Optical Amplifiers Placement in WDM Mesh Networks for Optical Multicasting Service Support

Ashraf M. Hamad and Ahmed E. Kamal

***Abstract*—The problem of placing the optical amplifiers (OAs) in wavelength-routing mesh networks has been studied in the literature in two contexts: *network provisioning* [1] and *connections provisioning* [2]. In this paper, we introduce optimal and heuristic solutions for the network provisioning problem. The solution is based on constructing a multicast forest for each multicast connection with the goal of minimizing the total number of OAs needed in the network, hence reducing its cost. The optimal solution is formulated as a Mixed Integer Linear Program (MILP). On the other hand, the heuristic solution is obtained by dividing the problem into subproblems and solving them separately while taking the interdependency between these subproblems into consideration. The results obtained from both solutions are compared and they are found to be in good match.**

***Index Terms*—Optical Amplifiers Placement Problem, All-Optical Multicasting, Power Aware Multicasting.**

I. INTRODUCTION

ROuting and Wavelength Assignment (RWA) is a fundamental problem in wavelength routing optical networks which has been investigated extensively in the literature [3]. However, most studies concentrate on the classical view of the problem in which the best routing structure and wavelength(s) are computed for all or some sessions while optimizing the network throughput, wavelength usage or blocking probability. With this classical view, most solutions do not consider practical issues, such as the power loss and noise. The importance of these issues stems from the fact that when a solution may exists for the classical RWA problem, it may not necessarily be feasible in practice.

We focus in this paper on the effect of power loss on RWA under All Optical Multicasting (AOM) scenario [4][5]. In a nutshell, AOM is about supporting multicast service in the optical domain by eliminating any conversion of the transport signal between the electronic and optical domains at the intermediate nodes. To achieve this goal, branching nodes of the multicast structures (called, light-trees or light-forests) are equipped with passive optical splitters [6] which are configured to split the power strength of the incoming signal into two or more outgoing links. This operation scheme has the advantage of:

1) Achieving signal transparency with respect to traffic type, bit rates, and protocol,
2) Simplifying the logical network stack structure, and
3) Reducing the cost of the switching nodes by eliminating the extra hardware of signal conversion.

A main source of power loss is the attenuation due to propagation in optical fiber; hence called the Propagation Loss. However, under AOM, the optical signal faces an additional source of power loss due to splitting at the branching nodes of the light-trees/forests. This is called Splitting Loss. Because of this extra power loss, the traditional Optical Amplifiers Placement (OAP) problem becomes more challenging.

OAP problem is investigated in the literature in two contexts. In the first context, namely, *Network Provisioning*, the problem is formulated as a network design problem with the objective of minimizing the network cost. In [1], we introduced a Mixed Integer Linear Program (MILP) that solves the Routing (**R**), Wavelength Assignment (**WA**) and Optical Amplifier Placement (**OAP**) subproblems in an integrated way for AOM traffic. The network cost in [1] is represented as the total power amplification needed in the network. For the unicast traffic, the authors in [7] addressed the OAP problem in the simpler broadcast-and-select architecture where no routing is performed. The problem was solved using Mixed Integer non-Linear Programming (MInLP) with the objective of minimizing the total number of OAs. The work in [8] generalized the problem in [7] by incorporating different layout topologies (stars, trees and/or rings) and by taking into account the fact that the cost of OAs is location depend. The proposed solution is based on simulation annealing. While the studies in [1], [7] and [8] considered the case of having unequally powered-signals at the entry point of the OAs, reference [9] solved the equal powered-signals instance of the problem proposed in [7] using MILP.

The second aspect of the OAP problem, namely, *Connection Provisioning*, studies the impact of power constraints on the operation of already provisioned networks. In this context, we proposed an MILP in [2] that provides the optimal RWA solutions for AOM. We also designed a heuristic algorithm that relies on a special link cost function that relates the routing decisions with various power constraints and produces comparable near optimal solutions. The authors in [10] followed an iterative heuristic approach in which an initial tree is modified by replacing a set of adjacent splitting nodes by a single splitting node. Another heuristic algorithm is proposed in [11] that ensures a minimum signal quality and fairness among all destinations. This is achieved using balanced light-trees.

The goal of this paper is to investigate the Network Provisioning aspect of the OAP. Our study takes two directions. In the first direction, we formulate the OAP problem as a MILP with the objective of minimizing the total number of OAs. The MILP solution provides the optimal solution for the problem

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by solving all the constituent subproblems jointly. However, the MILP formulation cannot provide fast solution for large instances of the problem. Therefore, in the second direction of our study, we solve the problem using a greedy heuristic that provides faster, yet near optimal solutions for the OAP problem. The algorithm is based on formulating link and path cost functions that enable us to base the routing decisions of the sessions on the required number of OAs.

The remainder of the paper is organized as follows. We first define the OAP problem and the system model under OAM. This is followed by the MILP formulation in Section III. Section III also includes an extension to our original formulation to handle the case of asymmetric splitting loss at the MC nodes. However, this extension results in a non-linear formulation. The heuristic is then introduced in Section IV. Some numerical results are presented in Section V. Finally, we conclude the paper in Section VI.

II. PROBLEM DESCRIPTION

A. Power Constraints and Optical Amplifier Model

There are two main system power constraints we consider in this paper. First, the individual wavelength's power level must be detectable at any point in the network by ensuring it does not fall below a certain threshold, called P_{Sen}. Second, the total power values of all wavelengths must not increase beyond an upper bound, called P_{MAX}.

However, an additional algorithm-driven power-constraint is considered in our model in which all the channels over any link must be equally powered. This symmetric constraint does not only simplify the OAP solution, it also ensures fair utilization of the deployed OAs. Ideally this can reduce the number of OAs by avoiding situations where delta between input powers is big and OAs are saturated by high-powered input signal(s) which results in small gain and yields to small span between OAs.

Also, we use a simple model for the OA gain which is determined by:

$$G(P_{in}) = MIN\{G_0, (P_{MAX} - P_{in})\} \tag{1}$$

where P_{in} represents the aggregate power of the input signal, and G_0 is the small-signal gain in dB. Both P_{MAX} and P_{in} are in dBm. Assuming flat gain over all the channels, this gain applies to all input wavelengths.

B. System Model and Assumptions

The network is an all-optical wavelength-routed WDM network and is modeled as a connected undirected graph. Each vertex represents an optical cross connect with Drop-and-Continue (DaC) capability. However, the nodal splitting capability is sparse such that nodes equipped with power splitters are called Multicast capable (MC) nodes; otherwise, they are called Multicast Incapable (MI) nodes. Moreover, the splitters have *complete* (i.e., maximum splitting fanout of the node equals, at least, its out-degree) and *fixed* splitting ratio (i.e., each copy of the signal acquires the same portion of the signal power) capabilities. On other hand, each undirected edge is equivalent to two fibers carrying traffic in opposite directions and all fibers support the same set of wavelengths.

The network does not support wavelength conversion; hence, wavelength continuity constraint should be maintained. Also, OAs can be placed either *on-site* or *on-link*. The on-site placement is sparse and it can be at the node's input, called Pre-Amplification, or node's output, called Post-Amplification. Accordingly, and based on the notation introduced in [4], our system model is characterized as $S^sF^cR^x$-M^s where the first term consists of three components that represent the sparse, complete and fixed splitting setting, respectively, while the second term represents the sparse on-site amplification.

In addition, our solutions assume the followings:

- The symmetric power constraint over each link is achieved by equipping each output port of all cross connects with an equalizer.
- Each node is equipped with an array of a sufficient number of fixed-tuned transceivers (transmitters/receivers).
- The general delivery structure of each multicast session is light forests where each light-tree is rooted at the source node using a separate transmitter.
- We employ the As Late As Possible (ALAP) OA placement policy [9], [7]; yet, any other policy can be used in our solutions.
- For sake of simplicity, our study deals with propagation, splitting and tapping losses only and it ignores other loss sources. Impairments due to non-linearities and noise are outside the scope of this work.
- All power levels are in dBm, while power gain/loss are in dB.
- The value of P_{sen} is assumed to be high enough to cope with the various types of noises and to guarantee an adequate Bit Error Rate (BER) [12].

C. Problem Definition

The OAP problem we are studying here is formally defined as follows:

Definition: Given the network topology, maximum number of wavelengths, maximum number of splitters, and static traffic demand matrix, the Optical Amplifiers Placement (OAP) problem is a network provisioning problem and its solution is a feasible allocation of OAs and splitters such that all traffic demands and power constraints are satisfied while minimizing the number of OAs.

III. MILP PROBLEM FORMULATION

A. Network Parameters

The following parameters are used in the formulation.

N, E, Λ	Sets of nodes, links, wavelengths, respectively.
i, j, k	Node identity, where $i, j, k \in N$
λ	Wavelength identity, where $\lambda \in \Lambda$
$e(i,j)$	Fiber link directed from node i to node j.
S	Maximum number of splitters.
β	Propagation loss ratio.
γ	Tapping power loss value at each node.
$L_{i,j}$	Length of $e(i,j)$ in Km.
K	Number of multicast sessions.
a	Multicast session identity, $0 \le a \le K-1$.
src_a	Multicast session source node.
D_a	Multicast session destination set.
Φ_i	Set of connections in which node i is a member.
Γ_i^a	Binary-indicator: 1 if $i \in D_a$; 0 otherwise.
P_{Sen}	Minimum detection power level per channel.
P_{Max}	Maximum aggregate power on a link.
P_1, P_2	Negative constants, where $P_1 < P_2$.
δ	Very small number used for SL linearization.
v, w	Integer constants, such that $v > w$.
Out_i	Degree of node i.

