Trajectory Optimization for Multiple UAVs Acting as Wireless Relays

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Abstract—This paper proposes a novel wireless relay selection scheme involving multiple mobile Unmanned Aerial Vehicles (UAVs) to support communicating ground users. The goal is to optimize the transmit power levels and trajectories of the relaying UAVs in order to maximize the data rate transmission of the ground users which are suffering from the absence of direct link. Assuming that each UAV is initially characterized by a predefined trajectory for a primary task, we propose to modify it whenever it is needed and the energy and trajectory boundaries constraints allow. We propose to solve this problem using an iterative two steps solution; first, a Mixed Integer Linear Programming (MILP) problem is formulated to optimally determine the users-UAVs associations and the UAVs’ corresponding transmit power levels. In the second step, an efficient algorithm based on a recursive shrink-and-realign process is proposed to optimize the UAV trajectories. The performance of the proposed method shows advantages in terms of average throughput compared to the predefined trajectories solution.

I. INTRODUCTION

Providing “connectivity from the sky” is a new and innovative trend in wireless communications. High and low altitude platforms, drones, aircrafts, and airships, known as Unmanned Aerial Vehicles (UAVs), are being considered as the candidates for deploying wireless communications complementing the terrestrial communication infrastructure. Recently, the use of UAVs as mobile relays to support ground cellular networks has received considerable attention. As reported by American Institute of Aeronautics and Astronautics (AIAA), the global market for commercial UAV applications will skyrocket to as much as $127 billion by 2020 [1].

The main advantage of using UAVs over static relays is that UAVs can collect data while flying, and can also follow mobile users more efficiently and communicate with them more optimally. Using UAVs as mobile relays can be also extremely helpful in different scenarios, such as military, public safety communication, and temporary/unexpected high traffic demand situations, due to their high flexibility, quick and dynamic deployment, and their ability to be equipped with communications and computational devices [2]. In fact, thanks to their mobility, UAVs are more robust against environmental changes and disasters. Several leading Information Technology companies have launched pilot projects, such as Aquila by Facebook [3] and Loon by Google [4], for providing ubiquitous Internet access worldwide by leveraging the UAV technology. The Third Generation Partnership Project (3GPP) is also studying aerial vehicles supported by Long Term Evolution (LTE) where the initial focus is on UAVs [5].

Recent studies in the literature investigated the UAVs’ deployment and its challenges. In [6], a placement technique that uses the UAVs as relays for cell overloading and outage compensation is proposed. Although they provided an analytical model for evaluating system performance in the downlink direction, the authors did not discuss the UAVs’ coverage performance and did not suggest any deployment method. The authors in [7] discussed the optimal deployment position for UAVs that maximizes the average data rate while keeping the symbol error rate under a certain level. However, their work is limited to only one relaying UAV. In [8], the authors analyzed the optimal altitude of one UAV for a certain coverage area that minimizes the transmit power of the UAVs. Moreover, they investigated the coverage of two UAVs positioned at a fixed altitude and their interference to each other over a certain coverage area.

Optimizing the UAVs trajectories and locations can significantly enhance the network performance either by reducing the load of other ground BSs or by covering areas with limited radio access. Therefore planning the trajectory of UAVs is very important to achieve these objectives. Few works in the literature discuss the trajectory optimization of the UAVs. For instance, the UAV trajectory optimization using sequential convex optimization technique has been studied in [9] for a point-to-point system model using only one UAV. In [10], the authors solve a one-dimensional placement problem and consider one UAV serving multiple ground users in a time sharing manner. This work simplifies the analysis but limits applicability in practice. However, the problem becomes more challenging when considering multiple UAVs serving multiple ground users.

