Consider the same 1.5 MW DFIG analyzed under unity power factor (data is repeated on the next slide). Once again, assume the generator operates with a maximum power point tracking (MPPT) system so that its mechanical torque $T_{em}$ is proportional to the square of the rotor speed.

(1) Assume the stator power factor is 0.95 leading. For each of the following speeds: 1750, 1650, 1500, 1350, and 1200 rpm, compute:

- Slip
- $T_{em}$ (kN-m)
- $I_s$ (use exact expression, i.e., with $R_s$)
- $V_r$ (volts)
- $I_r$ (amps)
- $R_{eq}$ (ohms)
- $X_{eq}$ (ohms)

(2) Repeat (1) except assume the stator power factor is 0.95 lagging.

(3) Repeat (1) except use approximate expression to obtain $I_s$.

<table>
<thead>
<tr>
<th>Generator Type</th>
<th>DFIG, 1.5 MW, 690 V, 50 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Mechanical Power</td>
<td>1.5 MW</td>
</tr>
<tr>
<td>Rated Stator Line-to-line Voltage</td>
<td>690 V (rms)</td>
</tr>
<tr>
<td>Rated Stator Phase Voltage</td>
<td>398.4 V (rms)</td>
</tr>
<tr>
<td>Rated Rotor Phase Voltage</td>
<td>67.97 V (rms)</td>
</tr>
<tr>
<td>Rated Stator Current</td>
<td>1068.2 A (rms)</td>
</tr>
<tr>
<td>Rated Rotor Current</td>
<td>1125.6 A (rms)</td>
</tr>
<tr>
<td>Rated Stator Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Rated Rotor Speed</td>
<td>1750 rpm</td>
</tr>
<tr>
<td>Nominal Rotor Speed Range</td>
<td>1200–1750 rpm</td>
</tr>
<tr>
<td>Rated Slip</td>
<td>0.1667</td>
</tr>
<tr>
<td>Number of Pole Pairs</td>
<td>2</td>
</tr>
<tr>
<td>Rated Mechanical Torque</td>
<td>8.185 kN-m</td>
</tr>
<tr>
<td>Stator Winding Resistance, $R_s$</td>
<td>2.65 mΩ</td>
</tr>
<tr>
<td>Rotor Winding Resistance, $R_r$</td>
<td>2.63 mΩ</td>
</tr>
<tr>
<td>Stator Leakage Inductance, $L_s$</td>
<td>0.1687 mH</td>
</tr>
<tr>
<td>Rotor Leakage Inductance, $L_r$</td>
<td>0.1337 mH</td>
</tr>
<tr>
<td>Magnetizing Inductance, $L_m$</td>
<td>5.4749 mH</td>
</tr>
<tr>
<td>Base Current, $I_b = 1.5 MW / (\sqrt{3} \times 690 \text{ V})$</td>
<td>1255.1 A (rms)</td>
</tr>
<tr>
<td>Base Flux Linkage, $A_\phi$</td>
<td>1.2681 Wb (rms)</td>
</tr>
<tr>
<td>Base Impedance, $Z_b$</td>
<td>0.3174 Ω</td>
</tr>
<tr>
<td>Base Inductance, $L_b$</td>
<td>1.0103 mH</td>
</tr>
<tr>
<td>Base Capacitance, $C_b$</td>
<td>10028.7 μF</td>
</tr>
</tbody>
</table>


Solution:
Note that:
\[ n_s = \frac{60f_s}{p} \text{ rpm} = 60 \times \frac{50}{2} = 1500 \text{ rpm} \]
So the synchronous speed is 1500 rpm.

1. Assume 0.95 leading. This means reactive power is being supplied to the grid. We want to make the below calculations for \( n_m = 1750, 1650, 1500, 1350, \) and \( 1200 \) rpm.

The equations to use for each of the calculations are provided below:

- **Slip**
  \[ slip = s = \frac{n_s - n_m}{n_s} \]

- **\( T_{em} \) (kN-m)**
  \[ T_{em} = -8185.1 \left( \frac{n_m}{1750} \right)^2 \]

- **\( I_s \) (use exact expression, i.e., with \( R_s \))**
  \[ I_s = \frac{V_s \cos \phi \pm \sqrt{(V_s \cos \phi)^2 - \frac{4R_s \omega_s I_{em}}{3p}}} {2R_s} \]
  Then, assuming that \( V_s \) is the reference (and has angle of 0 degrees), the phasor \( I_s \) is given by
  \[ I_s = I_s \angle 180 - \cos^{-1}(0.95) = I_s \angle 161.8^\circ \]

