Generator Response to Load Variation

The below table summarizes the way that load variation is allocated to generators in a power system. In these notes, we derive the proximity effect, the inertial effect, and the governor effect. The AGC effect is addressed in EE 553.

DELINEATION OF TIME PERIODS FOLLOW... ..NGE IN SYSTEM REAL POWER DEMAND
FOR RESPONSE OF A SINGLE GENERATOR

Time	Most Influential Factor for Determining a ΔP_{gen}	What is the Effect of this Factor on ΔP_{gen}?	Describing Equation
0^+	Electrical Proximity to disturbance of machine	The closer the generator to the disturbance, the more it will compensate for change in system real power demand, $P_{L\Delta}(0^+)$	$\Delta P_{gen_i} = \dfrac{P_{sik}}{\sum_{j=1}^{N} P_{sjk}} P_{L\Delta}(0^+)$ $P_{sjk} = \dfrac{V_j V_k}{X_{jk}} \cos(\delta_j - \delta_k)$
$0^+ \longrightarrow 3$ sec	Size (Inertia) of machine	The larger the generator, the more it will compensate for change in system real power demand, $P_{L\Delta}(0^+)$.	$\Delta P_{gen_i} = \left[\dfrac{H_i}{\sum_{j=1}^{N} H_j}\right] P_{L\Delta}(0^+)$ H_i = Inertia constant of machine i.
$3 \longrightarrow 20$ sec	Primary Speed Control (Governor)	Generation increases with frequency decline.	$\Delta P_{gen_i} = \dfrac{-\Delta f}{R}\left[\dfrac{S_{bi}}{60}\right]$ R = Steady state droop in p.u. S_{b_i} = MVA rating for machine i.
3 min $\longrightarrow$ 10 min	Supplementary Control (AGC)	Control Center Computer reallocates a new set point for each generator to return frequency to 60 Hz and tie lines to scheduled transfer.	$\Delta P_{gen_i} = \Delta P_{set_i}$ $\Delta P_{set_i} = \Delta P_{economic_i} + K_i\,[ACE]$ ACE = IA − IS + BΔf K = Constant, <1, for generator i. ACE = Area control error IA = Actual interchange IS = Scheduled interchange B = Frequency bias.

We derived last time the expression for synchronizing power coefficient for a multi-machine system with classical machine models, constant impedance loads, and the network reduced to generator internal nodes. This expression was:

$$P_{Sij} = \left.\frac{\partial P_{ij}}{\partial \delta_{ij}}\right|_{\delta_{ij0}} = |E_i||E_j|\left\{B_{ij}\cos\delta_{ij0} - G_{ij}\sin\delta_{ij0}\right\} \qquad (3.24)$$

We may interpret the synchronizing power coefficient in the following way:

> $P_{Sij}\,\Delta\delta_{ij}$ is the change in power flow between generator i and generator j at $t=0^+$ caused by:
>
> ➔A small change in powers at gens i and j, causing
> - → A small change in angular separation between generators i and j, $\Delta\delta_{ij}$ resulting from
>   - → Small changes in angle at gen i $\Delta\delta_i$, and gen j $\Delta\delta_j$, with the angles and voltages at all other generators fixed[1].

Some explanatory notes:

- Since $B_{ij}>>G_{ij}$ for transmission systems, and since δ_{ij0} is typically rather small, the dominant term is $B_{ij}\cos\delta_{ij0}$. From this, we see that P_{Sij} is a measure of
  - The transmission "strength" between generators i and j as reflected by the susceptance B_{ij} (here we assume a fully connected network so that each generator is connected to every other generator, which tends to be the case when you perform network reduction of the load buses resulting in nearly complete fill-in of the Y-bus).

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[1] We could conceive of the change in angle as being caused by a $\Delta\delta_j$ with $\Delta\delta_j=0$ or by $\Delta\delta_j$ with $\Delta\delta_i=0$ and it would not be wrong; indeed, it will be useful to do something similar when we derive the proximity effect below. The perspective that "in this case, the synchronizing power coefficient is dependent on only one bus," is addressed by the fact that we use B_{ij} and G_{ij} to compute the synchronizing power coefficient, and these terms B_{ij} and G_{ij} are very much dependent on two buses.

- The degree to which the angles of gens i and j are the same.

The Proximity Effect (See section 3.6-3.6.2 in text)
Consider that the network is reduced to its internal gen nodes (n of them), except for a single load bus, bus k, where we assign k=n+1.

At this load bus, the load is increased by ΔP_L at t=0.

Since all other nodes are generator nodes, their angles cannot change instantly at $t=0^+$ (since this would require an instantaneous mechanical movement).

Therefore, for any nodes i, j not k, $\Delta\delta_{ij}=0$.

