Representation of Saturation in Stability Studies (Sections 4.12.4, 4.12.3)

Kundur writes (pg 110) that

"A rigorous treatment of synchronous machine performance including saturation effects is a futile exercise. Any practical method of accounting for saturation effects must be based on semi-heuristic reasoning and judiciously chosen approximations, with due consideration to simplicity of model structure, data availability, and accuracy of results."

Some assumptions (see Kundur pg. 112-113) :

- 1. Leakage inductances are independent of saturation since the path of the leakage flux is mainly in the air. Therefore we may confine our analysis of saturation to the mutual inductances, represented by L_{AD} and L_{AQ} .
- 2. The leakage fluxes do not contribute to the iron saturation. This is reasonable because these fluxes are small (since their paths are mainly in air, and air has high permeability), and their paths coincide with that of the main flux for only a small part of its path. So we may determine saturation of the inductances as a function of λ_{AD} and λ_{AQ} .
- 3. The saturation relationship between the resultant air-gap flux and the mmf under loaded conditions is the same as under no-load conditions. This allows the saturation characteristics to be represented by the open-circuit saturation curve, which is usually the only saturation data readily available.

An additional assumption that is sometimes made is that L_{AQ} does not saturate, simply because the quadrature axis flux is usually quite small in comparison to the direct axis flux due to the effect of the main field winding. This assumption is quite good for salient pole machines but not so good for round-rotor machines.

Recall, from our equivalent circuit (shown below), that $\lambda_{AD} = (i_d + i_F + i_D)L_{AD}$.



Direct-axis equivalent circuit: The above is the same as Fig. 4.5 in your text

Define the following terms:

- Magnetization current: $i_M = (i_d + i_F + i_D) \rightarrow \lambda_{AD} = L_{AD} i_M$
- Maximum per-unit flux linkage without saturation: λ_{ADT}
- i_{M0} : current that would produce λ_{AD} if no saturation effects
- i_{MS} : current that produces λ_{AD} with saturation effects
- λ' : Flux linkage resulting from i_{MS} if no saturation effects

Define L_{AD0} as the inductance corresponding to the air-gap line. It is the inductance when i_M is small, i.e., it is the non-saturated inductance. Thus, $\lambda' = L_{AD0}i_{MS}$.

The magnetization curve appears as in the following figure:



From the figure, we can write that: $\frac{i_{M0}}{\lambda_{AD}} = \frac{i_{MS}}{\lambda'} \Longrightarrow \lambda_{AD} = \frac{i_{M0}}{i_{MS}} \lambda'$

But from the air-gap line equation, $\lambda' = L_{AD0}i_{MS}$, and substitution of this relation into the previous one yields:

$$\lambda_{AD} = \frac{\dot{i}_{M0}}{\dot{i}_{MS}} \lambda' = \frac{\dot{i}_{M0}}{\dot{i}_{MS}} L_{AD0} \dot{i}_{MS} \tag{*}$$

Define $K_S=i_{M0}/i_{MS}$. K_S is the fraction of the saturated current necessary to achieve the same flux linkage with no saturation (i.e., on the airgap line). Clearly, $0 < K_S \le 1$, where K_S close to 0 indicates a highly saturated i_{MS} ; K_S close to 1 indicates a non-saturated i_{MS} . So here we need to recognize a very important feature: K_S depends on the saturation level which depends on λ_{AD} . Substitution of K_S into (*) results in:

$$\lambda_{AD} = \frac{i_{M0}}{i_{MS}} \lambda' = K_S L_{AD0} i_{MS}$$

So K_S is a factor that we use to account for the difference between the magnetization curve and the air-gap line. Observation: We are trying to use K_S to compute λ_{AD} , yet K_S depends on λ_{AD} , that is, $\lambda_{AD}=K_S(\lambda_{AD})L_{AD0}i_{MS}$. This is why Kundur wrote, "A rigorous treatment of...saturation effects is a futile exercise" (see p. 1 of these notes). So how might we determine K_S?

Observe:

$$K_{S} = \frac{i_{M0}}{i_{MS}} = \frac{i_{M0}}{i_{M0} + \Delta i_{M}}$$

where $\Delta i_M = i_{MS} - i_{M0}$.

So evaluation of K_S requires evaluation of Δi_M , and our problem is now to get Δi_M .