In this paper, we consider a more practical scenario in a more dynamic environment in which the ground users are moving over time (variable location with time). Given a predefined trajectory of UAVs, we propose to quickly determine UAVs trajectory paths under boundary constraint. In other words, we assume that each UAV has a predefined trajectory...
related to its primary application, e.g., monitoring an area by taking videos, and, when needed, the UAV can also adjust its location and act as a relay to establish direct links and enhance the users’ throughput. Note that, it is not necessarily that all the UAVs will cooperate in the data transmission; some of them will be selected depending on several criteria that will be defined in the sequel. Hence, we propose to optimize the user-UAV association in addition to the UAVs’ transmit power levels while taking into consideration the channel quality of the communication channel and the UAVs’ battery levels. The contributions of this paper can be summarized as follows:

- Considering a dynamic and practical scenario where all ground users are aiming to connect with their destinations via multiple UAVs.
- Formulating an optimization problem that maximizes the average throughput of ground users by optimizing the user-UAV association and UAVs’ transmit power levels while taking into consideration the UAVs’ trajectories and energy budget constraints.
- Proposing an efficient two-step solution where, in the first step, we solve optimally a Mixed Integer Linear Programming (MILP) problem to determine the selected UAVs and their corresponding power levels then, we propose an efficient and quick deployment algorithm based on a recursive shrink-and-realign heuristic process to optimize the UAV trajectories.

II. SYSTEM MODEL

A. Network Model

In this study, we consider a wireless relaying system composed of multiple mobile users \( s = 1, \ldots, S \) aiming to transmit data to a single static destination, e.g., sink, as shown in Fig. 1. We assume that the users and destination are out of communication ranges of each other and they communicate using \( L \) mobile UAVs, \( l = 1, \ldots, L \), that can act as cooperative relays during a time period \( T \). Each selected UAV performs Decode-and-Forward (DF) relaying strategy that decodes the received signal before broadcasting it to the destination. Furthermore, the transmitted power level of each user is assumed to be fixed and denoted \( P_s \).

Let us consider a 3D coordinate system where the coordinate of destination is \( U_d = (0, 0, 0) \) and the coordinates of each user \( s \) and UAV \( l \) at time instant \( t = 1, \ldots, T \) are given, respectively, as \( U_s(t) = [x_s(t), y_s(t), 0]^t \) and \( J_l(t) = [x_l(t), y_l(t), z_l(t)]^t \), where \([ ]^t\) denotes the transpose operator.

We assume that the time horizon \( T \) is discretized into \( n = 1, \ldots, N \) equal time slots, such that \( T = N \tau \), where \( \tau \) is small enough that the movements of the users and the UAVs are negligible from our optimization problem’s point of view. Note that the choice of \( \tau \) depends on the speed of the users and UAVs. Without loss of generality, we assume that all users and UAVs are moving with constant speeds denoted by \( V_s \) and \( V_l \), respectively. Therefore, the following trajectory constraints should be satisfied

\[
||U_s[n+1]-U_s[n]||^2 \leq V_s \tau, \forall s = 1, \ldots, S, n = 1, \ldots, N-1,
\]

\[
||J_l[n+1]-J_l[n]||^2 \leq V_l \tau, \forall l = 1, \ldots, L, n = 1, \ldots, N-1,
\]

where \( V_s \tau \) and \( V_l \tau \) are the maximum distances that the user \( s \) and the UAV \( l \) can travel during each time slot \( n \). It is assumed that the trajectories of all users over the time \( T \) are pre-known.

Recall that the UAV trajectory is defined for another primary task, but it can be modified when needed to support cooperation under certain trajectory boundaries (i.e., \( J_{\min}[n] \) and \( J_{\max}[n] \)) that are defined as follows:

\[
J_{\min}[n] \leq J_l[n] \leq J_{\max}[n], \quad \forall l, \forall n
\]

where \( J_{\min}[n] \) and \( J_{\max}[n] \) are defined by the UAVs’ operator and define the tolerated region where the UAV can be located.

We consider that each UAV has initial and final locations that must leave from and arrive to within the time period \( T \). Thus, the UAV trajectories need to satisfy the following constraints too:

\[
J_1[1] = J_{1,0}, \quad \text{and} \quad J_k[N] = J_{k,f}, \quad \forall l = 1, \ldots, L.
\]

A binary variable \( \epsilon_{sl}[n] \) is introduced to indicate the association between user \( s \) and UAV \( l \) during time slot \( n \) and is given as follows:

\[
\epsilon_{sl}[n] = \begin{cases} 
1, & \text{if user } s \text{ associated to UAV } l \text{ during time slot } n. \\
0, & \text{otherwise}. 
\end{cases}
\]

We assume that each UAV can be associated to multiple users, however, it is assumed that each user can be associated with one UAV at most during time slot \( n \), therefore, the following condition should be respected:

\[
\sum_{l=1}^{L} \epsilon_{sl}[n] \leq 1, \quad \forall s, \forall n.
\]