B. MILP Variables

The following variables are used in the formulation:

$n_{i,j}$	Number of OAs on $e(i,j)$.
$T_{i,j}^{a,\lambda}$	Binary-indicator: 1 if $e(i,j)$ is used by session a over λ; 0 otherwise.
$\Im_i^{a,\lambda}$	Binary-indicator: 1 if λ is used by session a on any outgoing tree link from node i; 0 otherwise.
Υ_i^a	Binary-indicator: 1 if session a uses at least one output link from node i.
$H_i^{a,\lambda}$	Number of hops between src_a and node i over λ.
$SL_i^{a,\lambda}$	Splitting loss on λ at node i for session a.
$P_{i,j}^{\Omega,a,\lambda}$	Power level (in dBm) at the beginning ($\Omega = beg$) or end ($\Omega = beg$) of $e(i,j)$ for λ used by a.
f	Number of outgoing tree links.
A_f	Binary-indicator used for power loss linearlization.
α_i	Binary-indicator: 1 if node i is MC node.
M	Very large number.

C. MILP Formulation

The objective function is to minimize the network cost in terms of the number of OAs, and it is expressed as follows:

$$Minimize \sum_{e(i,j)\in E} n_{i,j} \tag{2}$$

The objective function is subject to the following constraints:

1. Routing and Wavelength Assignment Constraints:

$$\Im_i^{a,\lambda} \ge \sum_{j,j\neq i,e(i,j)\in E} \frac{T_{i,j}^{a,\lambda}}{M} \quad \forall i \in N; 0 \le a < K; \forall \lambda \in \Lambda \tag{3}$$

$$\Im_i^{a,\lambda} \le \sum_{j,j\neq i,e(i,j)\in E} T_{i,j}^{a,\lambda} \quad \forall i \in N; 0 \le a < K; \forall \lambda \in \Lambda \tag{4}$$

$$\Upsilon_i^a \ge \sum_{\lambda\in\Lambda} \frac{\Im_i^{a,\lambda}}{M} \quad \forall i \in N; 0 \le a < K; \forall \lambda \in \Lambda \tag{5}$$

$$\Upsilon_i^a \le \sum_{\lambda\in\Lambda} \Im_i^{a,\lambda} \quad \forall i \in N; 0 \le a < K; \forall \lambda \in \Lambda \tag{6}$$

Constraints (3) and (4) compute $\Im_i^{a,\lambda}$ as the disjunction between $T_{i,j}^{a,\lambda}$ variables of all the neighbor nodes j of node i. Similar disjunction relationship is maintained between $\Upsilon_i^{a,\lambda}$ and $\Im_i^{a,\lambda}$ variables using constraints (5) and (6) over all $\lambda' s$.

$$\sum_{i,i\neq src_a,e(i,src_a)\in E} \quad \sum_{\lambda\in\Lambda} T_{i,src_a}^{a,\lambda} = 0 \qquad 0 \le a < K \tag{7}$$

Equation (7) prevents any loop back to the source node from any of its neighbor nodes in the light-tree at any λ.

$$\sum_{\lambda\in\Lambda} \Im_i^{a,\lambda} = \sum_{\lambda\in\Lambda} \sum_{k,k\neq i,e(k,i)\in E} T_{k,i}^{a,\lambda} - \{\Gamma_i^a * (1 - \Upsilon_i^a)\}$$
$$\forall i \in N; i \neq src_a; 0 \le a < K \tag{8}$$

The above constraint guarantees that the number of the incoming channels equals the number of the *distinct* outgoing channels of node i, except for the case when node i is a leaf destination node.

$$\sum_{k,k\neq i,e(k,i)\in E} T_{k,i}^{a,\lambda} \le 1$$
$$\forall i \in N; i \neq src_a; 0 \le a < K; \forall \lambda \in \Lambda \tag{9}$$

Equation (9) prevents multiple traversals of nodes on each light-forest.

$$\sum_{0\le a<K,e(k,i)\in E} T_{i,j}^{a,\lambda} \le 1 \qquad \forall i,j \in N; i \neq j; \forall \lambda \in \Lambda \tag{10}$$

The above constraint guarantees that $e(i,j)$ is used by at most one light-tree over wavelength λ for each connection.

$$\sum_{j,i\neq j,e(i,j)\in E} T_{i,j}^{a,\lambda} \le M * \alpha_i + 1$$
$$\forall i \in N; i \neq src_a; 0 \le a < K; \forall \lambda \in \Lambda \tag{11}$$

Constraint (11) prevents branching at MI nodes. It ensures that node i has at most one outgoing tree link if it is an MI node.

$$\sum_{j,j\neq i,e(j,i)\in E} T_{j,i}^{a,\lambda} \ge \sum_{k,k\neq i,e(i,k)\in E} \frac{T_{i,k}^{a,\lambda}}{M}$$
$$\forall i \in N, i \neq src_a; 0 \le a < K \tag{12}$$

This last constraint guarantees wavelength continuity by ensuring that there is an incoming tree link incident on node i on wavelength λ if node i has at least one outgoing link employing the same wavelength.

$$\sum_i \alpha_i \le S \qquad \forall i \in N \tag{13}$$

Equation (13) is needed to ensure that the number of used splitters does not exceed the number of available splitters.

2. Loop Avoidance Constraints:

$$H^{a,\lambda}_{src_a} = 0 \qquad 0 \le a < K; \forall \lambda \in \Lambda \qquad (14)$$

$$1 - T^{a,\lambda}_{i,j} - \frac{H^{a,\lambda}_i + 1 - H^{a,\lambda}_j}{M} \ge 0$$
$$\forall e(i,j) \in E; 0 \le a < K; \forall \lambda \in \Lambda \qquad (15)$$

$$\Gamma^a_i + \frac{H^{a,\lambda}_i - (|N| - 1)}{M} \le 1$$
$$\forall e(i,j) \in E; 0 \le a < K; \forall \lambda \in \Lambda \qquad (16)$$

Initially, the number of hops from any source node to itself over any λ is zero. This is guaranteed by constraint (14). Then, constraint (15) ensures that if $e(i,j)$ is used by session a, then node j is one more hop away from source node than node i. Finally, constraint (16) ensures that a tree is generated by ensuring that the destination nodes are reached by at most $|N| - 1$ hops.

3. Power Constraints:

In order to ensure that the total power constraint is met, we assume that the power value of each wavelength cannot exceed $\frac{P_{Max}}{|\Lambda|}$, where $|\Lambda|$ is the number of wavelengths. Although this can result in using more OAs per link than needed (as more power can be used to reach longer distance over links with more free channels), this helps in simplifying the MILP formulation. Moreover, this does not contradict with the main purpose of the MILP formulation which is used to basically determine the goodness of the greedy solutions with respect to optimal counterparts.

$$P^{\Omega,a,\lambda}_{i,j} \ge P_{Sen} * T^{a,\lambda}_{i,j} + P_1 * (1 - T^{a,\lambda}_{i,j})$$
$$\forall e(i,j) \in E; \Omega \in \{beg, end\}; 0 \le a < K; \forall \lambda \in \Lambda \quad (17)$$

$$P^{\Omega,a,\lambda}_{i,j} \le \frac{P_{Max}}{K} * T^{a,\lambda}_{i,j} + P_2 * (1 - T^{a,\lambda}_{i,j})$$
$$\forall e(i,j) \in E; \Omega \in \{beg, end\}; 0 \le a < K; \forall \lambda \in \Lambda \quad (18)$$

Constraints (17) and (18) ensure that each power level at the beginning and end of $e(i,j)$ is within the valid ranges based on whether the light-forest uses $e(i,j)$ or not. In the former case, the power value should be between P_{Sen} and $\frac{P_{Max}}{|\Lambda|}$; otherwise, this value should equal to, *theoretically*, $-\infty$ dBm (i.e., 0 mW). In order to represent this case, we use two small negative constants, i.e., P_1 and P_2 with a value of $(-5) * M$ and $(-2) * M$, respectively, such that the power is a very small negative number. It is worth mentioning here that the value of $P^{beg,a,\lambda}_{i,j}$ is measured *before* on-site Post-Amplification, while $P^{end,a,\lambda}_{i,j}$ is measured *after* on-site Pre-Amplification, if any.

$$P^{beg,a,\lambda_1}_{i,j} - P^{beg,b,\lambda_2}_{i,j} = 3 \times M \times (T^{a,\lambda_1}_{i,j} - T^{b,\lambda_2}_{i,j})$$
$$\forall e(i,j); 0 \le a, b < K; \forall \lambda_1, \lambda_2, \in \Lambda, \lambda_1 \ne \lambda_2 \quad (19)$$

Constraints (19) are used to enforce the power symmetric constraint by ensuring that all the active signals at the beginning of each link have the same power strength. As the propagation loss and power gain are both linear, this condition is suffice to ensure that all the power signals over any link are symmetric. Please note that when light-forests links (of the same connection or different connections) use link $e(i,j)$, the right hand side of constraints (19) becomes zero and both power values at the beginning of the link should be equal. The same occurs in the case when both light-forests links do not use the same link. However, when only one of them exist over the link, the difference in their power values is guaranteed to be in the range between $|P1|$ and $|P2|$, which is in compliance with constraints (17) and (18).

$$(1 - T^{a,\lambda}_{i,j}) * M + P^{end,a,\lambda}_{i,j} =$$
$$(1 - T^{a,\lambda}_{i,j}) * (-M) + P^{beg,a,\lambda}_{i,j} + LG_{i,j} - \beta * L_{i,j}$$
$$\forall e(i,j) \in E; 0 \le a < K; \forall \lambda \in \Lambda \qquad (20)$$

