong correlation that arises with multicast traffic. This estimated load is in fact the reduced load which is used to calculate the probabilities of wavelength availabilities. The procedure is iterative, and in most cases converges[2].

In section II we introduce the network model. In section III we derive the blocking probabilities when wavelength conversion is used, while the probabilities of blocking without wavelength conversion are obtained in section IV. Section V presents some numerical examples and discussions, and section VI concludes with some remarks.

II Network Model

We consider a completely connected network with N nodes, such that each pair of nodes is connected by two fibers for communication in both directions.Each fiber (link) carries W wavelength channels. Calls are assumed to arrive at a node according to a Poisson process with rate a, and the call holding time is exponentially distributed with a mean of one time unit. Each of the arriving calls is directed to k destinations with probability r_k, and the destination nodes are uniformly chosen from all the nodes, excluding the source. The routing and wavelength selection assignment problem is done in two phases:

1. Each source node employs a path probing algorithm in which it sends path setup messages in order to determine whether all the destination nodes can be reached or not[3]. If a path from the source to all the destinations cannot be established, then the call is blocked; otherwise, proceed to phase 2.

2. One of the available multicast paths is selected at random, and also one of the available wavelength channels is selected at random.

[2]When convergence did not occur, i.e., oscillations are encountered, the two oscillating values are averaged and are used as a seed to the iterative process; this causes the procedure to converge.

[3]Since this paper is only concerned with the derivation of the blocking probabilities, the determination of the actual algorithm is beyond the scope of this paper.

It is assumed that each source node is able to transmit independently to different directly connected nodes using the same, or different wavelengths. It is also assumed that each of the links used by a session must lead to one of the destinations which have not been reached before, i.e., no node that is not a destination should be visited. The rationale behind this assumption is twofold: (1) to simplify the model, which is already two complex; and (2) to minimize the amount of bandwidth required to carry out that call[4]. If we consider the bandwidth unit to be one wavelength channel per link, then with the above assumption a call with k destinations will require exactly k bandwidth units. If the above assumption was not enforced, and nodes which are not destinations can be used as intermediate relay nodes, then the required bandwidth for any arbitrary session can be as large as $N-1$ units for an N node network.

This paper is concerned with the probability of call blocking. In order to obtain this probability, we require the channel occupancy distribution on the different links. The reduced load approach[5] will be employed as an approximation [20, 21]. We develop a model which employs a generalization of Erlang's fixed point equation. Straightforward application of Erlang's fixed point method in which the channel occupancy is modeled as a one-dimensional Markov process is not appropriate under multicasting conditions. The reason is that strong correlations exist between the channel occupancies on links originating from the same node. For example, assume that in a network with N nodes, calls are broadcast to all other $N-1$ nodes. If we use Erlang's one-dimensional Markov process model, and assume independence between links, then given that a target link, from node a to node b, has an available channel, an adjacent link, from node a to node c, will have a non-zero probability of being blocked. In reality, the adjacent link has the same number of available channels as the target link with probability 1.

We capture the correlation between adjacent links by considering the behavior of a target link in conjunction with the behavior of an adjacent link, i.e., two links originating from the same node (Figure (1.a)). We note that a similar correlation exists between this target link and the other

[4] Restricting the visited nodes to the set of destinations only simplifies the analysis significantly. However, the results will not represent the actual lower bounds on the blocking probabilities in this case. They can be regarded as approximations to the lower bounds, which is very accurate when the number of hops is small, e.g., two or three, or when the load is either light or heavy. It should be also noted that allowing any node to be visited will not necessarily result in better performance, since accommodating a call on a long path that contains nodes which are not destinations may result in blocking several other calls with shorter paths of destination nodes only.

[5] The reduced load on a link will be that load which is carried on the link; it will be denoted by λ, and is assumed to be Poisson distributed.

adjacent links. Therefore, we can use this correlation when considering the other links.

We employ a 3-dimensional Markov process, where the dimensions are:

- i: the number of available channels on the target link.
- j: the number of available channels on the adjacent link.
- k : the number of channels on the target and the adjacent link, which are used for common calls.

According to the above, k should not exceed the minimum of $W - i$ and $W - j$, where W is the total number of channels per link. Although the above process is three-dimensional, the number of states is limited to a moderate number since in practice the number of channels per link is small. Hence, the computational requirements imposed by this model are reasonable.

In part (a) of Figure 1 we show the target and adjacent links (a–b and a–c, respectively), while in part (b) the transitions between the states of the Markov process are shown. The horizontal transitions in the right and left directions respectively correspond to the use and release of a channel on the target link due to a call which does not use the adjacent link. The vertical transitions are the corresponding channel use and release on the adjacent link. The downward and upward diagonal transitions correspond to the initiation and termination of calls which use both links, respectively. The absent transitions indicate illegal transitions.

The transition rates (which are not shown in the figure) are given by the elements of the transition rate matrix, $\mathcal{P}$. This matrix has elements $P_{(i,j,k),(i',j',k')}$ which indicate the transition rate from state (i, j, k) to state (i', j', k'). The construction of the matrix $\mathcal{P}$ will depend on the routing method, and will be shown for each routing case separately. Once $\mathcal{P}$ is constructed, one can solve for the steady state probabilities, $\pi_{i,j,k}$, using:

$$\begin{aligned} \vec{\pi}\mathcal{P} &= 0 \text{ , and} \\ \sum_i \sum_j \sum_k \pi_{i,j,k} &= 1 \end{aligned}$$

where $\vec{\pi}$ is the vector of steady state probabilities. Once the steady state probabilities are obtained, one can obtain the marginal probability of having m available channels on the target link, $q(m)$.

We introduce the following definitions, noting that all links will be treated as identical and therefore no link number or index will be used:

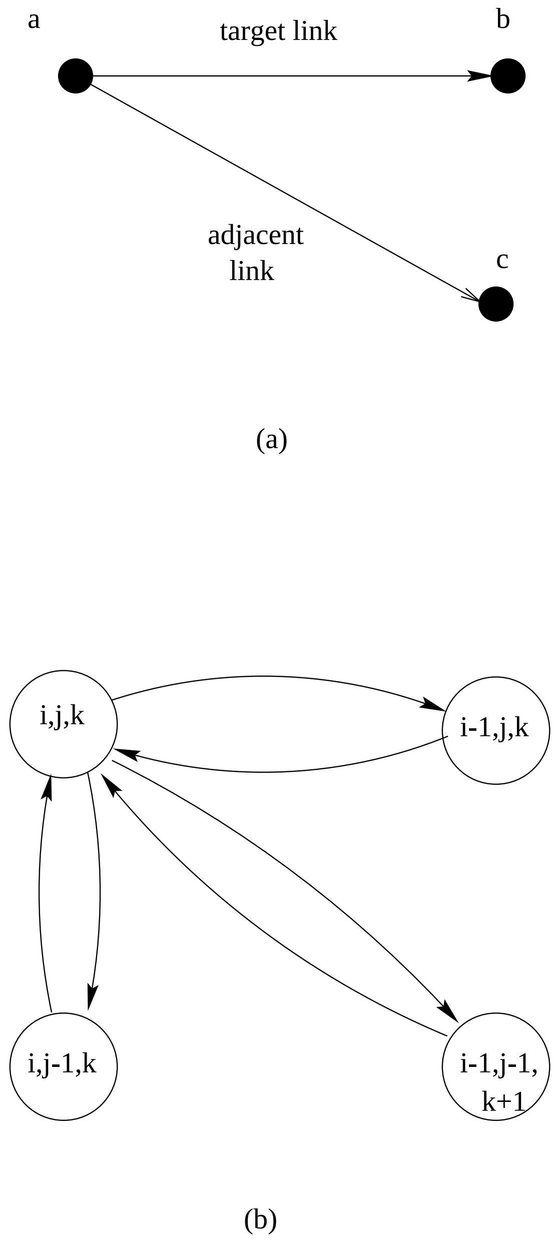

Figure 1: Capturing the correlation between two adjacent link: (a) the target and adjacent links; (b) transitions between states in the 3-dimensional Markov model.

