Linearized Analysis of the Synchronous Machine for PSS

Chapter 6 does two basic things:
1. Shows how to linearize the 7-state model (model #2, IEEE #2.1, called “full model without G-cct.”) of a synchronous machine connected to an infinite bus using the current-state-space model (sections 6.1-6.3) and using the flux-linkage state-space model (section 6.4). This material is useful for understanding the modeling required for power system eigenvalue calculation programs such as the EPRI programs MASS and PEALS contained within SSSP. Some information on these tools follow:
2. Linearizes the one-axis model of a synchronous machine connected to an infinite bus (sections 6.5-6.7). This material is useful for the conceptual understanding of why power system stabilizers are needed.

In these notes, we will address (2) and then return to (1) in the next class.

Some additional references for you on this issue are references [1,2] given at the end of chapter 6. These two references are:

Reference [1] came first and produced what is commonly referred to in the literature as the Heffron-Phillips model of the synchronous machine. Reference [2] extended the Heffron-Phillips model and is the most well known. Reference [2] is also viewed as the seminal work that motivated the need for power system stabilizers (PSS). This paper is on the web site for you to download, read, and place in your notebook. You will note that it contains material quite similar to what follows below.

Your text also provides background on this issue in several separate locations, found in the following sections:
- Section 3.5.1: Voltage regulator with one time lag
- Section 6.5: Simplified linear model
- Section 6.6: Block diagrams
- Section 6.7: State-space representation of simplified model
- Section 7.8: State-space description of the excitation system
- Section 8.4: Effect of excitation on dynamic stability
- Section 8.5: Root-locus analysis of a regulated machine connected to an infinite bus
- Section 8.7: Supplementary stabilizing signals

I will provide the minimal analysis necessary to see the basic issue.

The analysis uses the simplest model possible for which the excitation system may be represented – the one-axis model (model 7, IEEE #1.0), loaded through a connection to an infinite bus. The one-axis model is a 3-state model, developed based on the following main assumptions (there are others as well – see page 222):
1. Only the field winding is represented (so no G-circuit and no amortisseur windings).

2. \( \frac{d\lambda_d}{dt} = \frac{d\lambda_q}{dt} = 0 \)

The nonlinear equations for the one-axis model are given by eqs. (4.294) and (4.297), as follows:

\[
\begin{align*}
\dot{E}_q' &= \frac{1}{\tau_{do}'} E_{FD} - \frac{1}{\tau_{do}'} E_q, \text{ where } E_q = E'_q + (x'_d - x_d) I_d \\
\dot{\omega} &= \frac{1}{\tau_j} T_m - \frac{1}{\tau_j} [E'_q I_q + (x'_d - x_q) I_d I_q] - \frac{1}{\tau_j} D \omega \\
\dot{\delta} &= \omega - 1
\end{align*}
\]

To identify basic concepts, Concordia and deMello assumed a single machine connected to an infinite bus through a transmission line having series impedance of \( R_e + jX_e \), as illustrated in Fig. 1.

![Fig. 1](image)

The generator model connected to the infinite bus can be linearized and converted to the following state-space form, as given by eqs. (6.78) and (6.79) of Section 6.7:

\[
\begin{align*}
\Delta T_e &= K_1 \Delta \delta + K_2 \Delta E'_q + D \Delta \omega \\
\Delta V_t &= K_5 \Delta \delta + K_6 \Delta E'_q
\end{align*}
\] (**)
\[ \Delta \dot{E}_q' = -\left( \frac{1}{K_3' \tau'_{do}} \right) \Delta E'_q - \left( \frac{K_4'}{\tau'_{do}} \right) \Delta \delta + \left( \frac{1}{\tau'_{do}} \right) \Delta E_{FD} \]

\[ \Delta \dot{\omega} = -\left( \frac{1}{\tau_j} \right) \Delta T_e + \left( \frac{1}{\tau_j} \right) \Delta T_m = \left( -\frac{K_2}{\tau_j} \right) \Delta E'_q - \left( \frac{K_1}{\tau_j} \right) \Delta \delta - \frac{D}{\tau_j} (\Delta \omega) + \left( \frac{1}{\tau_j} \right) \Delta T_m \]

\[ \Delta \dot{\delta} = (\Delta \omega) \omega_B \]

(6.79)

Equations (**) and (6.79) are also provided in (7.72), (7.73), (7.74), (7.75), and (7.76).

The LaPlace transform of the above equations results in the following relations:

\[ \Delta T_e = K_1 \Delta \delta + K_2 \Delta E'_q + D \Delta \omega \]

\[ \Delta V_t = K_5 \Delta \delta + K_6 \Delta E'_q \]

\[ \Delta E'_q = \frac{K_3}{1 + K_3 \tau'_{do}} \Delta E_{FD} - \frac{K_3 K_4}{1 + K_3 \tau'_{do}} \Delta \delta \]

\[ \Delta \omega = \frac{1}{s \tau_j} \left( \Delta T_m - \Delta T_e \right) \]

(*)

\[ \Delta \delta = \frac{1}{s} \Delta \omega \]

where, in (*), the variables \( \Delta T_e, \Delta V_t, \Delta E'_q, \Delta \omega, \) and \( \Delta \delta \) represent LaPlace transforms of their corresponding time-domain functions (a slight abuse of notation).