We can also write that

$$\Delta\delta_{ik}=\Delta\delta_i-\Delta\delta_k=0-\Delta\delta_k=-\Delta\delta_k$$
$$\Delta\delta_{kj}=\Delta\delta_k-\Delta\delta_j=\Delta\delta_k-0=\Delta\delta_k$$

Recall eq. (3.23), which says that $\Delta P_{ei}=\sum_{\substack{j=1\\ j\neq i}}^{n} P_{Sij}\Delta\delta_{ij}$, and apply this equation to nodes i and k.

- For node i:

$$\Delta P_{ei}=P_{Sik}\underbrace{\Delta\delta_{ik}}_{-\Delta\delta_k}+\sum_{\substack{j=1\\ j\neq i\\ j\neq k}}^{n} P_{Sij}\underbrace{\Delta\delta_{ij}}_{0}=-P_{Sij}\Delta\delta_k \qquad \text{(eq. 3.51)}$$

- For node k:

$$\Delta P_{ek}=-\Delta P_L=\sum_{\substack{j=1\\ j\neq k}}^{n} P_{Skj}\Delta\delta_{kj}=\Delta\delta_k\sum_{j=1}^{n} P_{Skj}$$

$$\Rightarrow \Delta\delta_k=\frac{-\Delta P_L}{\sum_{j=1}^{n} P_{Skj}} \qquad \text{(eq. 3.54)}$$

Now substitute eq. (3.54) into eq. (3.51) to get:

$$\Delta P_{ei} = \frac{(-P_{Sik})(-\Delta P_L)}{\sum_{j=1}^{n} P_{Skj}} = \frac{P_{Sik}}{\sum_{j=1}^{n} P_{Skj}} \Delta P_L \qquad \text{(eq. 3.55)}$$

This equation is applicable at $t=0^+$. What does it mean?

Recalling the expression for the synchronizing power coefficient P_{Sik}, given by

$$P_{Sik} = \left.\frac{\partial P_{ik}}{\partial \delta_{ik}}\right|_{\delta_{ik0}} = |E_i||E_k|\{B_{ik}\cos\delta_{ik0} - G_{ik}\sin\delta_{ik0}\}$$

we see that eq. (3.55) indicates that at $t=0^+$,

- The closer, electrically, a generator is to the load change, and
- The closer the angles are (which means the less power flowing over lines connecting the gen to the load)

the more the gen will compensate for the load change at $t=0^+$.

It is called the "proximity effect." Observe that it is independent of generator "size" (inertia or MVA rating).

Inertial Effect (see Section 3.6.3 in text):

Recall the linearized swing equation for machine i (ignoring damping), **where all H_i are given on a common base.**

$$\frac{2H_i}{\omega_{\mathrm{Re}}} \frac{d^2\Delta\delta_i}{dt^2} = -\Delta P_{ei} \qquad \text{(eq. 3.56)}$$

For a load change ΔP_{Lk}, at $t=0^+$, consider using the initial ($t=0^+$) change in electrical power ΔP_{ei} as the difference between the mechanical power in and the electrical power out. We then substitute (3.55) into the right-hand-side of the above to obtain:

$$\frac{2H_i}{\omega_{\mathrm{Re}}} \frac{d^2\Delta\delta_i}{dt^2} = -\frac{P_{Sik}}{\sum_{j=1}^{n} P_{Skj}} \Delta P_L \qquad \text{(eq. 3.57)}$$

As an aside, *let's bring H_i to the right-hand-side and rearrange:*

$$\frac{2}{\omega_{\mathrm{Re}}}\frac{d^2\Delta\delta_i}{dt^2}=-\left[\frac{P_{Sik}}{H_i}\right]\frac{\Delta P_L}{\sum_{j=1}^{n}P_{Skj}}$$

This tells us that, for $\Delta P_L>0$, each machine will decelerate initially but at different rates, according to P_{Sik}/H_i. Gens having high P_{Sik} (close to load, least angular difference) and low inertia will initially decelerate the most.

Returning to (3.57), we rewrite it with H_i inside the differentiation, using $\Delta\omega_i$ instead of $\Delta\delta_i$, and writing it for all generators 1,…,n. Then add them up.

$$\frac{2}{\omega_{\mathrm{Re}}}\frac{dH_1\Delta\omega_1}{dt}=-\frac{P_{S1k}}{\sum_{j=1}^{n}P_{Skj}}\Delta P_L$$

$$\vdots$$

$$\frac{2}{\omega_{\mathrm{Re}}}\frac{dH_n\Delta\omega_n}{dt}=-\frac{P_{Snk}}{\sum_{j=1}^{n}P_{Skj}}\Delta P_L$$

$$\frac{2}{\omega_{\mathrm{Re}}}\sum_{i=1}^{n}\frac{dH_i\Delta\omega_i}{dt}=-\frac{\sum_{i=1}^{n}P_{Sik}}{\sum_{j=1}^{n}P_{Skj}}\Delta P_L=-\Delta P_L \qquad \text{(eq. 3.59)}$$

