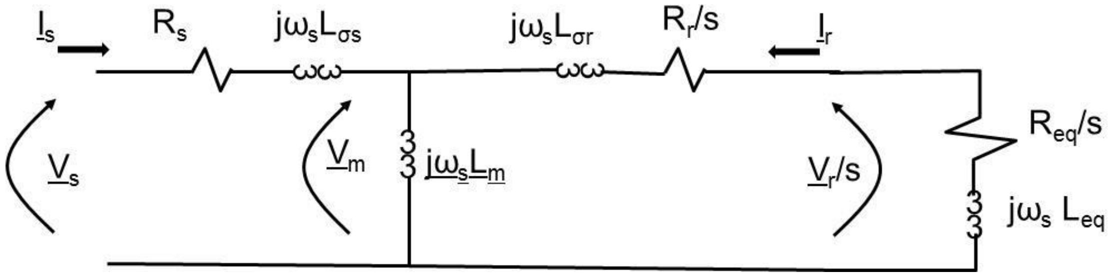
NAME: ____

EE 459/559, Exam 1, Fall 2016, Dr. McCalley, 75 minutes allowed (unless otherwise directed)
Closed Book, Closed Notes, Calculator Permitted, No Communication Devices.

The following information may or may not be useful for some parts of this exam.



$$\underline{I}_s = \frac{\lambda_s - L_m \underline{I}_r}{L_s}; \qquad \underline{I}_r = \frac{\lambda_s - L_s \underline{I}_s}{L_m}; \qquad \underline{I}_r = \frac{\lambda_r - L_m \underline{I}_s}{L_r}; \qquad \underline{I}_s = \frac{\lambda_r - L_r \underline{I}_r}{L_m}$$

$$\omega_s = \omega_m + \omega_r$$

1. (30 pts) A 1.0 MW, 575 v (line-to-line), 60 Hz 2160 rpm DFIG is used in a wind energy conversion system. The parameters of the generator are given below. The generator operates with an MPPT scheme.

Rated Rotor Speed	2160 rpm
Nominal Rotor Speed Range	1350–2160 rpm
Rated Slip	–0.2
Number of Pole Pairs	2
Rated Mechanical Torque	4.421 kN·m
Stator Winding Resistance, R_s	3.654 mΩ
Rotor Winding Resistance, R_r	3.569 mΩ
Stator Leakage Inductance, L_{ls}	0.1304 mH
Rotor Leakage Inductance, L_{lr}	0.1198 mH
Magnetizing Inductance, L_m	4.12 mH

Assuming the a-phase line-to-neutral stator voltage is the reference, determine the following when the DFIG operates in the generator mode at a speed of 2160 rpm, a torque of T_{em}=4421ntn-m, a stator power factor of 0.95 leading, and a stator current magnitude of $|I_s|$=874.38 amperes.

a. Compute the synchronous speed in electrical rad/sec ω_s and in rpm n_s.
<u>Solution</u>:
This is a 60 Hz machine; therefore, ω_s=2*π*60=376.99 rad/sec.
The rpm is given by
$$n_s = \left(\frac{\omega_s}{p}\frac{\text{rad}}{\text{sec}}\right)\frac{\text{rev}}{2\pi\text{rad}}\frac{60\,\text{sec}}{\text{min}} = \frac{376.99}{2}\frac{60}{2\pi} = 1800 rpm$$

b. Compute the slip, and identify whether this machine is operating sub-synchronously or super-synchronously.
<u>Solution</u>:
$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 2160}{1800} = -0.2$$
The slip is negative; therefore, the machine is operating in the <u>super-synchronous</u> mode.

c. Compute the frequency, in electrical rad/sec, of the rotor currents:
Solution:
$\omega_r = s\omega_s = -0.2*376.99 = -75.4$ rad/sec.

d. What is the speed of rotation of the rotor, in electrical rad/sec?
Solution:
One way to get it is to convert from rpm, as follows:

$$n_m = \left(\frac{\omega_m}{p}\frac{\text{rad}}{\sec}\right)\frac{\text{rev}}{2\pi\text{rad}}\frac{60\sec}{\min} = \frac{\omega_m}{p}\frac{60}{2\pi} \Rightarrow \omega_m = \frac{2\pi n_m p}{60} = \frac{2\pi(2160)2}{60} = 452.39 rad/\sec$$