N	number of nodes in the system
W	number of wavelength channels per link
a	offered load per node
r_k	probability that a call is destined to k destinations; the vector $\{r_1, r_2, \ldots, r_{N-1}\}$ is called the load vector
a_k	offered load per node with k destinations $= ar_k$
d_k	probability that the node at the end of a given link is chosen as a destination, given that the call is destined to k nodes out of $N-1$ nodes $= \frac{k}{N-1}$
e_k	probability that the two nodes connected by a given link are chosen as destinations, given that the call is destined to k nodes out of $N-1$ nodes $= \frac{k(k-1)}{(N-1)(N-2)}$
$t_i(k)$	probability that a given node is chosen as a destination for a call with k destinations, given i already known destinations $= \frac{k-i}{N-i-1}$
$\lambda^{(h)}(m,n)$	(reduced) load carried on the link, given that the target and the adjacent links have m and n available channels, respectively, and the load is experiencing its hth hop
$\pi_{m,n,l}$	the steady state probability that the target and the adjacent links have m and n available channels, respectively, and that l channels are being used for common calls
$q(m)$	probability that a given link has m idle wavelengths
$\beta(m)$	probability that a given adjacent link is blocked, given that the target link has m available channels $= \frac{\sum_{k=0}^{W-m} \pi_{0,m,k}}{q(m)}$
B	the unconditional link blocking probability $= q(0) = \sum_{m=0}^{W} \beta(m)q(m)$
$P_{a\&b}$	probability that a target link is not blocked, while its adjacent link is blocked $= \sum_{m=1}^{W} \beta(m)q(m)$
$P_{b\&b}$	probability that the target and the adjacent links are both blocked $= \beta(0)q(0)$
$\chi(i,j,p)$	probability of choosing j out of i events according to the binomial distribution with parameter $p = \begin{pmatrix} i \\ j \end{pmatrix} p^j(1-p)^{i-j}$
$H(i\|l)$	probability of reaching i out of i destinations, given that l nodes have been reached

Other definitions will be required in some special cases, and will be defined when needed.

In the following, all links will be treated as independent, and identical, and therefore no distinction between the links will be made, except between the target and adjacent links. It should be noted that the terms *target* and *adjacent* links can be used interchangeably.

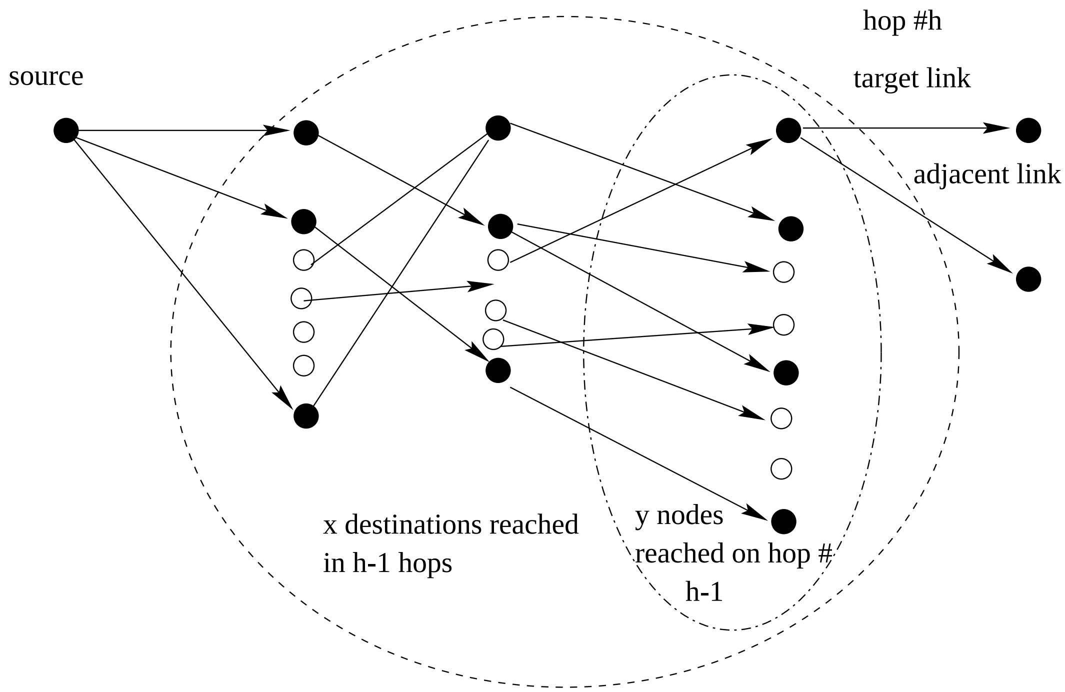

Figure 2: An illustration of steps involved in determining the carried load on the target link, on the hth hop.

We will derive the probability of call blocking by first evaluating the probability of call success, Pr(success), and then using

$$\Pr(\text{blocking}) = 1 - \Pr(\text{success}).$$

Pr(success) is dependent on the routing strategy, and will be evaluated for each of the considered cases.

Before going through the derivations of the reduced load in the different cases, we outline the general approach used in these derivations (see Figure (2)). To find the load on a target link, when the load is experiencing its hth hop, the following strategy will be followed:

- We must reach the node at the beginning of this target link on the *h-1*th hop. When this occurs, a total of x destinations would have already been reached, including y destinations reached on the *h-1*th hop itself. All of those x nodes must be destinations.

- Even if a group of the y nodes can reach the node at the end of the target link on the hth hop, that node should be reached using the target link. Selecting the target link in this case will be according to a uniform distribution.

- Destinations which could not be reached earlier may be reached on the hth hop through the y nodes.
- If after the hth hop, one or more destinations have still not been reached, they should be reached on further hops.
- The node at the end of the adjacent link, may or may not be reached, depending on whether it is a destination or not, and if a destination whether it has been reached before or not. Even if it has been reached earlier, it may be reached through other links, or may be even blocked on the hth hop and must be reached on subsequent hops.

The details of the derivations will be treated on a case by case basis.

III Networks with Wavelength Conversion

This section derives the session blocking probabilities under the assumption that nodes are capable of full wavelength conversion. We will consider the three cases in which routing is either direct, indirect with a maximum of two hops, and indirect with no restriction on the number of hops.

III.1 Direct Routing

In this subsection, we consider direct routing only. That is, the link between the source and each destination must have at least one available wavelength. Under these conditions, the load will be carried over one hop only, and $\lambda^{(h)}(m,n)$ for $h > 1$ must be equal to zero. Also, $\lambda^{(h)}(0,n)$ is equal to zero for all value of h.

Considering a target link, we further divide $\lambda^{(1)}(m,n)$ into two components: $\lambda_d^{(1)}(m,n)$ which is carried on the target link only, i.e., it is **d**isjoint from the adjacent considered link; and $\lambda_s^{(1)}(m,n)$ which is carried on both the target and adjacent link, i.e., it is **s**hared with the adjacent link. As such, these can be expressed as

$$\lambda_d^{(1)}(m,n) = \sum_{k=1}^{N-1} a_k d_k \times (1 - t_1(k))(1-\beta(m))^{k-1} \tag{1}$$

and

$$\lambda_s^{(1)}(m,n) = \sum_{k=2}^{N-1} a_k d_k \times t_1(k)(1-\beta(m))^{k-2}, \quad \text{for } n > 0 \tag{2}$$

The $a_k d_k$ factor in the above equations represents the load offered to the target link when the call is directed to k destinations. In equation (1) we must exclude the node at the end of the adjacent link, which occurs with probability $1 - t_1(k)$. The last term in that equation is the probability that the remaining $k-1$ destinations will be reached directly, given that the target link has m available channels. Equation (2) considers the case in which the node at the end of the adjacent link is a destination (with probability $t_1(k)$).

The elements of the transition rate matrix $\mathcal{P}$ are now given by:

$$
\begin{aligned}
P_{(i,j,k),(i,j-1,k)} &= \sum_{h=1}^{N-1} \lambda_d^{(h)}(j,i) \quad \text{for } i \neq 0 \\
P_{(i,j,k),(i-1,j,k)} &= \sum_{h=1}^{N-1} \lambda_d^{(h)}(i,j) \quad \text{for } j \neq 0 \\
P_{(i,j,k),(i-1,j-1,k+1)} &= \frac{\sum_{h=1}^{N-1} \lambda_s^{(h)}(j,i) + \lambda_s^{(h)}(i,j)}{2} \\
P_{(i,j,k),(i,j+1,k)} &= W - j - k \\
P_{(i,j,k),(i+1,j,k)} &= W - i - k \\
P_{(i,j,k),(i+1,j+1,k-1)} &= k \\
P_{(i,j,k),(i,j,k)} &= \sum_{(i',j',k') \neq (i,j,k)} P_{(i,j,k),(i',j',k')}
\end{aligned}
$$

The first two expressions above correspond to transitions caused by loads exclusive to the target and adjacent links, respectively. The third expression is the transition rate due to the load that is common to both links. The next three expressions correspond to channel release due to the end of calls using the target link only, the disjoint link only, or both links, respectively. Finally, the last expression is the normalizing transition rate.

Under direct routing, terms with $h > 1$ in the above summations will be zero. However, the above equations of the elements of $\mathcal{P}$ are written in general form so that they can be used in all cases, but with the respective values of λ, and with the maximum number of possible hops.

We note that the amount of traffic common to two adjacent links is not in general known[6]. In an exact characterization, since call arrivals are independent of the link states, then $\lambda_s^{(1)}(i,j)$ and $\lambda_s^{(1)}(j,i)$ should be equal. However, since our approach is approximate they may not be exactly the same[7]. We therefore average $\lambda_s^{(1)}(j,i)$ and $\lambda_s^{(1)}(i,j)$ in order to find that fraction of traffic corresponding to the downward diagonal transition rate.