B. Ground-to-Air Path Loss Model

The Path Loss (PL) of ground-to-air link is a weighted combination of two PL links: Line-of-Sight (LoS) and Non Line-of-Sight (NLoS) links. This is due to the mobility and ability of UAVs to serve users from high altitude as compared to ground users. In this case, there will be a probability to obtain the LoS link between the UAVs and users [11]. The PL between the UAV \( l \) at a position \( J_l[n] \) and a ground user \( s \) in a typical urban environment for LoS and NLoS is given, respectively, as [11]:

\[
PL_{\text{LoS}}^l[n] = \xi_{\text{LoS}} \left( \frac{4\pi\delta_{sl}[n]}{\lambda_0} \right), \quad PL_{\text{NLoS}}^l[n] = \xi_{\text{NLoS}} \left( \frac{4\pi\delta_{sl}[n]}{\lambda_0} \right),
\]

where \( \delta_{sl}[n] = ||J_l[n] - U_s[n]|| \) is the distance between the UAV \( l \) and the served user, \( \lambda_0 \) is the wavelength of the system. \( \xi_{\text{LoS}} \) and \( \xi_{\text{NLoS}} \) are the additional losses due to the free space propagation losses for LoS and NLoS links, respectively.
The LoS probability is given by [8], [12], [13]:
\[ p_{\text{LoS}}^s[n] = \frac{1}{1 + \nu_1 \exp(-\nu_2(\theta_{al}[n] - \nu_3))}, \]  
(8)
where \( \theta_{al}[n] = \frac{180}{\pi} \sin^{-1}(\frac{z_n[n]}{s_n[n]}) \) is the elevation angle between UAV \( l \) and the ground user \( s \) in (degree). The parameters \( \nu_1 \) and \( \nu_2 \) are constant values that depend on the environment.

The channel gain between the ground user \( s \) and UAV \( l \) during time slot \( n \) is given by
\[
PL_{sl}[n] = p_{\text{LoS}}^s[n] PL_{sl}[n] + (1 - p_{\text{LoS}}^s[n]) PL_{sl}[n].
\]
(9)

Finally, the channel gain between the ground user \( s \) and UAV \( l \) during time slot \( n \) is equal to
\[
h_{sl}[n] = \frac{1}{\rho_{sl}[n]}.
\]

C. UAV Power Model

In this paper, we consider both the transmission and operation power modes of the UAVs. For the transmission power level, each UAV can be either in an active mode if it is in communication with one of the users or in an idle mode otherwise. For simplicity, the total transmit power consumption of UAV \( l \) during a time slot \( n \) can be approximated by a linear model as follows [14]:
\[ P_l = \alpha_s \sum_{s=1}^{S} \epsilon_{sl}[n] P_{sl}[n] + \beta_l, \]
(10)
where \( \alpha_s \) corresponds to the power consumption that scales with the radiated power due to amplifier and feeder losses and \( \beta_l \) models an offset of site power which is consumed independently of the average transmit power. \( P_{sl}[n] \) is the transmitted power of UAV \( l \) during time slot \( n \) to forward the data of user \( s \). Besides the power consumed for the transmission, the UAV consumes additional hovering and hardware power, denoted by \( P_f \), and can be expressed as [15]:
\[ P_f = \left( \frac{(m_{rot}g)^3}{2mr_p^3\omega_p^2\rho} + P_{\text{har}} \right), \]
(11)
where \( m_{rot}, g, \) and \( \rho \) are the UAV mass in (Kg), earth gravity in (m/s²), and air density in (Kg/m³), respectively. The parameters \( r_p \) and \( \omega_p \) are the radius and the number of the UAV’s propellers, respectively. The power consumption due to the UAV hardware is denoted by \( P_{\text{har}} \).