Equation (20) is used on link $e(i,j)$ to express the power on wavelength λ at the end point of the link in terms of the power at the beginning of the link and the gain and loss due to amplification and attenuation, respectively. It should be noted that when the link is not part of the light-forest, i.e., the corresponding $T^{a,\lambda}_{i,j}$ equals 0, the power value at the end of the link is guaranteed to be between P_1 and $P2$.

$$(1 - T^{a,\lambda}_{i,j}) * v + P^{end,a,\lambda}_{i,j} - SL^{a,\lambda}_j - \gamma \ge$$
$$(1 - T^{a,\lambda}_{j,k}) * w + P^{beg,a,\lambda}_{j,k}$$
$$\forall e(i,j), e(j,k) \in E; 0 \le a < K; \forall \lambda \in \Lambda \qquad (21)$$

$$(1 - T^{a,\lambda}_{i,j}) * w + P^{end,a,\lambda}_{i,j} - SL^{a,\lambda}_j - \gamma \le$$
$$(1 - T^{a,\lambda}_{j,k}) * v + P^{beg,a,\lambda}_{j,k}$$
$$\forall e(i,j), e(j,k) \in E; 0 \le a < K; \forall \lambda \in \Lambda \qquad (22)$$

Constraints (21) and (22) are used to relate the values of the power levels between the end of an edge, say $e(i,j)$ and the beginning of the following hop, say $e(j,k)$, if any. In order to maintain consistency with equations (17)-(20), and to be able to handle all the cases of the usage of the links $e(i,j)$ and $e(j,k)$, the values of v and w are chosen such that $v \ge |P_1| + M$, and $w < 0$. The rationale of choosing these values is demonstrated with an aid of an example. Consider the case when both $T^{a,\lambda}_{i,j}$ and $T^{a,\lambda}_{j,k}$ equal 0. Recall that equations (17) and (18) ensure that $P^{beg,a,\lambda}_{i,j}$ and $P^{beg,a,\lambda}_{j,k}$ are between P_1 and P_2. The value of v is chosen to be $6 * M$ to guarantee that left hand side of inequality (21) is still greater than right hand side even with the case when both power values equal P_1. The same hold for the left hand side of inequality (21). Choosing the value of w to be negative helps in ensuring this too[1].

$$\frac{SL^{a,\lambda}_i}{M} \le \alpha_i \qquad \forall i \in N; 0 \le a < K; \forall \lambda \in \Lambda \qquad (23)$$

Equation 23 ensures that the splitting loss value at an MI node is 0 dB for any connection carried at any channel. What remains is to determine the value of this loss if signal splitting happens at an MC node, which is determined in dB by this

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[1] Proving that this criterion for choosing v and w hold for all the other cases of links usage is straightforward.

formula: $SL_i^{a,\lambda}$= $10 * \log_{10} f$. Since the splitting degree, f, is a variable, incorporating this loss directly in the formulation will make it non-linear. Here, we introduce an elegent way to find $SL_i^{a,\lambda}$ at MC nodes using a set of linear equations that are equivalent to the previous non-linear one.

$$\frac{\sum_{j,j\neq src_a,e(i,j)\in E} T_{i,j}^{a,\lambda} - f + \delta}{M} \leq A_f \qquad \forall i \in N; 0 \leq a < K; \forall \lambda \in \Lambda; 2 \leq f < Out_i \tag{24}$$

$$A_f * \{10 * \log_{10} f\} \leq SL_i^{a,\lambda} \qquad \forall i \in N; 0 \leq a < K; \forall \lambda \in \Lambda; 2 \leq f < Out_i \tag{25}$$

$$SL_i^{a,\lambda} \geq 0 \qquad \forall i \in N; 0 \leq a < K; \forall \lambda \in \Lambda \tag{26}$$

In this context, the value of A_f is 1 for all the values of f that are less than or equal the actual tree fanout at the node; otherwise it can be either 0 or 1. However, since the objective function minimizes the amplifier gain, it attempts to minimize the fanout, and hence the splitting loss, $SL_i^{a,\lambda}$. Therefore, the value of A_f in this case will be set to 0.

$$LG_{i,j} \leq g_{max} \times n_{i,j} \qquad \forall e(i,j) \tag{27}$$

$$LG_{i,j} > g_{max} \times (n_{i,j} - 1) \qquad \forall e(i,j) \tag{28}$$

These constraints determine the relation between the total gain and the needed number of OAs per link such that the minimum number of OAs are used. Similar constraints were used in [9] where g_{max} determined the maximum gain available at any amplifier which occurs when all the input signals are at P_{Sen}. However, as the number of occupied channels per link is determined by the MILP solution, we use an approximate approach to compute g_{max} in which we assume that all the channels over all the network links are occupied. This assumption enables us to precompute g_{max} using the OA model define in (1) such that the total input power (P_{in}) is calculated as $10 \times log10(\Lambda \times 10^{\frac{P_{Sen}}{10}})$. This approximation provides an exact value for g_{max} when Λ is small, which is the case with our numerical results.

D. Extension to the Asymmetric Splitting Case

In this subsection we introduce an extension to our optimal formulation to allow asymmetric splitting. That is, a node can split the signal unequally, provided that the sum of splitting ratios is equal to 1. While this extension can lead to a more optimal design, it has two problems. First, the implementation of asymmetric splitting may not be technically feasible, especially if the splitting ratio is arbitrary. It can, however, be approximated using a number of stages of splitters and combiners. Second, the formulation, as we will see below, becomes nonlinear, and this will further increase the complexity of solving it optimally. However, for the sake of completeness we introduce this formulation here, but we do not provide any results based on this formulation.

We define the following non-negative variables:

$SL_{i,j}^{a,\lambda}$	Splitting loss (in dB) of the signal on wavelength λ at node i for session a, which is then transmitted on the outgoing link $e(i,j)$.
$SR_{i,j}^{a,\lambda}$	The splitting ratio corresponding to $SL_{i,j}^{a,\lambda}$.
$S_i^{a,\lambda}$	A binary indicator which is 1 if the wavelength λ used by session a is split at node i (this includes the special case of a splitting ratio of 1, i.e., signal forwarding with no splitting).

Note that the set of $SL_{i,j}^{a,\lambda}$ variables, $\forall j$, replaces the $SL_i^{a,\lambda}$ variables. Note also that the relation between $SL_{i,j}^{a,\lambda}$ and $SR_{i,j}^{a,\lambda}$ variables is given by the following relation:

$$SL_{i,j}^{a,\lambda} = -10 \log_{10} SR_{i,j}^{a,\lambda}$$

which is the source of non-linearity iin the formulation.

In addition, based on the above definitions, constraints (21)-(26) will be replaced by the following constraints:

$$(1 - T_{i,j}^{a,\lambda}) * v + P_{i,j}^{end,a,\lambda} - SL_{j,k}^{a,\lambda} - \gamma \geq (1 - T_{j,k}^{a,\lambda}) * w + P_{j,k}^{beg,a,\lambda} \qquad \forall e(i,j), e(j,k) \in E; 0 \leq a < K; \forall \lambda \in \Lambda \tag{29}$$

$$(1 - T_{i,j}^{a,\lambda}) * w + P_{i,j}^{end,a,\lambda} - SL_{j,k}^{a,\lambda} - \gamma \leq (1 - T_{j,k}^{a,\lambda}) * v + P_{j,k}^{beg,a,\lambda} \qquad \forall e(i,j), e(j,k) \in E; 0 \leq a < K; \forall \lambda \in \Lambda \tag{30}$$

Constraints (29) and (30) are similar to constraints (21) and (22) and they relate the power levels between the end of a tree link and the beginning of the next tree link using the new variables $SL_{i,j}^{a,\lambda}$.

$$S_i^{a,\lambda} \geq \sum_{\forall e(i,j)\in E} \frac{T_{i,j}^{a,\lambda}}{M} \qquad \forall i \in N; 0 \leq a < K; \forall \lambda \in \Lambda \tag{31}$$

$$S_i^{a,\lambda} \leq \sum_{\forall e(i,j)\in E} T_{i,j}^{a,\lambda} \qquad \forall i \in N; 0 \leq a < K; \forall \lambda \in \Lambda \tag{32}$$

These constraints compute $S_i^{a,\lambda}$ as the disjunction between all the outgoing tree links of node i. They ensure that $S_i^{a,\lambda}$ is 1 if at least one of its outgoing links from node i is used by the tree of session a on λ.

$$\sum_{\forall e(i,j)\in E} SR_{i,j}^{a,\lambda} = S_i^{a,\lambda} \quad \forall i \in N; 0 \leq a < K; \forall \lambda \in \Lambda \tag{33}$$

$$SL_{i,j}^{a,\lambda} = -10 \log_{10} SR_{i,j}^{a,\lambda} \forall e(i,j); 0 \leq a < K; \forall \lambda \in \Lambda \tag{34}$$

Constraints (33) guarantee that the various splitting ratio does not exceed 1 when the right-hand side is 1, i.e., if node i is used to split the power of session a on channel λ. If not, all these power ratios are guaranteed to be zero. Finally, constraints (34) determine the relation between $SL_{i,j}^{a,\lambda}$ and $SR_{i,j}^{a,\lambda}$ as explained earlier.

IV. GREEDY ALGORITHM

The problem of Optical Amplifiers Placement (OAP) defined in subsection II-C consists of three main subproblems. These subproblems are: Routing (**R**), Wavelength Assignment (**WA**), and Power Assignment/Amplifiers Placement (**PAAP**) subproblems. The solution of one subproblem impacts the solutions of the other subproblems. Hence, the MILP formulation presented above solves the problem optimally by solving these subproblems jointly. Despite its optimality, the MILP formulation is not scalable as it cannot solve big-sized problem instances in a time efficient manner due to its high complexity. Such complexity is represented in terms of the numbers of constraints and variables which equal $O(|N|^4 \times |C| \times |\Lambda|)$ and $O(|N|^2 \times |C| \times |\Lambda|)$, respectively.

