1. (36 pts) Consider a 1.5 MW, 690 v, 50 Hz, 1750 rpm DFIG wind energy system. The parameters of the generator are given in the table to the right. The generator operates with a maximum power point tracking (MPPT) system so that its mechanical torque $T_{em}$ is proportional to the square of the rotor speed. The stator power factor is unity. For a rotor speed of 1750 rpm, determine:

   a. Slip
   b. $T_{em}$ (kN-m)
   c. $V_s$ (volts)
   d. $I_s$ (amps)
   e. $V_m$ (volts)
   f. $I_m$ (amps)
   g. $I_r$ (amps)
   h. $V_r$ (volts)
   i. $Z_{eq}$ (ohms)

You are provided with the following relationship and circuit:

$$ V_s = \frac{\sqrt{V_s^2 - \frac{4R_s \omega_s}{3p} T_{em}}}{2R_s} $$

**Solution:***

a. Slip

$$ \omega_s = 2\pi f = 2\pi(50) = 314.1592 \text{rad/sec} $$

$$ \Omega_m = \frac{1750 \text{rev}}{60 \text{sec}} \text{min} = 183.26 \text{rad/sec} $$

$$ \omega_m = p\Omega_m = 2*183.26 = 366.52 \text{rad/sec} $$

$$ s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{314.1592 - 366.52}{314.1592} = -0.1667 $$

b. $T_{em}$ (kN-m)

The “rated mechanical torque” in the table is for a speed of 1750rpm which is the speed given in this problem. So answer is 8.1851 kNm.

c. $V_s$ (volts)

$$ V_s = \frac{690}{\sqrt{3}} = 398.37 $$

d. $I_s$ (amps)
\[ V_s \pm \sqrt{V_s^2 - \frac{4R_s\omega_l}{3p} T_{em}} = \frac{398.37 \pm \sqrt{398.37^2 - \frac{4(0.00265)(314.1592)}{3(2)}}}{2(0.00265)} (-8185.1) \]
\[ I_s = -1068.2 \text{amps} \]

\[ V_m = V_s - I_s \left( R_s + j\omega_L L_{ss} \right) \]
\[ = 401.2 + j56.61 \]

\[ I_m = \frac{V_m}{jX_m} = \frac{401.2 + j56.61}{j(0.0054749)(314.1592)} = 32.9 - j233.2 \]

\[ I_r = I_m - I_s = (32.9 - j233.2) - (-1068.2) = 1101.1 - j233.2 \]

\[ V_r = sV_m - I_r \left( R_s + j\omega_L L_{sr} \right) = -0.1667(401.2 + j56.61) - (1101.1 - j233.34)(0.00263 + j(-0.1667)(314.1592)(0.0001337) \]
\[ = -65.62 - j17.76 \]

\[ Z_{eq} = \frac{V_r}{-I_r} = \frac{-65.62 - j17.76}{1101.1 - j233.2} = 0.0538 + j0.0275 \]

2. (22 pts) We developed the below qd0 voltage equations for the double-fed induction generator.

\[
\begin{bmatrix}
    v_{qs} \\
    v_{ds} \\
    v_{os} \\
    v_{qr} \\
    v_{dr} \\
    v_{or}
\end{bmatrix} = \begin{bmatrix}
    r_s & 0 & 0 & 0 & 0 & 0 \\
    0 & r_s & 0 & 0 & 0 & 0 \\
    0 & 0 & r_s & 0 & 0 & 0 \\
    0 & 0 & 0 & r_r & 0 & 0 \\
    0 & 0 & 0 & 0 & r_r & 0 \\
    0 & 0 & 0 & 0 & 0 & r_r
\end{bmatrix} \begin{bmatrix}
    i_{qs} \\
    i_{ds} \\
    i_{os} \\
    i_{qr} \\
    i_{dr} \\
    i_{or}
\end{bmatrix} + \begin{bmatrix}
    0 & -\omega & 0 & 0 & 0 & 0 \\
    \omega & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & -(\omega - \omega_m) & 0 & \lambda_{qr} \\
    0 & 0 & 0 & \omega - \omega_m & 0 & \lambda_{dr} \\
    0 & 0 & 0 & 0 & 0 & \lambda_{qr}
\end{bmatrix}
\]