Now define the "inertial center" of the system, in terms of angle and speed, as

- The weighted average of the angles:

$$\bar{\delta} \equiv \frac{\sum_{i=1}^{n} H_i \delta_i}{\sum_{i=1}^{n} H_i} \quad \text{or} \quad \overline{\Delta\delta} \equiv \frac{\sum_{i=1}^{n} H_i \Delta\delta_i}{\sum_{i=1}^{n} H_i}$$

- The weighted average of the speeds:

$$\bar{\omega} \equiv \frac{\sum_{i=1}^{n} H_i \omega_i}{\sum_{i=1}^{n} H_i} \quad \text{or} \quad \overline{\Delta\omega} \equiv \frac{\sum_{i=1}^{n} H_i \Delta\omega_i}{\sum_{i=1}^{n} H_i}$$

So we have an inertial speed center and an inertial angle center.

Differentiating $\overline{\Delta\omega}$ with respect to time, we get:

$$\frac{d\overline{\Delta\omega}}{dt} \equiv \frac{\sum_{i=1}^{n} \frac{d\left(H_i \Delta\omega_i\right)}{dt}}{\sum_{i=1}^{n} H_i}$$

Solving for the numerator on the right-hand-side results in:

$$\sum_{i=1}^{n} \frac{d\left(H_i \Delta\omega_i\right)}{dt} = \left[\sum_{i=1}^{n} H_i\right]\left[\frac{d\overline{\Delta\omega}}{dt}\right] \qquad \text{(eq. *)}$$

Recall (3.59):

$$\frac{2}{\omega_{\mathrm{Re}}} \sum_{i=1}^{n} \frac{dH_i \Delta\omega_i}{dt} = -\Delta P_L \qquad \text{(eq. 3.59)}$$

Now substitute eq. (*) into eq. (3.59) to get:

$$\frac{2}{\omega_{\mathrm{Re}}} \left[\sum_{i=1}^{n} H_i\right]\left[\frac{d\overline{\Delta\omega}}{dt}\right] = -\Delta P_L$$

Bringing the 2×(summation) over to the right-hand-side gives:

$$\frac{1}{\omega_{\text{Re}}}\left[\frac{d\overline{\Delta\omega}}{dt}\right]=\frac{-\Delta P_L}{2\sum_{i=1}^{n}H_i} \qquad \text{(eq. 3.60)}$$

Eq. (3.60) gives the *average deceleration of the system*.

But each individual machine will respond according to eq. (3.56), which is repeated here for convenience:

$$\frac{2H_i}{\omega_{\text{Re}}}\frac{d^2\Delta\delta_i}{dt^2}=-\Delta P_{ei}$$

or, in terms of ω,

$$2H_i\left[\frac{1}{\omega_{\text{Re}}}\frac{d\Delta\omega_i}{dt}\right]=-\Delta P_{ei} \qquad \text{(eq. 3.56)}$$

If there is no governor action on any machine, then **<u>after some time</u>**, all machine decelerations will converge to the average value given by eq. (3.60).

$$\frac{1}{\omega_{\text{Re}}}\left[\frac{d\overline{\Delta\omega}}{dt}\right]=\frac{-\Delta P_L}{2\sum_{i=1}^{n}H_i} \qquad \text{(eq. 3.60)}$$

In other words, at some time $t=t_1$, we have that

$$\frac{d\Delta\omega_i}{dt}=\frac{d\overline{\Delta\omega}}{dt} \quad \forall \quad i=1,\ldots,n$$

and then eq. (3.56) becomes:

$$2H_i\left[\frac{1}{\omega_{\text{Re}}}\frac{d\overline{\Delta\omega}}{dt}\right]=-\Delta P_{ei}$$

Substituting the right-hand-side of eq. (3.60) into the brackets of the last equation, we obtain:

$$2H_i\left[\frac{-\Delta P_L}{2\sum_{i=1}^{n} H_i}\right] = -\Delta P_{ei}$$

Canceling the "2" and the minus sign, we find that:

$$\Delta P_{ei} = \left[\frac{H_i}{\sum_{i=1}^{n} H_i}\right]\Delta P_L \qquad \text{(eq. 3.61)}$$

So at $t=t_1$, the machines compensate for the load change in proportion to their inertias.

If machines do not have turbine-governor speed control (or if you do not model it!), then the allocation of load change among generators, in the final steady-state, will be in proportion to the inertias, where the "heavier" machines get a larger proportion of the load change.

If you do represent turbine-governors, then the time t_1 is not very clear. One thing that is clear, however, is that the time t_1 should be before action of the turbine governor. Since most turbine governors do not typically have significant effect until about 2 seconds, we can safely say that t_1<2 seconds.

Fig. 3.9 in your text, pg. 82, illustrates the average speed deviation.