Another way to do it, given we know ω_r, is to use the equation at the top of the exam:

$$\omega_s = \omega_m + \omega_r \Rightarrow \omega_m = \omega_s - \omega_r = 376.99 - (-75.4) = 452.39 rad/\sec$$

e. What is the speed of rotation of the rotating magnetic field from the rotor, relative to the rotor, in electrical rad/sec?
Solution:
-75.4 rad/sec

f. What is the speed of rotation of the rotating magnetic field from the rotor, relative to the (fixed) stator, in electrical rad/sec?
Solution:
To have a constant torque, the speed of rotation of the magnetic field from the rotor and the speed of rotation of the magnetic field from the stator, both given relative to the (fixed) stator, must be the same. Therefore, the answer is 376.99 rad/sec.
One could also get this by reasoning that the speed of rotation of the rotating magnetic field from the rotor, relative to the (fixed) stator, is given by the speed of rotation of the rotor plus the speed of rotation of the magnetic field from the rotor relative to the rotor, that is:

$$\text{Speed of rotating mag field from rotor relative to stator}$$
$$= \omega_m + \omega_r = 452.39 + (-75.4) = 376.99 rad/\sec$$

g. Give the a-phase line-to-neutral stator voltage phasor (magnitude and angle):
Solution:
$|V_s|$ =575/sqrt(3)=331.98
➔V_s=331.98/_0°.

h. Give the stator current phasor (magnitude and angle):
Solution:
$|I_s|$=874.38 amperes.
pf=0.95 leading➔ acos(0.95)=18.2°. However, it is operating in the generator mode and so, with V_s as reference, generator mode with leading pf requires that I_s lie in the second quadrant. And so θ_i=180-18.2=161.8°.
Answer is therefore $|I_s|$=874.38 /_161.8° amperes.

i. Compute the magnetizing branch voltage phasor $\underline{V}_m$ (magnitude and angle).
Solution:
From the circuit:

$$\underline{V}_m = \underline{V}_s - \underline{I}_s\left(R_s + j\omega_s L_{\sigma s}\right) =$$
$$331.98 - 874.38\angle 161.8(0.003654 + j(376.99)(0.0001304))$$
$$= 349.91\angle 6.58°$$

j. The rotor current phasor $\underline{I}_r$ (magnitude and angle)
Solution:
From the circuit:

$$\underline{I}_r = \underline{I}_m - \underline{I}_s = \frac{\underline{V}_m}{j\omega_s L_m} - \underline{I}_s$$
$$= \frac{349.91\angle 6.58°}{j(376.99)(0.00412)} - 874.38\angle 161.8$$
$$= 990.17\angle -30.`12°$$

k. The rotor voltage phasor $\underline{V}_r$ (magnitude and angle)
Solution:
From the circuit:
$$\underline{V}_r = s\underline{V}_m + \underline{I}_r\left(R_r + js\omega_s L_{\sigma r}\right) =$$
$$-0.2(349.91\angle 6.58°) + 990.17\angle -30.`12° + j(-0.2)(376.99)(0.0001198)$$
$$= 73.03\angle -166.1°$$

2. (14 pts) Given $T_{em}=3pL_m \times \text{Imag}\{\underline{I}_s\, \underline{I}_r^*\}$, derive the following relation for torque:
$$T_{em} = 3p\,\text{Im}\left\{\underline{\lambda}_s^* \underline{I}_s\right\}$$
Solution:
Using $\underline{I}_r = \frac{\underline{\lambda}_s - L_s \underline{I}_s}{L_m}$;
we have that
$$T_{em} = 3pL_m\,\text{Im}\left\{\underline{I}_s\left(\frac{\underline{\lambda}_s - L_s\underline{I}_s}{L_m}\right)^*\right\} = 3p\frac{L_m}{L_m}\text{Im}\left\{\underline{I}_s\left(\underline{\lambda}_s - L_s\underline{I}_s\right)^*\right\} = 3p\,\text{Im}\left\{\underline{I}_s\underline{\lambda}_s^* - L_s\underline{I}_s\underline{I}_s^*\right\}$$
But note that $L_sI_sI_s^*$ is entirely real, and so since we are taking the imaginary part of what is inside the brackets, the second term can be eliminated. Therefore:
$$T_{em} = 3p\,\text{Im}\left\{\underline{\lambda}_s^* \underline{I}_s\right\}$$

3. (21 pts) The synchronous speed of a 60 Hz double-fed induction generator for a wind turbine is 1800 RPM. While operating at a speed of 2100 RPM, it sees a mechanical input power of 1 MW from the drive train.
   a. Compute the slip.
   Solution:
   $$s = \frac{n_s - n_m}{n_s} = \frac{1800-2100}{1800} = -0.1667$$
   b. Ignoring losses, compute the power through the stator and the power through the rotor.
   Solution:
   We assume positive directionality for P_s and P_r as into the machine and positive directionality for P_{mech} as out of the machine. Therefore:
   $$P_s = \frac{P_{mech}}{1-s} = \frac{-1}{1-(-0.1667)} = -0.8571\,\text{MW}$$
   $$P_r = -sP_s = -(-0.8571)(-0.1667) = -0.1429$$
   c. Identify whether the direction of flow of rotor power is into the rotor from the grid or into the grid from the rotor.
   Solution:
   From the last calculation, assuming positive directionality for P_r as into the machine, we see

that the direction of flow of rotor power must be into the grid from the rotor.
It is actually unnecessary to keep track of the signs of the various quantities (although it is helpful) because we know that the machine is operating in the supersynchronous mode, and in this case, the rotor is ALWAYS delivering power to the grid.