We also developed the following torque expression.

\[ T_{em} = \frac{3}{2} \frac{L_m}{L_{sr} + L_{sr}} p(\lambda_{qr} i_{dr} - \lambda_{ds} i_{qr}) \]

Using the above relations together with the implications of stator-voltage-oriented-control (SVOC), show that \( T_{em} \) is a function of d-axis rotor current and d-axis stator voltage only. Clearly state how SVOC plays a role in this development. State any other assumptions that are necessary.

**Solution:**

Extracting the relations for \( v_{qs} \) and \( v_{ds} \), we have:

\[ v_{qs} = r_s i_{qs} + \omega \lambda_{ds} \]
\[ v_{ds} = r_s i_{ds} - \omega \lambda_{qs} \]

Solving for \( \lambda_{ds} \) and \( \lambda_{qs} \), respectively, we get
\[ \lambda_{ds} = \frac{v_{qs} - r_{i}i_{qs}}{\omega} \quad \lambda_{qs} = \frac{-(v_{ds} - r_{i}i_{ds})}{\omega} \]

Now substitute these relations for \( \lambda_{ds} \) and \( \lambda_{qs} \) into the torque relation:

\[
T_{em} = \frac{3}{2} \frac{L_{m}}{L_{cr} + L_{m}} p(\lambda_{qs}i_{dr} - \lambda_{ds}i_{qr}) = \frac{3}{2} \frac{L_{m}}{L_{cr} + L_{m}} p\left(\frac{-(v_{ds} - r_{i}i_{ds})}{\omega}i_{dr} - \frac{v_{qs} - r_{i}i_{qs}}{\omega}i_{qr}\right)
\]

Now use SVOC which implies that \( v_{qs} = 0 \), and we obtain

\[
T_{em} = \frac{3}{2} \frac{L_{m}}{2\omega L_{cr} + L_{m}} p\left(-v_{ds}i_{dr} + r_{i}i_{ds}i_{dr} + r_{i}i_{qs}i_{qr}\right)
\]

Assume that \( r_{i} = 0 \), then

\[
T_{em} = -\frac{3}{2} \frac{L_{m}pV_{ds}i_{dr}}{2\omega L_{cr} + L_{m}}
\]

3. (24 pts) Consider a two-level voltage source inverter connected to a three-phase balanced RL load, and its DC voltage is 1220 volts. The inverter is modulated by space vector modulation and its simulation algorithm is given in the figure below. The switching frequency of the converter is 720Hz.

At a given instance of time, the three-phase reference voltages \( v_{a}^{*}, v_{b}^{*}, \) and \( v_{c}^{*} \), are found to be 0 volts, 487.9 volts, and -487.9 volts, respectively. For this problem, you may need the matrix and the equations provided below.

\[
\begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
\frac{\sqrt{3}}{2} & -\frac{1}{2} & -\frac{1}{2} \\
0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2}
\end{bmatrix}
\]

\[
T_a = \frac{\sqrt{3}v_{ref} T_S}{V_{DC}} \sin \left(\frac{\pi}{3} - \theta'\right)
\]

\[
T_b = \frac{\sqrt{3}v_{ref} T_S}{V_{DC}} \sin \theta'
\]

Determine:

a. The \( \alpha-\beta \) components of the three-phase reference voltages.
b. The specific switching sequence necessary to achieve the reference vector corresponding to your answer in (a). Here, you need to specify a sequence of particular states.
c. The 3 stationary vectors to be used in synthesizing the reference vector. Here, you need to identify appropriate vectors \( V_k \) in the space vector diagram shown below (no computation is required).
d. The dwell times necessary to achieve the reference vector.
Solution:

a. The α-β components of the three-phase reference voltages.
\[
\begin{bmatrix}
  v_a \\
  v_\beta
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix}
\begin{bmatrix}
  v_a \\
  v_\beta
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
  1 & -1/2 & -1/2 \\
  0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix} \begin{bmatrix}
  0 \\
  -487.9
\end{bmatrix} = \begin{bmatrix}
  0 \\
  563.38
\end{bmatrix}
\]