[6] In special cases, such as unicast traffic, and broadcast traffic, this load is known to be 0 and the total load, respectively.

[7] In the numerical examples, $\lambda_s^{(1)}(i,j)$ and $\lambda_s^{(1)}(j,i)$ were found to be very close.

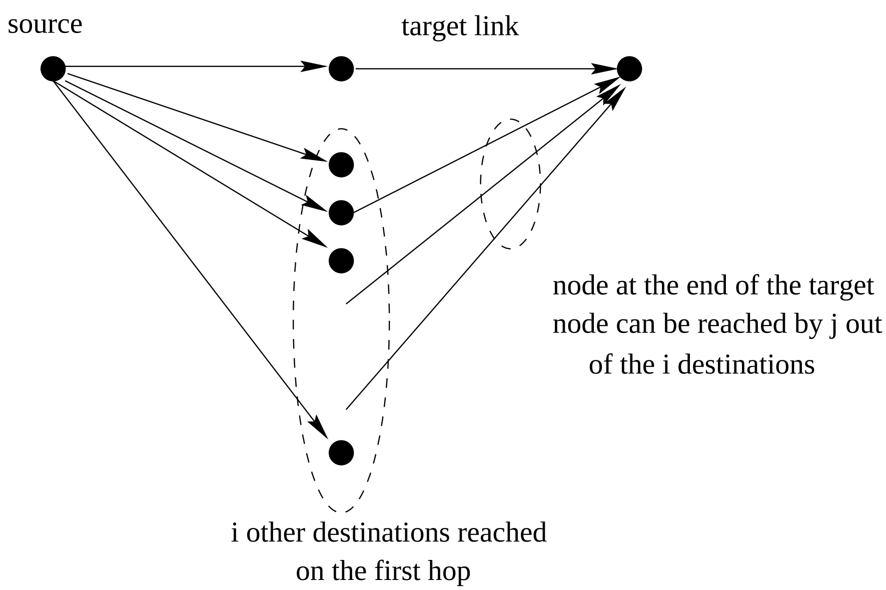

Figure 3: An illustration of the meaning of U: $i+1$ destinations nodes are reached on the first hop, and j out of the i, in addition to the node at the beginning of the target link, can reach the node at the end of the target link.

Once $q(m)$ is found, the probability of success is given by

$$\Pr(\text{success}) = \sum_{k=1}^{N-1} r_k \sum_{m=1}^{W} q(m)(1-\beta(m))^{k-1}$$

which is the fraction of traffic that will be carried successfully using the direct routing approach.

III.2 Indirect Routing with Two Levels

We now allow indirect routing with a maximum of two hops. That is, a destination can be reached either on the first or second hop. Also, only nodes which are destinations can be visited. For example, let node a be transmitting to both of nodes b and c. If all the channels on the $a-b$ link are occupied, when at least one channel on the $a-c$ link is available, then the only route available from a to b is through c.

To facilitate the derivation of the reduced loads for this case, we introduce the following additional definition which applies only if the target link is used on the second hop (see Figure 3).

$U(i)$	probability of using the target link to reach the node at its end, given that i other links are available for that purpose $= \sum_{j=0}^{i} \binom{i}{j} (1-B)^j B^{i-j} \times \frac{1}{j+1} = \frac{1-B^{i+1}}{(i+1)(1-B)}$

We can now write

$$\lambda^{(h)}(0,n) = 0, \quad \forall h, \text{ and}$$

$$\lambda^{(h)}(m,n) = 0, \quad \text{for } h > 2, \ \forall m, n.$$

The reduced load carried on a target link, which is not shared with the adjacent link when that adjacent link is not blocked (i.e., $n > 0$), depends on whether the target link is the first or the second hop. Expressions for this carried load are given by:

$$\lambda_d^{(1)}(m,n) = \sum_{k=1}^{N-1} a_k d_k (1 - t_1(k)) \sum_{i=0}^{k-1} \chi(k-1, i, \beta(m)) H(i|k-i) \quad \text{for } n > 0, \text{ and} \tag{3}$$

$$\begin{aligned}
\lambda_d^{(2)}(m,n) = {} & \sum_{k=2}^{N-1} a_k e_k (N-k) P_{a\&b} \\
& \times \Bigg\{ (1 - t_2(k)) \sum_{i=0}^{k-2} \chi(k-2, i, B) U(k-i-2)(1 - B^{k-i-2}\beta(m))^i \\
& + t_2(k) \Bigg[(1-B) \sum_{i=0}^{k-3} \chi(k-3, i, B) U(k-i-2)(1 - B^{k-i-2}\beta(m))^i \\
& + B \sum_{i=0}^{k-3} \chi(k-3, i, B)(1 - B^{k-3-i}\beta(m))^i \cdot U(k-i-3)(1 - U(k-i-3)) \Bigg] \Bigg\}
\end{aligned} \tag{4}$$

In deriving equation (3), which expresses the load carried on the first hop, we condition on having i destinations being blocked on the first hop. We then obtain the probability that they can be reached through the $k - i$ unblocked nodes, including the node at the end of the target link. The node at the end of the adjacent link in this case is not a destination. In equation (4), notice that the offered load to the target link, which connects two destination nodes, is now $a_k e_k$ when there are k destinations. Since we are concentrating on the second hop in equation (4), there are $N - k$ ways of choosing the source. Also, the probability of reaching the first node of the target link directly while not reaching the node at the end of the adjacent link directly is $P_{a\&b}$. The two terms inside the curly brackets correspond to the case in which the node at the end of the adjacent link is not a destination (with probability $1 - t_2(k)$), and a destination (with probability $t_2(k)$), respectively. Out of those two terms inside the square brackets, the first one corresponds to the case in which that node is reached on the first hop, while the second corresponds to the case in which it is reached on the second hop, but through another (destination) node. In each of these two terms, we also condition on having i blocked destinations on the first hop, but reach these destinations on the second hop. Notice the use of the probability $U()$ which allows the target link to be used to reach the node at its other end. Notice also that $(1-B^j\beta(m))^i$ in the above equations, with different values of j, is the probability that i blocked nodes can be reached on links through any of $j+1$ nodes, including the target link.

The expressions for the reduced load on the target link when the adjacent link is blocked are given by:

$$\lambda_d^{(1)}(m,0) = \lambda_d^{(1)}(m,1) + \sum_{k=2}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2,i,\beta(m)) H(i+1|k-i-1) \tag{5}$$

$$\begin{aligned}
\lambda_d^{(2)}(m,0) \quad = \quad & \sum_{i=0}^{k-2} a_k e_k (N-k) P_{a\&b} \\
& \Bigg\{ (1-t_2(k)) \sum_{i=0}^{k-2} \chi(k-2,i,B) U(k-i-2)(1-B^{k-2-i}\beta(m))^i \\
& + t_2(k) \Bigg[(1-B) \sum_{i=0}^{k-3} \chi(k-3,i,B) U(k-i-2)(1-B^{k-2-i}\beta(m))^i \\
& + B \sum_{i=0}^{k-3} \chi(k-3,i,B)(1-B^{k-3-i}\beta(m))^i \; U(k-i-3)(1-B^{k-3-i}) \Bigg] \Bigg\}
\end{aligned} \tag{6}$$

The above two equations are similar to equations (3) and (4), except that the node at the end of the adjacent link must be reached through other links if it is a destination. This is accounted for by the second term in equation (5), and $(1-B^{k-3-i})$ in equation (6).

Finally, the load shared between the target and the adjacent links, when both m and n are greater than zero is now expressed as:

$$\lambda_s^{(1)}(m,n) = \sum_{k=2}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2,i,\beta(m)) H(i|k-i) \tag{7}$$

$$\lambda_s^{(2)}(m,n) = \sum_{k=3}^{N-1} a_k e_k (N-k) t_2(k) Pa\&b \cdot \beta(0) \sum_{i=0}^{k-3} \chi(k-3,i,B)(1-B^{k-3-i}\beta(m))^i U(k-i-3)^2 \tag{8}$$

In both of the above equations, the node at the end of the adjacent link must be a destination. While in both equations we condition on having i nodes unreachable through the first hop, in equation (8) the node at the end of the adjacent link must be a destination that could not be reached on the first hop, but must be reachable on the second hop, and through the adjacent link. At the same time, the target link must be used to reach the node at its end. The last term in the equation accounts for these last two probabilities. Note that $P_{a\&b}$ in equation (8) is the probability that the nodes at the beginning and at the end of the target second hop link are respectively reachable and unreachable on the first hop. $\beta(0)$ is the probability that the node at the end of the adjacent link is blocked on the direct link given that one of the destinations was also unreachable.

The probability $H(i|l)$ in this case is equal to $(1-B^l)^i$

The elements of the transition matrix can be computed using the equations in the direct routing case.

