Linearized Analysis of the Synchronous Machine for PSS

Chapter 8 of VMAF does three basic things:

1. Shows how to linearize the 8-state model (model #1, IEEE #2.2, called "full model without G-cct.") of a synchronous machine connected to an infinite bus using the current-state-space model (sections 8.3-8.4) and using the flux-linkage state-space model (section 8.5). This material is useful for understanding the modeling required for power system eigenvalue calculation programs in SSAT. Some information on these tools follow:
   a. Kundur, Rogers, Wong, Wang, and Lauby, "A comprehensive computer program package for small signal stability analysis of power systems," IEEE Transactions on Power Systems, Vol. 5, No. 4, Nov., 1990 (on website).
   b. Wang, Howell, Kundur, Chung, and Xu, "A tool for small-signal security assessment of power systems," IEEE Transactions on Power Systems, …(on website).
   c. M. Crow, "Computational methods for electric power systems," chapter 7 on "Eigenvalue Problems," CRC Press, 2003.
2. Shows how to develop the A matrix for multimachine systems (Section 8.6).
3. Linearizes the one-axis model of a synchronous machine connected to an infinite bus (sections 8.7). This material is useful for the conceptual understanding of why power system stabilizers are needed.

In these notes, we will address (3) and then return to (1) in the next class.

Some additional references for you on this issue are references [10, 11] given at the end of chapter 8. These two references are:

[10] W. Heffron and R. Phillips, "Effect of modern amplidyne voltage regulators on under-excited operation of large turbine generators," AIEE Transactions, pt. III, vol. 71, pp. 692-696, 1952.

[11] F. deMello and C. Concordia, “Concepts of synchronous machine stability as affected by excitation control,” IEEE Transactions on Power Apparatus and Systems, PAS-88, pp 316-329, 1969.

Reference [10] came first and produced what is commonly referred to in the literature as the Heffron-Phillips model of the linearized synchronous machine. Reference [11] extended the Heffron-Phillips model and is the most well-known. Reference [11] is also viewed as the seminal work that motivated the need for power system stabilizers (PSS). This paper is on the web site for you to download, read, and place in your notebook. You will note that it contains material quite similar to what follows below.

VMAF also provides background on this issue in several separate locations, found in the following sections:
- Section 3.5.1: Voltage regulator with one time lag
- Section 8.7: Simplified linear model
- Section 8.8: Block diagrams
- Section 9.8.2: Continuously regulated systems
- Section 9.9: State-space representation of the excitation system
- Section 10.4: Effect of excitation on small-signal stability
- Section 10.5: Root-locus analysis of a regulated machine connected to an infinite bus
- Section 10.7: Supplementary stabilizing signals
- Section 10.8: Linear analysis of the stabilized generator
- Section 10.9: PSS tuning in multimachine power systems
- Section 10.10: Alternate types of PSS
- Section 10.11.2: Effect of the power system stabilizer

I will provide the minimal analysis necessary to see the basic issue.

The analysis uses the simplest model possible for which the excitation system may be represented – the one-axis model (model 7, IEEE #1.0), loaded through a connection to an infinite bus.

The one-axis model is a 3-state model, developed based on the following main assumptions (see p. 312 of VMAF):

1. Only the field winding is represented (so no G-circuit and no amortisseur windings).
2. No stator winding resistance.
3. Speed voltage terms assume ω is fixed at rated speed.
4. $d\lambda_d/dt = d\lambda_q/dt = 0$ (no stator transients).
5. Saturation is neglected.

The nonlinear equations for the one-axis model are given by eqs. (4.294) and (4.297) in VMAF, as follows (the below are in slightly different, but equivalent form to (4.294) and (4.297)):

$$\dot{E}'_q = \frac{1}{\tau'_{do}} E_{FD} - \frac{1}{\tau'_{do}} E_q, \text{ where } E_q = E'_q - (x'_d - x_d)I_d$$

$$\dot{\omega} = \frac{1}{\tau_j} T_m - \frac{1}{\tau_j}\left[E'_q I_q + (x'_d - x_q) I_d I_q\right] - \frac{1}{\tau_j} D\omega$$

$$\dot{\delta} = \omega - 1$$

To identify basic concepts, Concordia and deMello assumed a single machine connected to an infinite bus through a transmission line having series impedance of R_e+jX_e, as illustrated in Fig. 1.

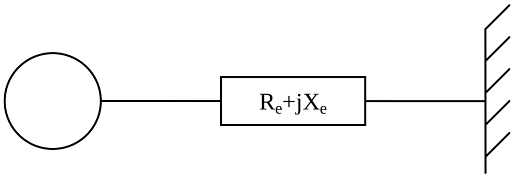

Fig. 1

VMAF, in Section 8.7, linearizes the above state equations for the one-axis model for the Fig. 1 case, resulting in

$$
\begin{aligned}
&\Delta\dot{E}'_q = -\left(\frac{1}{K_3\tau'_{do}}\right)\Delta E'_q - \left(\frac{K_4}{\tau'_{do}}\right)\Delta\delta + \left(\frac{1}{\tau'_{do}}\right)\Delta E_{FD} \\
&\Delta\dot{\omega} = -\left(\frac{1}{\tau_j}\right)\Delta T_e + \left(\frac{1}{\tau_j}\right)\Delta T_m \qquad (8.148) \\
&\Delta\dot{\delta} = (\Delta\omega)
\end{aligned}
$$

VMAF also provide the following linearized expressions for torque and terminal voltage of a generator with one-axis model connected to an infinite bus, resulting in Eqs. (8.137) and (8.141) of VMAF.

$$
\begin{aligned}
\Delta T_e &= K_1\Delta\delta + K_2\Delta E'_q + D\Delta\omega \\
\Delta V_t &= K_5\Delta\delta + K_6\Delta E'_q
\end{aligned}
\qquad (**)
$$

The first of equations (**) can be used to simplify the second one of the second one of (8.148), which results in

$$
\begin{aligned}
&\Delta\dot{E}'_q = -\left(\frac{1}{K_3\tau'_{do}}\right)\Delta E'_q - \left(\frac{K_4}{\tau'_{do}}\right)\Delta\delta + \left(\frac{1}{\tau'_{do}}\right)\Delta E_{FD} \\
&\Delta\dot{\omega} = \left(\frac{-K_2}{\tau_j}\right)\Delta E'_q - \left(\frac{K_1}{\tau_j}\right)\Delta\delta - \frac{D}{\tau_j}(\Delta\omega) + \left(\frac{1}{\tau_j}\right)\Delta T_m \qquad (8.149) \\
&\Delta\dot{\delta} = (\Delta\omega)
\end{aligned}
$$

In (**), (8.149), K_1-K_6 are described on the next page.

The LaPlace transform of the above equations (**) and (8.149), with some manipulation, results in the following relations:

$$
\begin{aligned}
\Delta T_e &= K_1\Delta\delta + K_2\Delta E'_q + D\Delta\omega \\
\Delta V_t &= K_5\Delta\delta + K_6\Delta E'_q \\
\Delta E'_q &= \frac{K_3}{1+K_3\tau'_{d0}\,s}\Delta E_{FD} - \frac{K_3K_4}{1+K_3\tau'_{d0}\,s}\Delta\delta \\
\Delta\omega &= \frac{1}{s\tau_j}\left(\Delta T_m - \Delta T_e\right) \qquad (*) \\
\Delta\delta &= \frac{1}{s}\Delta\omega
\end{aligned}
$$

where, in (*), the variables ΔT_e, ΔV_t, $\Delta E'_q$, $\Delta\omega$, and $\Delta\delta$ represent LaPlace transforms of their corresponding time-domain functions (a slight abuse of notation).

Finally, we note that E_{FD}, the stator EMF produced by the field current and corresponding to the field voltage v_F, is a function of the voltage regulator. Under linearized conditions, the change in E_{FD} is proportional to the difference between changes in the reference voltage and changes in the terminal voltage, i.e.,

$$\Delta E_{FD} = G_e(s)(\Delta V_{ref} - \Delta V_t) \qquad (***)$$

where $G_e(s)$ is the transfer function of the excitation system.

In the above equations (*) and (**), the various constants K_1-K_6 are:

$$K_1 = \left.\frac{\Delta T_e}{\Delta\delta}\right|_{E'_q=E'_{q0}} \qquad K_2 = \left.\frac{\Delta T_e}{\Delta E'_q}\right|_{\delta=\delta_0} \qquad K_4 = \left.\frac{-1}{K_3}\frac{\Delta E'_q}{\Delta\delta}\right|_{E_{FD}=constant}$$

$$K_5 = \left.\frac{\Delta V_t}{\Delta\delta}\right|_{E'_q=E'_{q0}} \qquad K_6 = \left.\frac{\Delta V_t}{\Delta E'_q}\right|_{\delta=\delta_0}$$

and K_3 is an impedance factor that accounts for the loading effect of the external impedance (see (8.128). Your text, on pages 313, 314, and 315, provides exact expressions for these constants for the case of the one-axis model we are analyzing, under the condition that the line connecting the generator to the infinite bus has impedance of $Z_e=R_e+jX_e$. I have attached an appendix to these notes that develop expressions for these constants under condition that $Z_e=jX_e$. The paper[1] on the website also provides these constants both ways, i.e., for $Z_e=R_e+jX_e$ and for $Z_e=jX_e$. There are two comments worth mentioning here:

1. K_1 is the synchronizing power coefficient and will be assumed positive in all that we do here.

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[1] P. de Mello and C. Concordia, “Concepts of synchronous machine stability as affected by excitation control,” IEEE Transactions on Power Apparatus and Systems, PAS-88: 316-329, 1969.