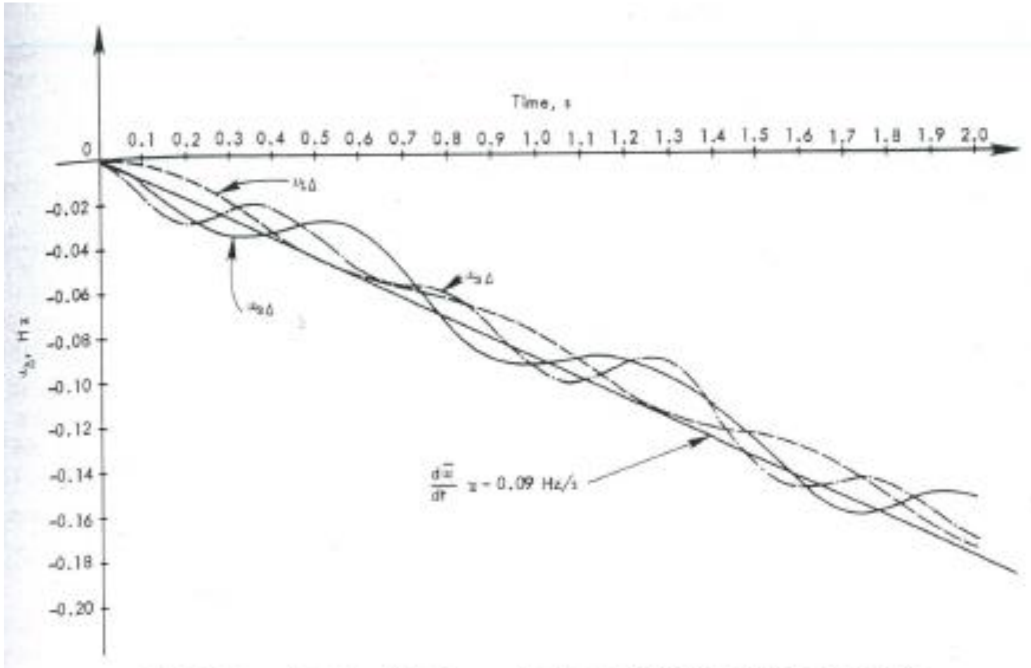


Fig. 3.9 Speed deviation following application of a 10 MW resistive load at bus 8.

Fig. 3.8 in your text, pg. 80, illustrates the change in electrical output power for several machines. There are two values of interest in Fig. 3.8: (1) the initial values of each machine, at $t=0^+$, which corresponds to ΔP_{ei} from the proximity effect; and (2) the average value at steady state, which corresponds to ΔP_{ei} from the inertial effect.

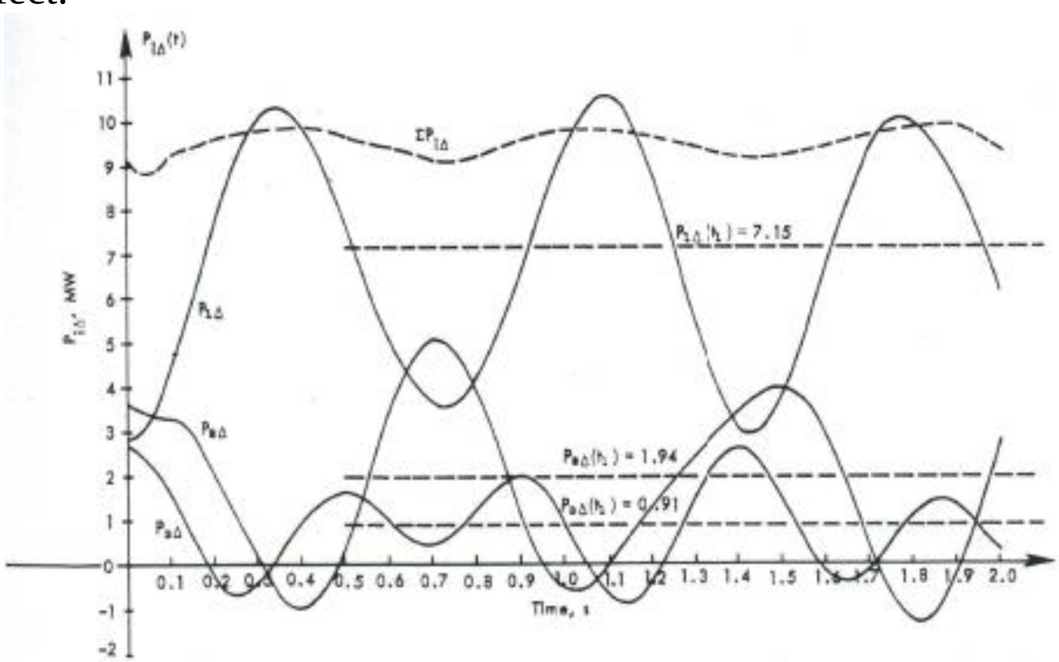


Fig. 3.8 $P_{i\Delta}$ versus t following application of a 10 MW resistive load at bus 8.