Finally, after solving for the steady state probabilities, and after obtaining $q(m)$, the probability of call success can be expressed as:

$$\begin{aligned} P(success) &= \sum_{k=1}^{N-1} r_k \left\{ B \sum_{i=0}^{k-2} \chi(k-1,i,\beta(0)) H(i+1|k-i-1) \right. \\ &\quad \left. + \sum_{m=1}^{W} q(m) \sum_{i=0}^{k-1} \chi(k-1,i,\beta(m)) H(i|k-i) \right\}. \end{aligned} \tag{9}$$

In equation (9) the first term between the curly brackets corresponds to the case in which a chosen target link is blocked, but the node at the end of this link is reachable through any of the $k-i-1$ other nodes which have been reached on the first hop. The second term corresponds to the case in which the chosen target link is not blocked, and has m available channels.

III.3 Indirect Routing with Any Number of Levels

We now relax the assumption of indirect routing of the last section by allowing any number of hops. According to these assumptions, the maximum number of hops, given there are k destinations, is equal to k. As will be shown in the numerical examples section, this allowance does not achieve much enhancement in performance. However, the added control cost can be considerable. To quantify this, consider an N node network, and with a k-destination session. The number of alternative routes which may have to be considered under the two-hop limit and no hop limit are $O(k^2)$ and $O(k!)$, respectively. The difference in complexity is evident for $k > 2$.

We define the following probabilities:

$\mathcal{B}(l)$	P(a destination node is not reachable on any of l links)= B^l
$R^{(h)}(i,j|k,l)$	P(reaching i out of k nodes, including j nodes on the hth hop including the first node of the target link, given that l nodes have been reached)

In this case, and for $m > 0$, the disjoint reduced load on the target link is given by:

$$\lambda_d^{(1)}(m,n) = \sum_{k=1}^{N-1} a_k d_k (1-t_1(k)) \sum_{i=0}^{k-1} \chi(k-1,i,\beta(m)) H(i|k-i) \quad \text{for } n > 0 \tag{10}$$

$$\lambda_d^{(h)}(m,n) = \sum_{k=2}^{N-1} a_k e_k (N-k) \sum_{i=h-1}^{k-1} \sum_{j=1}^{i-h+2} R^{(h-1)}(i,j|k,1) U(j-1)$$

$$
\begin{aligned}
&\times\left\{(1-t_2(k))\sum_{l=0}^{k-i-1}\chi(k-i-1,l,\beta(m))\sum_{s=0}^{l}\chi(l,s,\mathcal{B}(j-1))H(s|k-i-s)\right.\\
&+t_2(k)\left[\frac{i-1}{k-2}\sum_{l=0}^{k-i-1}\chi(k-i-1,l,\beta(m))\sum_{s=0}^{l}\chi(l,s,\mathcal{B}(j-1))H(s|k-i-s)\right.\\
&\left.\left.+\ \frac{k-i-1}{k-2}(1-U(j-1))\sum_{l=0}^{k-i-2}\chi(k-i-2,l,\beta(m))\sum_{s=0}^{l}\chi(l,s,\mathcal{B}(j-1))H(s|k-i-s)\right]\right\}
\end{aligned}
\tag{11}
$$

Equation (10) is identical to equation (5). Equation (11) is valid for $n > 0$ and $h > 1$. In this equation, we must reach the beginning of the target link in $h-1$ hops, and when we do, we would have already reached a total of i destinations including j destinations on the $h-1$th hop. $R^{(h-1)}(i,j|k,l)$ accounts for the probability of this event. We must also reach the node at the end of the target link, using the target link, which happens with probability $U(j-1)$. Part of the remaining nodes can be reached on the hth hop, while the rest will be reached on farther hops; the probability of this event is given by the two nested summations in all the terms inside the curly brackets. The two terms inside the curly brackets stand for the probabilities of reaching the rest of the nodes when the adjacent link does not lead to a destination, and leads to a destination, respectively. In the second case, we must take into account the two cases in which that end node has already been reached or not. These two cases correspond to the terms inside the square bracket. Having reached that node already has a probability of $(\frac{i-1}{k-2})$, while having not reached it, but reaching it through one of the other $j-1$ nodes happens with probability $\left(\frac{k-i-1}{k-2}\right)(1-U(j-1))$.

Similarly,

$$
\lambda_d^{(1)}(m,0)=\lambda_d^{(1)}(m,1)+\sum_{k=1}^{N-1}a_k d_k t_1(k)\sum_{i=0}^{k-2}\chi(k-2,i,\beta(m))H(i+1|k-i-1) \tag{12}
$$

$$
\begin{aligned}
\lambda_d^{(h)}(m,0) \quad &= \quad \sum_{k=2}^{N-1}a_k e_k(N-k)\sum_{i=h-1}^{k-1}\sum_{j=1}^{i-h+2}R^{(h-1)}(i,j|k,1)U(j-1)\\
&\cdot\sum_{l=0}^{k-i-1}\chi(k-i-1,l,\beta(m))\sum_{s=0}^{l}\chi(l,s,\mathcal{B}(j-1))H(s|k-i-s) \quad \text{for } h>1
\end{aligned}
\tag{13}
$$

Again, equation (12) is also identical to equation (5). In equation (13), after finding the probability of reaching a total of i nodes including j nodes on the $h-1$th hop (which includes the node at the start of the target link), we multiply by the probability of reaching the rest of the nodes, regardless of whether the node at the end of the adjacent link is a destination or not. We also have

$$\lambda_s^{(1)}(m,n) = \sum_{k=2}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2, i, \beta(m)) H(i|k-i) \quad \text{for } n > 0 \tag{14}$$

$$\lambda_s^{(h)}(m,n) = \sum_{k=2}^{N-1} a_k e_k (N-k) t_2(k) \sum_{i=h-1}^{k-2} \sum_{j=1}^{i-h+1} R^{(h-1)}(i,j|k,1) U(j-1)^2$$
$$\times \frac{k-i-1}{k-2} \sum_{l=0}^{k-i-2} \chi(k-i-2, l, \beta(m)) \sum_{s=0}^{l} \chi(l, s, \mathcal{B}(j-1) H(s|k-i-s) \quad \text{for } n > 0 \tag{15}$$

In equation (15), we find the probability of reaching the beginning of the target link on the $h-1$th hop, when the adjacent link also leads to a destination (with probability $t_2(k)$) which has not been reached yet. Then, we multiply by the probability of reaching the rest of the nodes, including the one at the end of the target link.

$R^{(h)}(i,j|k,l)$, as defined above, allows one to find the number of nodes reached in h hops, and the number of nodes reached on the hth hop (see Figure (2)). It can be obtained in a recursive manner as follows:

$$\begin{cases} P_{a\&b} \chi(k-2, k-j-1, \mathcal{B}(l)) & h = 1,\ i = j \\ P_{b\&b} \sum_{r=1}^{h+j-3} \chi(k-2, r, \mathcal{B}(l)) R^{(h-1)}(i-k+2+r, j|r+2, k-2-r) & h > 1,\ j+h-1 \leq i \leq k-1 \\ 0 & \text{otherwise} \end{cases}$$

for $l \geq 1$.

Also, $H(i|l)$

$$H(i|l) = \begin{cases} \sum_{j=0}^{i-1} \chi(i, j, \mathcal{B}(l)) \times H(j|i-j) & l, i > 0 \\ H(i|0) = 0 & i > 0 \\ H(0|l) = 1 & l \geq 0 \end{cases}$$

Once the elements of the transition rate matrix have been evaluated, the steady state probabilities be solved for. The probability of success can then be evaluated using equation (9).

It should be noted that when $k \leq 2$, the case considered in this section reduces to that considered in the previous section.

IV Networks without Wavelength Conversion

In this section we assume that the switches do not implement wavelength conversion. We also assume that to establish a call, all routes and wavelengths are exhaustively searched until either the call can be established, or all the routes and wavelengths have been exhausted. The outgoing links from the source may use the same or different wavelength channels.

The derivation of the reduced load equations in this section is more involved since one has to keep track of the number of commonly available wavelength channels on a given path. In cases such as indirect routing with any number of levels, it was found that even an approximate model is very involved, as will be explained in subsection IV.3. Therefore, we do not model this case, and will depend on simulation.

IV.1 Direct Routing

This case is identical to the case of networks with wavelength conversion.

IV.2 Indirect Routing with a Limit of Two Levels

With a limit of two hops, a call can be routed from the source to the destination either:

- on the direct link, in which case any available channel on this link can be used; or
- if the direct link is blocked, the call can be routed via a destination that has been reached via a direct link. This requires that the same wavelength be available on both the first and second links.