2. K_2, K_3, and K_6 are always positive.
3. VMAF express K_4 as (see eq. 8.130)

$$K_4 = \frac{1}{K_3} \frac{\Delta E'_q}{\Delta \delta}\bigg|_{E_{FD}=constant}$$

However, K_3, being an impedance factor, is positive. We want K_4 to also be positive; however, the above expression suggests that E'_q would increase with an increase in angle (or loading). This is counter to the idea of armature reaction, where the internal flux decreases as a result of stator current, as indicated by our conceptual analysis in the notes called "ExcitationSystems" per the below figure:

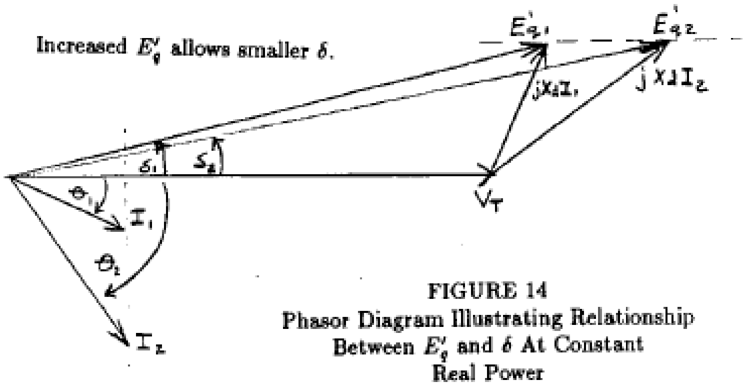

Increased E'_q allows smaller δ.

FIGURE 14
Phasor Diagram Illustrating Relationship Between E'_q and δ At Constant Real Power

In fact, the book itself indicates as much via eq. (eq. 3.11) where it says that "K_4 is the demagnetizing effect of a change in the rotor angle (at steady-state)," which is given by the following relationship:

$$K_4 = -\frac{1}{K_3} \lim_{t\to\infty} \Delta E'(t)\Big|_{\substack{\Delta v_F=0 \\ \Delta\delta=u(t)}} \qquad (3.11)$$

where we note the negative sign out front. Therefore, the book expression (eq. 8.130), needs a negative sign.

In eq. (***), $G_e(s)$ is the transfer function of the excitation system. Recall that there are several different kinds (DC, AC Alternator, and static), each requiring somewhat different modeling. One kind that has become quite common is the "static" excitation system, represented by Fig. 2a, where K_A is the exciter gain and T_A is the exciter time constant.

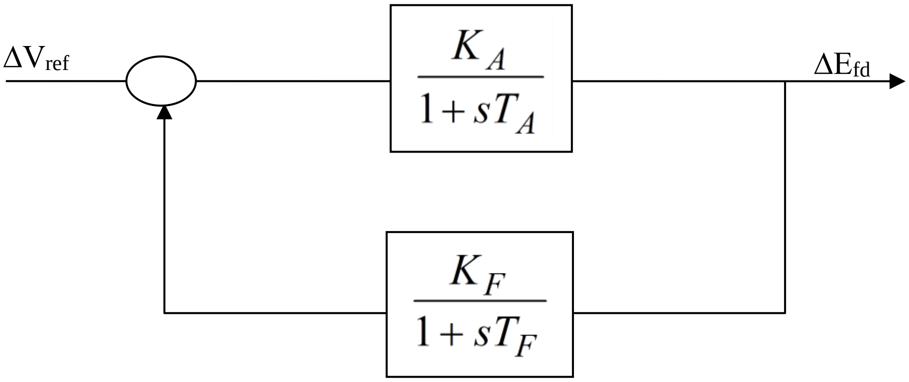


Fig. 2a

Fig. 2a is characterized by the following transfer function.

$$G_e(s) = \frac{(1+sT_F)K_A}{(1+sT_F)(1+sT_A)+K_F K_A} \qquad (****)$$

The static excitation is typically very fast (no rotating machine in the loop). Fast excitation response is beneficial for transient stability because generator terminal voltages see less voltage depression for less time during and after network faults. Such speed of excitation response can, however, cause problems for damping, as we shall see in what follows.

We repeat the equations (*) and (***) below.

$$\Delta T_e = K_1\Delta\delta + K_2\Delta E'_q + D\Delta\omega$$

$$\Delta V_t = K_5\Delta\delta + K_6\Delta E'_q$$

$$\Delta E'_q = \frac{K_3}{1+K_3\tau'_{d0}\,s}\Delta E_{FD} - \frac{K_3K_4}{1+K_3\tau'_{d0}\,s}\Delta\delta$$

$$\Delta\omega = \frac{1}{s\tau_j}(\Delta T_m - \Delta T_e) \qquad (*)$$

$$\Delta\delta = \frac{1}{s}\Delta\omega$$

$$\Delta E_{FD} = G_e(s)(\Delta V_{ref} - \Delta V_t) \qquad (***)$$

We may extract from the above equations (*) and (***) a block diagram relation, as seen in Fig. 2. Note that in this block diagram, τ_j=M (instead of τ_j=Mω_B as given in notes on “TorqueEquation”**)**. Careful comparison of this block diagram to Fig. 10.17 in your text will suggest they are the same.

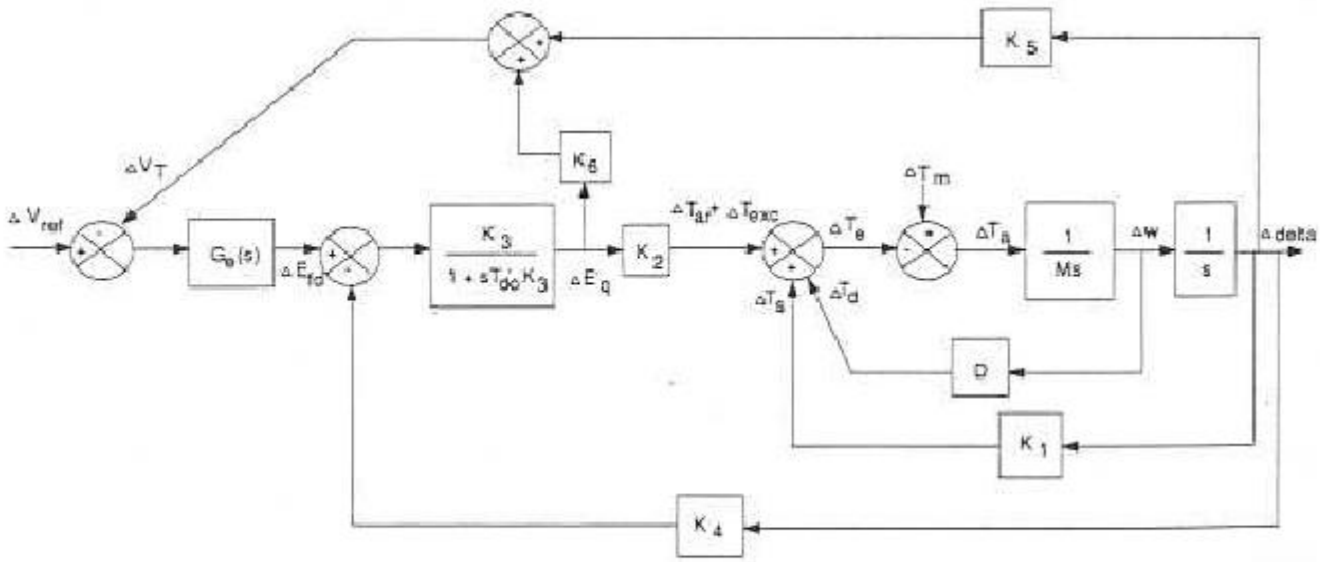


Fig. 2

We will use this block diagram to analyze the stability behavior of the machine. Although one can use a variety of methods to perform this analysis (Root locus, Routh’s criterion, eigenanalysis), we will resort to a rather unconventional but quite intuitive analysis procedure that conforms to that originally done in the deMello-Concordia paper. This analysis is based on the following observations made of the block diagram.

1. ΔT_d, the damping torque, is in phase with speed deviation $\Delta\omega$.
2. ΔT_s, synchronizing torque, is in phase with angle deviation $\Delta\delta$. We call this *synchronizing torque* because the higher it is, the more “stable” the machine will be with respect to loss of synchronism. This is confirmed by noting that high K_1 means low loading, as indicated by the fact that K_1 is the slope of the tangent to the power-angle curve at the operating point.

3. Because $\Delta\delta=(1/s)\ \Delta\omega$, we see that angle deviation lags speed deviation by 90 degrees in phase.

This leads to a "stability criterion"….

For stability, the composite electrical torque must have positive damping torque (it must have a component in phase with speed deviation) and positive synchronizing torque (it must have a component in phase with angle deviation).

So we can perform a qualitative analysis using the following ideas:

- Any electrical torque contribution in phase with angle deviation contributes positive synchronizing torque.
- Any electrical torque contribution in phase with speed deviation contributes positive damping torque.

Inertial torques:

Let's begin by just analyzing the "inertial" loops in the block diagram. These are the ones corresponding to D and K_1, as indicated by the two bold arrows in Fig. 3.