Table 3.5, pg. 81, compares the "actual" initial ($t=0^+$) change in powers for a 10 MW load change for each machine to that computed based on synchronizing power coefficients.

Table 3.5. Initial Power Change at Generators Due to 10-MW Load Added to Bus 8

(1) i	(2) $P_{i\Delta}$ (neglecting G_{ik})	(3) $P_{i\Delta}$ (with G_{ik})	(4) $P_{i\Delta}$ (computer study)	(5) $P_{i\Delta}$ [91% of (2)]	(6) $P_{i\Delta}$ [91% of (3)]
1	3.021	3.016	2.8	2.749	2.745
2	4.021	4.028	3.6	3.659	3.665
3	2.958	2.956	2.7	2.692	2.690
$\sum P_{i\Delta}$	10.000	10.000	9.1	9.100	9.100

VMAF explain the difference between the calculations of (2) and (3), and the results of the computer study in (4) in this way (p. 81): "Note that the actual load pickup is only 9.1 MW instead of the desired 10MW. This is due in part to the assumption of constant voltage V_k at bus 8 (actually, the voltage drops slightly) and to the assumed linearity of the system. If the computed $P_{i\Delta}$ are scaled down by 0.91, the results agree quite well with the values measured from the computer study. These values are also shown on the plot of Figure 3.8 at time t=0+ and are due only to the synchronizing power coefficients of the generators with respect to bus 8."

Now check the calculations for the inertial effect. From the data provided in Table 2.1 of VMAF, we see that

$MWS_1=2364$;

$MWS_2=640$;

$MWS_3=301$.

Then using (3.61), we have

$$\Delta P_{ei} = \left[\frac{H_i}{\sum_{i=1}^{n} H_i} \right] \Delta P_L = \begin{Bmatrix} \frac{2364}{3305} \\ \frac{640}{3305} \\ \frac{301}{3305} \end{Bmatrix} 10 = \begin{Bmatrix} 7.15 \\ 1.94 \\ 0.91 \end{Bmatrix}$$

which agrees with the values given in Fig. 3.8 above.

Governor load flow (see Section 2.3.2 in text)

Now what happens when we do model turbine-governors?

From eq. 2.28, we found that, following a load change, the steady-state change in pu gen mechanical power is related to the steady-state change in pu frequency according to:

$$\Delta P_{miu} = \frac{-\Delta\omega_u}{R_u} = \frac{-\Delta f_u}{R_u}$$

where R_u is given in per-unit on the machine base. Typically, R_u=0.05 in the US (5% droop).

Then we can express that the steady-state change in MW gen mechanical power is related to the steady-state change in pu frequency according to:

$$\Delta P_{mi} = \Delta P_{miu} S_{Bi} = \frac{-\Delta f_u}{R_u} S_{Bi} \qquad (**)$$

Then, relating pu frequency to frequency and substituting into (**)

$$\Delta f_u = \frac{\Delta f}{60} \Rightarrow \Delta P_{mi} = \frac{-\Delta f}{60 R_u} S_{Bi} = C S_{Bi}$$

So that

$$\Delta P_{mi} = C S_{Bi} \qquad (***)$$

where $C = \dfrac{-\Delta f}{60 R_u}$

Summing over all ΔP_{mi} yields ΔP_L:

$$\sum_{i=1}^{n} \Delta P_{mi} = C \sum_{i=1}^{n} S_{Bi} = \Delta P_L$$

so that we find:

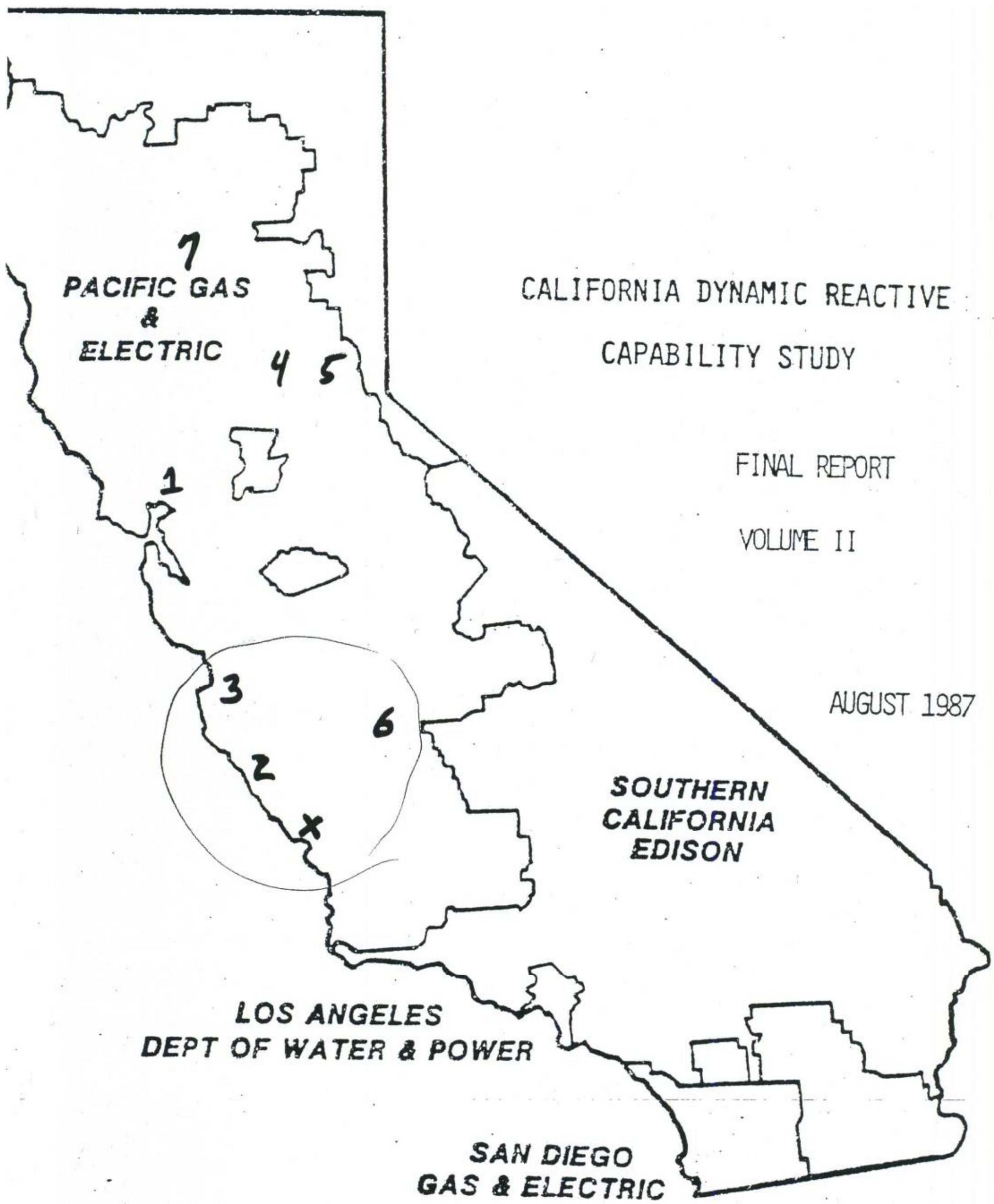
$$C = \frac{\Delta P_L}{\sum_{i=1}^{n} S_{Bi}}$$

Substituting C into eq. (***) above results in:

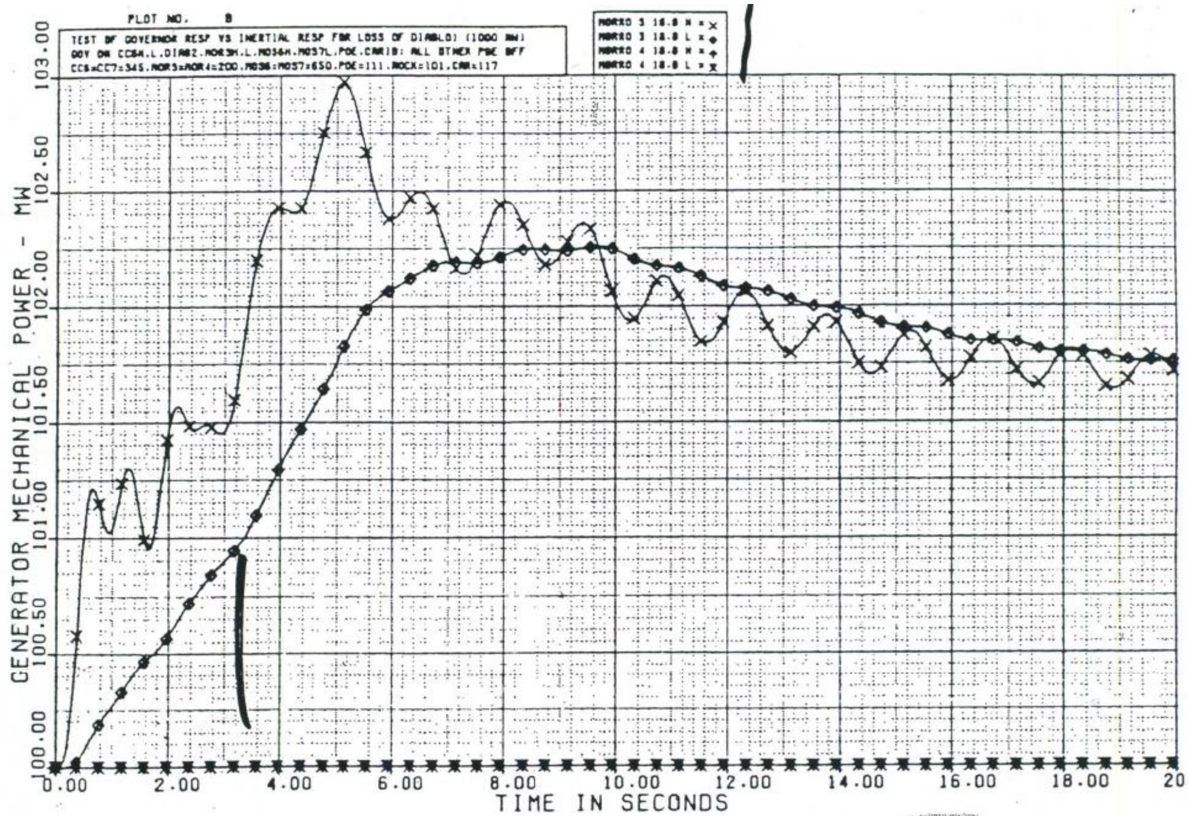
$$\Delta P_{mi} = \frac{S_{Bi}}{\sum_{i=1}^{n} S_{Bi}} \Delta P_L$$

