Wireless RF-based Energy Harvesting for Two-Way Relaying Systems

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***Abstract*—In this paper, we investigate the Energy Harvesting (EH)-based two-way relaying system using Amplify-and-Forward (AF) and Decode-and-Forward (DF) strategies. The relay is considered as an EH node that harvests the received Radio Frequency (RF) signal and uses this harvested energy to forward the information. Two relaying protocols based on Time Switching (TS) and Power Splitting (PS) receiver architectures are proposed to enable EH and information processing at the relay. Analytical throughput expressions are derived and optimized for both protocols. The goal is to find the optimal TS and PS ratios that maximize the total throughput. Numerical results illustrate the performance of TS and PS protocols for different strategies, and show that at high signal-to-noise ratio, PS is superior to TS, and AF is superior to DF in terms of achievable sum-rate.**

***Index Terms*—Energy harvesting, power splitting, time switching, two-way relaying.**

I. Introduction

Recently, the lifetime and sustainability of wireless networks have received significant attention from the research community [1]. Energy Harvesting (EH) is considered as a promising technique that addresses these challenges. Depending on the type of energy source, EH has different models. Apart from the traditional EH based on renewable energy such as solar, wind, or vibration [2], EH based on Radio Frequency (RF) has been considered as a promising source to power wireless terminals as it is widely available in the ambient atmosphere in all hours, day and night [3], [4].

Two RF EH protocols are proposed in literature: Time Switching-based (TS) protocol and Power Splitting-based (PS) protocol [5]. In the former, the receiver switches over time between EH and information processing, while in the latter, a portion of the received signal is used for EH and the remaining for the information processing.

Two-Way Relay (TWR), has lately attracted a lot of attention by reducing the total energy of the network and achieving higher data rate [6]. In conventional TWR, exchanging different messages between two sources takes place into two phases only instead of four phases in the traditional One-Way Relaying (OWR). In the first phase, which is known as the Multiple Access (MA) phase, the sources simultaneously transmit their signals to the relay. Subsequently, in the second phase, which is known as the Broadcasting (BC) phase, the relays broadcast the signal to the sources. Finally, the sources apply a self-interference cancellation operation to extract the desired data [7]–[9]. The main relaying strategies, that are used in the literature, are Amplify-and-Forward (AF) and Decode-and-Forward (DF) strategies, [10], [11]. In AF strategy, the relay amplifies the received signal before broadcasting it to the destinations, while in DF strategy, the relay decodes the source messages and transmits a function of them to the destinations. Although the amount of harvested energy from RF is limited compared to the other energy sources, many practical applications that combine the use of RF EH with cooperative relaying have been discussed recently, such as, emerging ultra-dense small cell deployments, point-to-point sensor networks, and dense wireless networks [12], [13]. For instance, in some cases, it might be difficult to replenish relays with a new battery or charge it using wired charging system (e.g., relay sensors located on mountains or in forests). Therefore, the communication between two nodes needs to rely on some external charging mechanisms in case of fail tolerance to remain active and hence, wireless EH at the relays becomes a necessity for such situations.

Several studies have discussed the RF EH with OWR where the relay harvests energy from the transmitted signal from the source and uses it to forward the signal to the destination [14], [15]. For instance, the authors in [14] proposed an interference-aided EH scheme for OWR systems using DF strategy where analytical expressions of the ergodic capacity and the outage capacity were derived. The work in [15] proposed AF delay-limited and delay-tolerant transmission modes and analyzed the outage probability and the ergodic capacity for each mode. TWR with TS using AF relaying is studied in [16]. The authors derived analytical throughput expressions for Multiple Access Broadcast (MABC) and time division broadcast schemes. Most of the aforementioned works on RF EH with cooperative communications focused on OWR systems.

Few works discussed the EH with TWR. The work in [16] focused on deriving the TS throughput using AF relay without optimization the total EH throughput. To the best of the authors' knowledge, the optimization of the achievable sum-rate of EH-TS and EH-PS protocols for TWR-AF and TWR-DF systems have not been discussed so far. In this paper, we study the EH-TS and EH-PS protocols for TWR using AF and DF. We firstly derive throughput expressions for both EH-TS and EH-PS protocols. Afterwards, we formulate optimization problems that maximize the throughput of EH TWR while taking into account the relay power limitation constraint. After proving their convexity, the problems are solved using CVX (other convex optimization solvers can be employed [17]) and the optimal solutions for the EH-TS and EH-PS are analyzed.

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II. System Model

We consider a half-duplex TWR system where two battery-powered sources, denoted by S_1 and S_2, exchange information through the help of an intermediate EH relay node, denoted by R as shown in Fig. 1. We assume that relay R is equipped with two components: an EH components, where the task is to convert the RF signal to direct current, and information processing components responsible in forwarding the received signal to the sources S_1 and S_2 using the harvested energy [18]. We assume that each node is equipped with a single antenna and that S_1 and S_2 are not within the communication range of each other.