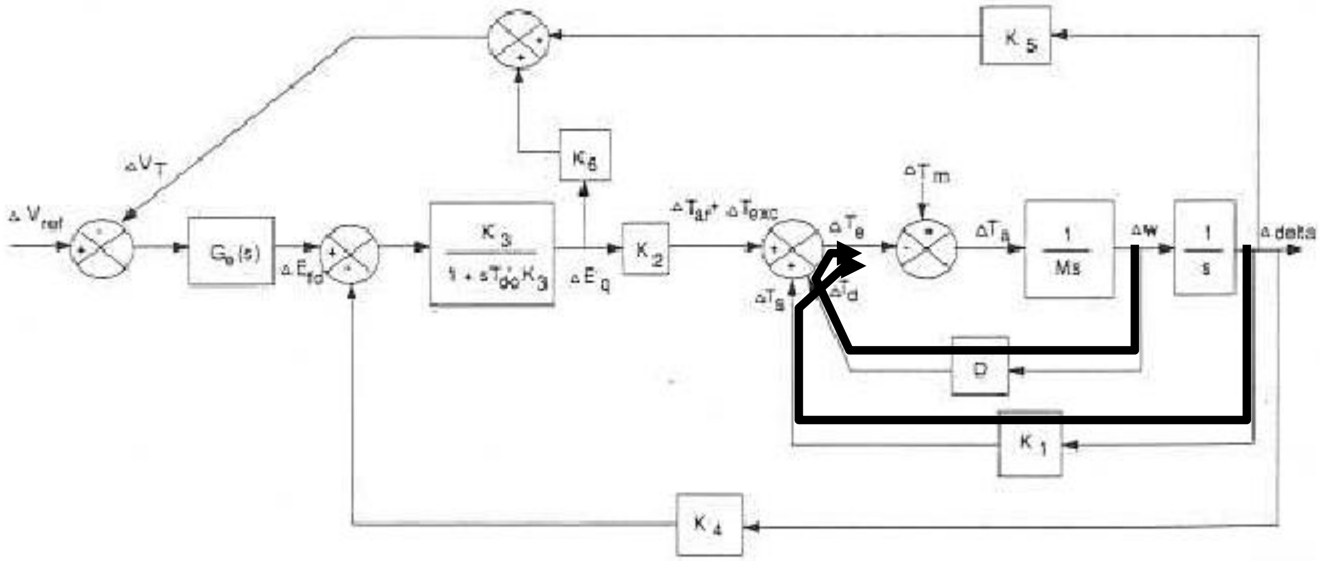


Fig. 3

- We see that the torque contribution through D, ΔT_D, is proportional to $\Delta\omega$ so it contributes positive damping, as expected.
- The torque contribution through K_1, ΔT_S, is proportional to $\Delta\delta$ so it contributes positive synchronizing torque, 90 degrees behind the damping torque. Figure 4 below illustrates.

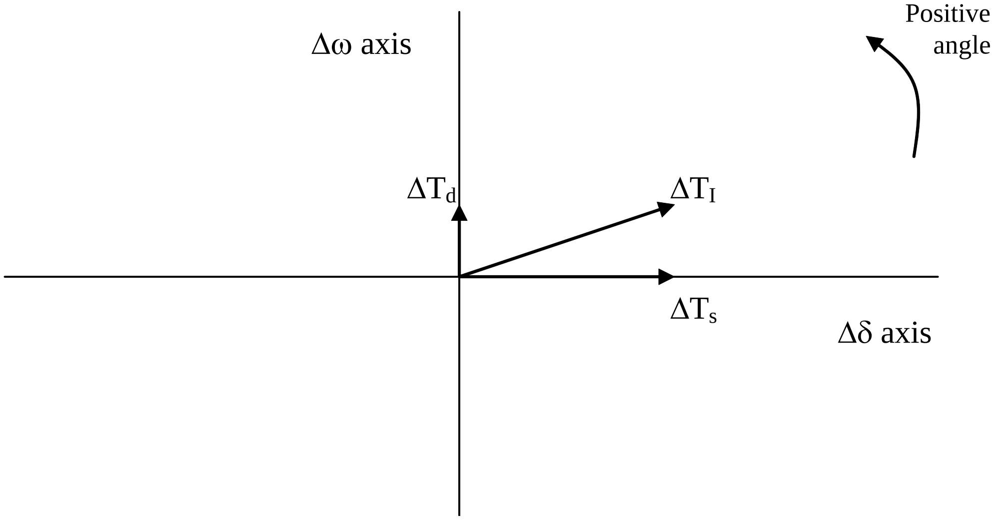


Fig. 4

So as long as D is positive and there are no other effects, we obtain positive damping contributions from the inertial torques.

Armature reaction torque:

But now let's consider the influence of armature reaction, when we get field weakening from the armature current. This effect is represented by the loop through K_4, K_3, and K_2, and is represented on the diagram by ΔT_{ar}, as indicated by the bold arrow in Fig. 5.

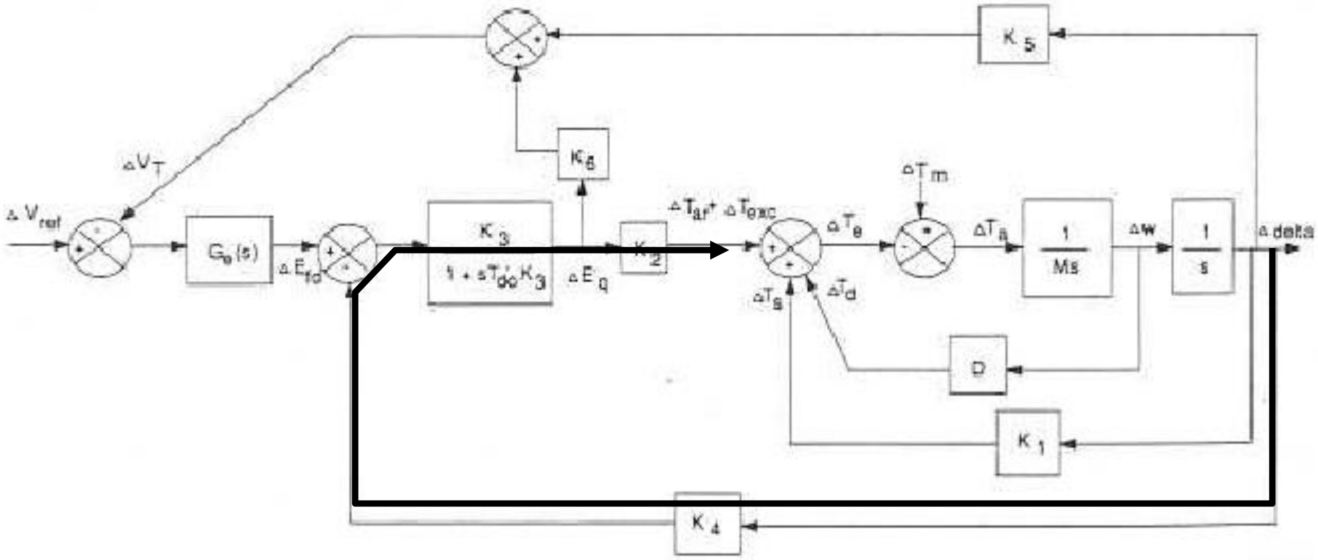


Fig. 5

The transfer function for ΔT_{ar} is given by:

$$\frac{\Delta T_{ar}}{\Delta\delta} = \frac{-K_2 K_3 K_4}{1 + sK_3\tau'_{d0}} = \frac{K_2 K_3 K_4 \angle -180^\circ}{1 + sK_3\tau'_{d0}}$$

Let's evaluate the phase of this transfer function at $s=j\omega_{osc}$ where ω_{osc} is the frequency corresponding to the weakly damped electromechanical modes of oscillation (from 0.2 Hz up to about 2.0 Hz). From this last transfer function, we can identify the denominator as $1+j\omega_{osc}K_3\tau'_{d0}$ which contributes a phase of $\tan^{-1}(\omega_{osc}K_3\tau'_{d0})$, and being on the denominator, is subtracted from the -180° phase of the numerator. Therefore the electrical torque contribution to phase, relative to $\Delta\delta$, is:

$$\phi_{ar} = -180 - \tan^{-1} K_3\tau'_{d0}\,\omega_{osc}$$

What does this do to the resulting torque? Since it is negative, we draw the vector with an angle measured opposite the positive angle.

We clearly get -180°, but we also get an additional negative angle from the $\tan^{-1}$ term. Since $t'_{d0}\omega_{osc}$ is positive, this additional angle must between 0 and 90°. The effect is shown in Fig. 6 below.

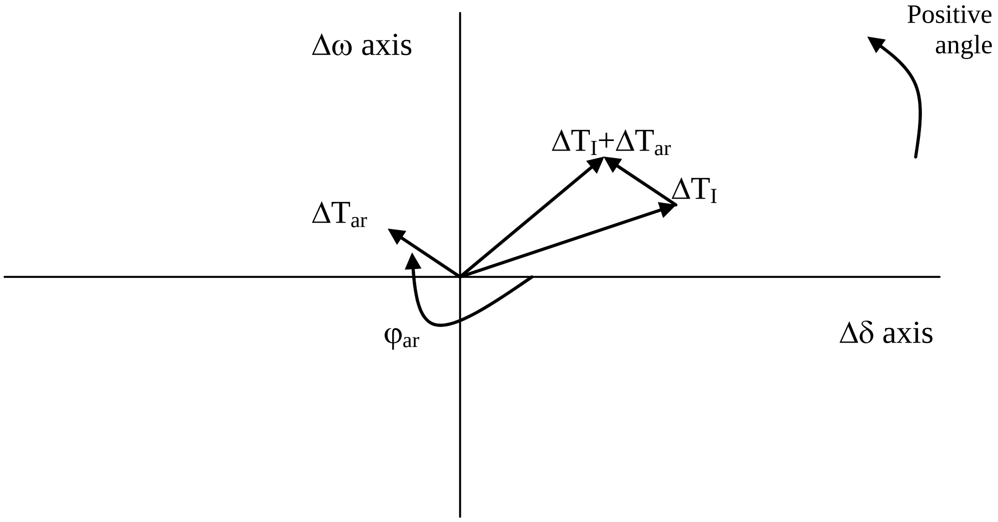


Fig. 6

Note that the effect of armature reaction on composite torque is to increase damping torque (in phase with $\Delta\omega$) and to decrease synchronizing torque (in phase with $\Delta\delta$).