III. PROBLEM FORMULATION

The transmission rate of the link between user \( s \) to UAV \( l \) during time slot \( n \) can be expressed as
\[ R_{sl}[n] = B \log_2 \left( 1 + \frac{\epsilon_{sl}[n] P_{sl}[n] h_{sl}[n]}{B N_0} \right), \]
(12)
where \( B \) is the transmission bandwidth and \( N_0 \) is the noise power. Note here, we assume that all users operate sparsely (allocate different bandwidths to different users, thus, no interference between users). The cross-interference case is left for a future extension of this work. Similarly, the transmission rate from UAV \( l \) to the destination during time slot \( n \) can be expressed as
\[ R_{lD}[n] = B \log_2 \left( 1 + \frac{\epsilon_{sl}[n] P_{sl}[n] h_{lD}[n]}{B N_0} \right), \]
(13)
Therefore, the end-to-end maximum transmission rate at the destination using DF strategy can be expressed as [16]
\[ R_s[n] = \min \left( R_{sl}[n], R_{lD}[n] \right), \]
(14)
where \( \min(\cdot) \) is the minimum function.

In the sequel, we formulate an optimization problem aiming to maximize the end-to-end system throughput at each time slot \( n \) by optimizing the following: 1) association between users and UAVs, 2) transmit power levels of the users and UAVs, and 3) trajectories of the UAVs. Therefore, the optimization problem can be formulated as follows:

\[
\text{maximize} \quad \frac{1}{N} \sum_{s=1}^{S} \sum_{t=1}^{L} \sum_{n=1}^{N} \min \left( R_{sl}[n], R_{lD}[n] \right),
\]
(15)
subject to:
\[
\sum_{l=1}^{L} \epsilon_{sl}[n] \leq 1, \quad \forall s, \forall n, \]
(16)
\[
\epsilon_{sl}[n] \in \{0, 1\}, \quad \forall s, \forall l, \forall n,
\]
(17)
\[
P_{sl}[n] \geq 0, \quad \forall s, \forall l, \forall n,
\]
(18)
\[
\sum_{s=1}^{S} \epsilon_{sl}[n] P_{sl}[n] \leq \bar{P}_l, \quad \forall l, \forall n,
\]
(19)
\[
\tau \sum_{n=1}^{N} \left( P_f[n] + \sum_{s=1}^{S} \epsilon_{sl}[n] P_{sl}[n] \right) \leq \bar{B}, \quad \forall l.
\]
(20)
\[
||J_{l}[n + 1] - J_{l}[n]||^2 \leq V_{l} \tau, \quad \forall l, \forall n,
\]
(21)
\[
J_{l}[1] = J_{l,0}, \quad J_{l}[N] = J_{l,f}, \quad \forall l.
\]
(22)
\[
J_{\text{min}}[n] \leq J_{l}[n] \leq J_{\text{max}}[n], \quad \forall l, \forall n.
\]
(23)
Constraints (18) and (19) ensure that the total transmit power of a UAV is between 0 and peak power level. Constraint (20) represents the UAV battery constraint, where \( \bar{B} \) is the maximum energy that can be stored in the UAVs. Constraints (21)-(23) indicate the trajectory constraints as explained in Section II.

IV. PROPOSED SOLUTION

The formulated optimization problem is a Mixed Integer Non-Linear Programming (MINLP), and solving it is a challenging task. In the sequel, we solve our optimization problem using a two-step iterative approach. In the first step, given a preplanned trajectory of UAVs, we transform the MINLP to a MILP problem that optimizes the user-to-UAV associations as well as UAVs’ transmit power levels. Then, in the second step, given these associations and power levels, we propose an efficient heuristic algorithm to modify the UAVs’ trajectories in order to shift some of them whenever it is possible and needed.

A. Association and Power Optimization with Fixed UAVs Trajectories

In this subsection, we aim to solve the optimization problem with fixed UAVs trajectories \( J_{l}[n], \forall l = 1, .., L, \forall n = 1, .., N \). We firstly linearize the objective function and hence the optimization problem by defining the new decision variable \( \tilde{\phi}_{sl}[n] \) as follows:
\[
\tilde{\phi}_{sl}[n] = \min \left( \frac{\epsilon_{sl}[n] P_{sl}[n] h_{lD}[n]}{B N_0}, \frac{\epsilon_{sl}[n] P_{sl}[n] h_{sl}[n]}{B N_0} \right),
\]
(24)
where \( \tilde{\phi}_{sl}[n] \) is the average PL for ground-to-air link in dB at a given time
Indeed, maximizing the rate given in (14) is equivalent to maximize $\phi_{sl}[n]$. Secondly, we introduce another decision variable $\rho_{sl}[n]$ for each UAV link power to linearize the product of binary and real decision variables as follows: 

$$\rho_{sl}[n] \leq \epsilon_{sl}[n] P_{sl}[n],$$

where the first two inequalities have to be respected

1) $P_{sl}[n] \geq \rho_{sl}[n] \geq 0$, 
2) $\rho_{sl}[n] \geq \bar{P}_l \epsilon_{sl}[n] - \bar{P}_l + P_{sl}[n]$,
3) $\rho_{sl}[n] \leq \bar{P}_l \epsilon_{sl}[n]$. 