We define the following probabilities which are needed in the subsequent derivations:

$c(n\|m,f,W)$	probability of n common free channels between two links, given m free channels on the first, f free channels on the second, and a total of W channels per link = $\dfrac{\binom{m}{n}\binom{W-m}{f-n}}{\binom{W}{f}}$
$b^{(l)}(n\|m)$	probability that a link with m free channels has n of those free channels in common with l other links originating from the node at the end of this particular link
$F(j\|i,m)$	P(reaching j destinations out of i destinations starting from the end of the first (direct) link, given m wavelengths on the link)
$G(j\|i,m)$	P(reaching j out of i, including the node at the end of a particular second link, given m wavelengths on that particular link, and the first link is not blocked)
$G_{\overline{a}}(j\|i,m,n)$	P(reaching j out of i nodes from the node at the end of the first link, and **not reaching** the node at the end of the adjacent link, given that m and n channels are available on the target and adjacent links, respectively)
$G_a(j\|i,m,n)$	P(reaching j out of i nodes from the node at the end of the first link, and **reaching** the node at the end of the adjacent link, given that m and n channels are available on the target and adjacent links, respectively)
$T(i,n,p\|l)$	P(reaching i out of i nodes over the second hop, via l nodes that have been reached directly, and n of the l nodes can also reach the destination at the end of the target link, while p of the l nodes will reach the destination at the end of the adjacent link)
$S(i,n\|l)$	P(reaching i out of i nodes over the second hop, via l nodes that have been reached directly, and n out of the l nodes can also reach the destination at the end of the target link) = $\sum_{p=0}^{l} T(i,n,p\|l)$

With the above definitions, we can now obtain the reduced load on the target link. Similar to subsection III.2, we have

$$\lambda^{(h)}(0,n) = 0, \quad \forall h, \text{ and}$$

$$\lambda^{(h)}(m,n) = 0, \quad \text{for } h > 2, \ \forall m,n.$$

The reduced load on the target link with m available channels, when that load is not shared with the adjacent link that has n, $n > 0$, free channels can be derived by considering the same situations considered under wavelength conversion. However, we have to involve the probabilities of wavelength continuity. The reduced load is given by

$$\lambda_d^{(1)}(m,n) = \sum_{k=1}^{N-1} a_k d_k (1-t_1(k)) \sum_{i=0}^{k-1} \chi(k-1,i,\beta(m)) \cdot \sum_{j=0}^{i} F(j|i,m) H(i-j|k-i-1) \quad \text{for } n>0, \text{ and} \tag{16}$$

$$\begin{aligned}\lambda_d^{(2)}(m,n) = {} & \sum_{k=2}^{N-1} a_k e_k (N-k) P_{a\&b} \times \\ & \Bigg\{ \Bigg[(1-t_2(k)) \sum_{i=0}^{k-2} \chi(k-2,i,B) \sum_{j=1}^{i+1} G(j|i+1,m) \sum_{r=0}^{k-i-2} S(i+1-j,r|k-i-2) \times \frac{1}{r+1} \Bigg] \\ & + t_2(k) \Bigg[(1-B) \sum_{i=0}^{k-3} \chi(k-3,i,B) \sum_{j=1}^{i+1} G(j|i+1,m) \sum_{r=0}^{k-i-2} S(i+1-j,r|k-i-2) \times \frac{1}{r+1} \\ & + B \sum_{i=0}^{k-3} \chi(k-3,i,B) \Bigg(\sum_{j=1}^{i+1} G_{\overline{a}}(j|i+1,m,n) \sum_{r=0}^{k-i-3} S(i+1-j,r|k-i-3) \times \frac{1}{r+1} \\ & + \sum_{j=1}^{i+1} G_a(j|i+1,m,n) \sum_{s=0}^{k-i-3} \sum_{r=0}^{k-i-3} T(i+1-j,r,s|k-i-3) \times \frac{1}{r+1} \times \frac{s}{s+1} \Bigg) \Bigg] \Bigg\}\end{aligned} \tag{17}$$

Equation (16) is straightforward. It is just to be noted that the last summation is the probability of reaching j out of i nodes from the end of the target link (which has m available channels), and reaching the remaining $i-j$ nodes via the $k-i-1$ nodes that have already been reached on the first hop.

Equation (17) is very similar to equation (4). The only difference is that in equation (17) we must make sure that when a wavelength is chosen on the first hop, it must also be employed on the second hop. Similar to equation (4), the two terms between the curly brackets in equation (17) correspond to the cases of the node at the end of the adjacent link not being a destination, and being a destination, respectively. When the node at the end of the adjacent link is not a destination, we need to only reach the node at the end of the target link, in addition to i other nodes which have been blocked on the first hop; hence, the double summation in the first term inside the curly brackets. The second term inside the curly brackets is also divided into the probability of that node being already reached on the first hop, and the probability of reaching it on the second hop, but through a different link, respectively. However, in all of the terms inside the curly brackets, we use the probability $G()$ that the node at the beginning of the target link can reach j other nodes, in addition to the node at the end of this link, and we multiply it by the probability $S()$ of reaching the rest of the destinations through other nodes. In doing so, $S()$ also caters for the probability that the node at the end of the target link can be reached by r other nodes. The $\frac{1}{r+1}$ factor is the probability that the target link will be used to reach that latter node in this case. In the case in which the node at the end of the adjacent link is a destination which will be reached on the second hop (last term which is multiplied by B for being blocked on the first hop), we distinguish between two cases: the case in which it cannot be reached on the adjacent link because of wavelength discontinuity; and the case in which it can be reached. In this last case we obtain the probability that the node at the end of the adjacent link can be reached by s other nodes, and one of those nodes will be used to reach it.

The case when the adjacent link is blocked can be considered similarly, but we take into account the fact that the node at the end of the adjacent link, if it is a destination, will be reached either on the first hop, or on the second hop but using another link. The expressions for the reduced load are given by

$$\begin{aligned}\lambda_d^{(1)}(m,0) &= \lambda_d^{(1)}(m,1)+ \\ &+ \sum_{k=1}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2,i,\beta(m)) \cdot \sum_{j=0}^{i+1} F(j|i+1,m) H(i+1-j|k-i-2)\end{aligned} \tag{18}$$

$$\begin{aligned}\lambda_d^{(2)}(m,0) &= \sum_{k=2}^{N-1} a_k e_k (N-k) P_{a\&b} \\ &\times \Bigg\{ (1-t_2(k)) \sum_{i=0}^{k-2} \chi(k-2,i,B) \sum_{j=1}^{i+1} G(j|i+1,m) \sum_{r=0}^{k-i-2} S(i+1-j,r|k-i-2) \times \frac{1}{r+1} \\ &+ t_2(k) \Bigg[(1-B) \sum_{i=0}^{k-3} \chi(k-3,i,B) \sum_{j=1}^{i+1} G(j|i+1,m) \sum_{r=0}^{k-i-2} S(i+1-j,r|k-i-2) \times \frac{1}{r+1} \\ &+ B \sum_{i=0}^{k-3} \chi(k-3,i,B) \sum_{j=1}^{i+1} G_{\overline{a}}(j|i+1,m,0) \sum_{r=0}^{k-i-3} S(i+2-j,r|k-i-3) \times \frac{1}{r+1} \Bigg] \Bigg\}\end{aligned} \tag{19}$$

Equation (18) is derived in a manner similar to that used in deriving equation (16), but taking into account the fact that the adjacent link has no available channels. This requires adding the second term which represents the case in which the node at the end of the adjacent link is a destination, but will be reached through other links. Equation (19) has been derived similar to equation (17), except that the node at the end of the adjacent node cannot be reached over the adjacent link ($G_a() = 0$).

Finally, the shared load between the target and the adjacent links can now be expressed as

$$\lambda_s^{(1)}(m,n) = \sum_{k=2}^{N-1} a_k d_k t_1(k) \sum_{i=0}^{k-2} \chi(k-2,i,\beta(m)) \sum_{j=0}^{i+1} F(i,j|m) H(i-j|k-i-1) \quad \text{, and} \tag{20}$$

$$
\begin{aligned}
\lambda_s^{(2)}(m,n) \;=\; & \sum_{k=3}^{N-1} a_k e_k (N-k) t_2(k) P_{a\&b}\beta(0) \\
& \times \left[\sum_{i=0}^{k-3} \chi(k-3,i,B) \sum_{j=1}^{i+1} G_a(j|i+1,m,n) \right. \\
& \left. \sum_{s=0}^{k-i-3} \sum_{r=0}^{k-i-3} T(i+1-j,r,s|k-i-3) \times \frac{1}{r+1} \times \frac{1}{s+1} \right]
\end{aligned} \tag{21}
$$

for $m, n > 0$. The derivation of the above equations follows earlier derivations, except that in equation (21) the node at the end of the adjacent link is reached over this adjacent link, even if that node can be reached through r other nodes.

The derivation of the auxiliary probabilities $b^{(l)}(n|m)$, $F(j|i,m)$, $G(j|i,m)$, $G_{\overline{a}}(j|i,m,n)$, $G_a(j|i,m,n)$, $S(i,n|l)$, $T(i,n,p|l)$ and $H(i|l)$ is carried out in the appendix.