Finally, we note that \( E_{FD} \), the stator EMF produced by the field current and corresponding to the field voltage \( v_F \), is a function of the voltage regulator. Under linearized conditions, the change in \( E_{FD} \) is proportional to the difference between changes in the reference voltage and changes in the terminal voltage, i.e.,

\[ \Delta E_{FD} = G_e(s) \left( \Delta V_{ref} - \Delta V_t \right) \]

(***)

where \( G_e(s) \) is the transfer function of the excitation system.
In the above equations (*) and (**), the various constants $K_1$-$K_6$ are defined as follows:

\[
K_1 = \left. \frac{\Delta T_e}{\Delta \delta} \right|_{E'_q = E'_{q0}} \quad K_2 = \left. \frac{\Delta T_e}{\Delta E'_q} \right|_{\delta = \delta_0} \quad K_4 = \left. -\frac{1}{K_3} \frac{\Delta E'_q}{\Delta \delta} \right|_{E_{FD} = \text{constant}}
\]

\[
K_5 = \left. \frac{\Delta V_t}{\Delta \delta} \right|_{E'_q = E'_{q0}} \quad K_6 = \left. \frac{\Delta V_t}{\Delta E'_q} \right|_{\delta = \delta_0}
\]

and $K_3$ is an impedance factor that accounts for the loading effect of the external impedance (see (6.58). Your text, on pages 223, 224, and 225, provides exact expressions for these constants for the case of the one-axis model we are analyzing, under the condition that the line connecting the generator to the infinite bus has impedance of $Z_e = R_e + jX_e$. I have attached an appendix to these notes that develop expressions for these constants under condition that $Z_e = R_e + jX_e$. The paper\(^1\) on the website also provides these constants both ways, i.e., for $Z_e = R_e + jX_e$ and for $Z_e = jX_e$. There are two comments worth mentioning here:

1. $K_1$ is the synchronizing power coefficient.
2. A&F express $K_4$ as (see eq. 6.60)

\[
K_4 = \left. \frac{1}{K_3} \frac{\Delta E'_q}{\Delta \delta} \right|_{E_{FD} = \text{constant}}
\]

However, $K_3$, being an impedance factor, is positive. We want $K_4$ to also be positive; however, the above expression suggests that $E'_q$ would increase with an increase in angle (or loading). This is counter to the idea of armature reaction, where the internal flux decreases as a result of stator current, as indicated by our conceptual analysis in the notes called “ExcitationSystems” per the below figure:

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In fact, the book itself indicates as much via eq. (3.11) where it says that “\(K_4\) is the demagnetizing effect of a change in the rotor angle (at steady-state),” which is given by the following relationship:

\[
K_4 = -\frac{1}{K_3} \lim_{t \to \infty} \Delta E'(t) \quad \Delta V_{ref} = 0, \quad \Delta \delta = u(t)
\]  

(3.11)

where we note the negative sign out front. Therefore, the book expression (eq. 6.60), without the negative sign, is incorrect.

In eq. (**), \(G_e(s)\) is the transfer function of the excitation system. Recall that there are several different kinds (DC, AC Alternator, and static), each requiring somewhat different modeling. One kind that has become quite common is the “static” excitation system, represented by Fig. 2a, where \(K_A\) is the exciter gain and \(T_A\) is the excited time constant.

![Phasor Diagram Illustrating Relationship Between \(E'_q\) and \(\delta\) At Constant Real Power](image-url)
Fig. 2a is characterized by the following transfer function.

\[
G_e(s) = \frac{(1 + sT_F)K_A}{(1 + sT_F)(1 + sT_A) + K_F K_A} \tag{****}
\]

The static excitation is typically very fast (no rotating machine in the loop). Fast excitation response is beneficial for transient stability because generator terminal voltages see less voltage depression for less time during and after network faults. Such speed of excitation response can, however, cause problems for damping, as we shall see in what follows.

We may extract from the above equations (\text{*}), (\text{**}), and (\text{***}) a block diagram relation, as seen in Fig. 2. Note that in this block diagram, \( \tau_j = M \). Careful comparison of this block diagram to Fig. 8.17 in your text will suggest they are the same.

![Block Diagram](image)

**Fig. 2**

We will use this block diagram to analyze the stability behavior of the machine. Although one can use a variety of methods to perform this analysis (Root locus, Routh’s criterion, eigenanalysis), we will resort to a rather unconventional but quite intuitive analysis
procedure that conforms to that originally done in the deMello-Concordia paper. This analysis is based on the following observations made of the block diagram.