Therefore, there is a need for a heuristic approach that produces fast solutions with high quality degree. Such heuristic must be able to capture the main characteristics of the problem under investigation. In this section, we present a heuristic solution, referred to as Optical-amplifiers Placement (**OP**) algorithm.

A. Greedy Algorithm Motivation and Main Characteristics

The main goal of the OP algorithm is to achieve the balance between the produced solution quality and the computation time. In order to achieve this goal, the operation of the OP algorithm relies on the Divide-and-Conquer concept by dividing the problem into its natural subproblems that are solved separately. However, the impact between these modules are taken into account by employing a special set of cost functions for the links, network and sessions. The significance of these cost functions stems from the fact that they are defined in terms of optical amplifiers numbers. As these cost functions are used for the light-forest construction and session routing, this allows us to capture the influence between **R** and **PAAP** subproblems and results in efficient solutions.

Moreover, the design of the OP algorithm realizes the influence of the *Power Sharing* concept [2] which is a result of sharing the available power by wavelengths at the entry point of the links and OAs. Such influence is translated as connection blocking (called Power Sharing Blocking) [2] during network operation phase. As a design problem, the power sharing concept still holds, but it has different influence as all the connections must be accommodated (i.e., no connection drops are allowed). In this context, power sharing concept results in changing the **Network Power Status** (**NPS**), which defines the network condition in terms of its power values at the beginning of each link[2], as well as the number, locations and gain values of the OAs. The change in the NPS is a result of any of following behaviors:

1) As optical signal hops from one link to another, its power strength may decay below P_{Sen} anywhere in the light-forest, even with the use of the source node's maximum available power. This results in adding more in-line, pre-amplification and/or post-amplification OAs, which increases the network cost. We refer to this behavior as *Power Shortage Behavior*.
2) Routing a new connection that shares links with some already provisioned connections in the network can change the NPS by either:
    a) Dropping the gain of at least one OA to a level that causes a service disruption for at least one session. Such service disruption occurs if the gain drop yields a sequence of changes in the optical signals strength and other OAs gains which results in violating the power constraints defined in subsection II-A. We refer to this behavior as *Gain Dropping Behavior*.
    b) Changing the power values assigned to an already provisioned light-forest(s) in order to maintain the power symmetric and maximum total power constraints defined by constraints (18) and (19), respectively. Therefore, we call this behavior, the *Power Adjustment behavior*.

The NPS is highly dynamic and sensitive to any change introduced to the NPS from these behaviors. This is because any adjustment made to the network condition at one point in the network may propagate to other network locations and can affect multiple connections. This can create a complicated management issue, especially if large number of connections are involved. Nevertheless, the OP algorithm is designed to tackle this dynamic nature by allowing changes to occur to the NPS while ensuring that their impacts are handled in an efficient manner.

Finally, the OP algorithm is designed to optimize the solutions at two levels. At the lowest level, namely, the light-forest construction level, it allows the destinations to be attached to the sub-forest using multiple alternative paths, instead of a single path. Using alternative routing in this manner allows the OP algorithm to explorer bigger solution space and the light-forest can expand to new destinations using the path of the minimum (present) path. In addition, the OP algorithm allows constructing more than one light-forest per session. The one with the least cost is then chosen to be placed in the network. At the light-forest placement level, the algorithm defines two operation modes, namely, *Fixed* and *Adaptive* Modes. In the Fixed mode, light-forests are constructed once based on the initial NPS and then they are placed in the network according to their costs. With the Adaptive mode, however, placing each light-forest in the network is followed by reconstructing the remaining light-forests that are not yet provisioned based on the latest NPS. Rerouting the remaining sessions allows the OP algorithm to account for the impact of light-forests on each other which improves the solution accuracy, yet, with the cost of extra computation.

B. Cost Functions Definitions

The following set of cost definitions are adopted by the OP algorithm:

- The current cost of link $e(i,j)$ is defined as the current number of OAs needed over the link, namely $c_{e(i,j)} =$

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[2] which is sufficient to determine the power values everywhere in the network.

$|n_{i,j}|$. This includes any Post-Amplification at node i, and any Pre-Amplification at node j.

- The current network cost is computed as the total number of OAs over all the links in the network. In other words, $C = \sum_{e(i,j)} c_{e(i,j)}$
- The cost of the path that connects a destination to the subtree-forest is defined as the change in the network cost that results if such a path is used for expanding the subtree-forest to that destination.
- The session cost is also calculated as the change in the network cost if its light-forest is placed in the network.

Using these cost functions has the following advantages:

1) It is possible to base the routing decisions of the light-forests on the system power budget. This establishes a connection between the R and PAAP subproblems and can result in better solutions.
2) These cost functions are dynamic as their definition is based on the the most recent NPS. This is important to ensure the correctness and goodness of the produced solutions.
3) Using the definitions of the path and session costs is effective in relating the routing and placement decisions to its future consequences. This post-influence scheme help in capturing the influence between the connections.
4) Finally, the link cost function is a positive increasing function. Therefore, link costs increase with the increase of the link usage. This is useful in balancing the load in the entire network.

C. OP Algorithm Details

We present the details of the OP algorithm in this subsection. The OP algorithm is designed as an iterative algorithm such that the final solution is the result of a set of optimized sub-solutions. Figure 1 depicts the basic operation of the algorithm which consists of three main stages. These stages are: the Light-Forests Construction (LFC) Stage, the Light-Forests Placement (LFP) Stage and the Light-Forest Reconstruction (LFR) Stage. The core operation of the LFC and LFR stages is the Light-Forest Construction Module which is depicted in Figure 2 while the core operation of the LFP stage is the the Light-Forest Placement Module which is depicted in Figure 3.

The LFC stage starts by initializing all the data structures, which include: the Network status (NS), the Network Power Status (NPS) and the set S. While NPS is defined earlier in subsection IV-A, NS defines the channels and links status in the network and set S determines the set of sessions which are not yet provisioned (placed). Initially, the set S includes all the multicast sessions.

Then, the Light-Forest Construction Module is invoked in order to construct the light forest for each multicast session in S. NS and NPS are then re-initialized and the set S is sorted according to its sessions' costs. The second (i.e., LFP) stage is then invoked for the first session, a, in the sorted S and its light-forest obtained from the LFC stage is placed in the network. Accordingly, the algorithm updates NS, NPS, and S and it proceeds with the remaining multicast sessions in S in two fashions based on its operational modes, i.e., Fixed or Adaptive. In the Fixed mode, the algorithm places all the initial light-forests constructed in the LFC Stage without changing them. On the other hand, the Adaptive mode involves the use of the LFR stage in which new light-forests are constructed for all the sessions in S based on the current NS and NPS. Therefore, the Light-Forest Construction Module is invoked again in this stage with the latest NS, NPS and S. After sorting S, the LFP stage is invoked with the first session in set S. The algorithm stops when all sessions are placed.

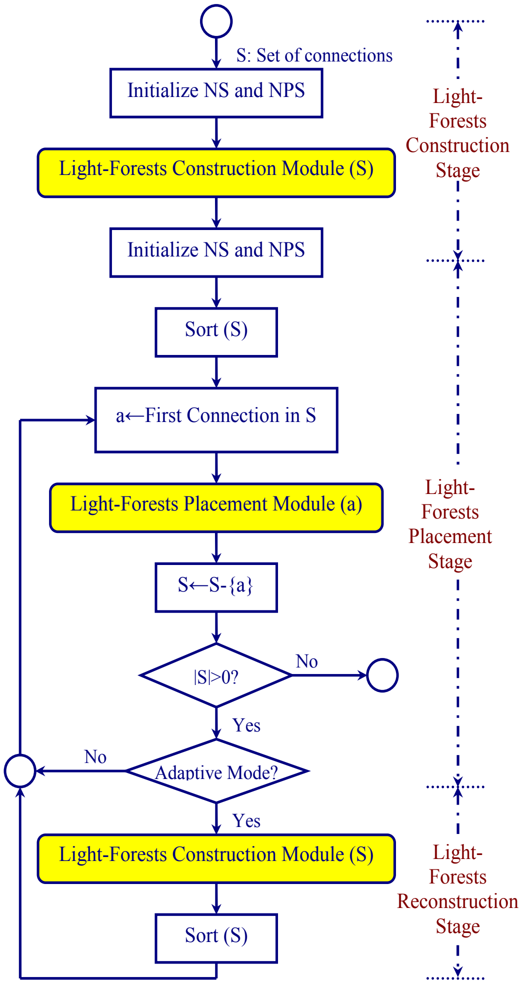

Fig. 1. Basic Operation of the OP Algorithm.

For sake of completeness, the details of all the Light-Forest Construction and Placement Modules are explained below.

D. *Light-Forest Construction Module*

As indicated by its name, the purpose of the Light-Forest Construction Module is to produce a light-forest for each multicast session according to the recent NS and NPS. Each light forest is constructed as if it is the only light-forest in the system. The purpose of this construction scheme is to determine the cost of each session (in terms of the change in the network cost) using the recent NS and NPS.