4. (15 pts): In the class notes, we developed the following relations:

$$\underline{I}_s = \frac{1}{\sigma L_s}\underline{\lambda}_s - \frac{L_m}{\sigma L_s L_r}\underline{\lambda}_r, \qquad \sigma = 1 - \frac{L_m^2}{L_s L_r}, \qquad \underline{V}_s \approx j\omega_s \underline{\lambda}_s, \qquad \underline{I}_s = \frac{Q_s}{3V_s} + j\frac{\omega_s T_{em}}{3pV_s}$$

Assuming $\underline{\lambda}_s$ as the reference, show that

$$\underline{\lambda}_r = \left[\frac{V_s}{\omega_s}\frac{L_r}{L_m} - \frac{Q_s}{3V_s}\frac{\sigma L_s L_r}{L_m}\right] - j\left[\frac{\omega_s T_{em}}{3pV_s}\frac{\sigma L_s L_r}{L_m}\right]$$

Solution:

Equate the two expressions for $\underline{I}_s$:

$$\underline{I}_s = \frac{Q_s}{3V_s} + j\frac{\omega_s T_{em}}{3pV_s} = \frac{1}{\sigma L_s}\underline{\lambda}_s - \frac{L_m}{\sigma L_s L_r}\underline{\lambda}_r \qquad \text{(i)}$$

Express $\underline{\lambda}_s$ as a function of $\underline{V}_s$:

$$\underline{\lambda}_s = \frac{\underline{V}_s}{j\omega_s}$$

But given that $\underline{\lambda}_s$ is the reference, we obtain

$$\underline{\lambda}_s = \frac{V_s}{\omega_s}\angle 0° = \frac{V_s}{\omega_s} \qquad \text{(ii)}$$

Substitute (ii) into (i):

$$\frac{Q_s}{3V_s} + j\frac{\omega_s T_{em}}{3pV_s} = \frac{1}{\sigma L_s}\frac{V_s}{\omega_s} - \frac{L_m}{\sigma L_s L_r}\underline{\lambda}_r$$

Solve for the term in $\underline{\lambda}_r$:

$$\frac{L_m}{\sigma L_s L_r}\underline{\lambda}_r = \frac{1}{\sigma L_s}\frac{V_s}{\omega_s} - \frac{Q_s}{3V_s} - j\frac{\omega_s T_{em}}{3pV_s}$$

Divide through by the coefficient of $\underline{\lambda}_r$:

$$\underline{\lambda}_r = \frac{\sigma L_s L_r}{\sigma L_m L_s}\frac{V_s}{\omega_s} - \frac{\sigma L_s L_r}{L_m}\frac{Q_s}{3V_s} - j\frac{\sigma L_s L_r}{L_m}\frac{\omega_s T_{em}}{3pV_s}$$

Simplifying:

$$\underline{\lambda}_r = \frac{L_r}{L_m}\frac{V_s}{\omega_s} - \frac{\sigma L_s L_r}{L_m}\frac{Q_s}{3V_s} - j\frac{\sigma L_s L_r}{L_m}\frac{\omega_s T_{em}}{3pV_s} \qquad \text{(iii)}$$

The above answers the exam question, and no more work is needed or expected. But just to complete the development, we also obtain an expression for $\underline{I}_r$, confirming relations on slide 22 of DFIG set 2.

From the flux-linkage equations, we may derive

$$\underline{I}_r = \frac{-L_m}{\sigma L_s L_r}\underline{\lambda}_s + \frac{1}{\sigma L_r}\underline{\lambda}_r \qquad \text{(iv)}$$

Substituting (ii) and (iii) into (iv), we obtain:

$$\underline{I}_r = \frac{-L_m}{\sigma L_s L_r}\frac{V_s}{\omega_s} + \frac{1}{\sigma L_r}\left[\frac{L_r}{L_m}\frac{V_s}{\omega_s} - \frac{\sigma L_s L_r}{L_m}\frac{Q_s}{3V_s} - j\frac{\sigma L_s L_r}{L_m}\frac{\omega_s T_{em}}{3pV_s}\right]$$

Distributing the $1/\sigma L_r$:

$$\underline{I}_r = \frac{-L_m}{\sigma L_s L_r}\frac{V_s}{\omega_s} + \left[\frac{1}{\sigma L_m}\frac{V_s}{\omega_s} - \frac{\sigma L_s L_r}{\sigma L_m L_r}\frac{Q_s}{3V_s} - j\frac{\sigma L_s L_r}{\sigma L_m L_r}\frac{\omega_s T_{em}}{3pV_s}\right]$$