b. The angle of the reference voltage vector & its corresponding sector number is given by
\[
\theta = \tan^{-1} \frac{v_\beta}{v_a} = \tan^{-1} \frac{-487.9}{0} = 90^\circ
\]
It will be in sector II, therefore the switching sequence is V₀, V₃, V₂, V₀, V₂, V₁, V₀

c. The three stationary vectors to be used in synthesizing the reference vector are V₂, V₃, and V₀.
d. This is in sector II, therefore:
\[
\theta' = \theta - (k-1) \frac{\pi}{3} \quad \text{for } 0 \leq \theta' < \frac{\pi}{3}
\]
\[
\theta' = \theta - (2-1) \frac{\pi}{3} = \theta - \frac{\pi}{3}
\]
So the reference vector, with θ=90deg, will have θ'=45deg.
\[
f_{sw} = 720Hz \Rightarrow T_s = \frac{1}{720} = 0.001389
\]
\[
T_a = \frac{\sqrt{3} v_{ref} T_s}{V_{DC}} \sin \left( \frac{\pi}{3} - \theta' \right) = \frac{\sqrt{3}(563.38)(0.001389)}{1220} \sin(60-45)
\]
\[
= 0.00028754 \text{sec}
\]
\[
T_b = \frac{\sqrt{3} v_{ref} T_s}{V_{DC}} \sin \theta' = \frac{\sqrt{3} \times 563.38 \times 0.0014}{1220} \sin(45) = 0.0007854
\]
\[
T_0 = T_s - T_a - T_b = 0.001389 - 0.00028754 - 0.0007854 = 0.00031606
\]

4. (18 pts) A 60 Hz power system has 1000 MW of conventional generation. Assume:
- The inertia constant for all conventional generation is H=6 seconds (on the base of the generator);
- A trip of a 100 MVA unit occurs with the unit producing 100 MW.
- The frequency nadir following generation trips occurs 1.5 second after the trip.

a. Estimate the nadir assuming all generation is conventional.
b. Estimate the nadir assuming 200 MW of conventional units are displaced by 500 MW of double-fed induction generators.
c. Give 3 “nested” reasons why it is important to avoid low frequency deviations.

Solution:

a. Total capacity of conventional generation following the trip is 1000MW-100MW=900MW. Using a 100 MVA base, the loss of the 100 MW results in
\[
\Delta P_L = 100/100=1 \text{ pu}
\]
\[
\sum_{i=1}^{n} H_i = \frac{900 \times 6}{100} = 54
\]

\[
m_f = \frac{d\Delta f}{dt} = -\frac{\Delta P_L \cdot f_{Re}}{2 \sum_{i=1}^{n} H_i} = -\frac{1(60)}{2 \times 54} = -0.5556 \text{Hz/sec}
\]

With \( t_1 = 1.5 \) seconds, then \( \Delta f = -0.5556 \times 1.5 = -0.8334 \) Hz, so that the nadir is 60 - 0.8334 = 59.17 Hz.

b. Total capacity of conventional generation following the trip is 800MW - 100MW = 700MW. Using a 100 MVA base, the loss of the 100 MW results in

\[
\Delta P_L = \frac{100}{100} = 1 \text{ pu}
\]

\[
\sum_{i=1}^{n} H_i = \frac{700 \times 6}{100} = 42
\]

\[
m_f = \frac{d\Delta f}{dt} = -\frac{\Delta P_L \cdot f_{Re}}{2 \sum_{i=1}^{n} H_i} = -\frac{1(60)}{2 \times 42} = -0.7143 \text{Hz/sec}
\]

With \( t_1 = 1.5 \) seconds, then \( \Delta f = -0.7143 \times 1.5 = -1.0714 \) Hz, so that the nadir is 60 - 0.7143 = 58.93 Hz.

c. Turbine failure or loss of life; turbine trip via under- or over-frequency relays; under-frequency load shedding.