In addition, due to the randomness involved in its operation, the Light-Forests Construction module generate more than one

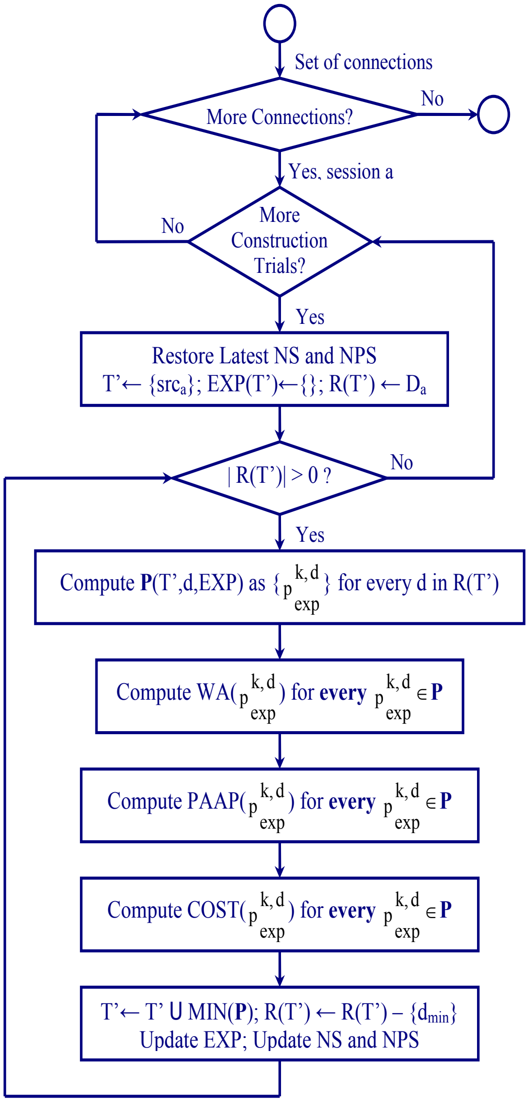

Fig. 2. Tree Construction Module.

light-forest for each session such that the best (least cost) light-forest is then chosen[3]. The input to the Light-Forest Construction module is the set S. As shown in Figure 2, each construction trial for the multicast session starts by restoring the latest NS and NPS in order to ensure the correctness of the light-forest instance construction.

The light-forest construction is performed iteratively using an extended version of the Member-Only Heuristic (MOH) [13]. Initially, the light-forest structure, called sub-forest or T', includes the source node only, scr_a. After each iteration, T' is expanded such that a new (i.e., unconnected) member is attached to T' via one light-forest node. The light-forest growth is permitted through specific set of nodes, called the expandable nodes or EXP(T') which consists of the source node, all the light-forest nodes which have Multicast Capability (MC nodes) or/and leaf nodes. Instead of a single path, the Light-Forest Construction Module computes k alternative paths to each remaining destination. Among all these computed paths, T' is then expanded using the path of the least cost. The module stops when all remaining nodes, R(T'), are included in the light-forest.

All relative data structures are updated at each iteration during which the following operations are performed:

1- Path Computation (PC):

In this step, the set of k-shortest paths from each unconnected destination, d, to each expandable node in T' is computed.

2- Path Wavelength Assignment (PWA):

The PWA step is performed for each computed path from step 1. Depending on the expandable node, two scenarios are possible in this operation. On one hand, we employ the First-Fit scheme (in which the first available common wavelength over all links in the path is chosen) if the path under investigation connects the destination node to the source node itself (i.e., new forest branch is created). On the other hand, if the attaching node is not the source node, the new forest segment from the expandable node to the destination should continue using the same channel (if available) used over the forest segment connecting the source node to the expandable node. If such a wavelength is not available, the path instance is ignored.

3- Path Power Assignment/Amplifiers Placement (P-PAAP):

For each path instance passed the PWA step, the P-PAAP operation is responsible for determining the power values and the OAs placement over each of its links. The P-PAAP module relies on using a queue structure, called Q, which consists of unique entities of the links identities and aims to separate the links identities from their power values. Using Q proves to significantly reduce the computation and management overheads in [2], hence, we adopt the same technique here.

The P-PAAP operation starts by marking the session under investigation as affected and adding the first link of the path to Q. Then P-PAAP runs iteratively such that at each iteration, the link at Q's head is processed by performing the following operations:

1) **Initial Power Determination Operation.**

   This operation is responsible for determining the power values at the beginning of the head link in Q such that no power constraint is violated and the power symmetry constraint is maintained. Two factors determine these power values, namely, whether the head link is connected from the source and/or it has more than one channel. For instance, if the link is launched from the session's source node and it is the only channel on the link, the maximum power value can be assigned to the channel. Otherwise, if the head link is not launched from the session's source, this step adjusts these power values according to the power and symmetric constraints.

2) **OA Placement Operation.**

   Using the power values at the beginning of the head link, the OAs are placed over the link based on the ALAP

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[3] The number of construction trials per connection is an input parameter.

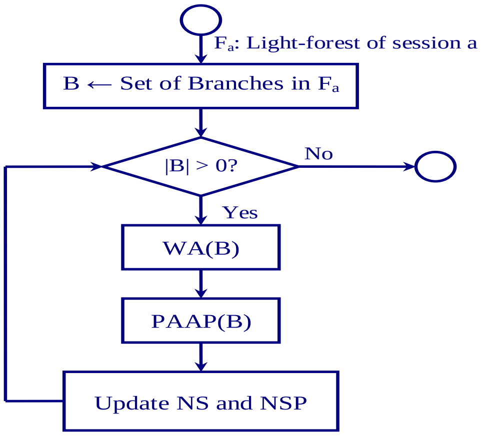

Fig. 3. Tree Placement Module.

policy. This step involves changing the previous OAs locations and maybe number over the link in order to accommodate for the change in NPS.

3) **Update Q Operation.**

Q is then updated by adding more links to it, if any. The set of potential links to be added to Q are chosen from the set of outgoing links, $\Re$, launched from the current head link's sink node. In order to prevent Q from being unnecessarily modified, Q is updated with those outgoing links that satisfy the following conditions:

- They are part of the light-forest of the current session under investigation. This should guarantee the continuation of power investigation for the current sub-forest.
- They are part of the light-forest of all the other sessions over the link, provided that a new power value is observed at the end of the head link which is different than the one from previous iteration. Such change in power values from iteration to iteration indicates a change in the system status that needs to be propagated. Hence, we use Q to trace such a change in the NSP.

Once processed fully, the link at the Q's head is removed and the P-PAAP operation continues with the next link in Q and it stops when Q becomes empty. Please note here that a link can be revisited more than once during the iteration lifetime[4]. This occurs because the same link can be at different depth of the various light-forests. During each link traversal, more power values over the link become available. P-PAAP deals with those power values that are currently available which enables it to work even with partial knowledge of the power values. However, allowing several traversals of the links ensures the complete availability of the power values at the link.

E. *Light-Forest Placement Module*

This module is responsible for placing the light-forest constructed in the Light-Forest Construction module in the network and then change the NS and NPS accordingly, as shown in Figure 3. Therefore, there is no routing effort in this module and it focuses on solving the WA and PAAP subproblems at the forest level. These two operations are similar to the corresponding ones introduced in the Light-Forest Construction Module. However, the delivery structure unit here is determined in terms of light-forest's branch instead of a path. For instance, the WA is performed for each branch of the light-forest such that it always finds the first available wavelength over all the branch's links (i.e., no continuation of usage of upstream channels involved). Similarly, the PAAP entity in the Light-Forest Placement Module operates like the PAAP entity in the Light-Forest Construction module (i.e., P-PAAP module). However, building Q starts from the first branch's link launched from the source node itself rather than from the first link of the path.

Please note that we can skip this module in the Adaptive operation mode as we can use the final NS and NPS information from the LFR stage when the chosen light-forest is placed in the network. However, it is essential to apply this module for the fixed operational mode in order to determine the WA and PAAP results for each placed light-forest in the network.

V. NUMERICAL RESULTS

We present some numerical results in this section. These results are obtained using CPLEX [14] for solving the MILP formulation and using C++ Programming Language for implementing the OP algorithm.

We first examine the quality of the solutions produced by the OP algorithm by comparing them with the optimal solutions of the MILP formulation using the sample 6-nodes mesh network shown in Figure 4. After establishing such quantitative comparison, we present various results that illustrate different aspects of the proposed OP algorithm using the 14-nodes NSFNET shown in Figure 5.

These results are obtained with the numerical values presented in Table I and under the following assumptions:

1) The multicast groups size follows a uniform distribution between 1 and N, where N is the number of nodes in the network.
2) Node membership in each multicast session is determined uniformly from all nodes after excluding the source node.
3) Descending-order policy is used in OP algorithm for sorting the set of connections constructed in the LFC stage.
4) The OP algorithm runs for at least 10 times per problem instance and the best solution that produces the minimum number of OAs is chosen.

A. *Comparative Results Between the Optimal and Suboptimal Numerical Results*

Tables II and III compare the results obtained from CPLEX to their counterparts obtained from the OP heuristic for the

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[4] Although multiple traversals of the link is permitted during the P-PAAP, Q contains at most one instance of the link at each algorithm step. Preventing multiple copies of the same link in Q eliminates any unneeded calculations since power values are separate from the link identities.

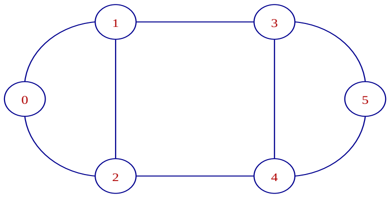

Fig. 4. Six Nodes Mesh Network.

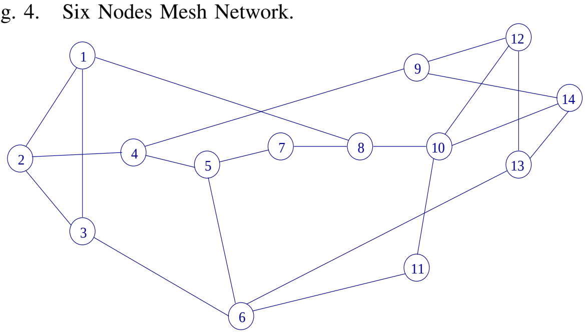

Fig. 5. NSFNET.