Note from Fig. 1 below that Δi_M grows exponentially larger with λ_{AD} - λ_{ADT} .



Fig. 1

So we reason that a good approximation to Δi_M is given by $\Delta i_M = A_S e^{B_S (\lambda_{AD} - \lambda_{ADT})}$

So that

$$K_{S} = \frac{i_{M0}}{i_{M0} + A_{S} e^{B_{S}(\lambda_{AD} - \lambda_{ADT})}}$$

Now it is clear from the above that K_S is a function of λ_{AD} , i.e.,

$$K_{S} = K_{S}(\lambda_{AD})$$

So that the mutual flux is given by

$$\lambda_{AD} = K_S(\lambda_{AD}) L_{AD0} i_{MS}$$

So how do we use it?

Assume that we have values for λ_d , λ_F , λ_D , λ_q , λ_Q , and λ_G . Then the steps for including saturation are:

- 1. Usually, two values of saturation are given that allow computation of A_s and B_s (these are usually called S1.0 and S1.2 more on that on p. 8 below and in Section 5.9 of VMAF).
- 2. Use the auxiliary equations to obtain the unsaturated values of λ_{AD} and λ_{AQ} :

$$\lambda_{AD} = \frac{L_{MD}}{l_d} \lambda_d + \frac{L_{MD}}{l_F} \lambda_F + \frac{L_{MD}}{l_D} \lambda_D \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q} \lambda_q + \frac{L_{MQ}}{l_Q} \lambda_Q + \frac{L_{MQ}}{l_G} \lambda_G$$

where

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \qquad \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} + \frac{1}{l_G}\right]$$

- 3. For a salient pole machine, let $\lambda = \lambda_{AD}$. For a round-rotor machine, let $\lambda = \operatorname{sqrt} \{\lambda_{AD}^2 + \lambda_{AQ}^2\}$
- 4. Check if $\lambda > \lambda_{ADT}$. If not, use the above unsaturated values. If so, proceed to step 5.
- 5. Obtain currents from 4.124, shown below:

$$\begin{bmatrix} i_{d} \\ i_{F} \\ i_{D} \\ i_{q} \\ i_{Q} \\ i_{Q} \end{bmatrix} = \begin{bmatrix} \frac{1}{l_{d}} & 0 & 0 & -\frac{1}{l_{d}} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{l_{F}} & 0 & -\frac{1}{l_{d}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{l_{D}} & -\frac{1}{l_{D}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{l_{q}} & 0 & 0 & -\frac{1}{l_{q}} \\ 0 & 0 & 0 & 0 & \frac{1}{l_{Q}} & 0 & -\frac{1}{l_{Q}} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{l_{Q}} & -\frac{1}{l_{Q}} \end{bmatrix} \begin{bmatrix} \lambda_{d} \\ \lambda_{d} \\ \lambda_{D} \\ \lambda_{d} \\ \lambda_{Q} \\ \lambda_{dQ} \end{bmatrix}$$
(4.124)

6. Compute the magnetizing current as

$$i_{M0} = i_d + i_F + i_D$$

7. Compute K_s according to:

$$\Delta i_M = A_S e^{B_S (\lambda - \lambda_{ADT})}$$
$$i_{MS} = i_{M0} + \Delta i_M$$
$$K_S = \frac{i_{M0}}{i_{MS}}$$

- 8. Update λ_{AD} and λ_{AQ} according to
 - a. Replace L_{AD} with $L_{AD} \leftarrow K_S L_{AD}$, and then compute:

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \twoheadrightarrow \lambda_{AD} = \frac{L_{MD}}{l_d} \lambda_d + \frac{L_{MD}}{l_F} \lambda_F + \frac{L_{MD}}{l_D} \lambda_D$$

b. If salient pole, then $\lambda_{AQ} = \lambda_{AQ}$ (i.e., no change), but if roundrotor, then replace L_{AQ} with $L_{AQ} \leftarrow K_S L_{AQ}$, and then compute:

$$\frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} + \frac{1}{l_G}\right] \twoheadrightarrow \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q + \frac{L_{MQ}}{l_G}\lambda_G$$

And then you can use the updated values of λ_{AD} and λ_{AQ} in the following to perform a numerical integration and get the next time step...