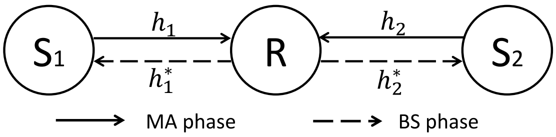

Fig. 1: System model of two-way relaying.

In the MA phase, both S_1 and S_2 transmit their messages x_1 and x_2 simultaneously to R with a power denoted by P_1 and P_2, respectively. In the BC phase, the relay R transmits the broadcast signal to the sources with a harvested power denoted by P_h. Let us define T as the total time block or epoch length to exchange messages between S_1 and S_2, and η as the energy conversion efficiency ratio when converting the RF signal to current $(0 \le \eta \le 1)$. Let us define $\bar{P}_s, \bar{P}_r, h_1$, and h_2 as the peak power at each source, the peak power at R, the channel gain between S_1 and R, and the channel gain between S_2 and R, respectively. The reverse channel gain between R and S_1, and between R and S_2 are denoted by h_1^* and h_2^*, respectively, where $(^*)$ denotes the conjugate operator. Without loss of generality, all channel gains are assumed to be constant during the two transmission phases. Also, all noise variances introduced by the receive antenna are assumed to be additive white Gaussian noise (AWGN) with zero mean and a variance equal to σ_r^2, and $\mathbb{E}\left[|x_1|^2\right] = \mathbb{E}\left[|x_2|^2\right] = 1$, where $\mathbb{E}\left[\cdot\right]$ denotes the expectation operator.

III. Energy Harvesting Protocols

In MA phase, the received signal at the relay is given by

$$y_r = \sqrt{P_1} h_1 x_1 + \sqrt{P_2} h_2 x_2 + n_r, \tag{1}$$

where n_r is the sum of the noises introduced by the relay antenna during reception with variance σ_r^2 and the noise introduced by the signal processing circuit from passband to baseband also assumed to be AWGN with zero mean and variance σ^2. In practice, the antenna noise has a negligible effect on the information signal and the average power of the received signal as well [19]. Hence, we ignore its impact in (1) (i.e., $\sigma_r^2 \ll \sigma^2$). Based on the relay receiver architecture, two practical protocols, named as TS relaying protocol and PS relaying protocol, can be adopted in the cooperative EH context.

A. Time Switching Relaying Protocol

In TS protocol, the relay spends a portion of time for EH and the remaining time for information processing. Let us assume α as the TS ratio where during αT time, the relay can harvest energy from the received signal $(0 < \alpha < 1)$, as shown in Fig. 2. The remaining time $(1-\alpha)T$ is used for communications such that the first $(1-\alpha)T/2$ is used for information transmission from S_1 and S_2 to the relay R (i.e., MA phase), while the second $(1-\alpha)T/2$ is used to broadcast the received signal from the relay to S_1 and S_2 (i.e., the BC phase). Later, it will be shown that the choice of the TS ratio affects the achievable sum-rate. In fact, increasing α allows the relay to harvest more energy that will be employed in forwarding the received signals to the destination. However, this will reduce the allocated time to perform the whole transmission and vice versa. Therefore, an optimal choice of α is required in order to enhance the TS sum-rate.

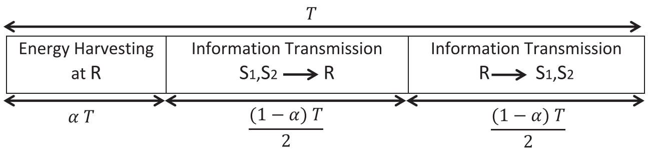

Fig. 2: Block diagram of the TS protocol.

The harvested energy and power (energy normalized) at the relay R, denoted by E_h^{TS} and P_hTS, are given, respectively, by

$$E_h^{\mathrm{TS}} = \eta \left(P_1|h_1|^2 + P_2|h_2|^2\right) \alpha T, \tag{2}$$

$$P_h^{\mathrm{TS}} = \frac{E_h^{\mathrm{TS}}}{(1-\alpha)T/2} = \frac{2\eta\alpha \left(P_1|h_1|^2 + P_2|h_2|^2\right)}{(1-\alpha)}. \tag{3}$$

1) Amplify-and-Forward: During the BC phase, R amplifies y_r by multiplying it by the relay amplification gain, denoted by w_{TS}, and broadcasts it to S_1 and S_2. Hence, the received signals at S_1 and S_2 are given, respectively, as

$$\begin{aligned} y_1 &= h_1^* w_{\mathrm{TS}} (h_1 \sqrt{P_1} x_1 + h_2 \sqrt{P_2} x_2 + n_r) + n_1, \\ y_2 &= h_2^* w_{\mathrm{TS}} (h_1 \sqrt{P_1} x_1 + h_2 \sqrt{P_2} x_2 + n_r) + n_2, \end{aligned} \tag{4}$$

where the amplification gain at the relay R using TS protocol can be expressed as [8]

$$w_{\mathrm{TS}} = \sqrt{P_h^{\mathrm{TS}} / \left(P_1|h_1|^2 + P_2|h_2|^2 + \sigma^2\right)}. \tag{5}$$