TABLE I
TYPICAL VALUES FOR THE SYSTEM PARAMETERS.

Symbol	β	P_{Sen}	P_{Max}	P_{sat}	G_0
Value	0.2 dB/km	-30 dB	0 dB	1.1298 mW	20 dB
Symbol	P_1	P_2	δ	v	w
Value	$-5 * M$	$-2 * M$	0.01	$20 * M$	$-M$

TABLE II
COMPARISON OF OAS NUMBERS OBTAINED BY CPLEX ($|OA|_C$) AND OP HEURISTIC ($|OA|_i$; WHERE $i = 1, 2, 3, 4$ REPRESENTS THE NUMBER OF ALTERNATIVE PATHS) FOR THE 6-MESH NETWORK. K AND Λ REPRESENT NUMBER OF SESSIONS AND AVAILABLE LAMBDA, RESPECTIVELY.

K	Λ	$\lvert OA\rvert_C$	$\lvert OA\rvert_1$	$\lvert OA\rvert_2$	$\lvert OA\rvert_3$	$\lvert OA\rvert_4$
1	2	157	158/158	158/158	158/158	158/158
2	2	158	160/158	158/158	158/158	158/158
3	2	159	160/160	159/159	159/159	159/159
4	2	159	160/160	159/159	159/159	159/159
5	2	159	160/160	159/159	159/159	159/159
6	2	159	160/160	159/159	159/159	159/159
7	2	159	161/161	161/161	160/160	160/160
8	2	159	161/161	161/160	161/160	161/160
1	3	159	159/159	159/159	159/159	159/159
2	3	159	159/159	159/159	159/159	159/159
3	3	160	160/160	160/160	160/160	160/160
4	3	160	160/160	160/160	160/160	160/160
5	3	160	160/160	160/160	160/160	160/160
6	3	160	160/160	160/160	160/160	160/160
7	3	160	160/160	160/160	160/160	160/160
8	3	161	161/161	161/161	161/161	161/161
9	3	161	161/161	161/161	161/161	161/161
1	4	159	159/159	159/159	159/159	159/159
2	4	159	159/159	159/159	159/159	159/159
3	4	159	160/160	160/160	159/159	159/159
4	4	160	160/160	160/160	160/160	160/160
5	4	160	160/160	160/160	160/160	160/160
6	4	160	160/160	160/160	160/160	160/160
7	4	161	161/161	161/161	161/161	161/161

6-nodes mesh network. It is important to note that the MILP formulation aims to solve the OA Placement (OAP) problem with more restrictive constraints than the OP heuristic. In this context, the optimal solutions from CPLEX are obtained while the number of splitters and their locations are not determined and the number of available channels in the network is predetermined. These constraints are relaxed in the OP heuristic solutions as the number/location of the splitters is fixed and no upper bound is imposed on the number of available channels.

Therefore, in order to make a meaningful comparison, the following three actions are taken into account:

1) The OP heuristic experiments are carried out with the same number/location of splitters obtained from CPLEX for the same problem instance.
2) The quality of the obtained solutions will be determined not only by the delta of the number of OAs (i.e., the objective function), but also by how much extra network resources (if any) needed by the OP algorithm.
3) Due to Constraints (18), we also make sure that the individual signal strength produced by OP algorithm does not exceed $\frac{P_{Max}}{\Lambda} dBm$ [5].

Table II determines the number of OAs obtained from CPLEX ($|OA|_C$) compared to those obtained from OP heuristic ($|OA|_i$). The index i determines the number of alternative paths used in constructing the light-forest in LFC stage. Each $|OA|_i$ solution takes the (x/y) format to determine the $|OA|$ when the Fixed and Adaptive schemes are employed, respectively. The same symbolic notation and (x/y) format is used in Table III to determine the network resources consumed by the produced solutions.

Basically, there are two network resources that are of interest to us and which are computed at the network-wide scale. These resources are:

1) The maximum number of distinct channels consumed over any link; referred to as ψ.
2) The number of links used in constructing all the light-forests (i.e., links with at least one used channel); referred to as Ł.

From the results in Table II, it is clear that the quality of the solutions produced by the OP heuristic is determined by its computation complexity. Two factors contribute to this complexity, namely, the number of alternative paths used for constructing each light-forest, and whether or not rerouting is performed to reconstruct the remaining unplaced light-forests. The OP heuristic permits the use of any combination of these factors such that the computation complexity ranges from **minimum computation** (namely, Fixed scheme with one alternative path for routing) to **maximum computation** (namely, Adaptive scheme with maximum number of alternative paths for routing).

The following conclusions can be drawn from the results in

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[5] As this action is taken for comparison purposes only, we do not take this limitation into account for all the results presented in Subsection V-B.

TABLE III

Comparison of used network resources for the 6-mesh network. ψ_C (Ł_C) and ψ_i (Ł_i) represent the maximum number of wavelengths (number of links) used by CPLEX, and OP heuristic, respectively. $i = 1, 2, 3, 4$ represents the number of alternative paths, while K and Λ represent number of sessions and available lambda, respectively.

K	Λ	ψ_C	ψ_1	ψ_2	ψ_3	ψ_4	Ł_C	Ł_1	Ł_2	Ł_3	Ł_4
1	2	2	2/2	1/1	1/1	1/1	4	6/5	5/5	5/5	5/5
2	2	2	2/2	1/1	1/1	2/2	6	7/7	7/7	7/7	7/6
3	2	2	2/2	2/2	2/2	2/2	11	12/12	12/12	12/11	12/11
4	2	2	2/2	2/2	2/2	2/2	11	12/12	12/12	12/12	12/12
5	2	2	2/2	2/2	2/2	2/2	12	12/12	12/12	12/12	12/12
6	2	2	2/2	2/2	2/2	2/2	12	13/12	12/12	12/12	12/12
7	2	2	3/3	3/2	3/2	3/2	14	13/12	13/12	12/12	12/12
8	2	2	4/4	4/3	4/3	4/3	14	13/13	13/13	13/13	13/13
1	3	1	1/1	1/1	1/1	1/1	5	5/5	5/5	5/5	5/5
2	3	2	2/2	2/2	2/2	2/2	8	8/8	8/8	8/8	7/7
3	3	3	2/2	2/2	2/2	2/2	11	12/12	12/12	12/12	12/12
4	3	3	2/2	2/2	2/2	2/2	11	12/12	12/12	12/12	10/10
5	3	3	2/2	2/2	2/2	2/2	11	12/12	12/12	12/12	10/10
6	3	3	2/2	2/2	2/2	2/2	12	12/12	12/12	12/12	10/10
7	3	3	3/3	3/3	3/3	3/3	12	11/11	11/10	11/10	10/10
8	3	3	4/4	4/4	4/4	4/4	13	12/11	12/11	11/11	11/11
9	3	3	5/5	5/5	5/5	5/5	14	13/13	13/13	13/13	13/13
1	4	1	1/1	1/1	1/1	1/1	5	5/5	5/5	5/5	5/5
2	4	3	2/2	2/2	2/2	1/1	12	11/11	11/11	11/11	11/11
3	4	2	2/2	2/2	2/2	2/2	12	11/11	11/11	11/11	11/11
4	4	3	3/2	2/2	2/2	2/2	13	11/11	10/10	11/11	10/10
5	4	2	2/2	2/2	2/2	2/2	13	12/10	11/10	10/10	12/10
6	4	2	2/2	2/2	2/2	2/2	13	12/10	11/10	11/10	10/10
7	4	3	3/3	3/3	3/3	3/3	13	11/10	11/10	10/10	10/10

Tables II:

- OP heuristic remarkably succeeds to obtain the optimal solution in most cases, even with minimum computation (e.g., check the results for all the cases when $\Lambda = 3$ and 4). For the other cases, on the other hand, the mismatch between the optimal solutions and the OP heuristics solution is 1 or 2 OAs only. This is relatively too small difference, especially for Wide Area Networks which is the focus of this study.
- Better solutions with lower mismatch are produced in most cases by increasing the computation complexity of the OP heuristic. For instance, the optimal solution for the case when $\Lambda = 2$ and $K = 7$ is 159 while the OP heuristic solutions improved from 161 to 160 when more alternative paths and Adaptive schemes are used. However, there is a trivial trade-off between the computation time/resources and the solution quality.
- Using more alternative paths alone (i.e., Fixed mode with multiple paths routing) or allowing rerouting scheme alone (i.e., Adaptive mode with single path routing) proves to provide good solution, especially when the system traffic is lightly loaded. For example, for the case of $\Lambda = 2$ and $K = 5$, using the Fixed mode with multiple paths routing improves the solution from 160 to 159 OAs (which is the optimal solution) starting from using two alternative paths and without the need for applying the Adaptive mode. On the other hand, for the case of $\Lambda = 2$ and $K = 2$, using the Adaptive mode with single path routing produces 158 OAs which is the optimal solution.
- The results also illustrate the fact that using alternative routing and allowing light-forest reconstruction do not conflict with each other when both are employed by the OP heuristic. On the contrary, they complement each other's work which results in saving more OAs, especially when the system traffic load is high. For example, when $\Lambda = 2$ and $K = 8$, the $|OA|_2$ equals 160 OAs when the Adaptive mode is employed, which is an improvement from the 161 OAs solution achieved when $i = 1$ (for both Fixed and Adaptive schemes) and when $i = 2$ (for the Fixed scheme). This means that using more alternative paths alone did not help improving the solution quality in this case until it was accompanied by using the Adaptive mode.