$$\dot{\lambda}_{d} = -\frac{r}{l_{d}}\lambda_{d} + \frac{r}{l_{d}}\lambda_{AD} - \omega\lambda_{q} - v_{d}$$
(4.126)

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F$$
(4.128)

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD}$$
(4.129)

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q$$
(4.130)

$$\dot{\lambda}_{Q} = -\frac{r_{Q}}{l_{Q}}\lambda_{Q} + \frac{r_{Q}}{l_{Q}}\lambda_{AQ}$$
(4.131a)

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ}$$
(4.131b)

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j} \lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j} \lambda_q\right] + \left[\frac{-D}{\tau_j}\right] \omega \qquad (4.133)$$
$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

Relation to input data to most commercial stability programs:

The input requirements for characterizing generator saturation for most commercial-grade stability programs are in terms of a parameter called S, defined by

$$S = \frac{\Delta i_M}{i_{M0}}$$

where $\Delta i_M = i_{MS} - i_{M0}$ as before.

Recall that

$$K_{S} = \frac{i_{M0}}{i_{MS}} = \frac{i_{M0}}{i_{M0} + \Delta i_{M}}$$

The relation between S and K_S is derived from the below:

$$S = \frac{\Delta i_M}{i_{M0}} \Longrightarrow S + 1 = \frac{\Delta i_M}{i_{M0}} + \frac{i_{M0}}{i_{M0}} = \frac{i_{M0} + \Delta i_M}{i_{M0}}$$
$$\Longrightarrow \frac{1}{S+1} = \frac{i_{M0}}{i_{M0} + \Delta i_M} = K_S \Longrightarrow S = \frac{1 - K_S}{K_S}$$

The specific data entry into most programs (including PSS/E) is

• S(1.0): value of S when open circuit terminal voltage is 1.0pu

• S(1.2): value of S when open circuit terminal voltage is 1.2pu Note that S(1.2) should always be larger than S(1.0). In the Diablo Canyon data, S(1.0) is 0.0769 and S(1.2) is 0.41. The corresponding values of K_S are 0.9286 and 0.7092, respectively. Use of S(1.0) and S(1.2) to compute A_S and B_S is provided in Section 5.9.1.

Final note on saturation. Section 4.12.3 develops a model where saturation is neglected. Such a model is useful for linearized analysis (although so is the current state-space model).

The approach for developing the flux-linkage model without saturation, as in Section 4.12.3) is simple – just substitute the auxiliary equations, i.e., the expressions for λ_{AD} and λ_{AQ} , i.e.,

$$\begin{split} \lambda_{AD} &= \frac{L_{MD}}{l_d} \lambda_d + \frac{L_{MD}}{l_F} \lambda_F + \frac{L_{MD}}{l_D} \lambda_D \\ \lambda_{AQ} &= \frac{L_{MQ}}{l_q} \lambda_q + \frac{L_{MQ}}{l_Q} \lambda_Q + \frac{L_{MQ}}{l_G} \lambda_G \end{split}$$

into the state equations:

$$\dot{\lambda}_{d} = -\frac{r}{l_{d}}\lambda_{d} + \frac{r}{l_{d}}\lambda_{AD} - \omega\lambda_{q} - v_{d}$$
(4.126)

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \tag{4.128}$$

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD}$$
(4.129)

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ}$$
(4.131a)

$$\dot{\lambda}_{Q} = -\frac{r_{Q}}{l_{Q}}\lambda_{Q} + \frac{r_{Q}}{l_{Q}}\lambda_{AQ}$$
(4.131b)
$$T \quad \begin{bmatrix} \lambda_{AQ} & \lambda_{AQ} \\ & & \end{bmatrix} \begin{bmatrix} -D \end{bmatrix}$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3 \tau_j} \lambda_d - \frac{\lambda_{AD}}{l_d 3 \tau_j} \lambda_q \right] + \left[\frac{-D}{\tau_j} \right] \omega \qquad (4.133)$$
$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

VMAF does this for the d-axis equations (λ_d , λ_F , and λ_D) on p. 121 but just gives the results for the q-axis equations (λ_q , λ_G , and λ_Q). And so we will do it for the q-axis equations, starting with the Gequation (4.131a); we also do it for the torque equation.