1. $\Delta T_d$, the damping torque, is in phase with speed deviation $\Delta \omega$.
2. $\Delta T_s$, synchronizing torque, is in phase with angle deviation $\Delta \delta$.
   We call this *synchronizing torque* because the higher it is, the more “stable” the machine will be with respect to loss of synchronism. This is confirmed by noting that high $K_1$ means low loading, as indicated by the fact that $K_1$ is the slope of the tangent to the power-angle curve at the operating point.
3. Because $\Delta \delta = (1/s) \Delta \omega$, we see that angle deviation lags speed deviation by 90 degrees in phase.

This leads to a “stability criterion”….

For stability, the composite electrical torque must have positive damping torque, which means that it must have a component in phase with speed deviation.

So we can perform a qualitative analysis using the following ideas:
- Any electrical torque contribution in phase with angle deviation contributes positive synchronizing torque.
- Any electrical torque contribution in phase with speed deviation contributes positive damping torque.

**Inertial torques:**
Let’s begin by just analyzing the “inertial” loops in the block diagram. These are the ones corresponding to $D$ and $K_1$, as indicated by the two bold arrows in Fig. 3.
Fig. 3
- We see that the torque contribution through D, $\Delta T_D$, is proportional to $\Delta \omega$ so it contributes positive damping, as expected.
- The torque contribution through $K_1$, $\Delta T_S$, is proportional to $\Delta \delta$ so it contributes positive synchronizing torque, 90 degrees behind the damping torque. Figure 4 below illustrates.

So as long as D is positive and there are no other effects, we obtain positive damping contributions from the inertial torques.
Armature reaction torque:

But now let’s consider the influence of armature reaction, when we get field weakening from the armature current. This effect is represented by the loop through $K_4$, $K_3$, and $K_2$, and is represented on the diagram by $\Delta T_{ar}$, as indicated by the bold arrow in Fig. 5.

![Diagram showing armature reaction torque](image)

**Fig. 5**

The transfer function for $\Delta T_{ar}$ is given by:

$$\frac{\Delta T_{ar}}{\Delta \delta} = - \frac{K_2 K_3 K_4}{1 + sK_3 \tau_{d0}'} = \frac{K_2 K_3 K_4}{1 + sK_3 \tau_{d0}'} \angle -180^\circ$$

Let’s evaluate the phase of this transfer function at $s = j\omega_{osc}$ where $\omega_{osc}$ is the frequency corresponding to the weakly damped electromechanical modes of oscillation (from 0.2 Hz up to about 2.0 Hz). From this last transfer function, we can identify the phase of the electrical torque contribution relative to $\Delta \delta$, which is:
\[ \phi_{ar} = -180 - \tan^{-1} K_3 \tau'_d 0 \omega_{osc} \]

What does this do to the resulting torque? Since it is negative, we draw the vector with an angle measured opposite the positive angle. We clearly get \(-180^\circ\), but we also get an additional negative angle from the \(\tan^{-1}\) term. Since \(\tau'_d 0 \omega_{osc}\) is positive, this additional angle must be between 0 and 90°. The effect is shown in Fig. 6 below.

**Fig. 6**

Note that the effect of armature reaction on composite torque is to increase damping torque (in phase with \(\Delta \omega\)) and to decrease synchronizing torque (in phase with \(\Delta \delta\)).
Excitation system torque:

This is the electrical torque that results from the $K_5$ and $K_6$ loops, as shown in Fig. 7 below.

![Fig. 7](image)

This torque may be expressed based on the block diagram as:

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s \tau'_d K_3} G_e(s) \left( \Delta V_{ref} - \Delta V_t \right)$$

Ignoring $\Delta V_{ref}$ (it represents manual changes in the voltage setting), and using (from eq. (**)): 

$$\Delta V_t = K_5 \Delta \delta + K_6 \Delta E'_q$$

we obtain

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s \tau'_d K_3} G_e(s) \left( -K_5 \Delta \delta - K_6 \Delta E'_q \right)$$

We want to express each torque as a function of $\Delta \delta$ or $\Delta \omega$. But the last expression has a $\Delta E'_q$. We can address this by noticing the
relation (from the block diagram) that \( \Delta T_{\text{exc}}=K_2 \Delta E'_{q} \Rightarrow \Delta E'_{q}=\Delta T_{\text{exc}}/K_2 \), and so we can write that

\[
\Delta T_{\text{exc}} = \frac{K_2 K_3}{1 + s \tau'_d K_3} G_e(s) \left( -K_5 \Delta \delta - \frac{K_6}{K_2} \Delta T_{\text{exc}} \right)
\]

Solving for \( \Delta T_{\text{exc}} \)

\[
\Delta T_{\text{exc}} = \frac{K_2 K_3}{1 + s \tau'_d K_3} G_e(s) \left( -K_5 \Delta \delta - \frac{K_6}{K_2} \Delta T_{\text{exc}} \right) = \frac{-K_2 K_3 K_5 \Delta \delta}{1 + s \tau'_d K_3} G_e(s) - \frac{K_3 K_6 \Delta T_{\text{exc}} G_e(s)}{1 + s \tau'_d K_3}
\]

\[
\Delta T_{\text{exc}} = \frac{-K_2 K_3 K_5}{1 + s \tau'_d K_3} G_e(s) \Delta \delta = \frac{-K_2 K_3 K_5 G_e(s) \Delta \delta}{1 + s \tau'_d K_3 + K_3 K_6 G_e(s)}
\]

\[
\Delta T_{\text{exc}} = \frac{-K_2 K_3 K_5}{1 + s \tau'_d K_3 + K_3 K_6 G_e(s)} G_e(s) (\Delta \delta)
\]