  Please note that the nature of 6-nodes network as being a small network with limited number of links and nodal degrees, makes it hard to distinguish between the individual impact of each of these improvement schemes on the quality of the final solution. Such a distinction will be addressed in Subsection V-B when the results from NSFNET are presented.
- Finally, it is worth noting that because of Constraints (18), the ($|OA|_C$) can increase for the same number of sessions, K, when more number of channels becomes available in the system as the maximum power of each channel will decrease. For instance, $|OA|_C = 159, 160$, and 161 for $K = 7$, while $\Lambda = 2, 3$, and 4, respectively.

In order to complete our comparative investigation, we focus on Table III to determine the amount of resources used by the OP heuristic compared to those consumed by CPLEX. With respect to ψ, we note that OP heuristic succeeded to use the same maximum number of wavelengths over any link is most cases (e.g., for all the cases when $\Lambda = 2$ and $K = 3$ to 6) which gives more credibility to these solutions as they are produced with similar experimental conditions as CPLEX. Interestingly, the value of ψ_i can be even less than ψ_C which is absolutely fine as it is not the objective of the MILP formulation to reduce ψ. This can happen especially when the number of available channels, Λ, is high enough with respect to the traffic load as it is the case when $\Lambda = 3$ and $K = 2$ to $K = 6$.

On the other hand, especially when the traffic load is high with respect to the number of available wavelengths, we note that the OP heuristic tends to use more wavelength channels per link than CPLEX. This is true for example when $\Lambda = 2$ and $K = 8$ and when $\Lambda = 3$ and $K = 8$ and 9. However, these extra resources are still within acceptable ranges (i.e., 1 or 2 extra wavelengths only) especially in Wide Area Network environments where the cost of adding extra channels to the system is comparatively much less than adding extra OAs.

With respect to Ł, we note that there is no direct relation between the CPLEX solutions and the OP heuristic solutions

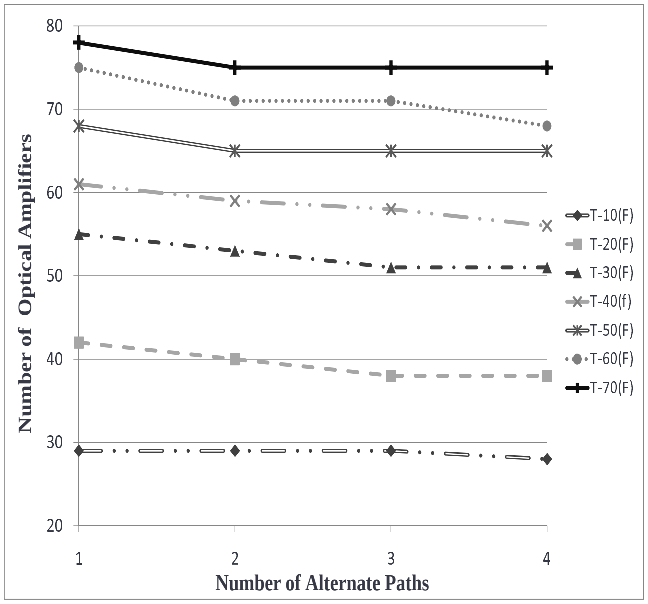

Fig. 6. Impact of Using Alternative Routing on the Number of OAs for the Fixed Routing Scheme at Different Traffic Loads. The notation $T-i(F)$ indicates that the system traffic load (T) is i sessions for the Fixed (F) scheme.

as the problem is not formulated to take this parameter into consideration. Generally, the results indicate that Ł used in both methods are comparable.

It is important to note that ψ_i and Ł$_i$ decrease when more computation power is added to the OP heuristic. This is very significant result and will be addressed further in Subsection V-B.

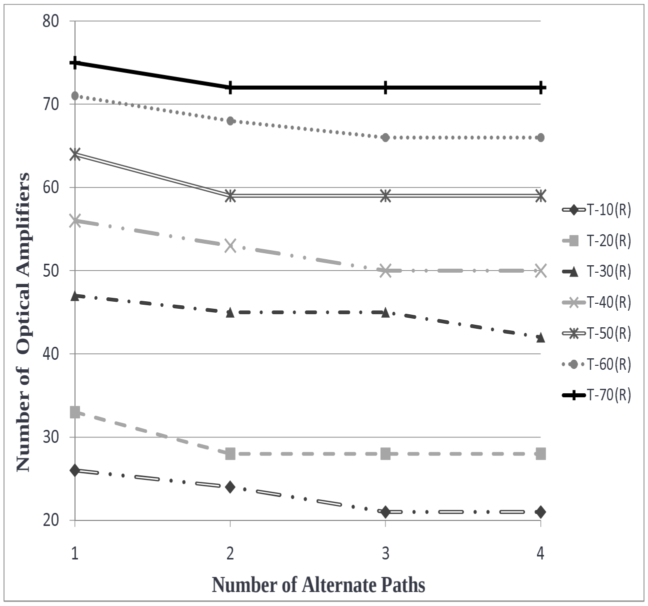

Fig. 7. Impact of Using Alternative Routing on the Number of OAs for the Adaptive Routing Scheme at Different Traffic Loads. The notation $T-i(R)$ indicates that the system traffic load (T) is i sessions for the Adaptive (or Rerouting, R) scheme.

B. OP Heuristic Results

In this subsection, we use the NSFNET shown in Figure 5 to report various numerical results for the OP heuristic. In these experiments, we use two splitters placed at nodes 5 and 8, and we assume that there is upper limit on the number of available system channels. These results address the following issues:

1. Impact of OP Heuristic on the Number of OAs

Figures 6 and 7 depict the impact of the OP heuristic on the number of OAs for the Fixed and Adaptive operation modes, respectively, at different traffic loads.

From these figures, we find that the number of needed OAs, $|OA|$, increases as the traffic load increases. However, for each traffic load, $|OA|$ improves (i.e., decreases) as more computation power is employed. In this context, and in agreement with the results in Subsection V-A, we notice that both solution improvement schemes (namely, using alternative routing and allowing light-forest reconstruction) contribute to this solution improvement in a constructive manner by joining their forces together to find better solutions.

In order to illustrate this with some numerical results, we consider the case when traffic load is 40 sessions. The initial solution with minimum power computation is 61 OAs. By using more alternative paths only (i.e., using the Fixed mode), Figure 6 shows that the solution improved from 61 to 59, 58 and 56, when the number of alternative paths increased from 1 to 4, respectively. On the other hand, by using the Adaptive scheme only (i.e., no alternative paths), $|OA|$ drops from 61 to 56 too, as shown in Figure 7. Yet, more improvement could be achieved when alternative routing is employed with the Adaptive scheme and $|OA|$ drops as low as 50 OAs when 4 alternative paths are used.

2. Individual Contribution of Using Alternative Routing and Adaptive Rerouting Schemes on $|OAs|$

The results also show the relative contribution of the individual operation scheme to the solution quality. In order to demonstrate this individual contribution, we concentrate on the results obtained by applying each scheme alone in the OP heuristic. We define Δ as the difference in $|OA|$ obtained by subtracting the solution produced when alternative routing scheme with 4 paths is used alone from the solution produced when Adaptive rerouting scheme is used alone. Table IV shows Δ at different traffic loads.

From Table IV, we can see that the values of Δ are non-negative. Therefore, the improvement achieved by using the Adaptive scheme alone is as good as or even better than the solution produced by using the maximum number (i.e., 4) of alternative paths under the Fixed mode. For example, when $K = 30$, Δ equals 4 OAs as the solution obtained using the Fixed scheme with 4 alternative paths is 51 OAs while 47 OAs is obtained by using the Adaptive scheme alone.

Also, it is clear from Table IV that Δ is highly affected by the network traffic load. When traffic load is small (i.e., 10 sessions), Δ is relatively small as Fixed scheme alone seems to

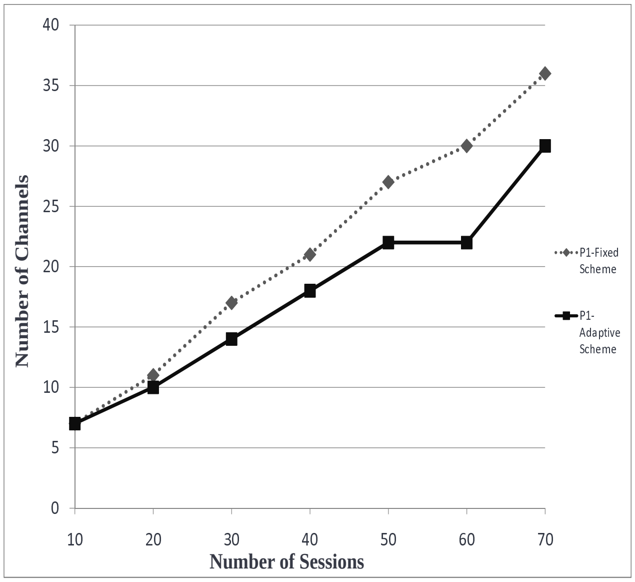

Fig. 8. Maximum number of used channels (ψ) at different traffic loads for the Fixed scheme and the Adaptive scheme when number of alternative paths is 1.