First, we handle the state equation for λ_G .

$$\begin{split} \dot{\lambda}_{G} &= -\frac{r_{G}}{l_{G}} \lambda_{G} + \frac{r_{G}}{l_{G}} \left(\frac{L_{MQ}}{l_{q}} \lambda_{q} + \frac{L_{MQ}}{l_{Q}} \lambda_{Q} + \frac{L_{MQ}}{l_{G}} \lambda_{G} \right) \\ &= -\frac{r_{G}}{l_{G}} \lambda_{G} + \frac{r_{G}L_{MQ}}{l_{G}l_{q}} \lambda_{q} + \frac{r_{G}L_{MQ}}{l_{G}l_{Q}} \lambda_{Q} + \frac{r_{G}L_{MQ}}{l_{G}^{2}} \lambda_{G} \\ &= -\frac{r_{G}}{l_{G}} \left(1 - \frac{L_{MQ}}{l_{G}} \right) \lambda_{G} + \frac{r_{G}L_{MQ}}{l_{G}l_{q}} \lambda_{q} + \frac{r_{G}L_{MQ}}{l_{G}l_{Q}} \lambda_{Q} \end{split}$$

Now for the λ_q equation (4.130):

$$\begin{split} \dot{\lambda}_{q} &= -\frac{r}{l_{q}}\lambda_{q} + \frac{r}{l_{q}}\lambda_{AQ} + \omega\lambda_{d} - v_{q} = -\frac{r}{l_{q}}\lambda_{q} + \frac{r}{l_{q}}\left(\frac{L_{MQ}}{l_{q}}\lambda_{q} + \frac{L_{MQ}}{l_{Q}}\lambda_{Q} + \frac{L_{MQ}}{l_{G}}\lambda_{G}\right) + \omega\lambda_{d} - v_{q} \\ &= -\frac{r}{l_{q}}\lambda_{q} + \frac{r}{l_{q}}\frac{L_{MQ}}{l_{q}}\lambda_{q} + \frac{r}{l_{q}}\frac{L_{MQ}}{l_{Q}}\lambda_{Q} + \frac{r}{l_{q}}\frac{L_{MQ}}{l_{G}}\lambda_{G} + \omega\lambda_{d} - v_{q} \\ &= -\frac{r}{l_{q}}\left(1 + \frac{L_{MQ}}{l_{q}}\right)\lambda_{q} + \frac{r}{l_{q}}\frac{L_{MQ}}{l_{Q}}\lambda_{Q} + \frac{r}{l_{q}}\frac{L_{MQ}}{l_{G}}\lambda_{G} + \omega\lambda_{d} - v_{q} \end{split}$$

And now for the λ_Q equation (4.131b):

$$\begin{split} \dot{\lambda}_{Q} &= -\frac{r_{Q}}{l_{Q}} \lambda_{Q} + \frac{r_{Q}}{l_{Q}} \lambda_{AQ} = -\frac{r_{Q}}{l_{Q}} \lambda_{Q} + \frac{r_{Q}}{l_{Q}} \left(\frac{L_{MQ}}{l_{q}} \lambda_{q} + \frac{L_{MQ}}{l_{Q}} \lambda_{Q} + \frac{L_{MQ}}{l_{G}} \lambda_{G} \right) \\ &= -\frac{r_{Q}}{l_{Q}} \lambda_{Q} + \frac{r_{Q}}{l_{Q}} \frac{L_{MQ}}{l_{q}} \lambda_{q} + \frac{r_{Q}}{l_{Q}} \frac{L_{MQ}}{l_{Q}} \lambda_{Q} + \frac{r_{Q}}{l_{Q}} \frac{L_{MQ}}{l_{G}} \lambda_{G} \\ &= -\frac{r_{Q}}{l_{Q}} \left(1 - \frac{L_{MQ}}{l_{Q}} \right) \lambda_{Q} + \frac{r_{Q}}{l_{Q}} \frac{L_{MQ}}{l_{q}} \lambda_{q} + \frac{r_{Q}}{l_{Q}} \frac{L_{MQ}}{l_{Q}} \lambda_{q} + \frac{r_{Q}}{l_{Q}} \frac{L_{MQ}}{l_{G}} \lambda_{G} \end{split}$$

