HW#3, EE 653, Fall 2002, Dr. McCalley
Due: Tuesday, 10/1/02

1. A generating unit is composed of a large number of components, and a failure of any of these components results in an outage of the unit. Assume that there are n components of interest in a generator and that each can fail independently of the others. Further assume that the time to failure of component i can be descried by an exponential distribution with parameter λ_i, i=1,2,...,n. Determine the reliability of the unit, neglecting the possibility that the failed component can be replaced or repaired.

The time to failure of the ith component, T_i, is described by a density function:

$$f_{T_i}(t) = \lambda_i e^{-\lambda_i t}$$

The probability that this component will be operational at time t is thus

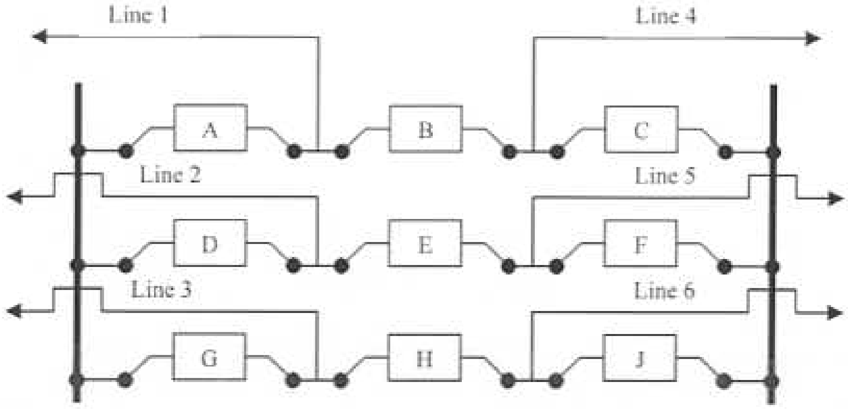
$$R_i(t) = P(T_i \geq t) = 1 - F_{T_i}(t) = e^{-\lambda_i t}$$

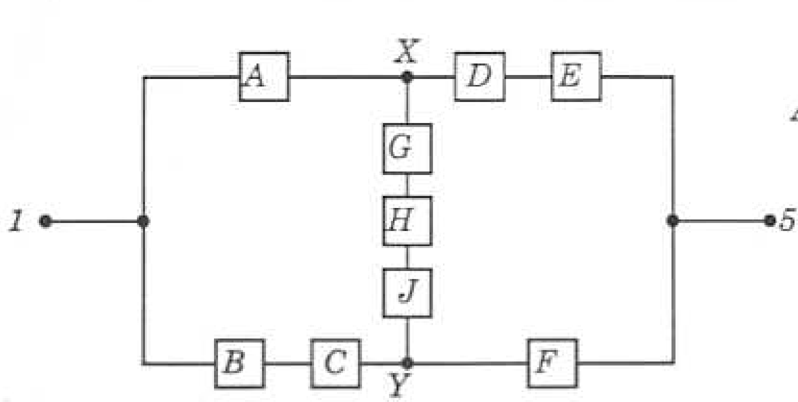
For n components connected in series (in reliability sense) whose failures are independent events, we have, from Eq. 7.41,

$$P_S = \prod_{i=1}^{n} R_i(t) = e^{-\Sigma \lambda_i t} \qquad (7.42)$$

$$\lambda = \sum_{i=1}^{n} \lambda_i$$

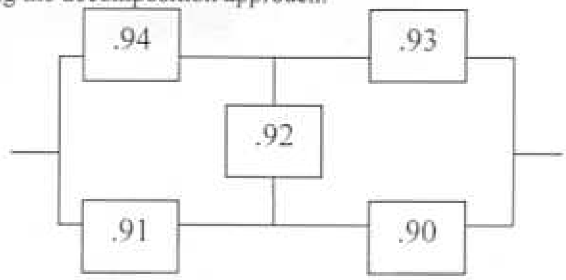
2. Consider a breaker-and-a-half switching station shown below. The station designer is particularly interested in the reliability of a particular power transmission through the station, which normally enters the station on line 1 and exits on line 5. Since the power transfers are usually large, a high financial penalty accompanies loss of this path. Construct a reliability block diagram of the station components between terminals 1 and 5.





Similar Arrangements:
1-5
1-6
2-4
2-6
3-4
3-5

3. A system of independently operating elements is represented by the bridge network below. The numbers are component reliabilities. Use the delta-star transformation to compute system reliability. Verify your answer using the decomposition approach.