Since the channels are known perfectly at S_i, $i = \{1, 2\}$, S_i can remove the self interference. Note that in the case when imperfect channel estimation is considered, self interference can still be applied, however, it will introduce an error related to the channel estimation that can be included in the noise. The investigation of the impact of the imperfect channel estimation on the system performance is left for a future extension of this work. Therefore, Signal-to-Noise Ratios (SNRs) at S_1 and S_2 can be, respectively, given as follows

$$\gamma_1^{\mathrm{AF}}(\alpha) = \frac{P_2 w_{\mathrm{TS}}^2 |h_1|^2 |h_2|^2}{\sigma^2(1 + w_{\mathrm{TS}}^2 |h_1|^2)}, \gamma_2^{\mathrm{AF}}(\alpha) = \frac{P_1 w_{\mathrm{TS}}^2 |h_1|^2 |h_2|^2}{\sigma^2(1 + w_{\mathrm{TS}}^2 |h_2|^2)}. \tag{6}$$

By substituting (5) into (6), the SNRs can be re-written as

$$\gamma_1^{\mathrm{AF}}(\alpha) = \frac{a_1 \alpha}{\sigma^2 + b_1 \alpha}, \quad \gamma_2^{\mathrm{AF}}(\alpha) = \frac{a_2 \alpha}{\sigma^2 + b_2 \alpha}, \tag{7}$$

where $a_1 = 2\eta Q |h_1|^2 |h_2|^2 P_2$, $b_1 = \sigma^2(2\eta Q|h_1|^2 - 1)$, $a_2 = 2\eta Q |h_1|^2 |h_2|^2 P_1$, $b_2 = \sigma^2(2\eta Q|h_2|^2 - 1)$, and $Q = \frac{P_1|h_1|^2 + P_2|h_2|^2}{P_1|h_1|^2 + P_2|h_2|^2 + \sigma^2}$. Thus, the TWR sum-rate can be expressed

$$R_{\mathrm{TS}}^{\mathrm{AF}}(\alpha) = (1-\alpha)\Big(C(\gamma_1^{\mathrm{AF}}(\alpha)) + C(\gamma_2^{\mathrm{AF}}(\alpha))\Big), \tag{8}$$

$$R_{\text{TS}}^{\text{DF}}(\alpha) = (1-\alpha)\min\left[\min\left[C\left(\frac{P_1|h_1|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{TS}}|h_2|^2}{\sigma^2}\right)\right] + \min\left[C\left(\frac{P_2|h_2|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{TS}}|h_1|^2}{\sigma^2}\right)\right], C\left(\frac{P_1|h_1|^2+P_2|h_2|^2}{\sigma^2}\right)\right]. \quad (9)$$

where $C(x) = \frac{T}{2}\log_2(1+x)$.

2) Decode-and-Forward: The achievable rate region of TS using DF strategy for the two phases, where MABC scheme is considered, must satisfy the following sum-rate constraints [20]

$$R_{1,\text{TS}}^{\text{DF}} \leq (1-\alpha)\min\left(C\left(\frac{P_1|h_1|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{TS}}|h_2|^2}{\sigma^2}\right)\right), \quad (10a)$$

$$R_{2,\text{TS}}^{\text{DF}} \leq (1-\alpha)\min\left(C\left(\frac{P_2|h_2|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{TS}}|h_1|^2}{\sigma^2}\right)\right), \quad (10b)$$

$$R_{1,\text{TS}}^{\text{DF}} + R_{2,\text{TS}}^{\text{DF}} \leq (1-\alpha)C\left(\frac{P_1|h_1|^2+P_2|h_2|^2}{\sigma^2}\right). \quad (10c)$$

From (10a), we can see that the rate from S_1 to S_2 should be less than the rates achieved in the links between S_1 to R and R to S_2. The same remark is noticed for the rate from S_2 to S_1 in (10b). Equation (10c) represents the maximum achievable region of the MA channel. Hence, the TS sum-rate using DF is given by (9).

3) Optimization Problem for EH-TWR with TS Protocol: Let us denote by $R_{\text{TS}}^{\text{S}}(\alpha)$ the sum-rate of EH-TS TWR using AF (i.e., S=AF) or DF (i.e., S=DF) strategies. Hence, the optimization problem that maximizes the sum-rate for TS protocol while satisfying relay power limitation constraint can now be formulated as

$$\underset{0<\alpha<1}{\text{maximize}} \quad R_{\text{TS}}^{\text{S}}(\alpha) \quad (11)$$

$$\text{subject to: } \alpha\left(2\eta\left(P_1|h_1|^2+P_2|h_2|^2\right)+\bar{P}_r\right) \leq \bar{P}_r. \quad (12)$$

Constraint (12) is equivalent to the relay harvested power constraint given in (3) (i.e., $P_h^{\text{TS}} = \eta\alpha\left(P_1|h_1|^2+P_2|h_2|^2\right)/(1-\alpha) \leq \bar{P}_r$) and indicates that the harvested power has to be less than the relay power limitation. Indeed, exceeding $\bar{P}_r$ will affect the relay performance or burn the relay. This can be applied in the case where energy storage is not available at the relay. In this case, if the harvested power is higher than $\bar{P}_r$, the system will decide to reduce α in order to avoid the loss of energy (i.e., the extra harvested energy that will not be used by the relay).