Excitation system torque:

This is the electrical torque that results from the K_5 and K_6 loops, as shown in Fig. 7 below.

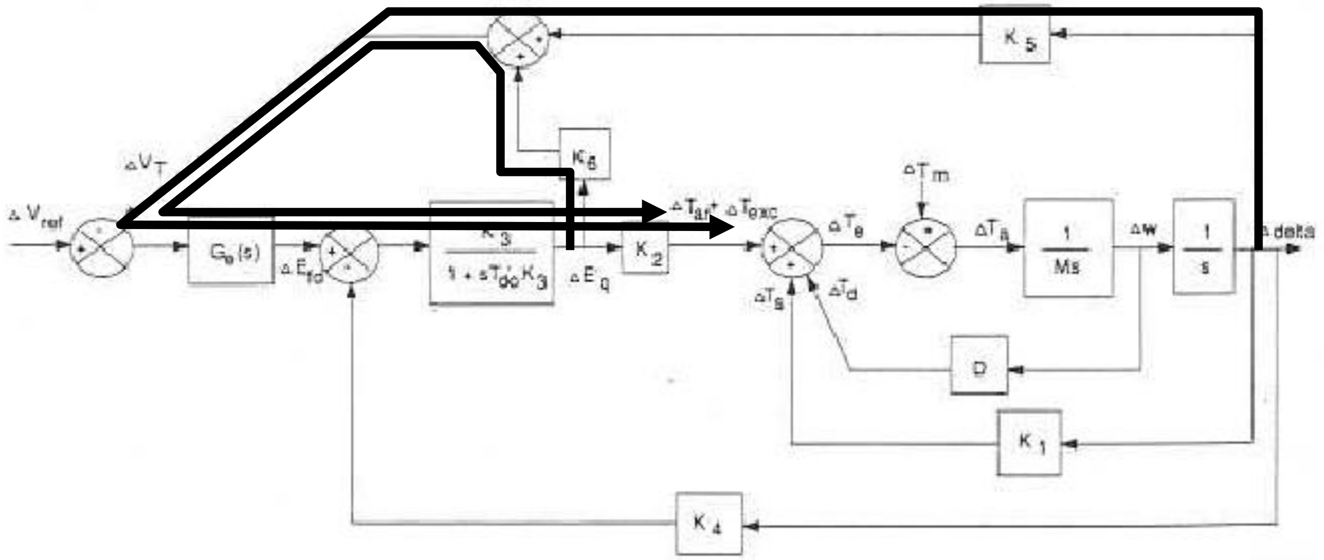


Fig. 7

This torque may be expressed based on the block diagram as:

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s\tau'_{d0} K_3} G_e(s)\left(\Delta V_{ref} - \Delta V_t\right)$$

Ignoring ΔV_{ref} (it represents manual changes in the voltage setting), and using (from eq. (**)):

$$\Delta V_t = K_5 \Delta\delta + K_6 \Delta E'_q$$

we obtain

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s\tau'_{d0} K_3} G_e(s)\left(-K_5 \Delta\delta - K_6 \Delta E'_q\right)$$

We want to express each torque as a function of Δδ or Δω. But the last expression has a $\Delta E'_q$. We can address this by noticing the relation (from the block diagram) that $\Delta T_{exc} = K_2 \Delta E'_q \Rightarrow \Delta E'_q = \Delta T_{exc}/K_2$, and so we can write that

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s\tau'_{d0} K_3} G_e(s)\left(-K_5 \Delta\delta - \frac{K_6}{K_2} \Delta T_{exc} \right)$$

Solving for ΔT_{exc}

$$\Delta T_{exc} = \frac{K_2 K_3}{1 + s\tau'_{d0} K_3} G_e(s)\left(-K_5 \Delta\delta - \frac{K_6}{K_2} \Delta T_{exc} \right) = \frac{-K_2 K_3 K_5 \Delta\delta}{1 + s\tau'_{d0} K_3} G_e(s) - \frac{K_3 K_6 \Delta T_{exc} G_e(s)}{1 + s\tau'_{d0} K_3}$$

$$\Delta T_{exc}\left(1 + \frac{K_3 K_6 G_e(s)}{1 + s\tau'_{d0} K_3} \right) = \frac{-K_2 K_3 K_5 \Delta\delta}{1 + s\tau'_{d0} K_3} G_e(s)$$

$$\Delta T_{exc} = \frac{\dfrac{-K_2 K_3 K_5}{1 + s\tau'_{d0} K_3} G_e(s)\Delta\delta}{1 + \dfrac{K_3 K_6 G_e(s)}{1 + s\tau'_{d0} K_3}} = \frac{-K_2 K_3 K_5 G_e(s)\Delta\delta}{1 + s\tau'_{d0} K_3 + K_3 K_6 G_e(s)}$$

$$\Delta T_{exc} = \frac{-K_2 K_3 K_5}{1 + s\tau'_{d0} K_3 + K_3 K_6 G_e(s)} G_e(s)(\Delta\delta)$$

Now substitute equation (****) for the static excitation transfer function $G_e(s)$, repeated here for convenience,

$$G_e(s) = \frac{(1 + sT_F)K_A}{(1 + sT_F)(1 + sT_A) + K_F K_A} \qquad (****)$$

$$\Delta T_{exc} = \frac{-K_2 K_3 K_5 \left(\dfrac{(1 + sT_F)K_A}{(1 + sT_F)(1 + sT_A) + K_F K_A} \right)}{1 + s\tau'_{d0} K_3 + K_3 K_6 \left(\dfrac{(1 + sT_F)K_A}{(1 + sT_F)(1 + sT_A) + K_F K_A} \right)} (\Delta\delta)$$

Multiply top and bottom by $(1 + sT_F)(1 + sT_A) + K_F K_A$:

$$\Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + sT_F)}{[(1 + sT_F)(1 + sT_A) + K_F K_A](1 + s\tau'_{d0} K_3) + K_3 K_6 K_A (1 + sT_F)} (\Delta\delta)$$

The above relation appears quite challenging to analyze, but we can simplify the task greatly by observing that the denominator is third order. Thus, it will be possible to write the above relation as:

$$\Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + sT_F)}{(s + p_1)(s + p_2)(s + p_3)} (\Delta\delta)$$

where p_i are the poles. We may have 3 real poles or 1 real with 2 complex. We are aware that static excitation systems generally contribute 1 real with 2 complex. We express the real pole as $p_1=\sigma_1$ and the two complex poles as $p_2=\sigma_2+j\omega_2$, and $p_3=\sigma_3+j\omega_3$, where $\sigma_i>0$ (otherwise $s=-p_i$ will have a right-half-plane pole). Thus, the transfer function becomes:

$$\Delta T_{exc} = \frac{-K_2K_3K_5K_A(1+sT_F)}{\left(s+\sigma_1\right)\left(s+\sigma_2+j\omega_2\right)\left(s+\sigma_3+j\omega_3\right)}\left(\Delta\delta\right)$$

We want to evaluate the transfer function at $s=j\omega_{osc}$, where ω_{osc} is the frequency of oscillation of concern (we assume this frequency to be an interarea oscillation between groups of generators). Therefore,

$$\Delta T_{exc} = \frac{-K_2K_3K_5K_A(1+j\omega_{osc}T_F)}{\left(j\omega_{osc}+\sigma_1\right)\left(j\omega_{osc}+\sigma_2+j\omega_2\right)\left(j\omega_{osc}+\sigma_3+j\omega_3\right)}\left(\Delta\delta\right)$$

On combining imaginary terms in the denominator, we get:

$$\Delta T_{exc} = \frac{-K_2K_3K_5K_A(1+j\omega_{osc}T_F)}{\left(\sigma_1+j(\omega_{osc})\right)\left(\sigma_2+j(\omega_2+\omega_{osc})\right)\left(\sigma_3+j(\omega_3+\omega_{osc})\right)}\left(\Delta\delta\right)$$

We are interested in the phase of ΔT_{exc} relative to $\Delta\delta$.

<u>Fact</u>: When the generator is heavily loaded, it is possible for K_5 to be negative. See Ex 8.7, Fig. 8.1 (copied below), and section 10.4.3 in VMAF. This makes the numerator of the previous transfer function positive.