Similar to equation (9),

$$
\begin{aligned}
P(success) \;=\; & \sum_{k=1}^{N-1} r_k \left\{ B \sum_{i=0}^{k-2} \chi(k-1,i,\beta(0)) H(i+1|k-i-1) \right. \\
& \left. + \sum_{m=1}^{W} q(m) \sum_{i=0}^{k-1} \chi(k-1,i,\beta(m)) \sum_{j=0}^{i} F(j|i,m) H(i-j|k-i-1) \right\}
\end{aligned} \tag{22}
$$

The only difference is that in the second term (the case in which the target link has m available channels) we must make sure that when we reach j nodes out of i from the end of that link, they must be reached on the same wavelength. This is accounted for by the probability $F()$.

IV.3 Indirect Routing with Any Number of Levels

This case is more involved than all the cases considered in this paper. The difficulty stems from the fact that the spanning trees starting from the nodes which are reached at any stage are not independent. This follows since a particular node may possess free channels that belong to more than one spanning tree. This should be expected since different wavelengths can be used on different first (direct) hops.

An example to illustrate the problem is shown in Figure 4. In this example, the source is node s and the set of destinations is given by $\{a,b,c,d,e,f\}$. Node a is the only node that is reachable directly from s on either of wavelengths μ_1 or wavelength μ_2. Also, node b is reachable from node a on the same wavelengths. Nodes c and d are both reachable from node b on both wavelengths. Node e is reachable from node c on wavelength μ_1, while node f is reachable from node d on wavelength

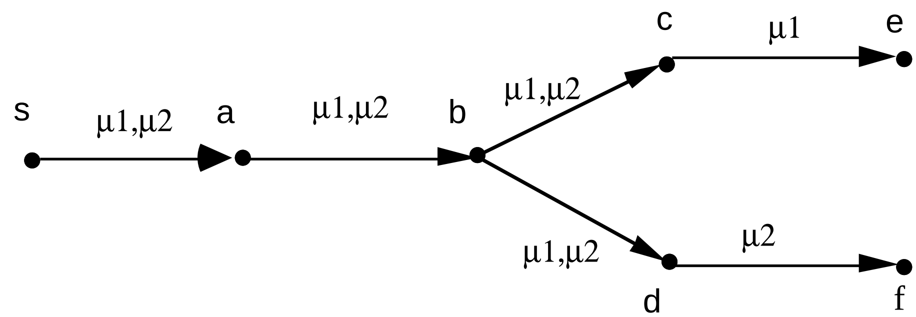

Figure 4: Example to illustrate the problem in modeling indirect routing with any number of levels

μ_2. If one considers only the number of wavelengths available on each of the links, then this call can be successful according to the given spanning tree. However, if we consider the identities of the wavelength channels, then the call will be blocked, also using the same spanning tree. This means that one has to take into account, not only how many nodes have been reached on how many wavelength channels, but also the identities of the wavelength channels used to reach those nodes. This increases the complexity of the model substantially.

Because of the complexity involved, we rely on simulation to assess the blocking probabilities in this case, and we will not derive expressions for blocking probabilities.

V Numerical Results

The expressions for the reduced load derived in the previous section assume the knowledge of the channel availability distribution. Since these probabilities depend in turn on the knowledge of the reduced loads, through the matrix $\mathcal{P}$, an iterative solution procedure must be used. In this procedure, we start by assuming arbitrary values of $\lambda(m,n)$, and we use them to solve for the steady state probabilities and $q(m)$. Using the computed values of $q(m)$, we recalculate $\lambda(m,n)$, and solve again for the steady state probabilities. The procedure is repeated until the relative error in two successive computations of $\lambda(m,n)$ does not exceed a certain error value. The error value used in all the examples is 10^{-3}.

We first validate the model by comparing the analytical results against results from simulation. In Table 1 we consider a network with a total of three completely interconnected nodes. Each of the interconnecting fibers carries three wavelength channels, and wavelength conversion is used. Two load scenarios are considered: 1 Erlang per node, and 2 Erlangs per node. For each of these two scenarios, three different load vectors are considered which correspond to unicast service, broadcast

Network parameters					Blocking prob.:model			Blocking prob.:simulation		
N	W	Load per node	Load vector	Load per channel	direct routing	alternate with 2 hops	alternate with no limit	direct routing	alternate with 2 hops	alternate with no limit
3	3	1	1 0	.1667	.0126	.0126	.0126	.0126	.0126	.0126
3	3	1	.5 .5	.25	.0410	.0250	.0250	.0409	.0253	.0253
3	3	1	0 1	.333	.0625	.0625	.0625	.0625	.0625	.0625
3	3	2	1 0	.333	.0625	.0625	.0625	.0625	.0625	.0625
3	3	2	.5 .5	.5	.1528	.1164	.1164	.1529	.1194	.1193
3	3	2	0 1	.667	.2105	.2105	.2105	.2105	.2105	.2105

Table 1: Blocking probabilities for a network with wavelength conversion and 3 nodes

service, and a uniform distribution of the number of destinations. The offered load per channel is calculated for each of the cases and is shown in the table. The table contains the blocking probabilities for the three routing strategies. The table also shows the offered load per channel. It is shown that the analytical results match the simulation results very closely. It is also shown that since the maximum number of destinations is two, then both indirect routing strategies yield exactly the same results. Table 2 considers the case of a network with six nodes, and three channels per fiber. Different load vectors are used, and the corresponding blocking probabilities are shown in the table. Most of the analytical results shown in the table are very close to simulation results. Only one case showed a lower level of accuracy. This is the case in which the load vector corresponds to a uniform distribution of destinations[8] Tables 3 and 4 show the corresponding examples for the case of no wavelength conversion. Similar conclusions can be drawn from these tables.

From the above tables we conclude two things. First, the use of indirect routing rather than direct routing enhances the performance of the system. Indirect routing with no hop limit results

[8]Unlike the unicast and broadcast cases, in which the transition probabilities are either to the adjacent states and the extreme states, respectively, and in which the number of hops is typically one, the uniform load cases shows more variability. The transition rate matrix is more dense, and the state probabilities are expressed in terms of so many other state probabilities. These expressions assume a certain degree of independence between links. For example, in the probability $H()$ used in equations (10)-(15), capturing the effect of the previously traced links assumes independence between those links. Therefore, as the transition rate matrix becomes more dense, and as the number of hops increases, the effect of this independence becomes more apparent.

Network parameters					Blocking prob.: model			Blocking prob.: simulation		
N	W	Load per node	Load vector	Load per channel	direct routing	alternate with 2 hops	alternate with no limit	direct routing	alternate with 2 hops	alternate with no limit
6	3	1	1 0 0 0 0	.06667	.00109	.00109	.00109	.00103	.00103	.00103
6	3	1	.2 .2 .2 .2 .2	.2	.0371	.0069	.0067	.0343	.0079	.0078
6	3	1	0 0 0 0 1	.3333	.0623	.0619	.06267	.0625	.0625	.0624
6	3	1	.5 .5 0 0 0	.1	.0049	.0017	.0017	.0049	.0017	.0017
6	3	2	1 0 0 0 0	.1333	.007156	.007156	.007156	.007268	.007268	.007268
6	3	2	.2 .2 .2 .2 .2	.4	.1378	.0423	.038	.1320	.0514	.048
6	3	2	0 0 0 0 1	.6667	.21056	.2105	.2098	.21169	.2106	.2106
6	3	2	.5 .5 0 0 0	.2	.0279	.01129	.01129	.0279	.0114	.0114

Table 2: Blocking probabilities for a network with wavelength conversion and 6 nodes

Network parameters					Blocking prob.:model		Blocking prob.:simulation		
N	W	Load per node	Load vector	Load per channel	direct routing	alternate with 2 hops	direct routing	alternate with 2 hops	alternate with no limit
3	3	1	1 0	.1667	.0126	.0126	.0126	.0126	.0216
3	3	1	.5 .5	.25	.0410	.02636	.0409	.0253	.0253
3	3	1	0 1	.333	.0625	.0624	.0625	.0625	.0625
3	3	2	1 0	.333	.0625	.0625	.0625	.0625	.0625
3	3	2	.5 .5	.5	.1528	.12266	.1529	.1194	.1194
3	3	2	0 1	.667	.2105	.2103	.2105	.2105	.2105

Table 3: Blocking probabilities for a network without wavelength conversion and 3 nodes

Network parameters					Blocking prob.: model		Blocking prob.: simulation		
N	W	Load per node	Load vector	Load per channel	direct routing	alternate with 2 hops	direct routing	alternate with 2 hops	alternate with no limit
6	3	1	1 0 0 0 0	.06667	.00109	.00109	.00107	.00107	.00107
6	3	1	.2 .2 .2 .2 .2	.2	.0371	.00777	.0343	.01016	.00875
6	3	1	0 0 0 0 1	.3333	.0623	.06263	.0625	.06248	.06244
6	3	1	.5 .5 0 0 0	.1	.0049	.001775	.0049	.0019	.0019
6	3	2	1 0 0 0 0	.1333	.007156	.007156	.007268	.007067	.007067
6	3	2	.2 .2 .2 .2 .2	.4	.1378	.055355	.1320	.06653	.05817
6	3	2	0 0 0 0 1	.6667	.21056	.21052	.21169	.021052	.021051
6	3	2	.5 .5 0 0 0	.2	.0279	.01196	.0279	.01328	.01328

Table 4: Blocking probabilities for a network without wavelength conversion and 6 nodes

in a performance which is marginally better than that under indirect routing with a two hop limit. Therefore, it is not really justifiable to use a significantly more complex control function in order to achieve that marginal enhancement. Second, the use of wavelength conversion reduces the probability of call blocking as compared to the absence of wavelength conversion.