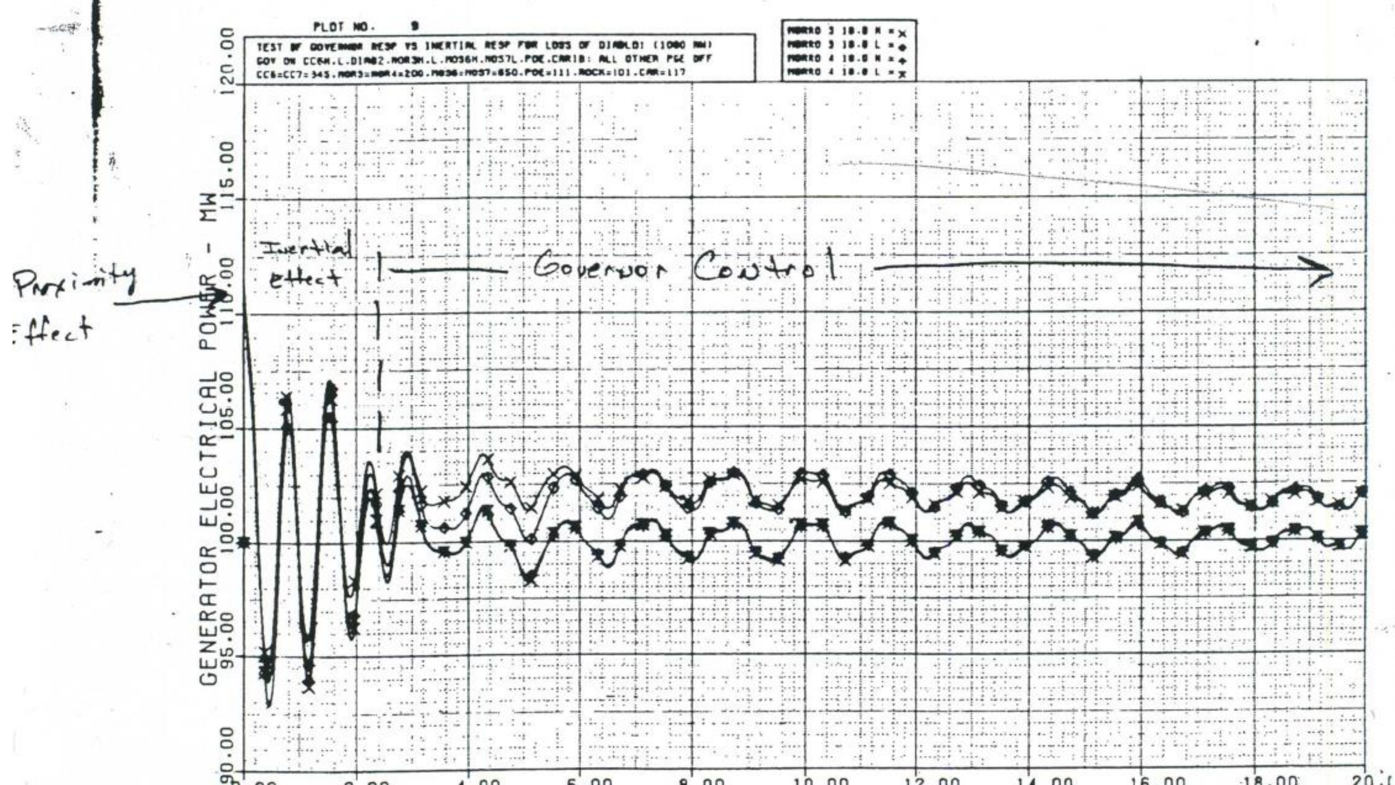
So during the 2-20s period, gens pick up according to their rating.