We obtain the equivalent star configuration values by using (3.55), (3.56), and (3.57).

$$\therefore R_A = 0.9948$$

$$R_B = 0.9930,\ R_C = 0.9954$$

The network shown in Figure 3.8 may be expressed as its equivalent as shown in Figure 3.9. The reliability equation for this structure is

$$R_T = [1-(1-R_1 R_A)(1-R_2 R_B)]\,R_C$$

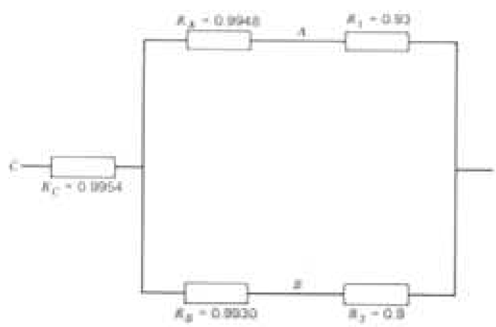


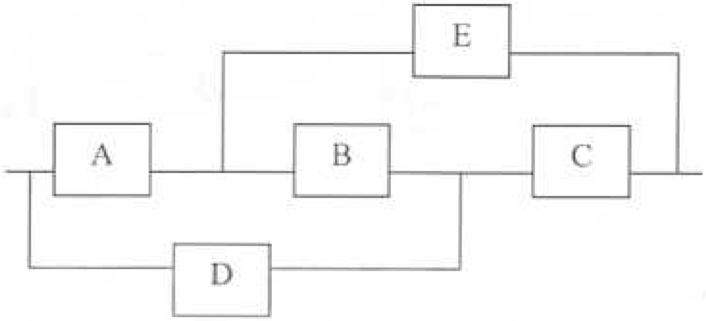
Figure 3.9 A transformed two-state device structure.

Numerically the value of the total bridge reliability is

$$R_T = 0.987$$

for the given component reliability values.

4. Consider a system represented by the following diagram. Use the tie-set and cut-set methods to estimate the system reliability.



The minimum tie-sets of the system are

$$T_1 = AE$$

$$T_2 = DC$$

$$T_3 = ABC.$$

The reliability of the system is the union of the tie-sets:

$$R = P(AE \cup DC \cup ABC)$$ Eqt 2.36

$$= P(AE) + P(DC) + P(ABC) - P(AEDC) - P(AEBC) - P(DCAB) + P(AEDCB).$$

Assuming independence of probabilities, then Eq. (2.36) can be written as

$$R = P(A)P(E) + P(D)P(C) + P(A)P(B)P(C) - P(A)P(E)P(D)P(C) - P(A)P(E)P(B)P(C) - P(D)P(C)P(A)P(B) + P(A)P(E)P(D)P(C)P(B). \quad (2.37)$$

If all units are identical and each has a probability p of functioning properly, then Eq. (2.37) becomes

$$R = 2p^2 + p^3 - 3p^4 + p^5. \quad (2.38)$$

We can also apply the cut-set method to determine R. The minimum cut-sets are

$$C_1 = \bar{A}\bar{D}$$

$$C_2 = \bar{E}\bar{C}$$

$$C_3 = \bar{A}\bar{C}$$

$$C_4 = \bar{B}\bar{E}\bar{D}.$$

The reliability of the system is

$$R = 1 - P(\bar{A}\bar{D} \cup \bar{E}\bar{C} \cup \bar{A}\bar{C} \cup \bar{B}\bar{E}\bar{D}). \quad (2.39)$$