B. Power Splitting Relaying Protocol

In PS protocol, the relay splits the received signal into two components with a ratio β, defined as the relay PS ratio, where $0<\beta<1$ such that $\sqrt{\beta}(\sqrt{P_1}h_1x_1+\sqrt{P_2}h_2x_2)$ corresponds to the part of RF signal that will be converted to a current, and $\sqrt{(1-\beta)}(\sqrt{P_1}h_1x_1+\sqrt{P_2}h_2x_2)$ is used for information processing as shown in Fig. 3. Note that the power splitting operation holds before converting the signal from the passband to the baseband, Hence, after signal conversion, the received signal at the relay can be expressed as:

$$\tilde{y_r} = \sqrt{(1-\beta)}(\sqrt{P_1}h_1x_1+\sqrt{P_2}h_2x_2)+n_r. \quad (13)$$

In this protocol, the transmission in each phase is performed during $T/2$.

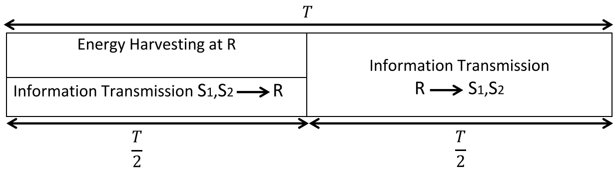

Fig. 3: Block diagram of the PS protocol.

As will be shown in the sequel, the choice of the PS ratio affects the achievable sum-rate. Indeed, high values of β will provide more input RF signal to the energy harvester receiver. However, this will reduce the quality of the signal that will be forwarded by the relay and vice versa. Therefore, an optimized choice of β will enhance the achievable sum-rate. The harvested energy and power (energy normalized) at the relay, denoted by E_h and P_h, are given, respectively, by

$$E_h^{\text{PS}} = \eta\left(P_1|h_1|^2+P_2|h_2|^2\right)\beta\frac{T}{2}, \quad (14)$$

$$P_h^{\text{PS}} = \frac{E_h^{\text{PS}}}{T/2} = \eta\beta\left(P_1|h_1|^2+P_2|h_2|^2\right). \quad (15)$$

1) Amplify-and-Forward: During the BC phase, the relay amplifies the received signal y_r by the relay amplification gain denoted as w_{PS} and broadcasts it to S_1 and S_2. Finally, the received signals at S_1 and S_2 are given respectively as

$$y_1 = h_1^* w_{\text{PS}}\left(\sqrt{1-\beta}\left(h_1\sqrt{P_1}x_1+h_2\sqrt{P_2}x_2\right)+n_r\right)+n_1,$$
$$y_2 = h_2^* w_{\text{PS}}\left(\sqrt{1-\beta}\left(h_1\sqrt{P_1}x_1+h_2\sqrt{P_2}x_2\right)+n_r\right)+n_2, \quad (16)$$

where the amplification gain at the relay using PS protocol can be expressed as

$$w_{\text{PS}} = \sqrt{P_h^{\text{PS}}/\left((1-\beta)\left(P_1|h_1|^2+P_2|h_2|^2\right)+\sigma^2\right)}. \quad (17)$$

After removing the self interference, the SNRs at S_1 and S_2 can be given, respectively, as follows

$$\gamma_1^{\text{AF}}(\beta) = \frac{(1-\beta)P_2 w_{\text{PS}}^2|h_1|^2|h_2|^2}{\sigma^2(1+w_{\text{PS}}^2|h_1|^2)},$$
$$\gamma_2^{\text{AF}}(\beta) = \frac{(1-\beta)P_1 w_{\text{PS}}^2|h_1|^2|h_2|^2}{\sigma^2(1+w_{\text{PS}}^2|h_2|^2)}. \quad (18)$$