<u>Example</u>: This is an example from investigation I did on the California system where I modeled governors on most (but not all) machines, dropped one of the Diablo units at t=0, and then looked at the proximity and governor effects described above. In below map, observe that Diablo is located at the "X." Electrically close machines include 1, 2, and 3. Number 6 is geographically close but electrical proximity is diminished due to the fact that it is on the 230 kV system (Diablo, and machines 1, 2, 3 are on the 500 kV system). Machines 4, 5, and 7 are geographically & electrically far.



The below plots show mechanical power and electrical power out for four synchronous machines. Two of them that are plotted, Morro 3 H and Morro 3 L, are on governor control. The other two that are plotted are not. All four machines were initially at 100 MW.


PLOT NO. 8
TEST OF GOVERNOR RESP VS INERTIAL RESP FOR LOSS OF DIABLO1 (1000 MW)
GOV ON CC6H.L.DIAB2.MOR3H.L.MO36H.MO37L.POE.CAR1B: ALL OTHER PGE OFF
CC6=CC7=345.MOR3=MOR4=200.MO36=MO37=650.POE=111.ROCK=101.CAR=117
MORRO 3 18.0 H = X
MORRO 3 18.0 L = ◆
MORRO 4 18.0 H = +
MORRO 4 18.0 L = X
GENERATOR MECHANICAL POWER - MW
103.00
102.50
102.00
101.50
101.00
100.50
100.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
14.00
16.00
18.00
20
TIME IN SECONDS


PLOT NO. 9
TEST OF GOVERNOR RESP VS INERTIAL RESP FOR LOSS OF DIABLO1 (1000 MW)
GOV ON CC6H.L.DIAB2.MOR3H.L.MO36H.MO37L.POE.CAR1B: ALL OTHER PGE OFF
CC6=CC7=345.MOR3=MOR4=200.MO36=MO37=650.POE=111.ROCK=101.CAR=117
MORRO 3 18.0 H = X
MORRO 3 18.0 L = ◆
MORRO 4 18.0 H = +
MORRO 4 18.0 L = X
GENERATOR ELECTRICAL POWER - MW
120.00
115.00
110.00
105.00
100.00
95.00
90.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
14.00
16.00
18.00
20.0
TIME IN SECONDS
Inertial effect
Governor Control
Proximity Effect

The table below summarizes machine response. Observe that machines 1, 2, 3, and 6, which we indicated were electrically closest to Diablo, pick up the most at $t=0^+$. Machines 4, 5, and 7, electrically farthest, pick up the least at $t=0^+$.

Note also that at t=20, of the units on governor control, machine 3 (Moss 6 H and L) picked up the most, because they had the largest MVA ratings (445 and 375 MVA).

Drop Diablo #1 (1000 MW) at t=0. (20 sec simulation)

Unit	#	MWS	MVA	P_{max}	Gov Status	$P_g(0)$	$P_g(0^+)$	$P_g(2)$	$P_m(20)$	$P_g(20)$	$\Delta P_g(0^+)$	$\Delta P_g(2)$	$\Delta P_m(20)$	$\Delta P_g(20)$
1 CC	H	596	192	172.5	ON	125	129.4		126.7		4.4		.009 1.7	
CC #6	L	650	192	172.5	ON	125	129.4		126.75		4.4		.009 1.75	
CC #7	H	596	192	—	OFF	125	129.4		125		4.4		0.0	
CC #7	L	650	192	—	OFF	125	129.4		125		4.4		0.0	
2 MOR #3	H	596	192	172.5	ON	100	111		101.7		11.0		.009 1.7	
MOR #3	L	650	192	172.5	ON	100	111		101.75		11.0		.009 1.75	
MOR #4	H	596	192	—	OFF	100	111		100		11.0		0.0	
MOR #4	L	650	192	—	OFF	100	111		100		11.0		0.0	
3 MOSS #6	H	639	445	411.0	ON	351	371		355.2		20.0		.009 4.2	
MOSS #6	L	3383	375	339.0	ON	300	320		302.3		20.0		.006 2.3	
MOSS #7	H	639	445	—	OFF	351	371		351		20.0		0.0	
MOSS #7	L	3383	375	—	OFF	300	320		300		20.0		0.0	
4 PD		450	138	120	ON	111	112.0		111.8		1.0		.006 0.8	
5 ROCK CR		612	126	—	OFF	101	101.8		101		0.8		0.0	
6 HAAS		421	150	72	ON	142	147.4		143		5.4		.007 1.0	
7 CARIBOU		356	122		ON	117	117.9		117.7		0.9		.006 0.7	