- **\( I_r \) (amps)**
  \[ I_r = I_m - I_s \left( R_s + j \omega_s L_{cm} \right) \]
  \[ I_r = I_m - I_s \left( R_s + j \omega_s L_{cm} \right) \]

- **\( V_r \) (volts)**
  \[ V_r = V_m + I_r \left( \frac{R_s}{s} + j \omega_s L_{eq} \right) \]
  \[ = V_m - I_s \left( R_s + j \omega_s L_{cm} \right) + I_r \left( \frac{R_s}{s} + j \omega_s L_{eq} \right) \]

- **\( Z_{eq} \) (ohms)**
  \[ Z_{eq} = R_{eq} + j \omega_s L_{eq} = \frac{V_r}{-I_r} = \left( \frac{sV_m + I_r (R_s + j \omega_s L_{eq})}{I_r} \right) \]

- **\( R_{eq} \) (ohms)**
  \[ R_{eq} \]
  \[ \Rightarrow R_{eq} \text{ is the real part of } Z_{eq} \]

- **\( X_{eq} \) (ohms)**
  \[ X_{eq} \]
  \[ \Rightarrow X_{eq} \text{ is the imaginary part of } Z_{eq} \]

Observe here that the equation to obtain \( Z_{eq} \) has negative \( -I_r \) on the denominator. This is as it should be, given our directionality of \( I_r \).
Applying these expressions (see matlab code at end of this document), we obtain the following for the 0.95 leading condition:

<table>
<thead>
<tr>
<th>( n_m )</th>
<th>1200</th>
<th>1350</th>
<th>1500</th>
<th>1650</th>
<th>1750</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s ) (slip)</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>-0.1</td>
<td>-0.1667</td>
</tr>
<tr>
<td>( T_{em} ) (nTn-m)</td>
<td>-3848.7</td>
<td>-4871.0</td>
<td>-6013.5</td>
<td>-7276.4</td>
<td>-8185.1</td>
</tr>
<tr>
<td>( I_s ) (amps)</td>
<td>530.5\textdegree 161.8°</td>
<td>670.8\textdegree 161.8°</td>
<td>827.2\textdegree 161.8°</td>
<td>999.7\textdegree 161.8°</td>
<td>1123.6\textdegree 161.8°</td>
</tr>
<tr>
<td>( I_r ) (amps)</td>
<td>657.4\textdegree 37.8°</td>
<td>795.1\textdegree 34.3°</td>
<td>951.1\textdegree 31.6°</td>
<td>1124.9\textdegree 29.6°</td>
<td>1250.4\textdegree 28.4°</td>
</tr>
<tr>
<td>( V_r ) (volts)</td>
<td>86.9\textdegree 5.65°</td>
<td>45.0\textdegree 6.25°</td>
<td>2.5\textdegree 31.6°</td>
<td>42.81\textdegree 165.8°</td>
<td>73.6\textdegree 165.4°</td>
</tr>
<tr>
<td>( R_{eq} ) (ohms)</td>
<td>-0.0959</td>
<td>-0.0430</td>
<td>-0.0026</td>
<td>0.0275</td>
<td>0.0431</td>
</tr>
<tr>
<td>( X_{eq} ) (ohms)</td>
<td>-0.0909</td>
<td>-0.0368</td>
<td>0</td>
<td>0.0263</td>
<td>0.0402</td>
</tr>
</tbody>
</table>

(2) Repeat (1) except assume the stator power factor is 0.95 lagging.

- **Slip**
  \[ s = \frac{n_s - n_m}{n_s} \]

- **\( T_{em} \) (kN-m)**
  \[ T_{em} = -8185.1 \left( \frac{n_m}{1750} \right)^2 \]

- **\( I_s \) (use exact expression, i.e., with \( R_s \))**
  \[ I_s = \frac{V_s \cos \phi \pm \sqrt{(V_s \cos \phi)^2 - 4R_s \omega_s T_{em}}}{2R_s} \]

Then, assuming that \( V_s \) is the reference (and has angle of 0 degrees), the phasor \( I_s \) is given by

\[ I_s = I_s \angle -180 + \cos^{-1}(0.95) = I_s \angle -161.8° \]

- **\( I_r \) (amps)**
  \[
  V_m = V_s - L_r (R_r + j\omega_s L_{cr}) \\
  I_m = \frac{V_m}{j\omega_s L_m} \\
  I_r = I_m - I_s = \frac{V_r - L_r (R_r + j\omega_s L_{cr})}{j\omega_s L_m} - I_s
  \]