Now substitute equation (****) for the static excitation transfer function \( G_e(s) \), repeated here for convenience,

\[
G_e(s) = \frac{(1+sT_F)K_A}{(1+sT_F)(1+sT_A)+K_F K_A}
\]

\[
\Delta T_{\text{exc}} = \frac{-K_2 K_3 K_5 \left( \frac{(1+sT_F)K_A}{(1+sT_F)(1+sT_A)+K_F K_A} \right)}{1 + s \tau'_d K_3 + K_3 K_6 \left( \frac{(1+sT_F)K_A}{(1+sT_F)(1+sT_A)+K_F K_A} \right)} (\Delta \delta)
\]

Multiply top and bottom by \((1+sT_F)(1+sT_A)+K_F K_A\):

\[
\Delta T_{\text{exc}} = \frac{-K_2 K_3 K_5 K_A(1+sT_F)}{\left[(1+sT_F)(1+sT_A)+K_F K_A\right]\left[1 + s \tau'_d K_3 + K_3 K_6 K_A(1+sT_F)\right]} (\Delta \delta)
\]

The above relation appears quite challenging to analyze, but we can simplify the task greatly by observing that the denominator is third order. Thus, it will be possible to write the above relation as:
\[ \Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + s T_F)}{(s + p_1)(s + p_2)(s + p_3)} (\Delta \delta) \]

where \( p_i \) are the poles. We may have 3 real poles or 1 real with 2 complex. We are aware that static excitation systems generally contribute 1 real with 2 complex. We express the real pole as \( p_1 = \sigma_1 \) and the two complex poles as \( p_2 = \sigma_2 + j \omega_2 \), and \( p_3 = \sigma_3 + j \omega_3 \), where \( \sigma_i > 0 \) (otherwise \( s = -p_i \) will have a right-half-plane pole). Thus, the transfer function becomes:

\[ \Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + s T_F)}{(s + \sigma_1)(s + \sigma_2 + j \omega_2)(s + \sigma_3 + j \omega_3)} (\Delta \delta) \]

We want to evaluate the transfer function at \( s = j \omega_{osc} \), where \( \omega_{osc} \) is the frequency of oscillation of concern (we assume this frequency to be an interarea oscillation between groups of generators). Therefore,

\[ \Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + j \omega_{osc} T_F)}{(j \omega_{osc} + \sigma_1)(j \omega_{osc} + \sigma_2 + j \omega_2)(j \omega_{osc} + \sigma_3 + j \omega_3)} (\Delta \delta) \]

On combining imaginary terms in the denominator, we get:

\[ \Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + j \omega_{osc} T_F)}{(\sigma_1 + j(\omega_{osc}))(\sigma_2 + j(\omega_2 + \omega_{osc}))(\sigma_3 + j(\omega_3 + \omega_{osc}))} (\Delta \delta) \]

We are interested in the phase of \( \Delta T_{exc} \) relative to \( \Delta \delta \).

**Fact**: When the generator is heavily loaded, it is possible for \( K_5 \) to be negative. See Ex 6.6, Fig. 6.1, and section 8.4.3 in A&F. This makes the numerator of the previous transfer function positive.

A simulation of such a case is shown in Fig. 8 below. The solid curve represents generators with fast high-gain excitation systems, but no PSS. The other two curves represent significantly fewer of such generators.
Repeating our transfer function:

$$\Delta T_{\text{exc}} = \frac{-K_2 K_3 K_5 K_A (1 + j\omega_{\text{osc}} T_F)}{(\sigma_1 + j(\omega_{\text{osc}}))(\sigma_2 + j(\omega_2 + \omega_{\text{osc}}))(\sigma_3 + j(\omega_3 + \omega_{\text{osc}}))}(\Delta \delta)$$

Assuming $K_5 < 0$ (so that the negative sign of the transfer function cancels the negative sign of $K_5$), the phase of $\Delta T_{\text{exc}}$ relative to $\Delta \delta$ is given by

$$\phi_{\text{exc}} = \tan^{-1} \frac{\omega_{\text{osc}} T_F}{\sigma_1} - \tan^{-1} \frac{\omega_{\text{osc}}}{\sigma_1} - \tan^{-1} \frac{\omega_2 + \omega_{\text{osc}}}{\sigma_2} - \tan^{-1} \frac{\omega_3 + \omega_{\text{osc}}}{\sigma_3}$$