(25)

The first two inequalities ensure that $\rho_{sl}[n]$ value is between $\epsilon_{sl}[n]$ and $P_{sl}$. The third inequality guarantees that $\rho_{sl}[n] = 0$ if $\epsilon_{sl}[n] = 0$, and $\rho_{sl}[n] = P_{sl}[n]$ if $\epsilon_{sl}[n] = 1$. Therefore, the optimization problem can be reformulated as MILP as follows:

$$\begin{align*}
\text{maximize} & \quad \frac{1}{N} \sum_{s=1}^{S} \sum_{l=1}^{L} \sum_{n=1}^{N} \bar{\phi}_{sl}[n] \\
\text{subject to:} & \\
& \sum_{s=1}^{S} \rho_{sl}[n] \leq \bar{P}_l, \quad \forall l, \forall n, \\
& \tau \sum_{n=1}^{N} \left( P_f[n] + \frac{1}{S} \sum_{s=1}^{S} \rho_{sl}[n] \right) \leq \bar{B}, \quad \forall l, \\
& P_{sl}[n] \geq \rho_{sl}[n] \geq 0, \quad \forall s, \forall l, \forall n, \\
& \rho_{sl}[n] \geq \bar{P}_l \epsilon_{sl}[n] - \bar{P}_l + P_{sl}[n], \quad \forall s, \forall l, \forall n, \\
& \rho_{sl}[n] \leq \bar{P}_l \epsilon_{sl}[n], \quad \forall s, \forall l, \forall n, \\
& \frac{P_{sl}[n]}{B N_0} \geq \bar{\phi}_{sl}[n], \quad \forall s, \forall l, \forall n, \\
& \frac{\rho_{sl}[n]}{B N_0} \geq \bar{\phi}_{sl}[n], \quad \forall s, \forall l, \forall n, \\
\end{align*}$$

(26)

Notice that the solution for such an MILP problem can be determined optimally using on-the-shelf software such as Gurobi/CVX interface [17].

B. Trajectory Optimization with Fixed Power and Association

In this subsection, we consider optimizing the trajectories of the UAVs for fixed associations and UAVs’ transmit power levels. Due to the non-convexity of the problem even with fixed associations and UAVs’ transmit power levels, we introduce a quick and efficient algorithm based on shrink-and-realign process. This algorithm presents several advantages over other heuristic algorithms. The main advantages are summarized as follows: (i) it is easy to implement by using a simple search process with few parameters to manipulate, (ii) it has low computational cost, and (iii) it provides a fast convergence to a close-to-optimal solution.

We propose a Recursive Uniform Search (RUS) algorithm to optimize the UAV trajectories. Our algorithm starts by generating initial $Q$ high efficiency next position candidates $J_q^n[n+1], q = 1 \cdots Q$ of size $L \times 1$ to identify promising candidates and to form initial populations $Q$ as shown in Fig. 2. These candidates need to satisfy the trajectory constraints given in (21)-(23). Then, it determines the objective function achieved by each candidate by solving the MILP optimization problem described in Section IV-A. After that, it finds the initial best local candidate $q_i^{1, \text{local}}[n]$ that provides the highest solution for iteration $i$. Then, we start recursive sampling with uniform distribution in these areas. Using shrink-and-realign sample spaces process to find the best solution $q_i^n[n]$ and the corresponding trajectory $J_q^{q_i^n}[n+1], l = 1 \cdots L$ as shown in Fig. 2. This shrink-and-realign procedure is repeated until the size of the sample space decreases below a threshold.

Note that RUS is a modified version of Recursive Random Search (RRS) algorithm described in [18], [19], where it has been tested on a suite of well-known and difficult benchmark functions. The results showed that in terms of quickly locating a “good” solution, RRS outperforms other search algorithms, such as multistart pattern search and controlled random search. The details of the joint optimization approach are given in Algorithm 1.