By substituting (17) into (18) and after simplifications, the SNRs can be written as

$$\gamma_1^{\text{AF}}(\beta) = \frac{c_1\beta(1-\beta)}{\sigma^2+d_1\beta}, \quad \gamma_2^{\text{AF}}(\beta) = \frac{c_2\beta(1-\beta)}{\sigma^2+d_2\beta}, \quad (19)$$

where, $c_1 = \eta Q|h_1|^2|h_2|^2P_2$, $d_1 = \sigma^2 Q(\eta|h_1|^2-1)$, $c_2 = \eta Q|h_1|^2|h_2|^2P_1$, $d_2 = \sigma^2 Q(\eta|h_2|^2-1)$, and Q is already defined under (7). Thus, the TWR sum-rate is expressed as

$$R_{\text{PS}}^{\text{AF}}(\beta) = C(\gamma_1^{\text{AF}}(\beta)) + C(\gamma_2^{\text{AF}}(\beta)). \quad (20)$$

$$R_{\text{PS}}^{\text{DF}}(\beta) = \min\left[\min\left[C\left(\frac{(1-\beta)P_1|h_1|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{PS}}|h_2|^2}{\sigma^2}\right)\right] + \min\left[C\left(\frac{(1-\beta)P_2|h_2|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{PS}}|h_1|^2}{\sigma^2}\right)\right], C\left(\frac{(1-\beta)(P_1|h_1|^2+P_2|h_2|^2)}{\sigma^2}\right)\right]. \tag{21}$$

2) Decode-and-Forward: When the PS protocol is employed using DF relaying strategy, the relay node will first decode the information based on the received information signal $(1-\beta)y_r$. In the BC phase, the relay node forwards the decoded information to their corresponding sources using the energy harvested. Thus, the achievable rate region of PS using DF strategy for the two phases must satisfy the following sum-rate constraints

$$R_{1,\text{PS}}^{\text{DF}} \le \min\left(C\left(\frac{(1-\beta)P_1|h_1|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{PS}}|h_2|^2}{\sigma^2}\right)\right), \tag{22a}$$

$$R_{2,\text{PS}}^{\text{DF}} \le \min\left(C\left(\frac{(1-\beta)P_2|h_2|^2}{\sigma^2}\right), C\left(\frac{P_h^{\text{PS}}|h_1|^2}{\sigma^2}\right)\right), \tag{22b}$$

$$R_{1,\text{PS}}^{\text{DF}} + R_{2,\text{PS}}^{\text{DF}} \le C\left(\frac{(1-\beta)(P_1|h_1|^2+P_2|h_2|^2)}{\sigma^2}\right). \tag{22c}$$

Hence, the TS sum-rate using DF is given by (21).

3) Optimization Problem for EH-TWR with PS Protocol: Let us denote by $R_{\text{PS}}^{\text{S}}(\beta)$ the sum-rate of EH-PS TWR using AF (i.e., S=AF) or DF (i.e., S=DF) strategies. Hence, the optimization problem that maximizes the sum-rate for PS protocol while satisfying relay power limitation constraint can be now formulated as

$$\underset{0<\beta<1}{\text{maximize}} \quad R_{\text{PS}}^{\text{S}}(\beta) \tag{23}$$

$$\text{subject to:} \eta\beta\left(P_1|h_1|^2+P_2|h_2|^2\right) \le \bar{P}_r, \tag{24}$$

where constraint (24) corresponds to the relay harvested power imitation constraint.

IV. Time Switching and Power Splitting Convexity Lemmas

To prove the convexity of the optimization problems formulated in (11)-(12) and (23)-(24), we refer to the following Lemmas in [17];

- **Lemma A**: $\sum_{j=1}^{J}\log_2(m_j(x))$ is a concave if all $m_j's$ are concave and positive.
- **Lemma B**: if f and g are concave, positive, with one non-decreasing and the other non-increasing, then fg is concave.

A. Convexity Lemma of Time Switching Protocol

Lemma 1. *Let $R_{TS}^{S}(\alpha)$ the sum-rate of EH-TS TWR using AF (i.e., S=AF) or DF (i.e., S=DF), then the formulated optimization problem given in* (11)-(12) *is a convex optimization problem in α, since $R_{TS}^{S}(\alpha)$ is a concave and constraints* (12) *and $0<\alpha<1$ are convex.*

Proof. It is straightforward to see that the constraints are affine functions in α, thus, convex. In order to prove the concavity of $R_{\text{TS}}^{\text{S}}(\alpha)$, we investigate the AF and DF separately.

Firstly, for AF strategy, the goal is to prove that $R_{\text{TS}}^{\text{AF}}(\alpha)$ given in (8) is concave in α. By assuming $f=(1-\alpha)$ and $g=C(\gamma_1^{\text{AF}}(\alpha))+C(\gamma_2^{\text{AF}}(\alpha))$ We can notice the following;