- **\( V_r \) (volts)**
  \[
  \frac{V_r}{\sqrt{s}} = V_m + L_r \left( \frac{R_r}{s} + j\omega_s L_{cr} \right) \\
  = V_s - L_r (R_r + j\omega_s L_{cr}) + I_r \left( \frac{R_r}{s} + j\omega_s L_{cr} \right)
  \]

- **\( Z_{eq} \) (ohms)**
  \[
  Z_{eq} = R_{eq} + js\omega_s L_{eq} = \frac{V_r}{-L_r} = \left( \frac{sV_m + L_r (R_r + js\omega_s L_{cr})}{I_r} \right)
  \]

- **\( R_{eq} \) (ohms)**
  \( \Rightarrow R_{eq} \) is the real part of \( Z_{eq} \)

- **\( X_{eq} \) (ohms)**
  \( \Rightarrow X_{eq} \) is the imaginary part of \( Z_{eq} \)
Applying these expressions (see matlab code at end of this document), we obtain the following for the 0.95 lagging condition:

<table>
<thead>
<tr>
<th>n_m</th>
<th>1200</th>
<th>1350</th>
<th>1500</th>
<th>1650</th>
<th>1750</th>
</tr>
</thead>
<tbody>
<tr>
<td>s (slip)</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>-0.1</td>
<td>-0.1667</td>
</tr>
<tr>
<td>T_{em} (nN-m)</td>
<td>-3848.7</td>
<td>-4871.0</td>
<td>-6013.5</td>
<td>-7276.4</td>
<td>-8185.1</td>
</tr>
<tr>
<td>I_s (amps)</td>
<td>530.5∠-161.8°</td>
<td>670.8∠-161.8°</td>
<td>827.2∠-161.8°</td>
<td>999.7∠-161.8°</td>
<td>1123.6∠-161.8°</td>
</tr>
<tr>
<td>I_r (amps)</td>
<td>523.4∠-6.8°</td>
<td>657.4∠-1.4°</td>
<td>811.1∠2.4°</td>
<td>983.5∠5.2°</td>
<td>1108.3∠6.7°</td>
</tr>
<tr>
<td>V_r (volts)</td>
<td>80.6∠6.9°</td>
<td>41.2∠8.6°</td>
<td>2.1∠2.4°</td>
<td>36.6∠-165.8°</td>
<td>62.3∠-164.2°</td>
</tr>
<tr>
<td>R_{eq} (ohms)</td>
<td>-0.1498</td>
<td>-0.0616</td>
<td>-0.0026</td>
<td>0.0368</td>
<td>0.0555</td>
</tr>
<tr>
<td>X_{eq} (ohms)</td>
<td>-0.0363</td>
<td>-0.0109</td>
<td>0</td>
<td>0.0059</td>
<td>0.0089</td>
</tr>
</tbody>
</table>

(3) Repeat (1) except use approximate expression to obtain \( I_s \).

- **Slip**
  \[
  s = \frac{n_s - n_m}{n_s}
  \]

- **\( T_{em} \) (kN-m)**
  \[
  T_{em} = -8185.1 \left( \frac{n_m}{1750} \right)^2
  \]

- **\( I_s \) (use exact expression, i.e., with R_s)**
  \[
  I_s = \frac{\omega_s T_{em}}{3pV_s \cos \phi}
  \]

Then, assuming that \( V_s \) is the reference (and has angle of 0 degrees), the phasor \( I_s \) is given by
\[
I_s = I_s \angle 180 - \cos^{-1}(0.95) = I_s \angle 161.8°
\]

- **\( I_r \) (amps)**
  \[
  V_m = V_s - I_r (R_s + j\omega_s L_{m\omega})
  \]
  \[
  I_m = \frac{V_m}{j\omega_s L_m}
  \]
  \[
  I_r = I_m - I_s = \frac{V_s - I_s (R_s + j\omega_s L_{m\omega}) - I_s}{j\omega_s L_m}
  \]

- **\( V_r \) (volts)**
  \[
  \frac{V_r}{s} = V_m + I_r \left( \frac{R_s}{s} + j\omega_s L_{m\omega} \right)
  \]
  \[
  = V_s - I_s (R_s + j\omega_s L_{m\omega}) + I_r \left( \frac{R_s}{s} + j\omega_s L_{m\omega} \right)
  \]

- **\( Z_{eq} \) (ohms)**
  \[
  Z_{eq} = R_{eq} + js\omega_s L_{eq} = \frac{V_r}{-I_r} = \left( \frac{sV_m + I_r (R_s + js\omega_s L_{m\omega})}{I_r} \right)
  \]

- **\( R_{eq} \) (ohms)**
  \( R_{eq} \) is the real part of \( Z_{eq} \)

- **\( X_{eq} \) (ohms)**
  \( X_{eq} \) is the imaginary part of \( Z_{eq} \)
Applying these expressions (see matlab code at end of this document), we obtain the following for the 0.95 leading condition. Comparing to the solutions obtained in (1), we observe that the approximate evaluation of stator current magnitude $I_s$ seems to incur little error.