In Figure 5 we study the effect of the number of channels per fiber on the call blocking probability for all routing strategies. We assume that the network has 10 nodes, with each node offering a total load of 2 Erlangs. Since in most networks, unicast applications are expected to dominate, we assume that the probability of a unicast call is 0.5, while the probability of a call with k destinations for $k = 2, 3, \ldots, 9$ is equal to 0.0625. As expected, increasing the number of channels per fiber significantly reduces the call blocking probability. Adding one more channel per fiber reduces the call blocking probability by almost one order of magnitude for all cases. For example, under wavelength conversion, and with indirect routing with a limit of two hops, increasing the number of channels per fiber from 6 to 7 reduces the call blocking probability from 4.9×10^{-5} to 5×10^{-6}. We notice also that direct routing results in the worst performance, while the other indirect routing strategies result in almost the same performance. This confirm our earlier conclusion that the complexity of indirect routing with a number of hops beyond two is not justifiable.

Since the above enhancement in performance is expected to be mainly due to the reduction in

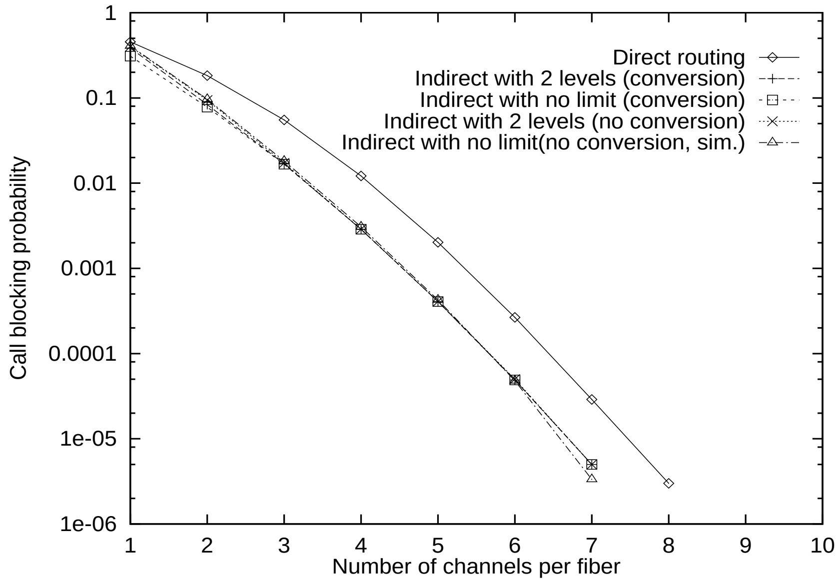

Figure 5: Blocking probabilities versus the number of wavelength channels per fiber, when the node offered load is 2 Erlangs

the load on the different channels when the number of channels is increased, in Figure 6 we study the performance of the system under a given load per channel regardless of the number of channels.

We keep the same parameters as before, except that the offered load per node is changed in order to make the offered load per channel equal to 0.5. It is noticed from the figure that the call blocking probabilities with indirect routing strategies are much better than those achieved with direct routing, and they respond to increasing the number of channels even faster. At the same time, using wavelength conversion results in a slightly better performance. With a small number of channels, increasing the limit on the number of hops slightly improves the performance. This improvement disappears by increasing the number of channels.

In the last two examples, we study the effect of the number of destinations per call. We consider a network with 8 nodes, and 4 channels per fiber. Seven experiments are performed in which the number of destinations is fixed at k, where $k = 1, 2, \ldots, 7$. We consider two cases: a fixed offered load per node that is equal to 2 Erlangs, and a constant load per channel that is equal to 0.5. The results are shown in Figures 7 and 8, respectively.

First, it should be noticed that in both the unicast and the broadcast cases, all routing strategies yield exactly the same performance. This is true for the broadcast case since all links leaving any of the sources will be either all blocked, or all unblocked. It is also noticed that with direct routing, increasing the number of destinations increases the blocking probability. However, the

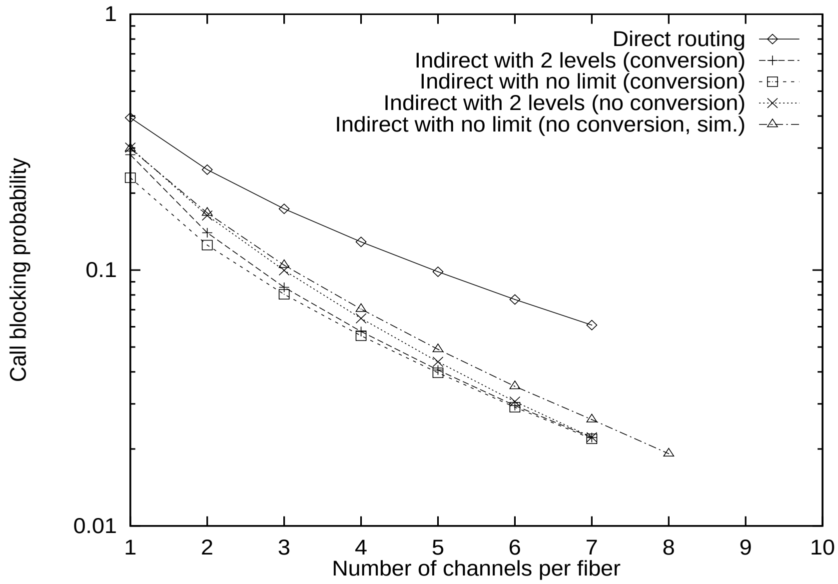

Figure 6: Blocking probabilities versus the number of wavelength channels per fiber, when the channel offered load is 0.5 Erlangs

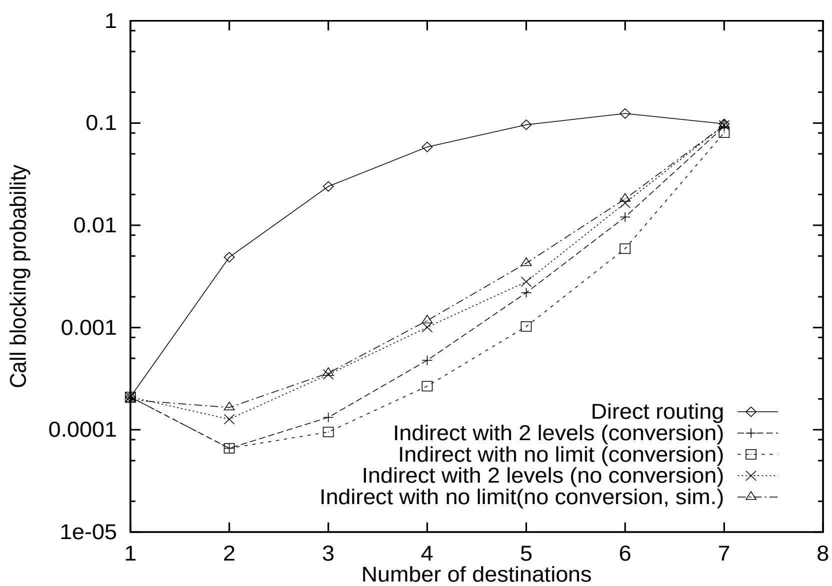

Figure 7: Blocking probabilities versus the number of wavelength channels per fiber, when the node offered load is 2 Erlangs