Consider some typical data, were $\omega_{\text{osc}} = 4.396 \text{rad/sec}$ (0.7Hz), $\sigma_1 = 0.2$, $\sigma_2 + j\omega_2 = 5 + j4.5$, $\sigma_3 + j\omega_3 = 5 - j4.5$, $T_F = 0.5$. Then

$$\phi_{\text{exc}} = \tan^{-1} 4.396(0.5) - \tan^{-1} \frac{4.396}{0.2} - \tan^{-1} \frac{8.896}{5} - \tan^{-1} \frac{-0.104}{5}$$

$$\phi_{\text{exc}} = \tan^{-1} 2.198 - \tan^{-1} 21.98 - \tan^{-1} 1.7792 - \tan^{-1} -0.0208$$

$$= 65.536 - 87.395 - 60.662 + 1.192 = -81.329$$
Using some typical data, the above identifies the phase lag to be -81.329°. In this case, our diagram will appear as in Fig. 9 below.

![Fig. 9](image)

and we see that the damping can go negative for fast (small $T_A$)-high gain (large $K_A$) excitation systems under heavy loading conditions! And this explains the effect observed in Fig. 8.

So what do we do about this?

**Solution 1**: Limit $K_A$ to as high as possible without causing undamped oscillations. This limits the magnitude (length) of the $\Delta T_{exc}$ vector (see Fig. 9). But high-gain, fast response excitation systems are good for transient (early-swing) instability! This is indicated by the fact that, in Fig. 9, the $\Delta T_{exc}$ vector increases the synchronizing torque (i.e., it causes the resultant torque to be further to the right along the $\Delta \delta$ axis). And so we would rather not do this. This is a “conflicting problem” in that increasing $K_A$ helps transient (early swing) stability but hurts oscillatory (damping). In the words of de Mello & Concordia (pg. 6 of the paper posted on the website):
Solution 2: Provide a supplementary torque component that offsets the negative damping torque caused by the excitation system. Again, in the words of de Mello and Concordia:

> Basically, the idea is to push (rotate forward) our torque vector back into the upper-right quadrant. Thus we need to phase-advance the torque vector by between 20 to 90 degrees. We will introduce a supplementary torque that does this, denoted by $\Delta T_{\text{pss}}$, as indicated in Fig. 10 below.
Fig. 10
The transfer function $K_SG_{\text{lead}}(s)$ is intended to provide the supplementary signal $\Delta T_{\text{PSS}}$ as illustrated in Fig. 11 below.

Fig. 11
We will take $\Delta \omega$ as the feedback signal for our control loop to provide $\Delta T_{\text{PSS}}$ (we could also use angle deviation, but speed deviation is easier to obtain as a control signal).
We may provide “shaping” networks to process the feedback signal in providing it with the proper amount of phase (lead or lag). For example (see Dorf, pg. 362-363), a network to provide phase lead is shown in Fig. 12.

![Phase Lead Network Diagram](image)

**Fig. 12**

(One can alternatively use digital signal processing techniques.) In the phase lead network above, we get that

\[
G_{lead}(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \left\{ \frac{R_1}{Cs} \right\}/\{R_1 + 1/ Cs\}} = \frac{1 + \alpha \tau s}{\alpha (1 + \tau s)}
\]

where

\[
\alpha = \frac{R_1 + R_2}{R_2}, \quad \tau = \frac{R_1 R_2}{R_1 + R_2} C
\]

Dorf shows that the maximum value of phase lead given by the above network occurs at a frequency of

\[
\omega_m = \frac{1}{\tau \sqrt{\alpha}}
\]

and the corresponding phase lead you get at this frequency is given by
\[
\sin \phi_m = \frac{\alpha - 1}{\alpha + 1}
\]

So the idea is that you can know the frequency \(\omega_m\) that you want to provide the maximum phase lead. This is the frequency of your most troublesome electromechanical mode and is considered to be the PSS tuning mode.

Note from the above diagram that the desired supplementary signal \(\Delta T_{\text{PSS}}\) is actually *lagging* \(\Delta \omega\), so one might think that we should provide phase lag, not phase lead, to the input signal (which is actuated by \(\Delta \omega\)). This would in fact be the case if we could introduce the “shaped” signal (the output of \(G_{\text{lead}}\)) directly at the machine shaft.

However, this is not very easy to do because we cannot produce a mechanical torque directly from an electrical signal transduced from rotor speed.

In fact, the only place we can introduce an electrical signal is at the voltage regulator, i.e., the input to the excitation system, \(G_e(s)\).

This causes a problem in that we now incur the phase lag introduced by \(G_e(s)\) and the \(\tau_{d0}\) block, which is typically around \(\phi_{\text{exc}} = -80\) degrees as discussed previously.

So this means that we must think about it in the following way:
1. We start with the \(\Delta \omega\) signal.
2. We introduce a phase lead of an amount equal to \(X\). What is \(X\)?
3. We incur \(-80\) degrees of phase lag from \(\phi_{\text{exc}}\).
4. We provide \(\Delta T_{\text{PSS}}\) lagging \(\Delta \omega\) by, say \(-25\) degrees. This means that \(X-80\approx -25\) degrees \(\Rightarrow\) \(X=55\) degrees.