- f is a concave function since it is linear, positive ($\alpha<1$), and non-increasing function.
- Trivially, g is a positive function since $\gamma_1^{\text{AF}}(\alpha)>0$ and $\gamma_2^{\text{AF}}(\alpha)>0$ and the $\log_2$ function is a non-decreasing function. To prove the convexity of g, we consider 3 cases of b_i according to (7); when b_i is positive (i.e., $2\eta Qh_i>1$), when b_i is negative (i.e., $2\eta Qh_i<1$), and when b_i equals to 0 (i.e., $2\eta Qh_i=1$).
  1) When b_i is positive, then the second derivative of $\gamma_i^{\text{AF}}(\alpha)$ is always negative and equals to $\frac{-2a_ib_i\sigma^2}{(\sigma^2+b_i\alpha)^3}$ since the denominator of the SNR to power 3 is positive and a_i is positive. Therefore, based on Lemma A $\log_2(\gamma_i^{\text{AF}}(\alpha))$ is concave.
  2) When b_i is negative, $\log_2(\gamma_1^{\text{AF}}(\alpha))=\log_2(a_i\alpha)+\log_2(\frac{1}{\sigma^2+b_i\alpha})$, It easy to see that the first term is concave and the second term is concave for positive α.
  3) When b_i equals to 0, it is also straightforward to show that $\gamma_1^{\text{AF}}(\alpha)$ is concave.

Therefore, for any value of b_i, g is always concave since the sum of concave functions is also a concave function [17]. Based on Lemma B, $R_{\text{TS}}^{\text{AF}}(\alpha)=fg$ is a concave function in α.

For DF strategy case, we introduce new decision variables $R_{\text{a,TS}}^{\text{DF}}, R_{\text{b,TS}}^{\text{DF}}, R_{\text{min,TS}}^{\text{DF}}$ that simplify the expressions of the sum-rate region, such that $R_{a,\text{TS}}^{\text{DF}} \le C\left(\frac{P_1|h_1|^2}{\sigma^2}\right)$, $R_{a,\text{TS}}^{\text{DF}} \le C\left(\frac{P_h^{\text{TS}}|h_2|^2}{\sigma^2}\right)$, $R_{b,\text{TS}}^{\text{DF}} \le C\left(\frac{P_2|h_2|^2}{\sigma^2}\right)$, $R_{b,\text{TS}}^{\text{DF}} \le C\left(\frac{P_h^{\text{TS}}|h_1|^2}{\sigma^2}\right)$, $R_{\text{min,TS}}^{\text{DF}} \le C\left(\frac{P_1|h_1|^2+P_2|h_2|^2}{\sigma^2}\right)$, and $R_{\text{min,TS}}^{\text{DF}} \le R_{a,\text{TS}}^{\text{DF}}+R_{b,\text{TS}}^{\text{DF}}$. These inequalities are added to the problem as constraints and the objective becomes the maximization of $(1-\alpha)R_{\text{min,TS}}^{\text{DF}}$. The convexity of these new constraints can be proved using the same proof steps of AF and the concavity of the objective function is proved using Lemma B where $(1-\alpha)$ and $R_{\text{min,TS}}^{\text{DF}}$ are concave, positive, with one non-decreasing function and the other is a non-increasing one. Therefore, $R_{\text{TS}}^{\text{DF}}(\alpha)$ is always concave in α. □

B. Convexity Lemma of Power Splitting Protocol

Lemma 2. *Let $R_{PS}^{S}(\beta)$ the sum-rate of EH-PS TWR using AF (i.e., S=AF) or DF (i.e., S=DF) , then the formulated optimization problem given in* (23)-(24) *is a convex optimization problem in β, since $R_{PS}^{S}(\beta)$ is a concave and constraints* (24) *and $0<\beta<1$ are convex.*

Proof. Constraints (24) and $0<\beta<1$ are affine, thus are convex. For AF strategy case, the sum-rate is given by $C(\gamma_1^{\text{AF}}(\beta))+C(\gamma_2^{\text{AF}}(\beta))$, where $\gamma_1^{\text{AF}}(\beta)=\frac{c_1\beta(1-\beta)}{\sigma^2+d_1\beta}$, $\gamma_2^{\text{AF}}(\beta)=\frac{c_2\beta(1-\beta)}{\sigma^2+d_2\beta}$. Similar to the previous proof in Lemma 2, we consider 3 cases of d_i; when d_i is positive (i.e., $\eta|h_i|^2>1$), when d_i is negative (i.e., $\eta|h_i|^2 < 1$), and when d_i equals to 0 (i.e., $\eta|h_i|^2 = 1$).

1) When d_i is positive, the second derivative of $\gamma_i^{\text{AF}}(\beta)$ will be always negative and equals to $\frac{-2c_i\sigma^2(d_i\sigma^2)}{(\sigma^2+d_i\alpha)^3}$ since the denominator of the SNR to power 3 is positive and c_i is positive. Therefore, based on Lemma A, $\log_2(\gamma_i^{\text{AF}}(\beta))$ is concave.
2) When d_i are negative, $\log_2(\gamma_i^{\text{AF}}(\beta)) = \log_2(c_i\beta(1-\beta)) + \log_2(\frac{1}{\sigma^2+d_i\beta})$, It easy to see that the first term is a concave function (a polynomial of degree 2 with negative coefficient of β^2) and the second term is also concave. Therefore, the sum is a concave since the sum of concave functions is also a concave function [17].
3) When d_i equals to 0, it is straightforward to show that the sum-rate is concave.