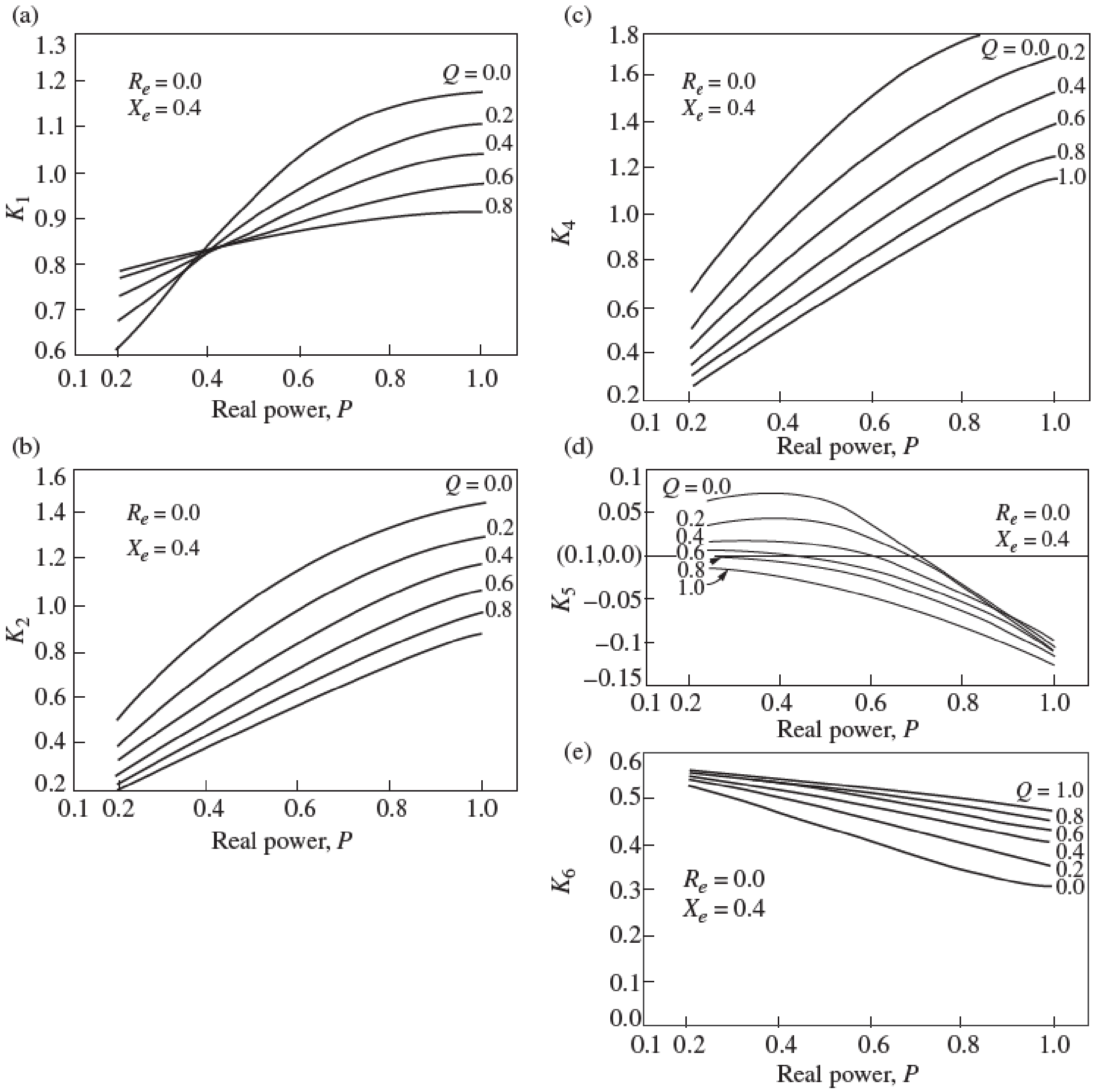


Figure 8.1 Variation of parameters $K_1, \ldots, K_6$ with loading: (a) K_1 versus P (real power) and Q (reactive power) as parameter, (b) K_2 versus P and Q, (c) K_4 versus P and Q, (d) K_5 versus P and Q, (e) K_6 versus P and Q. (Source: © IEEE. Reprinted from [12].)

A simulation of such a case is shown in Fig. 8 below. The solid curve represents generators with fast high-gain excitation systems, but no PSS. The other two curves represent significantly fewer of such generators.

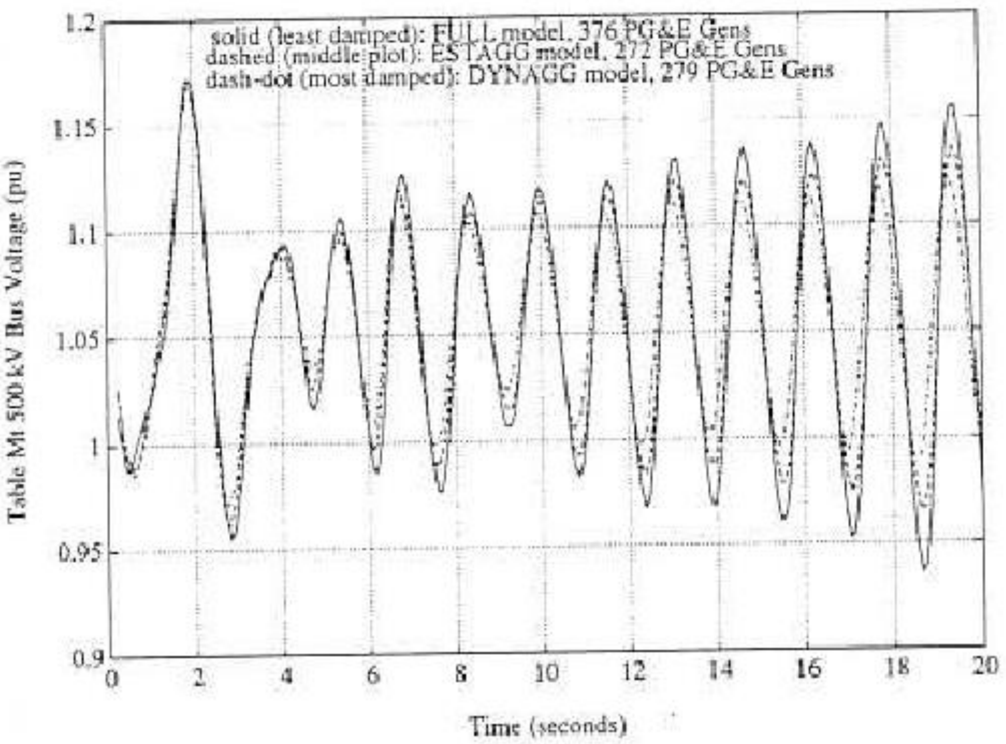


Fig. 8

Repeating our transfer function:

$$\Delta T_{exc} = \frac{-K_2 K_3 K_5 K_A (1 + j\omega_{osc} T_F)}{\left(\sigma_1 + j(\omega_{osc})\right)\left(\sigma_2 + j(\omega_2 + \omega_{osc})\right)\left(\sigma_3 + j(\omega_3 + \omega_{osc})\right)}\left(\Delta\delta\right)$$

Assuming $K_5<0$ (so that the negative sign of the transfer function cancels the negative sign of K_5), the phase of ΔT_{exc} relative to $\Delta\delta$ is given by

$$\varphi_{exc} = \tan^{-1}\omega_{osc}T_F - \tan^{-1}\frac{\omega_{osc}}{\sigma_1} - \tan^{-1}\frac{\omega_2 + \omega_{osc}}{\sigma_2} - \tan^{-1}\frac{\omega_3 + \omega_{osc}}{\sigma_3}$$

Consider some typical data, where ω_{osc}=4.396rad/sec (0.7Hz), σ_1=0.2, $\sigma_2+j\omega_2$=5+j4.5, $\sigma_3+j\omega_3$=5-j4.5, T_F=0.5. Then

$$\varphi_{exc} = \tan^{-1} 4.396(0.5) - \tan^{-1}\frac{4.396}{0.2} - \tan^{-1}\frac{8.896}{5} - \tan^{-1}\frac{-0.104}{5}$$

$$\varphi_{exc} = \tan^{-1} 2.198 - \tan^{-1} 21.98 - \tan^{-1} 1.7792 - \tan^{-1} - 0.0208$$

=65.536-87.395-60.662+1.192=-81.329

Using some typical data, the above identifies the phase lag to be -81.329°. In this case, our diagram will appear as in Fig. 9 below.

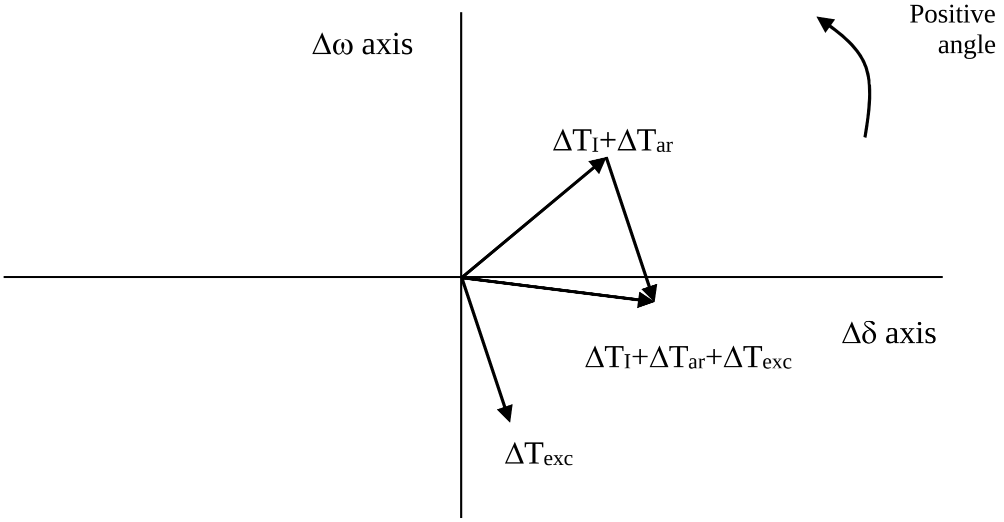
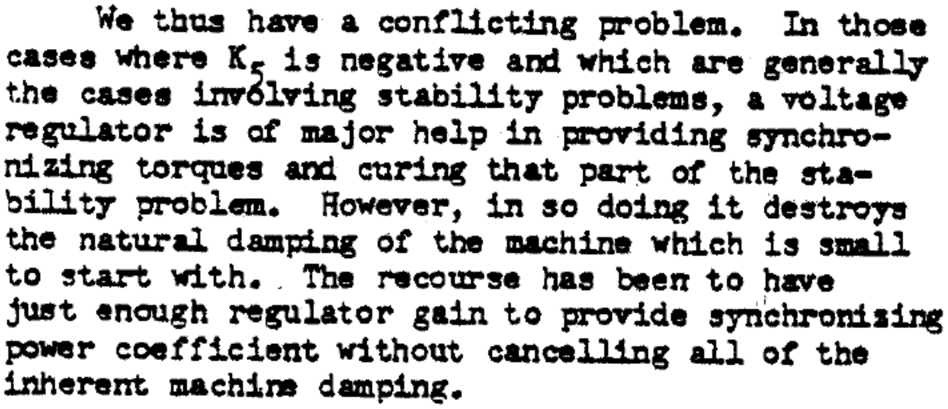