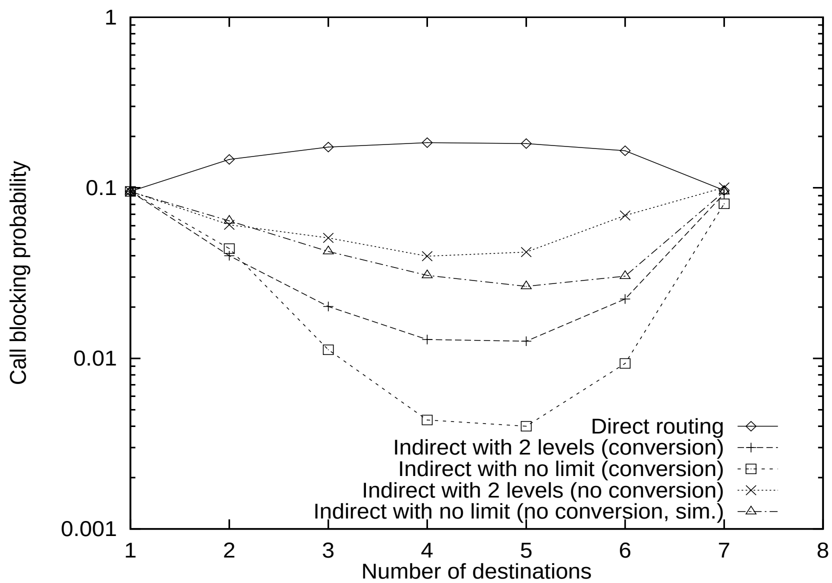

Figure 8: Blocking probabilities versus the number of wavelength channels per fiber, when the channel offered load is 0.5 Erlangs

broadcast case, $k = 7$, results in a slightly better performance because of the above mentioned correlation. With the indirect routing strategies, increasing the number of destinations from the unicast case improves the performance first, before it starts to degrade. This can be explained since with two destinations, a source has three alternative routes of reaching all the destinations compared to just one route for the case of one destination. This reduces the probability of call blocking. After that, the increased loading on the links degrades the performance. Fixing the offered load per channel (Figure 8) shows that for a given load per channel, increasing the number of destinations, which increases the number of alternative routes to reach the destinations, results in an even improved performance. However, the performance starts to degrade when the number of destinations increases because of correlation.

VI Conclusions

In this paper we derived expressions for the call blocking probabilities in optical networks under multicast service in a completely connected network topology. We considered the case in which full wavelength conversion is allowed, and the case in which no wavelength conversion is used. For both cases, we considered the direct routing method, and the indirect routing method in which the maximum number of hops can be restricted to two hops, or can be unrestricted. This last case under no wavelength conversion was found to be very involved, and the blocking probabilities were obtained using simulation only. The solution method was based on the reduced load approximation technique, which is iterative in nature.

Based on the numerical examples, it was found that although indirect routing improved the blocking probabilities, the incremental improvement in performance that was achieved by using more than two hops is marginal. Therefore, it is advantageous to use indirect routing with multicast service, but restricting the maximum number of hops to two usually yields a sufficiently good performance. It was also shown that increasing the number of destinations does not necessarily result in a degraded performance. On the contrary, since increasing the number of destinations, when employing indirect routing, increases the number alternative routes from the source to all the destinations, a better performance can be achieved.

Appendix

In this appendix we derive the auxiliary probabilities $b^{(l)}(n|m)$, $F(j|i,m)$, $G(j|i,m)$, $G_{\overline{a}}(j|i,m,n)$, $G_a(j|i,m,n)$, $S(i,n|l)$, $T(i,n,p|l)$ and $H(i|l)$ which are required in the derivation of the blocking probability under multicast service without wavelength conversion, and when a limit of two hops is imposed.

We first define $\eta_{i,j}$ and $\delta_{i,j}$ in the exactly the same was e_i and d_i are defined, except that we condition on having a total of j possible destinations. Therefore, we have

$$\eta_{i,j} = \frac{i}{j}$$

$$\delta_{i,j} = \frac{i(i-1)}{j(j-1)}$$

The probability $b^{(l)}(n|m)$ can be obtained recursively as follows:

$$\begin{aligned} b^{(1)}(n_1|m) &= \sum_{f_1=n_1}^{W-m+n_1} c(n_1|m,f_1,W)q(f_1) \\ b^{(l)}(n_l|m) &= \sum_{n_{l-1}=n_l}^{m} \sum_{f_l=n_l}^{W-n_{l-1}+n_l} c(n_l|n_{l-1},f_l,W)b^{(l-1)}(n_{l-1}|m)q(f_l) \quad \text{for } l>1 \end{aligned}$$

These expressions are based on adding one link at a time, and restricting the number of common free channels, based on the newly added link.

$F(j|i,m)$ can now be computed by starting from the end of the target link and reaching exactly j destinations on links which will have n common free channels with the target link. It can be expressed as:

$$F(j|i,m) = \binom{i}{j} \sum_{n=1}^{m} b^{(j)}(n|m) F(0|i-j,n) \tag{23}$$

with the initial condition

$$F(0|i,m) = \left[\sum_{f=0}^{W-m} c(0|m,f,W) q(f)\right]^i = \left[b^{(1)}(0|m)\right]^i$$

To compute $G(j|i,m)$ it is required that out of the i destinations which must be reached on the second hop, j will be reached, including the one at the end of the target link which has m available wavelength channels. $G(j|i,m)$ can therefore be expressed as

$$G(j|i,m) = \sum_{f=1}^{W} \sum_{n=1}^{\min(m,f)} c(n|m,f,W) \frac{q(f)}{1-B} F(j-1|i-1,n) \tag{24}$$

In equation (24), $c(n|m,f,W)$ is the probability that, given that the considered second link has m available channels, and the direct link has f channels, they both have n common available channels. $F(j-1|i-1,n)$ indicates the probability of reaching $j-1$ destinations out of the remaining $i-1$ destinations given those available n channels.

Once $G(j|i,m)$ has been calculated, one can proceed to calculate

$$G_{\overline{a}}(j|i,m,n) = c(0|m,n,W) G(j|i,m), \quad \text{and}$$

$$G_a(j|i,m,n) = \sum_{r=1}^{\min(m,n)} c(r|m,n,W) G(j|i,r)$$

The above two expressions are simply obtained by multiplying $G()$ by the probabilities that the adjacent link has no common channels with the j destinations, or has at least one common channel, respectively.

Finally, $S(i,n|l)$ can also be obtained recursively by counting the number of destinations that can be reached from each of the l nodes at the end of the first hop, one at a time:

$$\begin{aligned} S(i,n|l) &= \sum_{j=0}^{i} \left(\sum_{f=1}^{W} F(j|i+1,f) \frac{q(f)}{1-B} \right) \cdot \frac{i+1-j}{i+1} S(i-j,n|l-1) \\ &+ \sum_{j=0}^{i} \left(\sum_{f=1}^{W} F(j+1|i+1,f) \frac{q(f)}{1-B} \right) \cdot \frac{j+1}{i+1} S(i-j,n-1|l-1) \end{aligned} \tag{25}$$

with $S(0,0|0) = 1$, $S(i,n|l) = 0$ for $n > l$, $S(i,n|0) = 0$ for $i > 0,\ n \geq 0$, $S(0,0|l) = 1$ for $l \geq 0$, $S(0,n|l) = 0$ for $l,n > 0$ and

$$S(i,0|l) = \sum_{j=0}^{i} \left(\sum_{f=1}^{W} F(j|i+1,f) \frac{q(f)}{1-B} \right) \cdot \frac{i+1-j}{i+1} S(i-j,0|l-1) \quad . \tag{26}$$

Similar to $S(i, n|l)$, we can also obtain$T(i, n, p|l)$ by incorporating the probability that p out of the l nodes can reach the node at the end of the target link.

$$
\begin{aligned}
T(i,n,p|l) = \\
&\sum_{j=0}^{i}(\sum_{f=1}^{W} F(j|i+2,f)\frac{q(f)}{1-B}(1-\eta_{j,i+2}-2\delta_{j,i+2}(1-\delta_{j-1,i+1})T(i-j,n,p|l-1) \\
&+\sum_{j=0}^{i}(\sum_{f=1}^{W} F(j+1|i+2,f)\frac{q(f)}{1-B})g_{j+1,i+2}[T(i-j,n-1,p|l-1)+T(i-j,n,p-1|l-1)] \\
&+\sum_{j=0}^{i}(\sum_{f=1}^{W} F(j+2|i+2,f)\frac{q(f)}{1-B})\eta_{j+2,i+2}T(i-j,n-1,p-1|l-1) \qquad (27)
\end{aligned}
$$

with the following initial conditions:

$$
\begin{aligned}
T(i,n,p|0) &= 0 \quad \text{for } i \text{ or } n \text{ or } p > 0 \\
T(i,n,p|l) &= 0 \quad \text{for } n \text{ or } p > l \\
T(0,0,0|l) &= 1 \quad \text{for } l \geq 0 \\
T(0,n,p|l) &= 0 \quad \text{for } n \text{ or } p > 0
\end{aligned}
$$

Notice that equation (25) can be obtained from equation (27) by summing over p from 0 to l.

Similarly, but with less involvement we can compute the following:

$$
H(i|l) = \sum_{j=0}^{i}\left(\sum_{f=1}^{W} F(j|i,f)\frac{q(f)}{1-B}\right)H(i-j|l-1) \qquad (28)
$$

with $H(0|l) = 1$ for $l \geq 0$, and $H(i|0) = 0$ for $i > 0$.

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