Algorithm 1 Joint optimization algorithm

1: for $n = 1 \cdots N$ do
2: \hspace{.5cm} $i = 1$.
3: \hspace{.5cm} Generate an initial population $Q$ composed of $Q$ candidates $J_q^n[n+1], q = 1 \cdots Q$ that satisfied constraints (21)-(23).
4: \hspace{.5cm} while Not converged or reaching certain threshold do
5: \hspace{.5cm} \hspace{.5cm} for $q = 1 \cdots Q$ do
6: \hspace{.5cm} \hspace{.5cm} \hspace{.5cm} Compute the corresponding objective function.
7: \hspace{.5cm} \hspace{.5cm} \hspace{.5cm} end for
8: \hspace{.5cm} \hspace{.5cm} Find $\epsilon_{sl}[n]$ and $P_{sl}[n]$ by solving the MILP optimization problem for candidate $q$ using Gurobi/CVX interface [17].
9: \hspace{.5cm} \hspace{.5cm} Compute the corresponding objective function.
10: \hspace{.5cm} \hspace{.5cm} end for
11: \hspace{.5cm} \hspace{.5cm} Find $(q_i^{1, \text{local}}[n]) = \arg \max_{q_i} \sum_{s=1}^{S} \sum_{l=1}^{L} \sum_{n=1}^{N} \bar{\phi}_{sl}[n]$ (i.e., $q_i^{1, \text{local}}[n]$ indicates the index of the best local candidate that results in the highest objective function for iteration $i$ during time slot $n$).
12: \hspace{.5cm} \hspace{.5cm} Start recursive sampling with uniform distribution in these areas.
13: \hspace{.5cm} \hspace{.5cm} Use shrink-and-realign sample spaces process to find the best solution.
14: \hspace{.5cm} \hspace{.5cm} $i = i+1$.
15: \hspace{.5cm} end while
16: end for

V. SELECTED NUMERICAL RESULTS

In this section, selected numerical results are provided to demonstrate the benefits of using UAVs as relays. We consider a system with $L = 3$ UAVs flying at a fixed altitude $z_l = 60$ m $\forall l = 1, \cdots, L$ and $S = 10$ ground users distributed randomly within an area of $200m \times 400m$. We use $N = 30$ in our simulations. For simplicity, we assume that the users are in
Table I: System parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<td>( \lambda ) (m)</td>
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<td>( \rho_1 )</td>
<td>5.6</td>
<td>( \rho_2 )</td>
<td>7.29</td>
</tr>
<tr>
<td>( B ) (kJ)</td>
<td>20</td>
<td>( L_{\text{tot}} ) (dB)</td>
<td>1</td>
<td>( L_{\text{tot}} ) (dB)</td>
<td>12</td>
</tr>
<tr>
<td>( V_r ) (m/s)</td>
<td>4</td>
<td>( T_L ) (dB)</td>
<td>6.8</td>
<td>( B_2 ) (Hz)</td>
<td>180000</td>
</tr>
<tr>
<td>( T_p ) (cm)</td>
<td>20</td>
<td>( m )</td>
<td>4</td>
<td>( \bar{P}_{\mu} ) (W)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Fig. 3: Preplanned trajectories for the three UAVs in our experiments.

Fig. 4: The proposed trajectory algorithm and preplanned trajectory for UAV 2.

fixed locations during \( N \) time slots. The maximum UAVs’ transmit power is \( P_1 = 30 \) dBm. The noise power \( N_0 \) is assumed to be \( 7.2 \times 10^{-16} \) W/Hz. In Table I, we present the values of the remaining environmental parameters used in the simulations from the previous work in [15], [20].

As shown in Fig. 3, we assume that each UAV can move with a tolerance of +/- 20m (\( J_{\text{min}}[n] \) and \( J_{\text{max}}[n] \)) away from this preplanned trajectory as explained in (3) (trajectory boundaries). Note that some simple algorithms can be used to avoid collisions between UAVs, but it is out of the scope of this paper. Fig. 4 plots the UAV trajectories calculated by our proposed heuristic and the preplanned trajectory for one UAV. It shows that our proposed algorithm has more degrees of freedom by modifying the trajectory of the UAV to be close to ground users as much as possible to enhance the channel gain and the total throughput.