Therefore \(X\) must be about 55 degrees. So we must provide an appropriate shaping network. This shaping network is referred to as \(G_{\text{lead}}(s)\).
Therefore we can write
\[ \sin 55 = \frac{\alpha - 1}{\alpha + 1} \]
And solve for \( \alpha \):
\[ \sin 55 = 0.819 = \frac{\alpha - 1}{\alpha + 1} \Rightarrow 0.819(\alpha + 1) = \alpha - 1 \]
\[ 0.819\alpha + 0.819 = \alpha - 1 \Rightarrow -0.181\alpha = -1.819 \]
\[ \Rightarrow \alpha = 10.05 \]
Then choose \( \tau \) based on
\[ \omega_{osc} = \omega_m = \frac{1}{\tau\sqrt{\alpha}} \]
where \( \omega_m = 2\pi(f_{osc}) \), and \( f_{osc} \) is the frequency of the oscillation “problem mode.” That is,
\[ \omega_{osc} = \frac{1}{\tau\sqrt{\alpha}} \Rightarrow \tau\sqrt{\alpha} = \frac{1}{\omega_{osc}} \Rightarrow \tau = \frac{1}{\omega_{osc}\sqrt{\alpha}} \]
and for \( \omega_{osc} = 4.396 \text{rad/sec (0.7Hz)} \), we have:
\[ \tau = \frac{1}{4.396\sqrt{10.05}} = 0.0718 \]
Then, if you are using an RC phase lead network, you can choose \( R \) and \( C \) according to
\[ \alpha = \frac{R_1 + R_2}{R_2}, \quad \tau = \frac{R_1R_2}{R_1 + R_2}C \]
Note the **principle** behind the power system stabilizer:
- Cancel the phase lags introduced by the excitation system with
  - the right amount of lead compensation so that
    - the total torque exerted on the shaft by the excitation control effect will be
      - **in phase** with speed deviation and
    - thus provide positive damping.
So the PSS introduces a supplementary signal into the voltage regulator with proper phase and gain adjustments to produce a component of damping that will be sufficient to cancel the negative damping from the exciters.

Final Exam Question #1:
Your book provides expressions for $K_1$-$K_6$ on pp. 223-225 for the case that the transmission line connecting the generator to the infinite bus has impedance of $Z_e=R_e+jX_e$. Also, at the end of these notes, the same constants $K_1$-$K_6$ are derived for the case that the transmission line connecting the generator to the infinite bus has impedance of $Z_e=jX_e$ (i.e., $R_e=0$). Starting from the expressions given in your book, set $R_e=0$ and show that those expressions collapse to the expressions given at the end of these notes.

Final Exam Question #2:
Work problem 8.1 in your text. Observe the transfer function $G_e(s)$ associated with equation (8.14) differs from the transfer function that we used in the above notes.

Final Exam Question #3:
Work problem 8.2 in your text.
APPENDIX 3
DERIVATION OF BLOCK DIAGRAM MODEL
FOR LOADED GENERATOR CONDITIONS

Under loaded conditions, a disturbance will cause the power angle, $\delta$, between the voltage behind the transient reactance, $E'_d$ and the infinite bus voltage, $V_{in}$, to deviate from its non-zero steady-state value and then oscillate. The oscillations die out in the damped or stable case, and they grow in the undamped or unstable case. From the phasor diagram in Figure A3-1, it is apparent that the change in $\delta$ is related to the change in $E'_d$ and the change in $V_{in}$.

We derive the relationships between these quantities in what follows, assuming a synchronous generator is connected to an infinite bus, as above, and that the armature and external resistances are zero.

Derivation of Relationship Between $E_{fd}$ and $\delta$.

We define all reactances as the generator reactance plus the line reactance, i.e.,

$$X_d = X_{d,gen} + X_L$$

$$X'_d = X'_{d,gen} + X_L$$

$$X_f = X_{f,gen} + X_L$$

With these definitions, the phasor diagram indicates that

$$\frac{E'_d - V_{in} \cos \delta}{E'_d - V_{in}} = \frac{X_d}{X'_d} = \frac{1}{K_3}$$

where

$$K_3 = \frac{X'_d}{X_d}$$

Solution of the above equation for $E_d$ in terms of $K_3$ yields

$$E_d = \frac{E'_d}{K_3} + V_{in}[1 - \frac{1}{K_3} \cos \delta]$$

(1)

The equation for the generator field winding in terms of field quantities is

$$v_f = i_f + \frac{dl_f}{dt}$$

(2)

To get this equation in terms of stator quantities, we define a constant, $k/r_f$, analogous to a transformer turns ratio, that allows us to refer quantities from the rotor side to the stator side. The field voltage referred to the stator side is therefore

$$E_{fd} = \frac{k}{r_f} v_f \Rightarrow v_f = \frac{k}{r_f} E_{fd}$$

Because $i_f r_f$ is equal to the field voltage $v_f$ under steady-state conditions, it follows that $E_f$ is

$$E_f = \frac{k}{r_f} i_f r_f \Rightarrow i_f r_f = \frac{r_f}{k} E_f$$
This leaves only the voltage behind the transient reactance, which may be defined as

\[ E_t' = \frac{k}{L_f} \lambda_f \Rightarrow \frac{dE_t'}{dt} = \frac{L_f}{k} \frac{dE_t'}{dt} \]