Based on Lemma A, $R_{\text{PS}}^{\text{AF}}(\beta)$ is a concave function in β.

For the DF strategy, similarly to the TS case, we introduce new decision variables $R_{\text{a,PS}}^{\text{DF}}, R_{\text{b,PS}}^{\text{DF}}, R_{\text{min,PS}}^{\text{DF}}$ that represent the rate region of PS protocol using DF strategy, in which $R_{a,\text{PS}}^{\text{DF}} \le C\left(\frac{(1-\beta)P_1|h_1|^2}{\sigma^2}\right)$, $R_{a,\text{PS}}^{\text{DF}} \le C\left(\frac{P_h^{\text{PS}}|h_2|^2}{\sigma^2}\right)$, $R_{b,\text{PS}}^{\text{DF}} \le C\left(\frac{(1-\beta)P_2|h_2|^2}{\sigma^2}\right)$, $R_{b,\text{PS}}^{\text{DF}} \le C\left(\frac{P_h^{\text{PS}}|h_1|^2}{\sigma^2}\right)$, $R_{\text{min,PS}}^{\text{DF}} \le C\left(\frac{(1-\beta)(P_1|h_1|^2+P_2|h_2|^2)}{\sigma^2}\right)$, and $R_{\text{min,PS}}^{\text{DF}} \le R_{a,\text{PS}}^{\text{DF}} + R_{b,\text{PS}}^{\text{DF}}$, where the convexity of these new constraints can be proved using the same steps in AF case proof. Accordingly, the objective function becomes $R_{\text{min,PS}}^{\text{DF}}$ is linear and thus concave. Therefore, $R_{\text{PS}}^{\text{DF}}(\beta)$ is always concave in β. $\square$

V. Simulation Results

In this section, numerical results are presented to demonstrate the performance of the investigated protocols for RF-based EH in TWR systems. The transmission between S_1 and S_2 is generated periodically with time length $T = 1$. All the fading channel gains adopted in the framework are assumed to be independent and identically distributed (i.i.d). The noise variance and the conversion ratio are set to $\sigma^2 = 0.001$ and $\eta = 0.9$, respectively. In the sequel, we assume that $P_1 = P_2 = P_s$. A Monte Carlo simulation is performed to determine the average achievable sum-rate and the corresponding TS and PS ratios.

We start by investigating the sum-rate for EH-TWR system using the TS protocol. Fig. 4 plots the achievable throughput and the time switching ratio α versus the power relay limitation $\bar{P}_r$ for different values of source power $P_s = \{10, 20, 30\}$ dBm. It is noticed that by increasing the relay power budget, the relay is allowed to harvest more RF power and hence increase the total sum-rate till a certain maximum value where the sum-rate achieves a steady-state region limited by P_s. Indeed, if P_s is low, the relay cannot harvest energy greater than a certain level. This can also be deduced from the variations of the TS ratio α that remains constant from a certain value of $\bar{P}_r$. At this point, α reaches the maximum value that corresponds to the maximum harvested time αT and maximum information processing time $(1-\alpha)T/2$. Notice also that the lower P_s, the higher α. This is because the relay decides to compensate the low received power of the RF signal by spending more time to harvest more energy. For instance,

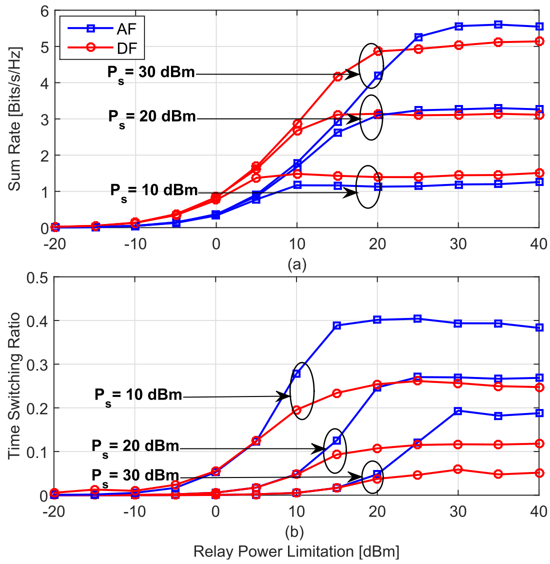

Fig. 4: The achieved rate and α for TS protocol versus $\bar{P}_r$.

for high $\bar{P}_r$ with AF strategy, when the RF power is reduced by 10 dBm from 20 dBm to 10 dBm, α varies from 27% to 40%. Comparing the employed forwarding strategies: AF and DF, it is shown that DF outperforms the AF in terms of sum-rate in low and mid $\bar{P}_r$ regions. However, for high $\bar{P}_r$, it is noticed that AF achieves higher sum-rate mainly for high values of P_s. This can be justified by the fact that the sum-rate in TWR of the DF strategy becomes a bottleneck for the MA phase in the high $\bar{P}_r$ regime. Therefore, to achieve better performance, it is advised to select the suitable strategy according to the system parameters.