Fig. 5 plots the achieved average throughput per user versus users’ transmit power with UAV peak transmit power \( P_1 = 30 \) dBm for different trajectory optimization solutions; using 1- the preplanned trajectory, 2- the proposed shrink-and-realign algorithm given in Section. IV-B, and 3- the exhaustive search where we assume we have very large number of candidates

\( K \) combinations for the next positions of the UAVs. This figure shows that our proposed algorithm is close in performance from exhaustive search solution (optimal). It is shown that the proposed trajectory algorithm reaches near optimal solution with complexity of order \( O(NLQI_{\max}) \) compared to \( O(LK^N) \) complexity order for exhaustive search. Furthermore, the improvement of our proposed trajectory algorithm over the preplanned trajectory is shown clearly. For instance, using \( P_S = 20 \) dBm, our proposed algorithm can improve the throughput by around 20% over the preplanned trajectory by achieving 0.33 MBits/s instead of 0.28 MBits/s. This figure also shows that the achievable throughput is improving with the increase of \( P_S \) up to a certain point, due to the fact that starting from this value of \( P_S \) the SNR can not be improved because it depends on the value of \( P_{\text{num}}[n] \) as shown in (24) which is limited by \( P_1 \). Also, one can see that the gap between different solutions is reduced as \( P_S \) increased, this is due to the fact that the increasing the user transmit power improves the received signals at the UAVs. Furthermore, we plot the 95% confidence interval for the proposed trajectory algorithm for different values of the users’ power (i.e., \( P_S = \{20,30\} \)) dBm.

In Fig. 6, we compare the average user throughput (i.e., \( 1/(NS) \sum_{s=1}^S \sum_{l=1}^L \sum_{n=1}^N R_s[n] \)) with the minimum (i.e., \( \min(1/N \sum_{l=1}^L \sum_{n=1}^N R_s[n]) \)) and maximum (i.e., \( \max(1/N \sum_{l=1}^L \sum_{n=1}^N R_s[n]) \)) throughputs can be achieved by users. This figure gives us an overview of the throughput ranges among all users.

Fig. 7 shows the convergence speed of our algorithm for \( P_S = 20 \) dBm and \( P_1 = 30 \) dBm. We plot the throughput of one user versus the number of iterations. Note that an iteration in Fig. 7 corresponds to one iteration of the “while loop” given in Algorithm 1 (i.e., line 3-12). In other words, it corresponds to one iteration of shrink-and-realign but it includes the execution of the MILP. The figure shows that the proposed algorithm achieves its solution with few iterations only (i.e., 6-9 iterations).

Finally, in Table II, we show an example of the association and throughput values of our proposed trajectory algorithm and preplanned trajectory for \( P_S = 20 \) dBm and \( P_1 = 30 \) dBm for a certain time slot \( n = N/3 \). This table validates our analysis of trying to assign users to UAVs in optimal way, and UAVs can update their trajectories to establish direct

![Table I: System parameters](https://example.com/table.png)

![Fig. 3: Preplanned trajectories for the three UAVs in our experiments.](https://example.com/f3.png)

![Fig. 4: The proposed trajectory algorithm and preplanned trajectory for UAV 2.](https://example.com/f4.png)

![Fig. 5: Average throughput range versus users’ transmit power for \( P_1 = 30 \) dBm.](https://example.com/f5.png)

![Fig. 6: Average user throughput comparison.](https://example.com/f6.png)

![Fig. 7: Convergence speed of our algorithm.](https://example.com/f7.png)
throughput performance by assigning most of users to UAVs. Our proposed algorithm tries to keep users assigned to UAVs unlike the traditional case with fixed trajectories where it could have some users unassigned for a certain time slot (users $S_2, S_5$, and $S_8$).

VI. SUMMARY AND FUTURE WORK
In this paper, we proposed an efficient optimization framework for Unmanned Aerial Vehicles (UAVs) to perform as links and increase the throughput performance by assigning most of users to UAVs. Our proposed algorithm tries to keep users assigned to UAVs unlike the traditional case with fixed trajectories where it could have some users unassigned for a certain time slot (users $S_2, S_5$, and $S_8$).

REFERENCES