Substitution of the above three expressions into equation 2 yields

\[ \frac{r_f}{k} E_{fd} = \frac{r_f}{k} E_t + \frac{L_f}{k} \frac{dE_t'}{dt} \]

Multiplying through by \( k/r_f \) gives

\[ E_{fd} = E_t + T_{A0} \frac{dE_t'}{dt} \Rightarrow E_t = E_{fd} - T_{A0} \frac{dE_t'}{dt} \]  

(3)

where \( T_{A0} = L_f/r_f \) is the open circuit transient time constant. Substitution of equation 3 into equation 1, multiplying by \( K_3 \) and rearranging, we have

\[ K_3 T_{A0} \frac{dE_t'}{dt} + E_t = K_3 E_{fd} + V_{m0}[1 - K_3] \sin \delta \]  

(4)

We assume that all voltages in equation 4 are normalized to the same base voltage.

Recalling the swing equation,

\[ M \frac{d^2 \delta}{dt^2} + D \frac{d\delta}{dt} + P_s = P_m \]  

(5)

where \( P_s \) and \( P_m \) are the electrical and mechanical powers, respectively, and \( P_s \) may be expressed as

\[ P_s = \frac{E_t' V_{m0}}{X_t} \sin \delta + \frac{V_{m0}^2}{2} \frac{1}{X_t} - \frac{1}{X_t} \sin 2\delta \]

Linearizing equations 4 and 5 about the steady-state operating point, \( E_{t0}, \delta_0 \), we have

\[ K_3 T_{A0} \frac{d\Delta E_t'}{dt} + \Delta E_t' = K_3 \Delta E_{fd} - V_{m0}(1 - K_3) \Delta \sin \delta_0 \]  

(6)

\[ M \frac{d^2 \Delta \delta}{dt^2} + D \frac{d\Delta \delta}{dt} + P_{s0} \frac{\partial P_s}{\partial \delta} \mid_{\delta_0} \Delta \delta + \frac{P_{s0}}{P_{s0}} \frac{\partial P_s}{\partial \delta} \mid_{\delta_0} \Delta E_t' = \Delta P_m \]  

(7)

Defining the constants \( K_1 \) through \( K_4 \) as

\[ K_1 = \frac{\partial P_s}{\partial \delta} \mid_{\delta_0} \Delta E_t' = \frac{E_t V_{m0}}{X_t} \cos \delta_0 + \frac{V_{m0}^2(X_t - X_t')}{X_t X_t'} \cos 2\delta_0 \]

\[ K_2 = \frac{\partial P_s}{\partial \delta} \mid_{\delta_0} \Delta \delta = \frac{V_{m0}}{X_t} \cos \delta_0 \]

\[ K_3 = \frac{V_{m0}}{X_t} \]

(previously defined)

\[ K_4 = \left[ K_4 - 1 \right] V_{m0} \sin \delta_0 = \frac{X_t - X_t'}{X_t} \frac{X_t - X_t'}{X_t} \frac{X_t - X_t'}{X_t} \]

Substituting these constants into linearized equations 6 and 7,

\[ K_3 T_{A0} \Delta E_t' + \Delta E_t' = K_3 \Delta E_{fd} - K_3 K_4 \Delta \delta \]  

(8)

\[ M \Delta \delta + D \Delta \delta + K_1 \Delta \delta + K_3 \Delta E_t' = \Delta P_m \]  

(9)

LaPlace transforming equations 8 and 9, and noting that \( \Delta E_t'(0^-) = \Delta \delta(0^-) = 0 \), we have

\[ K_3 T_{A0} s \Delta E_t'(s) + \Delta E_t'(s) = K_3 \Delta E_{fd}(s) - K_3 K_4 \Delta \delta(s) \]  

(10)

\[ (M s^2 + D s + K_1) \Delta \delta(s) + K_3 \Delta E_t'(s) = \Delta P_m(s) \]  

(11)

Rearranging equations 10 and 11, we have

\[ (K_3 T_{A0}s + 1) \Delta E_t'(s) = K_3 \Delta E_{fd}(s) - K_2 K_4 \Delta \delta(s) \]  

(12)

\[ (M s^2 + D s + K_1) \Delta \delta(s) = \Delta P_m - K_2 \Delta E_t' \]  

(13)

\[ \delta = \]  

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Inspecting equations 12 and 13, we can draw the associated block diagram, given in Figure A3-2.

Derivation of Relationship Between δ and \( V_T \).