Again, assuming independence of probabilities, Eq. (2.39) becomes

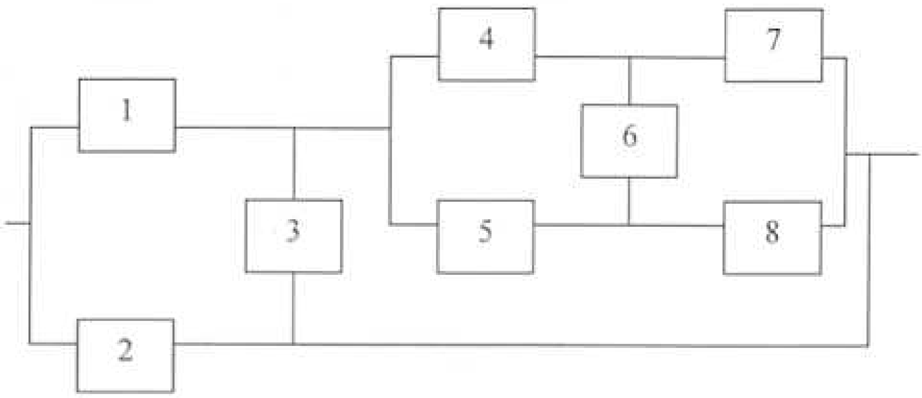
$$R = 1 - [P(\bar{A}\bar{D}) + P(\bar{E}\bar{C}) + P(\bar{A}\bar{C}) + P(\bar{B}\bar{E}\bar{D}) - P(\bar{A}\bar{D}\bar{E}\bar{C}) - P(\bar{A}\bar{D}\bar{C}) - P(\bar{A}\bar{D}\bar{B}\bar{E}) - P(\bar{E}\bar{C}\bar{A}) - P(\bar{E}\bar{C}\bar{B}\bar{D}) - P(\bar{A}\bar{C}\bar{B}\bar{E}\bar{D}) + P(\bar{A}\bar{D}\bar{E}\bar{C}) + P(\bar{A}\bar{D}\bar{C}\bar{B}\bar{E}) + P(\bar{A}\bar{D}\bar{C}\bar{B}\bar{E}) + P(\bar{E}\bar{C}\bar{A}\bar{B}\bar{D}) - P(\bar{A}\bar{B}\bar{C}\bar{D}\bar{E})].$$

Substituting $P(\bar{A}) = [1 - P(A)]$ and $P(A) = P(B) = P(C) = P(D) = P(E) = p$ in the above equation, we obtain

$$R = 2p^2 + p^3 - 3p^4 + p^5. \quad (2.40)$$

Equations (2.38) and (2.40) are identical.

5. Consider the reliability block diagram shown below. Use the cutset approach to obtain a lower bound on the system reliability, for t=1000 hours. Failure times for all components are exponentially distributed with parameters: $\lambda_1=1\times10^{-6}\text{hr}^{-1}$, $\lambda_2=1\times10^{-5}\text{hr}^{-1}$, $\lambda_3=2\times10^{-5}\text{hr}^{-1}$, $\lambda_4=\lambda_5=\lambda_6=\lambda_7=\lambda_8=1\times10^{-4}\text{hr}^{-1}$.



the minimal path sets are: $P_1 = (2)$, $P_2 = (1, 3)$, $P_3 = (1, 4, 7)$, $P_4 = (1, 5, 8)$, $P_5 = (1, 4, 6, 8)$, $P_6 = (1, 5, 6, 7)$. The minimal cut sets are: $C_1 = (1, 2)$, $C_2 = (4, 5, 3, 2)$, $C_3 = (7, 8, 3, 2)$, $C_4 = (4, 6, 8, 3, 2)$, $C_5 = (5, 6, 7, 3, 2)$.

Using the system cut sets discussed earlier and (4.30),

$$R_s(t) \geq 1 - \left[\Pr(C_1) + \Pr(C_2) + \cdots + \Pr(C_5)\right]$$

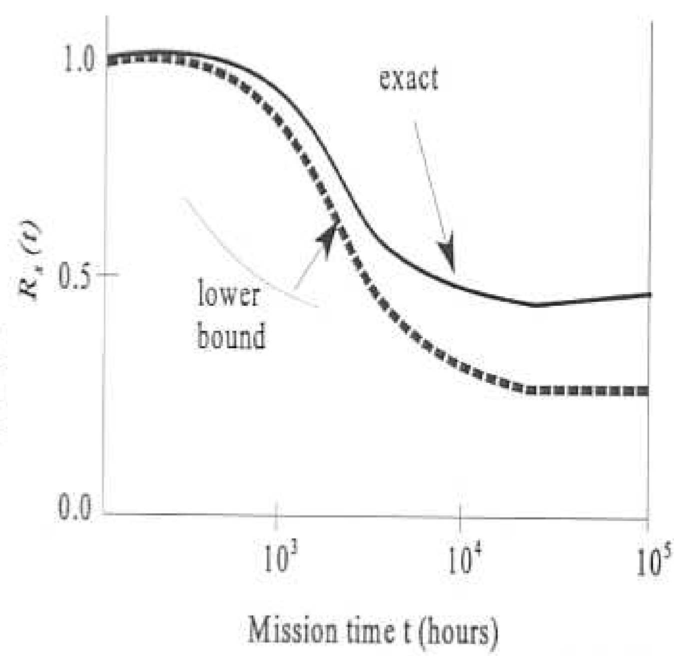
assuming C_1 and C_2 are independent, and