<table>
<thead>
<tr>
<th>$n_m$</th>
<th>1200</th>
<th>1350</th>
<th>1500</th>
<th>1650</th>
<th>1750</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$ (slip)</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
<td>-0.1</td>
<td>-0.1667</td>
</tr>
<tr>
<td>$T_{em}$ (ntn-m)</td>
<td>-3848.7</td>
<td>-4871.0</td>
<td>-6013.5</td>
<td>-7276.4</td>
<td>-8185.1</td>
</tr>
<tr>
<td>$I_p$ (amps)</td>
<td>$532.5 \angle 161.8^\circ$</td>
<td>$673.9 \angle 161.8^\circ$</td>
<td>$832.0 \angle 161.8^\circ$</td>
<td>$1006.7 \angle 161.8^\circ$</td>
<td>$1132.5 \angle 161.8^\circ$</td>
</tr>
<tr>
<td>$I_r$ (amps)</td>
<td>$659.3 \angle -37.8^\circ$</td>
<td>$798.3 \angle -34.3^\circ$</td>
<td>$956.0 \angle -31.6^\circ$</td>
<td>$1132.0 \angle -29.5^\circ$</td>
<td>$1259.4 \angle -28.3^\circ$</td>
</tr>
<tr>
<td>$V_r$ (volts)</td>
<td>$86.9 \angle 5.67^\circ$</td>
<td>$45.0 \angle 6.28^\circ$</td>
<td>$2.5 \angle -31.6^\circ$</td>
<td>$42.84 \angle 165.7^\circ$</td>
<td>$73.7 \angle 165.3^\circ$</td>
</tr>
<tr>
<td>$R_{eq}$ (ohms)</td>
<td>-0.0957</td>
<td>-0.0428</td>
<td>-0.0026</td>
<td>0.0273</td>
<td>0.0428</td>
</tr>
<tr>
<td>$X_{eq}$ (ohms)</td>
<td>-0.0906</td>
<td>-0.0367</td>
<td>0</td>
<td>0.0262</td>
<td>0.0399</td>
</tr>
</tbody>
</table>

MATLAB CODE FOR CALCULATIONS

Note that:
- one must change “phi” to set the right power factor and to indicate whether it is leading or lagging;
- one must change “n” to set the speed;
- one must remove the “%” from the code to use the exact evaluation of $I_s$ (and then add the “%” to the code for the approximate relation).

```
Vsll=690;
Rs=0.00265;
Rr=0.00263;
Lls=0.0001687;
Llr=0.0001337;
Lm=0.0054749;
phi=161.8*pi/180;
pp=2;
Vs=Vsll/sqrt(3);
n=1200;
omega_m=pp*n*2*pi/60;
omega_s=2*pi*50;
Tem=-8185.1*(n/1750)^2;
s=(omega_s-omega_m)/omega_s

%EXACT EVALUATION OF Is
%Isroot=sqrt((Vs*cos(phi))^2-4*Rs*Tem*omega_s/(3*pp));
%Isplus=(Vs*cos(phi)+Isroot)/(2*Rs);
%Isminus=(Vs*cos(phi)-Isroot)/(2*Rs);

%APPROXIMATE EVALUATION OF Is
Isplus=omega_s*Tem/(3*pp*Vs*cos(phi));
Is=abs(Isplus)*(cos(phi)+i*sin(phi));
Vm=Vs-Is*(Rs+i*omega_s*Lls);
Im=Vm/(i*omega_s*Lm);
Ir=Im-Is;
Irmag=abs(Ir)
Irangle=atan2(imag(Ir), real(Ir))*180/pi
Vr=s*Vm+Ir*(Rr+i*s*omega_s*Llr);
Vrmag=abs(Vr)
Vrangle=atan2(imag(Vr), real(Vr))*180/pi
```
\[ Z_{eq} = \frac{V_r}{(-1 \cdot I_r)} \]