Fig. 9

and we see that the damping can go negative for fast (small T_A)-high gain (large K_A) excitation systems under heavy loading conditions! And this explains the effect observed in Fig. 8.

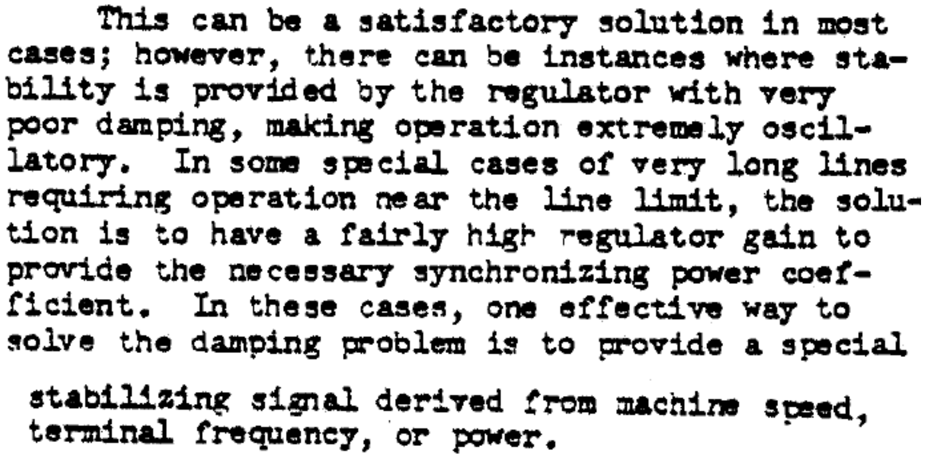
So what do we do about this?

<u>Solution 1</u>: Limit K_A to as high as possible without causing undamped oscillations. This limits the magnitude (length) of the ΔT_{exc} vector (see Fig. 9). But high-gain, fast response excitation systems are good for transient (early-swing) instability! This is indicated by the fact that, in Fig. 9, the ΔT_{exc} vector increases the synchronizing torque (i.e., it causes the resultant torque to be further to the right along the Δδ axis). And so we would rather not do this. This is a “conflicting problem” in that increasing K_A helps transient (early swing) stability but hurts oscillatory (damping). In the words of de Mello & Concordia (pg. 6 of the paper posted on the website):



> We thus have a conflicting problem. In those cases where K_5 is negative and which are generally the cases involving stability problems, a voltage regulator is of major help in providing synchronizing torques and curing that part of the stability problem. However, in so doing it destroys the natural damping of the machine which is small to start with. The recourse has been to have just enough regulator gain to provide synchronizing power coefficient without cancelling all of the inherent machine damping.

Solution 2: Provide a supplementary torque component that offsets the negative damping torque caused by the excitation system. Again, in the words of de Mello and Concordia:



> This can be a satisfactory solution in most cases; however, there can be instances where stability is provided by the regulator with very poor damping, making operation extremely oscillatory. In some special cases of very long lines requiring operation near the line limit, the solution is to have a fairly high regulator gain to provide the necessary synchronizing power coefficient. In these cases, one effective way to solve the damping problem is to provide a special stabilizing signal derived from machine speed, terminal frequency, or power.

➔Basically, the idea is to push (rotate forward) our torque vector back into the upper-right quadrant. Thus we need to phase-advance the torque vector by between 20 to 90 degrees. We will introduce a supplementary torque that does this, denoted by ΔT_{pss}, as indicated in Fig. 10 below.

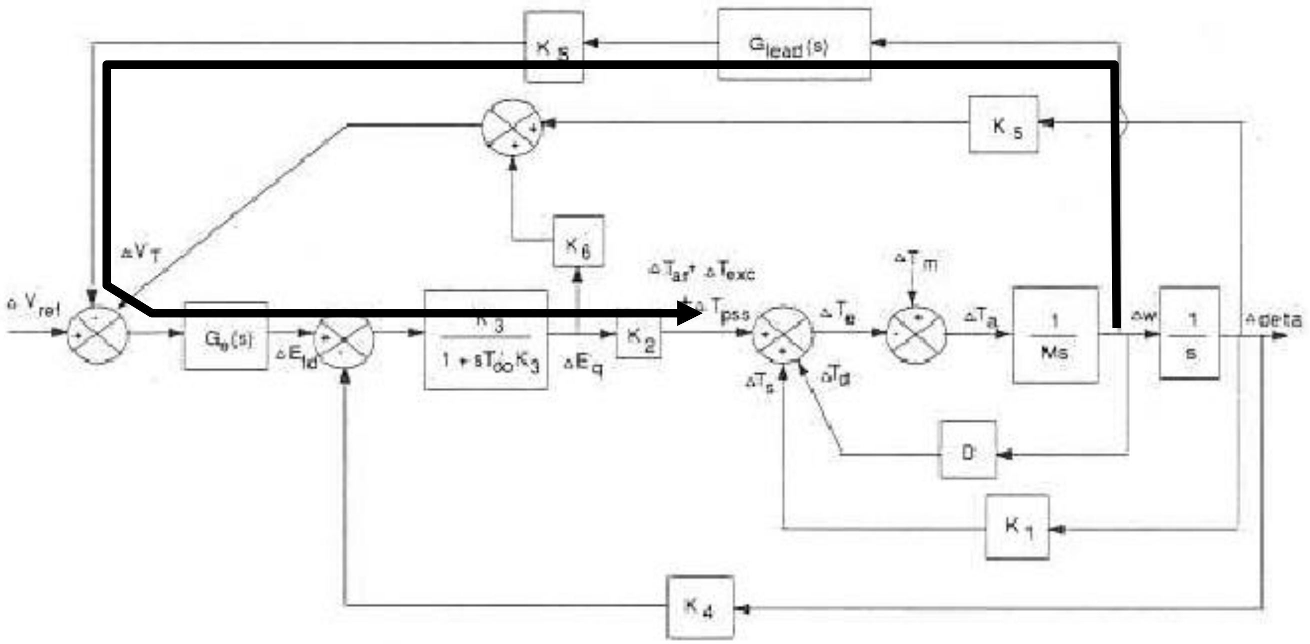


Fig. 10

The transfer function $K_S G_{lead}(s)$ is intended to provide the supplementary signal ΔT_{PSS} as illustrated in Fig. 11 below.

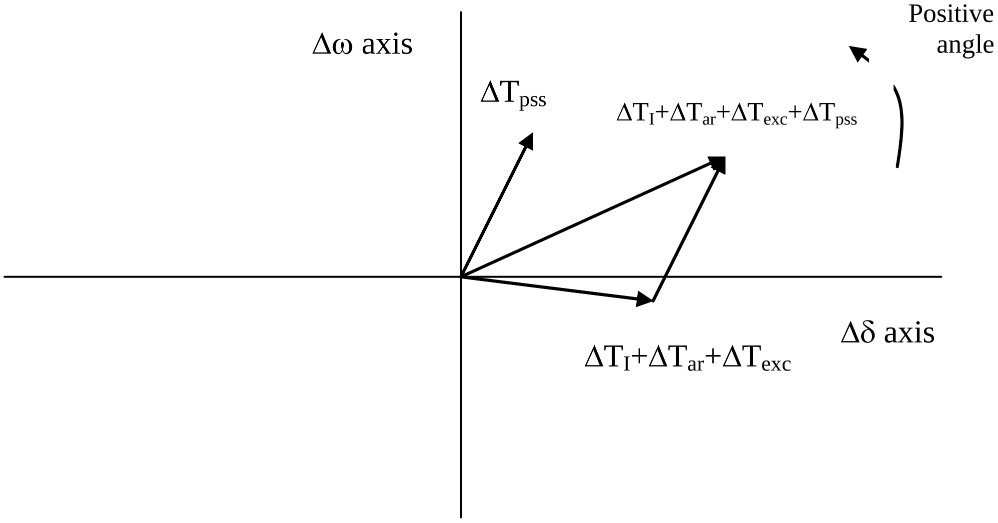


Fig. 11

We will take $\Delta\omega$ as the feedback signal for our control loop to provide ΔT_{pss} (we could also use angle deviation, but speed deviation is easier to obtain as a control signal).