The performances of the PS protocol are illustrated in Fig. 5 where the throughput and the time switching ratio α versus power relay limitation $\bar{P}_r$ are plotted for different values of source power $P_s = \{10, 20, 30\}$ dBm. Almost the same remarks in terms of achieved sum-rate of the TS protocol can be deduced for the PS one. However, some differences can be noticed in the variations of the ratios. Although both of them are inversely proportional to P_s, β follows a different profile with AF. In fact, we notice that when AF sum-rate reaches the steady-state region for high values of $\bar{P}_r$, β becomes almost the same for the various values of P_s (around 60%) unlike the AF and DF case with TS and the DF case with PS where α and β depend on the value of P_s, respectively. This is due to the existence of the parameter β in the expression of the amplification gain of the AF-PS strategy given in (17) which forces the relay to use the maximum allowed w_{PS} expressed as $\sqrt{\frac{\bar{P}_r}{(1-\beta)(P_s(|h_1|^2+|h_2|^2)+\sigma^2)}}$ in this region. Although β is the same, the sum-rate remains different because of the different values of P_s.

It is also shown that PS protocol achieves greater sum-rates with both relaying strategies mainly at high $\bar{P}_r$ values. This can be explained by the fact that the PS protocol performs its transmission over the total interval T, while TS transmits the signals during $(1-\alpha)T$. Indeed, at high SNR regime, the transmission time duration is more influential than the SNR effect in the $\log_2$ function due to the availability of harvested power for both protocols.

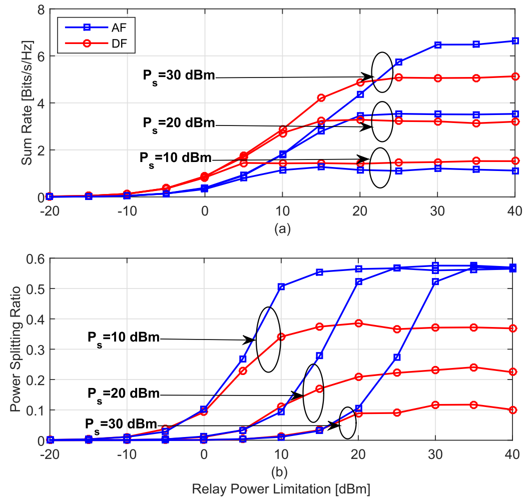

Fig. 5: The achieved rate and β for PS protocol versus $\bar{P}_r$.

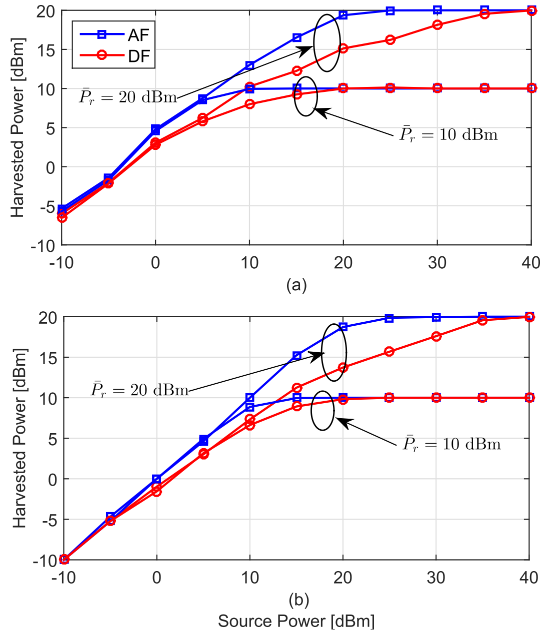

Fig. 6: The harvested power at the relay versus source power P_s for (a) TS protocol, (b) PS protocol.

Fig. 6 illustrates the harvested power at the relay versus the source power P_s for TS and PS protocols, where the energy conversion efficiency $\eta = 0.9$ and the relay power limitation $\bar{P}_r = \{10, 20\}$ dBm. We can notice that the the relay harvests more energy as P_s increases up to a certain point, due to the fact that starting from this value of P_s, the system needs to reduce α and β for TS and PS protocols, in order to respect constraints (12) and (24), respectively. Also, we notice that the amount of harvested power is almost the same for AF and DF for low and high values of P_s. However, in the intermediate region, where DF outperforms AF in terms of sum-rate for both protocols, AF consumes more power.

VI. Conclusion

In this paper, an RF-based energy harvesting two-way relaying system was investigated, where the relay harvests the RF received signal, and uses the harvested energy to forward the signal to the sources. Two EH protocols at the relay were adopted, namely time-switching and power-splitting protocols. Analytical throughput expressions were derived and optimized for both protocols using two relaying strategies: amplify-and-forward and decode-and-forward. After proving the convexity of the optimization problems, optimal time switching and power splitting ratios were obtained using CVX.

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