The block diagram model above produces only \( \Delta \delta \) and \( \Delta E' \). The excitation system, however, senses the terminal voltage \( V_T \) and feeds it back to the excitier input. We now determine how changes in \( \delta \) and \( E'_q \) affect the feedback quantity \( V_T \). In other words, we desire to find two constants \( K_\delta \) and \( K_E \) such that

\[
\Delta V_T = K_\delta \Delta \delta + K_E \Delta E'_q
\]

From the phasor diagram in Figure A3-1, we can write

\[ E'_q e^{j\delta} = V_{\infty} + jX'_d jI_d e^{j\delta} + jX_q I_q e^{j\delta} \]

Multiplying by \( e^{-j\delta} \) and converting to trig functions,

\[ E'_q = V_{\infty} \cos \delta - jV_{\infty} \sin \delta - X'_d I_d + jX_q I_q \]

Equating real and imaginary parts,

\[
E'_q = V_{\infty} \cos \delta - X'_d I_d \Rightarrow I_d = \frac{V_{\infty} \sin \delta}{X'_d} \quad (15)
\]

\[
V_{\infty} \sin \delta = X_q I_q \Rightarrow I_q = \frac{V_{\infty} \sin \delta}{X_q} \quad (16)
\]

The generator terminal voltage \( V_T \) is related to the infinite bus voltage \( V_{\infty} \) by the equation

\[
\bar{V}_T = V_{\infty} + jX_L I
\]

where the bar over \( V_T \) and \( I \) imply phasor quantities. Now we introduce the \( d \) and \( q \) quantities \( V_d, V_q, I_d, \) and \( I_q \) as

\[
\bar{V}_T = (V_d + jV_q)e^{j\delta} \quad (18)
\]

\[
\bar{I} = (I_d + jI_q)e^{j\delta} \quad (19)
\]

Substituting equations 18 and 19 into 17, we have

\[
(V_d + jV_q)e^{j\delta} = V_{\infty} + jX_L(I_d + jI_q)e^{j\delta}
\]

Multiplying through by \( e^{-j\delta} \) gives

\[
V_d + jV_q = V_{\infty} \cos \delta - jV_{\infty} \sin \delta + jX_L I_q - X_L I_d
\]

Equating real and imaginary parts, we have

\[
V_d = V_{\infty} \cos \delta - X_L I_q \quad (20)
\]

\[
V_q = X_L I_d - V_{\infty} \sin \delta \quad (21)
\]

Substituting equations 15 and 16 into equations 20 and 21, respectively, we have

\[
V_d = \frac{X_L}{X_q} V_{\infty} \sin \delta = \frac{V_{\infty} \sin \delta (X_L/X_q - 1)}{X_q} \quad (22)
\]

\[
V_q = \frac{X_L}{X_q} V_{\infty} \cos \delta - V_{\infty} \sin \delta = \frac{V_{\infty} \sin \delta (X_L/X_q - 1)}{X_q} \quad (23)
\]
Noting that,

\[ 1 - \frac{X_L}{X_d} = \frac{X_L - X_d}{X_d} = \frac{X_d \tan \delta}{X_d} \]
\[ \frac{X_L}{X_d} - 1 = \frac{X_L - X_d}{X_d} = \frac{X_d \tan \delta}{X_d} \]

equations 22 and 23 may be simplified to

\[
V_t = \frac{X_d \tan \delta}{X_d} V_{vo} \cos \delta + \frac{X_L}{X_d} E_q^f
\]
\[
V_2 = -\frac{X_d \tan \delta}{X_d} V_{vo} \sin \delta
\]

Also,

\[
V_t^2 = V_t V_2 = (V_t + jV_2)e^{j\delta} (V_t - jV_2)e^{-j\delta}
\]
\[
\Rightarrow V_t = \sqrt{V_t^2 + V_2^2}
\]

Equations 24, 25, and 26 give the dependence of \(V_t\) on \(E_q^f\) and \(\delta\), i.e., \(V_t = V_t(E_q^f, \delta)\). We now find the linearized dependence of \(\Delta V_t\) on \(\Delta E_q^f\) and \(\Delta \delta\), i.e.,

\[
\Delta V_t = \frac{\partial V_t}{\partial E_q^f} \Delta E_q^f + \frac{\partial V_t}{\partial \delta} \Delta \delta
\]

Using equations 24, 25, and 26, and the chain rule for differentiation, we can compute \(K_v\) and \(K_E\) as

\[
K_v = \left. \frac{\partial V_t}{\partial \delta} \right|_{E_q^f = 0} = \frac{X_d \tan \delta}{X_d} \sin \delta
\]
\[
K_E = \left. \frac{\partial V_t}{\partial E_q^f} \right|_{\delta = 0} = \frac{X_d \tan \delta}{X_d} \cos \delta
\]

where \(V_{vo}, V_{vo},\) and \(V_{vo}\) are computed using equations 24, 25, and 26 with the steady-state values \(\delta\) and \(E_q^f\) substituted is for \(\delta\) and \(E_q^f\), respectively.

Modeling equation 27 in block diagram form, together with the block diagram of \(\Delta E_q^f/\Delta \delta\) (Figure A3-2) and the block diagram of the excitation system (Figure 18), we have the block diagram for the entire excitation control system, Figure A3-3.