We may provide “shaping” networks to process the feedback signal in providing it with the proper amount of phase (lead or lag). For example (see Dorf, pg. 362-363), a network to provide phase lead is shown in Fig. 12.

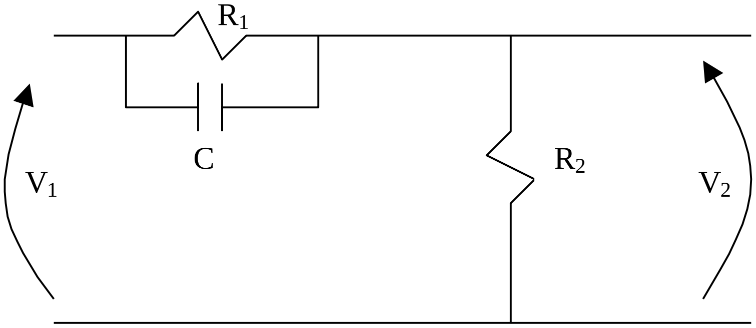


Fig. 12

(One can alternatively use digital signal processing techniques.)
In the phase lead network above, we get that

$$G_{lead}(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \left\{R_1 \frac{1}{Cs}\right\} \Big/ \left\{R_1 + 1/Cs\right\}} = \frac{1+\alpha\tau s}{\alpha(1+\tau s)}$$

where

$$\alpha = \frac{R_1 + R_2}{R_2}, \quad \tau = \frac{R_1 R_2}{R_1 + R_2} C$$

Dorf shows that the maximum value of phase lead given by the above network occurs at a frequency of

$$\omega_m = \frac{1}{\tau\sqrt{\alpha}} \qquad \text{(eqt. a)}$$

and the corresponding phase lead you get at this frequency is given by

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1} \qquad \text{(eqt. b)}$$

So the idea is that you know how much phase lead you want (this is φ_m), and from that you can solve for α in (eqt. b). You also know the frequency ω_m that you want to provide the maximum phase lead (the frequency of your most troublesome electromechanical mode and is considered to be the PSS tuning mode), and therefore with α and ω_m, you can use (eqt. a) to solve for τ.

Note from the above diagram that the desired supplementary signal ΔT_{PSS} is actually *lagging* $\Delta\omega$, so one might think that we should provide phase lag, not phase lead, to the input signal (which is actuated by $\Delta\omega$). This would in fact be the case if we could introduce the "shaped" signal (the output of G_{lead}) directly at the machine shaft.

However, this is not very easy to do because we cannot produce a mechanical torque directly from an electrical signal transduced from rotor speed.

In fact, the only place we can introduce an electrical signal is at the voltage regulator, i.e., the input to the excitation system, $G_e(s)$.

This causes a problem in that we now incur the phase lag introduced by $G_e(s)$ and the τ'_{d0} block, which is typically around ϕ_{exc}= -80 degrees as discussed previously.

So this means that we must think about it in the following way:

1. We start with the $\Delta\omega$ signal.
2. We introduce a phase lead of an amount equal to X. What is X?
3. We incur ~80 degrees of phase lag, caused by ϕ_{exc}.
4. We provide ΔT_{pss} lagging $\Delta\omega$ by, say ~25 degrees. This means that X-80≈-25 degrees ➔ X=55 degrees.

Therefore X must be about 55 degrees. So we must provide an appropriate shaping network. This shaping network is referred to as $G_{lead}(s)$.

Therefore we can write

$$\sin 55 = \frac{\alpha - 1}{\alpha + 1}$$

And solve for α:

$$\sin 55 = 0.819 = \frac{\alpha - 1}{\alpha + 1} \Rightarrow 0.819(\alpha + 1) = \alpha - 1$$

$$0.819\alpha + 0.819 = \alpha - 1 \Rightarrow -0.181\alpha = -1.819$$

$$\Rightarrow \alpha = 10.05$$

Then choose τ based on

$$\omega_{osc} = \omega_m = \frac{1}{\tau\sqrt{\alpha}}$$

where $\omega_m=2\pi(f_{osc})$, and f_{osc} is the frequency of the oscillation "problem mode." That is,

$$\omega_{osc} = \frac{1}{\tau\sqrt{\alpha}} \Rightarrow \tau\sqrt{\alpha} = \frac{1}{\omega_{osc}} \Rightarrow \tau = \frac{1}{\omega_{osc}\sqrt{\alpha}}$$

and for ω_{osc}=4.396rad/sec (0.7Hz), we have:

$$\tau = \frac{1}{4.396\sqrt{10.05}} = 0.0718$$

Then, if you are using an RC phase lead network, you can choose R and C according to

$$\alpha = \frac{R_1 + R_2}{R_2}, \quad \tau = \frac{R_1 R_2}{R_1 + R_2} C$$

Note the ***<u>principle</u>*** behind the power system stabilizer:

- Cancel the phase lags introduced by the excitation system with
    - the right amount of lead compensation so that

- the total torque exerted on the shaft by the excitation control effect will be
    - *in phase* with speed deviation and
        - thus provide positive damping.

So the PSS introduces a supplementary signal into the voltage regulator with proper phase and gain adjustments to produce a component of damping that will be sufficient to cancel the negative damping from the exciters.

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Final Exam Question #1:

Your book provides expressions for K_1-K_6 on pp. 313-314 for the case that the transmission line connecting the generator to the infinite bus has impedance of $Z_e=R_e+jX_e$. Also, at the end of these notes, the same constants K_1-K_6 are derived for the case that the transmission line connecting the generator to the infinite bus has impedance of $Z_e=jX_e$ (i.e., $R_e=0$). Starting from the expressions given in your book, set $R_e=0$ and show that those expressions collapse to the expressions given at the end of these notes.

Final Exam Question #2:

Work problem 10.1 in your text. Observe the transfer function $G_e(s)$ associated with equation (10.14) differs from the transfer function that we used in the above notes.

Final Exam Question #3:

Work problem 10.2 in your text.

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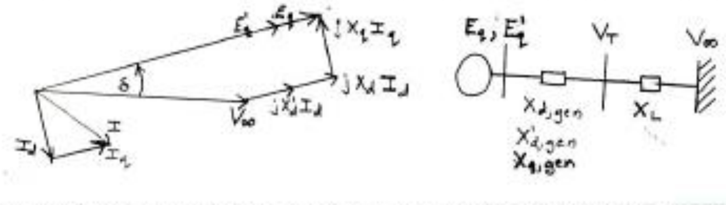
APPENDIX 3
DERIVATION OF BLOCK DIAGRAM MODEL FOR LOADED GENERATOR CONDITIONS

Under loaded conditions, a disturbance will cause the power angle, δ, between the voltage behind the transient reactance, E_q' and the infinite bus voltage, V_∞, to deviate from its non-zero steady-state value and then oscillate. The oscillations die out in the damped or stable case, and they grow in the undamped or unstable case. From the phasor diagram in Figure A3-1, it is apparent that the change in δ is related to the change in E_q' and the change in V_∞.



We derive the relationships between these quantities in what follows, assuming a synchronous generator is connected to an infinite bus, as above, and that the armature and external resistances are zero.

Derivation of Relationship Between E_{fd} and δ.

We define all reactances as the generator reactance plus the line reactance, i.e.,

$$X_d = X_{d,gen} + X_L$$

$$X_d' = X_{d,gen}' + X_L$$

$$X_q = X_{q,gen} + X_L.$$

With these definitions, the phasor diagram indicates that

$$\frac{E_q - V_\infty \cos\delta}{E_q' - V_\infty} = \frac{X_d}{X_d'} = \frac{1}{K_3}$$

where

$$K_3 \doteq \frac{X_d'}{X_d}.$$

← Agrees with 6.5B + 6.56 in A+F under cdt that Re = 0, given above definitions

Solution of the above equation for E_q in terms of K_3 yields

$$E_q = \frac{E_q'}{K_3} + V_\infty[1 - \frac{1}{K_3}]\cos\delta \tag{1}$$

The equation for the generator field winding in terms of field quantities is

$$v_f = r_f i_f + \frac{d\lambda_f}{dt} \tag{2}$$

To get this equation in terms of stator quantities, we define a constant, k/r_f, analagous to a transformer turns ratio, that allows us to refer quantities from the rotor side to the stator side. The field voltage referred to the stator side is therefore

$$E_{fd} = \frac{k}{r_f} v_f \Rightarrow v_f = \frac{r_f}{k} E_{fd}$$

Because $i_f r_f$ is equal to the field voltage v_f under steady-state conditions, it follows that E_q is

$$E_q = \frac{k}{r_f} i_f r_f \Rightarrow i_f r_f = \frac{r_f}{k} E_q$$

This leaves only the voltage behind the transient reactance, which may be defined as

$$E'_q = \frac{k}{L_f}\lambda_f \Rightarrow \frac{d\lambda_f}{dt} = \frac{L_f}{k}\frac{dE'_q}{dt}$$

Substitution of the above three expressions into equation 2 yields

$$\frac{r_f}{k}E_{fd} = \frac{r_f}{k}E_q + \frac{L_f}{k}\frac{dE'_q}{dt}$$

Multipling through by k/r_f gives

$$E_{fd} = E_q + T'_{do}\frac{dE'_q}{dt} \Rightarrow E_q = E_{fd} - T'_{do}\frac{dE'_q}{dt} \tag{3}$$

where $T'_{do} = L_f/r_f$ is the open circuit transient time constant. Substitution of equation 3 into equation 1, multipling by K_3 and rearranging, we have

$$K_3T'_{do}\frac{dE'_q}{dt} + E'_q = K_3E_{fd} + V_\infty[1 - K_3]\cos\delta \tag{4}$$

We assume that all voltages in equation 4 are normalized to the same base voltage.