And finally for the torque equation (4.133):

$$\begin{split} \dot{\omega} &= \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j} \lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j} \lambda_q\right] + \left[\frac{-D}{\tau_j}\right] \omega \\ &= \frac{T_m}{\tau_j} + \left[\frac{\frac{L_{MQ}}{l_q} \lambda_q + \frac{L_{MQ}}{l_Q} \lambda_Q + \frac{L_{MQ}}{l_G} \lambda_G}{l_q 3\tau_j} \lambda_d - \frac{\frac{L_{MD}}{l_d} \lambda_d + \frac{L_{MD}}{l_F} \lambda_F + \frac{L_{MD}}{l_D} \lambda_D}{l_d 3\tau_j} \lambda_q\right] + \left[\frac{-D}{\tau_j}\right] \omega \\ &= \frac{T_m}{\tau_j} + \frac{L_{MQ}}{l_q^2 3\tau_j} \lambda_q \lambda_d + \frac{L_{MQ}}{l_Q l_q 3\tau_j} \lambda_Q \lambda_d + \frac{L_{MQ}}{l_G l_q 3\tau_j} \lambda_G \lambda_d - \frac{L_{MD}}{l_d^2 3\tau_j} \lambda_d \lambda_q - \frac{L_{MD}}{l_d 3\tau_j} \lambda_F \lambda_q - \frac{L_{MD}}{l_Q l_d 3\tau_j} \lambda_D \lambda_q + \left[\frac{-D}{\tau_j}\right] \omega \\ &= \frac{T_m}{\tau_j} + \left(\frac{L_{MQ}}{l_q^2 3\tau_j} - \frac{L_{MD}}{l_d^2 3\tau_j} \right) \lambda_q \lambda_d + \frac{L_{MQ}}{l_Q l_q 3\tau_j} \lambda_Q \lambda_d + \frac{L_{MQ}}{l_Q l_q 3\tau_j} \lambda_Q \lambda_d + \frac{L_{MQ}}{l_Q l_q 3\tau_j} \lambda_G \lambda_d - \frac{L_{MD}}{l_Q l_q 3\tau_j} \lambda_F \lambda_q - \frac{L_{MD}}{l_D l_d 3\tau_j} \lambda_D \lambda_q + \left[\frac{-D}{\tau_j}\right] \omega \end{split}$$

The above state equations are included in the state equations as follows:

$$\begin{bmatrix} \dot{\lambda}_{d} & \lambda_{F} & \lambda_{D} & \lambda_{q} & \lambda_{Q} & \lambda_{Q} & \omega & \delta \\ \hline -\frac{r}{\ell_{d}} \left(1 - \frac{L_{MD}}{\ell_{d}} \right) & \frac{r}{\ell_{d}} \frac{L_{MD}}{\ell_{F}} & \frac{r}{\ell_{d}} \frac{L_{MD}}{\ell_{D}} & -\omega & 0 & 0 & 0 & 0 \\ \hline -\frac{r}{\ell_{d}} \left(\frac{1 - \frac{L_{MD}}{\ell_{d}} \right) & \frac{r}{\ell_{d}} \frac{L_{MD}}{\ell_{F}} & \frac{r}{\ell_{d}} \frac{L_{MD}}{\ell_{D}} & 0 & 0 & 0 & 0 \\ \hline \frac{r_{F}}{\ell_{F}} \frac{L_{MD}}{\ell_{d}} & -\frac{r_{F}}{\ell_{F}} \left(1 - \frac{L_{MD}}{\ell_{F}} \right) & \frac{r}{\ell_{F}} \frac{L_{MD}}{\ell_{D}} & 0 & 0 & 0 & 0 \\ \hline \frac{r_{D}}{\ell_{D}} \frac{L_{MD}}{\ell_{d}} & \frac{r_{D}}{\ell_{D}} \frac{L_{MD}}{\ell_{F}} & -\frac{r_{D}}{\ell_{D}} \left(1 - \frac{L_{MD}}{\ell_{D}} \right) & 0 & 0 & 0 & 0 \\ \hline \frac{r_{D}}{\ell_{D}} \frac{L_{MD}}{\ell_{d}} & \frac{r_{D}}{\ell_{D}} \frac{L_{MD}}{\ell_{F}} & -\frac{r_{D}}{\ell_{D}} \left(1 - \frac{L_{MD}}{\ell_{D}} \right) & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{d}} & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{d}} & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{Q}} & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{r_{G}}{\ell_{G}} \frac{L_{MQ}}{\ell_{q}} & -\frac{r_{G}}{\ell_{G}} \left(1 - \frac{L_{MQ}}{\ell_{Q}} \right) & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{Q}} & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{Q}} & 0 & 0 \\ \hline \frac{r_{D}}{\ell_{D}} \frac{L_{MD}}{\ell_{d}} & \frac{r}{\ell_{d}} \frac{L_{MD}}{\ell_{d}} & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{Q}} & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{Q}} & \frac{r}{\ell_{d}} \frac{L_{MQ}}{\ell_{Q}} & 0 & 0 \\ \hline 0 & 0 & 0 & \frac{r_{G}}{\ell_{G}} \frac{L_{MQ}}{\ell_{q}} & \frac{r_{O}}{\ell_{G}} \frac{L_{MQ}}{\ell_{G}} & \frac{r_{O}}{\ell_{Q}} \left(1 - \frac{L_{MQ}}{\ell_{Q}} \right) & 0 & 0 \\ \hline \frac{r_{D}}{\ell_{D}} \frac{L_{MD}}{\ell_{d}} \frac{r_{d}}{\ell_{d}} \frac{L_{MD}}{\ell_{D}} \lambda_{q} & \frac{r_{D}}{\ell_{d}} \frac{L_{MQ}}{\ell_{d}} \lambda_{d}} & \frac{r_{D}}{\ell_{Q}} \frac{L_{MQ}}{\ell_{Q}} \lambda_{d}} & \frac{r_{MQ}}{\ell_{Q}} \lambda_{d} & \frac{r_{MQ}}{\ell_{Q}} \lambda_{d} & -\frac{r_{D}}{\ell_{D}} \\ \hline \frac{r_{MD}}{\ell_{D}} \frac{r_{d}}{\ell_{d}} \frac{r_{d}}{\ell_{d}} \frac{r_{d}}{\ell_{d}} \frac{r_{d}}{\ell_{d}} \lambda_{d}} & \frac{r_{MQ}}{\ell_{d}} \lambda_{d}} & \frac{r_{MQ}}{\ell_{Q}} \lambda_{d}} & \frac{r_{MQ}}{\ell_{Q}} \lambda_{d} & \frac{r_{MQ}}{\ell_{Q}} \lambda_{d} & -\frac{r_{M}}{\ell_{D}} \\ \hline \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \lambda_{d}} & \frac{r_{M}}{\ell_{d}} \lambda_{d}} & \frac{r_{M}}{\ell_{d}} \lambda_{d}} & \frac{r_{M}}{\ell_{d}} \lambda_{d}} & \frac{r_{M}}{\ell_{d}} \lambda_{d}} \\ \hline \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \lambda_{d}} & \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \frac{r_{M}}{\ell_{d}} \lambda_{d}} & \frac{r_{M}}{\ell_{d}} \lambda_{d}} \\ \hline \frac{$$



Comments:

- 1. There is a very good treatment of saturation in IEEE Std 1110-2002, "IEEE Guide for Synchronous Generator Modeling Practices and Applications in Power System Stability Analyses," see chapter 6. Indeed, this is a very good standard to complement our entire course, and I encourage you to download, print, and include in your binder/folder.
- 2. Note the presence of v_d and v_q on the right-hand-side. Recall that $\underline{v}_{0dq} = \underline{P} \underline{v}_{abc}$ and so v_d and v_q come from the phase voltages v_a , v_b , and v_c . Since the phase voltages are affected by the load currents, so are v_d and v_q . So, we need to represent the load in order to complete the model. This is the subject of section 4.13. It is done in Section 4.13.2 for the current-state space model and in Section 4.13.3 for the flux linkage state space model. I have already addressed it in class, in the notes called "LoadEquations," for the current state space model only. I will leave you to read Section 4.13.3 for the flux linkage state space model. There is also a nice example (Ex 4.4, p. 128) which illustrates integration of the load equations.
- 3. Section 4.12.4 is the first instance in VMAF where saturation is addressed, but not the last. It is also addressed in Section 5.9.1 and in Appendix D.1.2.