Simplifying within the brackets:

$$\underline{I}_r = \frac{-L_m}{\sigma L_s L_r}\frac{V_s}{\omega_s} + \frac{1}{\sigma L_m}\frac{V_s}{\omega_s} - \frac{L_s}{L_m}\frac{Q_s}{3V_s} - j\frac{L_s}{L_m}\frac{\omega_s T_{em}}{3pV_s}$$

Factoring $V_s/L_m\sigma\omega_s$ from the first two terms:

$$\underline{I}_r = \frac{V_s}{L_m\sigma\omega_s}\left[\frac{-L_m^2}{L_s L_r} + 1\right] - \frac{L_s}{L_m}\frac{Q_s}{3V_s} - j\frac{L_s}{L_m}\frac{\omega_s T_{em}}{3pV_s}$$

Noting that the term in the brackets is σ, we have:

$$\underline{I}_r = \frac{V_s}{L_m\omega_s} - \frac{L_s}{L_m}\frac{Q_s}{3V_s} - j\frac{L_s}{L_m}\frac{\omega_s T_{em}}{3pV_s}$$

5. (20 pts) True/false:

a.	F
b.	T
c.	T
d.	F
e.	F
f.	T
g.	F
h.	T
i.	T
j.	F

_____a. The electric generation technology with the largest share of new capacity (MW) built in the US during the 2015 year was solar-photovoltaic.

_____b. For any US wind plant built in 2016, the federal production tax credit provides an additional $22 for every MWhr produced over the first 10 years of the plant's life.

_____c. The unsubsidized levelized cost of energy for wind plants is among the lowest of all electric generating technologies in 2016 mainly because, relative to other technologies, (i) it has very low operations & maintenance cost (fixed and variable, including fuel), and (ii) its investment (capital) cost has steadily declined over the past few years.

_____d. LCOE is the right indicator for deciding whether to build a wind plant or not.

_____e. The main reason why most wind turbines are built with a vertical axis is that vertical axis wind turbines (VAWTs) have a higher wind energy conversion efficiency than horizontal axis wind turbines (HAWTs).

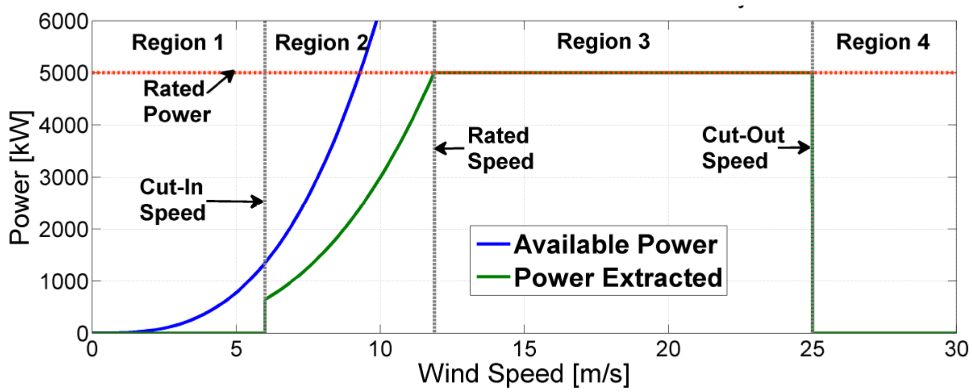
_____f. The video of the wind energy laboratory included illustration and discussion of a table-top wind turbine, a load box, the gear box, the pitch motors, the hub assembly, the shaft and bearings, several different types of generators, and data acquisition equipment.

_____g. The Betz limit requires that the power of the wind downstream from a wind-turbine must be less than 59.26% of the power of the wind upstream from the turbine.

_____h. A type 1 generator, i.e., a squirrel-cage induction generator, must operate super-synchronously when in generator mode; it has almost constant speed; it has a wide torque range and so wind gusts are passed on directly as varying power injections to the grid.

_____i. When a DFIG generates power in the sub-synchronously mode, the shaft must receive power from the rotor.

_____j. The paper titled "A tutorial of wind turbine control for supporting grid frequency through active power control," provides the following description: "*In standard Region 3 control, blade pitch is typically held constant at the value β^* that produces the peak C_p. The goal is then to maintain the TSR at the optimal level λ^*; hence, the tip-speed, and therefore rotor speed, must vary proportionally to the wind speed. This is achieved by varying the generator torque*."


Region 1
Region 2
Region 3
Region 4
Rated Power
Cut-In Speed
Rated Speed
Cut-Out Speed
Available Power
Power Extracted
Power [kW]
Wind Speed [m/s]
0
1000
2000
3000
4000
5000
6000
0
5
10
15
20
25
30