Recalling the swing equation,

$$M\frac{d^2\delta}{dt^2} + D\frac{d\delta}{dt} + P_e = P_m \tag{5}$$

where P_e and P_m are the electrical and mechanical powers, respectively, and P_e may be expressed as

$$P_e = \frac{E'_qV_\infty}{X'_d}\sin\delta + \frac{V_\infty^2}{2}[\frac{1}{X_q} - \frac{1}{X'_d}]\sin 2\delta$$

Linearizing equations 4 and 5 about the steady-state operating point, E'_{qo}, δ_o, we have

$$K_3T'_{do}\frac{d\Delta E'_q}{dt} + \Delta E'_q = K_3\Delta E_{fd} - V_\infty(1 - K_3)\Delta\delta\sin\delta_o \tag{6}$$

$$M\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + \frac{\partial P_e}{\partial\delta}\mid_{\delta_o,E'_{qo}}\Delta\delta + \frac{\partial P_e}{\partial E'_q}\mid_{\delta_o,E'_{qo}}\Delta E'_q = \Delta P_m \tag{7}$$

Defining the constants K_1 through K_4 as

$K_1 = \frac{\partial P_e}{\partial\delta}\mid_{\delta_o,E'_{qo}} = \frac{E'_{qo}V_\infty}{X'_d}\cos\delta_o + V_\infty^2[\frac{X_q - X'_d}{X_qX'_d}]\cos 2\delta_o$

$K_2 = \frac{\partial P_e}{\partial E'_q}\mid_{\delta_o,E'_{qo}} = \frac{V_\infty}{X'_d}\sin\delta_o$

$K_3 = \frac{X'_d}{X_d}$

(previously defined) $K_4 = [\frac{1}{K_3} - 1]V_\infty\sin\delta_o = \frac{X_d - X'_d}{X'_d}V_\infty\sin\delta_o$

Substituting these constants into linearized equations 6 and 7,

$$K_3T'_{do}\Delta\dot{E}'_q + \Delta E'_q = K_3\Delta E_{fd} - K_3K_4\Delta\delta \tag{8}$$

$$M\Delta\ddot{\delta} + D\Delta\dot{\delta} + K_1\Delta\delta + K_2\Delta E'_q = \Delta P_m \tag{9}$$

LaPlace transforming equations 8 and 9, and noting that $\Delta E'_q(0^-) = \Delta\delta(0^-) = 0$, we have

$$K_3T'_{do}s\Delta E'_q(s) + \Delta E'_q(s) = K_3\Delta E_{fd}(s) - K_3K_4\Delta\delta(s) \tag{10}$$

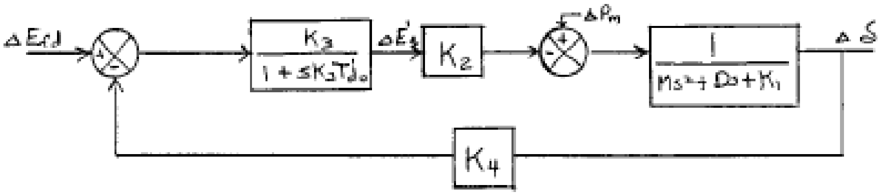
$$(Ms^2 + Ds + K_1)\Delta\delta(s) + K_2\Delta E'_q(s) = \Delta P_m(s) \tag{11}$$

Rearranging equations 10 and 11, we have

$$(K_3T'_{do}s + 1)\Delta E'_q(s) = K_3\Delta E_{fd}(s) - K_3K_4\Delta\delta(s) \tag{12}$$

$$(Ms^2 + Ds + K_1)\Delta\delta = \Delta P_m - K_2\Delta E'_q \tag{13}$$

Inspecting equations 12 and 13, we can draw the associated block diagram, given in Figure A3-2.



Derivation of Relationship Between δ and V_T.

The block diagram model above produces only $\Delta\delta$ and $\Delta E'_q$. The excitation system, however, senses the terminal voltage V_T and feeds it back to the exciter input. We now determine how changes in δ and E'_q affect the feedback quantity V_T. In other words, we desire to find two constants K_5 and K_6 such that

$$\Delta V_T = K_5\Delta\delta + K_6\Delta E'_q \tag{14}$$

From the phasor diagram in Figure A3-1, we can write

$$E'_q e^{j\delta} = V_\infty + jX'_d jI_d e^{j\delta} + jX_q I_q e^{j\delta}$$

Multipling by $e^{-j\delta}$ and converting to trig functions,

$$E'_q = V_\infty \cos\delta - jV_\infty \sin\delta - X'_d I_d + jX_q I_q$$

Equating real and imaginary parts,

$$E'_q = V_\infty \cos\delta - X'_d I_d \Rightarrow I_d = \frac{V_\infty \sin\delta}{X'_d} \tag{15}$$

$$V_\infty \sin\delta = X_q I_q \Rightarrow I_q = \frac{V_\infty \sin\delta}{X'_q} \tag{16}$$

The generator terminal voltage V_T is related to the infinite bus voltage V_∞ by the equation

$$\overline{V_T} = V_\infty + jX_L\overline{I} \tag{17}$$

where the bar over V_T and I imply phasor quantities. Now we introduce the d and q quantities V_d, V_q, I_d, and I_q as

$$\overline{V_T} = (V_q + jV_d)e^{j\delta} \tag{18}$$

$$\overline{I} = (I_q + jI_d)e^{j\delta} \tag{19}$$

Substituting equations 18 and 19 into 17, we have

$$(V_q + jV_d)e^{j\delta} = V_\infty + jX_L(I_q + jI_d)e^{j\delta}$$

Multipling through by $e^{-j\delta}$ gives

$$V_q + jV_d = V_\infty \cos\delta - jV_\infty \sin\delta + jX_L I_q - X_L I_d$$

Equating real and imaginary parts, we have

$$V_q = V_\infty \cos\delta - X_L I_d = V_\infty \cos\delta - \frac{X_L V_\infty \sin\delta}{X'_d} \tag{20}$$

$$V_d = X_L I_q - V_\infty \sin\delta = \frac{X_L V_\infty \sin\delta}{X'_d} - V_\infty \sin\delta \tag{21}$$

Substituting equations 15 and 16 into equations 20 and 21, respectively, we have

$$V_q = V_\infty \cos\delta - \frac{X_L}{X'_d}[V_\infty \cos\delta - E'_q] = V_\infty \cos\delta[1 - \frac{X_L}{X'_d}] + \frac{X_L}{X'_d}E'_q \tag{22}$$

$$V_d = \frac{X_L}{X_q}V_\infty^{\sin\delta} - V_\infty \sin\delta = V_\infty \sin\delta[\frac{X_L}{X_q} - 1] = \frac{X_L - X_q}{X_q} \tag{23}$$

Noting that

$$1 - \frac{X_L}{X'_d} = \frac{X'_d - X_L}{X_d} = \frac{X_{d,gen}}{X'_d}$$

and

$$\frac{X_L}{X_q} - 1 = \frac{X_L - X_q}{X_q} = -\frac{X_{q,gen}}{X_q}$$

equations 22 and 23 may be simplified to

$$V_q = \frac{X_{d,gen}}{X'_d} V_\infty \cos\delta + \frac{X_L}{X'_d} E'_q \tag{24}$$

$$V_d = -\frac{X_{q,gen}}{X_q} V_\infty \sin\delta \tag{25}$$

Also,

$$V_T^2 = V_T V_T^* = (V_q + jV_d)e^{j\delta}(V_q - jV_d)e^{-j\delta}$$

$$\Rightarrow V_T = \sqrt{V_q^2 + V_d^2} \tag{26}$$

Equations 24, 25, and 26 give the dependence of V_T on E'_q and δ, i.e., $V_T = V_T(E'_q, \delta)$. We now find the linearized dependence of ΔV_T on $\Delta E'_q$ and $\Delta\delta$, i.e.,

$$\Delta V_T = \frac{\partial V_T}{\partial \delta} |_{\delta_o, E'_{qo}} \Delta\delta + \frac{\partial V_T}{\partial E'_q} |_{\delta_o, E'_{qo}} \Delta E'_q = K_5 \Delta\delta + K_6 \Delta E'_q \tag{27}$$

Using equations 24, 25, and 26, and the chain rule for differentiation, we can compute K_5 and K_6 as

$$K_5 = V_{infty}[\tfrac{\partial V_T}{\partial \delta} |_{\delta_o, E'_{qo}} = \tfrac{X_{q,gen} V_{do}}{X_q V_{To}} \cos\delta_o - \tfrac{X'_{d,gen} V_{qo}}{X'_d V_{To}} \sin\delta_o]$$

$$K_6 = \tfrac{\partial V_T}{\partial E'_q} |_{\delta_o, E'_{qo}} = \tfrac{X_L V_{qo}}{X'_d V_{To}}$$

where V_{qo}, V_{do}, and V_{To} are computed using equations 24, 25, and 26 with the steady-state values δ_o and E'_{qo} substituted in for δ and E'_q, respectively.

Modeling equation 27 in block diagram form, together with the block diagram of $\Delta E_{fd}/\Delta\delta$ (Figure A3-2) and the block diagram of the excitation system (Figure 15), we have the block diagram for the entire excitation control system, Figure A3-3.