$$\Pr(C_1) = \left[1 - \exp(-\lambda_1 t)\right]\left[1 - \exp(-\lambda_2 t)\right]$$

and so on. Therefore,

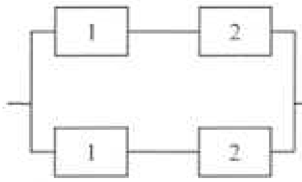
$$R_s(t) \geq = 1 - \Big[\left(1 - e^{-\lambda_1 t}\right)\left(1 - e^{-\lambda_2 t}\right) + \left(1 - e^{-\lambda_2 t}\right)\left(1 - e^{-\lambda_3 t}\right)\left(1 - e^{-\lambda_4 t}\right)$$
$$\left(1 - e^{-\lambda_5 t}\right) + \left(1 - e^{-\lambda_2 t}\right)\left(1 - e^{-\lambda_3 t}\right)\left(1 - e^{-\lambda_7 t}\right)\left(1 - e^{-\lambda_8 t}\right)$$
$$+ \left(1 - e^{-\lambda_2 t}\right)\left(1 - e^{-\lambda_3 t}\right)\left(1 - e^{-\lambda_4 t}\right)\left(1 - e^{-\lambda_6 t}\right)\left(1 - e^{-\lambda_8 t}\right)$$
$$+ \left(1 - e^{-\lambda_2 t}\right)\left(1 - e^{-\lambda_3 t}\right)\left(1 - e^{-\lambda_5 t}\right)\left(1 - e^{-\lambda_6 t}\right)\left(1 - e^{-\lambda_7 t}\right)\Big]$$

For some typical values of λ, the lower bound for $R_s(t)$ can be compared to the exact value of $R_s(t)$. Here, "exact" means the cut sets are not assumed disjoint. For example, Figure 4.7 shows the exact and the lower probability bound of system reliability for $\lambda_1 = 1 \times 10^{-6}\ \text{hr}^{-1}$, $\lambda_2 = 1 \times 10^{-5}\ \text{hr}^{-1}$, $\lambda_3 = 2 \times 10^{-5}\ \text{hr}^{-1}$, and $\lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 1 \times 10^{-4}\ \text{hr}^{-1}$.

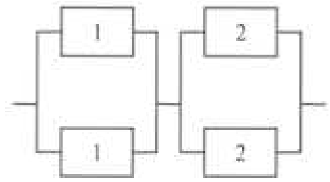


6. A classic question is whether redundancy is better at the system level or the component level. To answer this question, consider the two logic diagrams below, consisting of two #1 components and two #2 components, with each pair having exponentially distributed lifetimes such that λ_1=0.0005 hr^{-1}, and λ_1=0.001 hr^{-1}. Develop expressions for the two configurations and plot the reliability as a function of time for t=0 to t=1000 hours.

(i)



(ii)



Calculating the reliability of both systems involves application of (6.4) and (6.5) but in different orders. For (i)

$$R(t) = 1 - [1 - R_1(t)R_2(t)]^2 = 2R_1(t)R_2(t) - [R_1(t)R_2(t)]$$

$$= R_1R_2[2 - R_1R_2].$$

For (ii),

$$R(t) = [1 - (1 - R_1(t))^2][1 - (1 - R_2(t))^2]$$

$$= 2R_1(t)R_2(t)[2 - R_1(t) - R_2(t)] + [R_1(t)R_2(t)]^2$$

$$= R_1R_2[2 - R_1]\,[2 - R_2].$$

Figures 6.10 and 6.11 show these two system reliabilities for exponential units with parameters $\lambda_1 = 0.0005$ and $\lambda_2 = 0.001$, and for units having Weibull distributed lifetimes, both with shape parameter 5 and scale parameter 1060. In each case the 'mission' time is 1000.

System (ii) is always more reliable than system (i) but this will in practice usually be subject to the reliability of the extra failure sensing/switching mechanism required in (ii). This could be modelled by using a slightly higher hazard rate for unit 2 in system (ii) than in system (i). Alternative approaches are considered in Section 6.10.