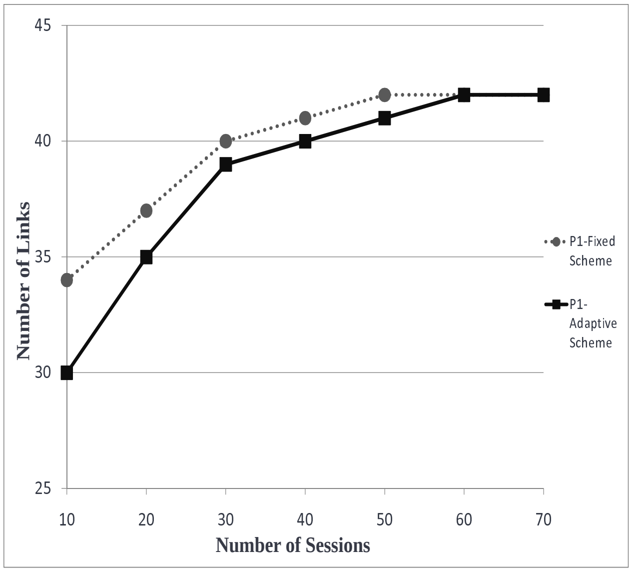

Fig. 9. Number of used links (Ł) at different traffic loads for the Fixed scheme and the Adaptive scheme when number of alternative paths is 1.

provide a comparable solution. Adaptive scheme performs best when the traffic load is moderate (i.e., 20-30 sessions). Yet, Δ decreases with more traffic load until it becomes zero when the system traffic load is high. For example, both schemes produce the same number of OAs (i.e., 75 OAs) when traffic load is 70 multicast sessions.

Moreover, Δ is also affected by the network topology, hence, the above observations with respect to Δ do not always apply to the 6-mesh network results. For example, the Fixed scheme outperforms the alternative scheme when $\Lambda = 2$ and $K = 2$ to $K = 8$.

3. Impact of Using Alternative Routing and Adaptive Rerouting Schemes on the Network resources

In this subsection, we present the amount of network resources used by each operation scheme. Figures 8 and 9 depict the values of ψ and Ł, respectively, consumed under the Fixed and Adaptive scheme at various traffic loads when the number of alternative paths is one. Similar results are obtained with more alternative paths, however, due to space limitation they are not presented here.

Generally, we find out that the Adaptive scheme has better utilization of the network resources than the Fixed scheme. Such utilization is affected directly by the traffic load. As shown by 8, the Adaptive scheme succeeds to use less number of channels per link and the saving in ψ increases with the increase in traffic load until traffic load reaches 60 sessions. At this point, the gap in ψ between the two schemes is the maximum (i.e., 8 channels) and it then starts to decrease. This behavior can be understood with the aid of Figure 9 in which we see that the Adaptive scheme also consumes less Ł. However, the amount of saving in Ł between the two schemes decreases when traffic load increases. This means that the Adaptive scheme tends to use more links to accommodate more traffic which directly results in consuming less number of channels per link. This behavior continues until all the available network links (i.e., the 42 links) are used. As shown in Figure 9, this occurs starting when traffic load equals 60 sessions. beyond this point, the Adaptive scheme starts to utilize the available channels and the amount of saving in ψ starts to decrease.

TABLE IV
THE RELATIVE PERFORMANCE OF USING ADAPTIVE METHOD ALONE WITH RESPECT TO ALTERNATIVE ROUTING AT DIFFERENT TRAFFIC LOAD.

Traffic load (sessions)	10	20	30	40	50	60	70
$\Delta(OAs)$	2	5	4	1	0	0	0

4. Impact of OP Heuristic on the Network Bandwidth Utilization

The Network Bandwidth Utilization (NBU) is computed as the total number of occupied (busy) channels used over all the network links. In Figure 10 we compare the NBU achieved by the Fixed and Adaptive schemes when the number of alternative paths is one at various traffic loads. We note that the NBU achieved by the Adaptive scheme is always higher than the one achieved by the Fixed scheme. This result is best understood when read in conjunction with Figures 8 and 9. In this context, we find out that while the Adaptive scheme is using less network resources in terms of ψ and Ł than the Fixed scheme, its NBU is higher which means that it is utilizing these resources more efficiently by reducing the waste due to channel fragmentation within each link. This is mainly achieved in the Adaptive scheme by its ability to base its routing decisions on the most recent network status which enables it to utilize more contiguous channels per link.

After studying the impact of each operation scheme on the NBU, we also investigate the impact of using different alternative paths on the NBU when Fixed and alternative

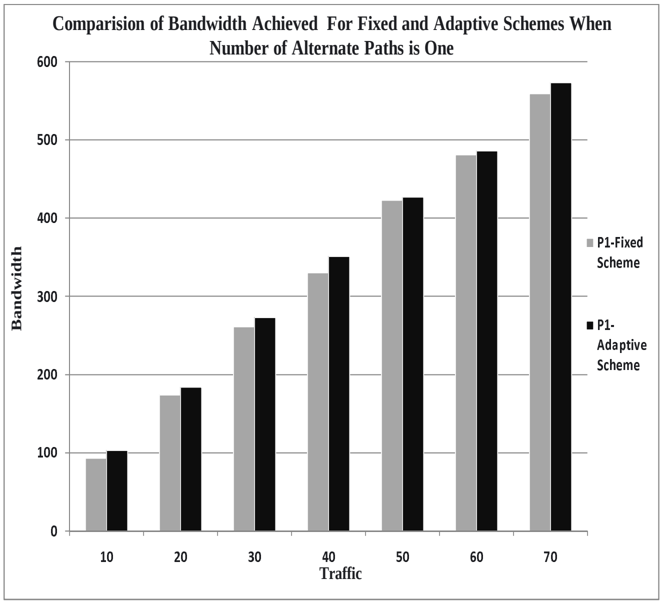

Fig. 10. Network Bandwidth Utilization (NBU) achieved at various traffic loads by the Fixed Scheme and Adaptive Scheme when one Alternative path is used.

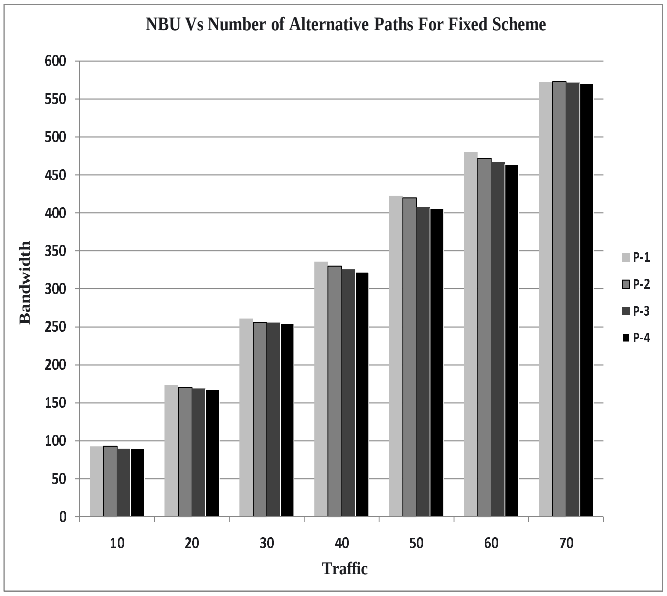

Fig. 11. Comparison of the Network Bandwidth Utilization (NBU) achieved with different alternative paths When Fixed Scheme is used at various traffic loads.

schemes are used in Figures 11 and 12, respectively. From these figures, we find out that for each traffic load, the NBU decreases as the number of alternative paths increases. For example, for $K = 40$, the NBU decreases from 351 to 341 channels when number of alternative paths increases from 1 to 4, respectively.

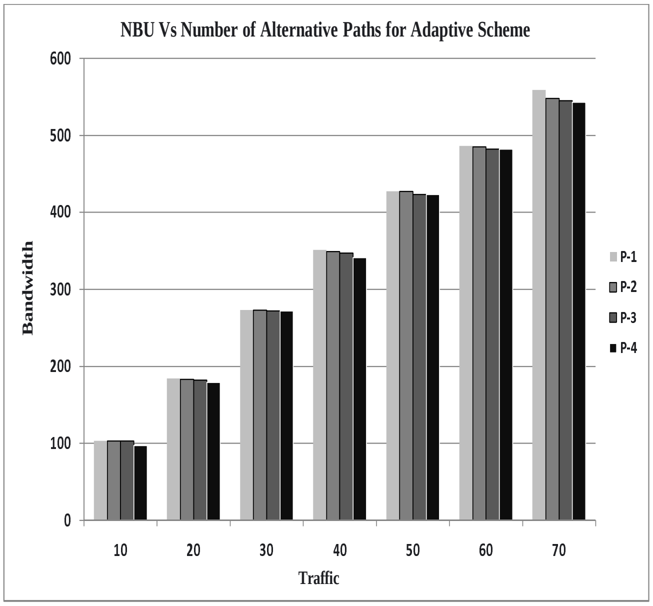

Fig. 12. Comparison of the Network Bandwidth Utilization (NBU) achieved with different alternative paths When Adaptive Scheme is used at various traffic loads.

VI. CONCLUSIONS

We studied the problem of Optical Amplifiers placement (OAP) and proposed three solution schemes for it in this paper. The first scheme produces optimal solutions using MILP formulation. The other two schemes are proposed as two modes of operation for the same heuristic scheme (called OP Algorithm), namely Fixed and Adaptive modes. The design of this heuristic consists of several appealing characteristics that make it able to produce near optimal solutions in an efficient manner. The results also show that the Adaptive scheme outperforms the Fixed scheme in terms of number of OAs and network resources; yet, it requires extra computation.

The optimal formulation in this paper can be extended to include more practical issues. One extension is to include the case of asymmetric splitting at MC nodes. This extension has been included in Subsection III-D in the paper. However, it has resulted in a non-linear formulation. A second extension is to take the impact of the noise induced by the amplified spontaneous emission in optical amplifiers, and the need to include regenerators, which will increase the cost of the system. Also, this will introduce another factor of non-linearity in the problem formulation. As part of our future work, we plan to include these factors in the formulation, while exploring the use of linear formulations, which may include approaches to transform the non-linear equations into linear ones.

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