Per-unitization and Equivalent Circuits

1.0 Normalization of voltage equations

We desire to normalize the voltage equations, that is, we desire to express them in per-unit. The advantages of doing so are:

- The per-unit system offers computational simplicity:
    - by eliminating units and expressing system quantities as dimensionless ratios;
    - by eliminating the need for using arbitrary constants and simplifying some of the mathematical expressions so that they may be expressed in terms of equivalent circuits.
- The numerical values of currents and voltages are related to their rated values irrespective of machine size[1].
- Impedances, when given on the machine base, lie on a relatively narrow range so that errors can be easily detected.

There are several different possible normalization schemes. What VMAF does (see Appendix C in VMAF) is to carefully compare the merits of all of these schemes. In doing so, they developed some criteria, guidelines, the most important of which is that the form of the voltage equations and the power expression must be independent of whether they are in pu or MKS.

Note that machine manufacturers, when expressing their machine data in per unit, may use a different system that does not satisfy the power invariance property - they use Park's original transformation (called "*Q*" in VMAF, eq. (4.22), instead of our "*P*"). Below is a typical machine data sheet[2].

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[1] K. Padiyar, "Power system dynamics: stability and control," 2nd edition, BS Publications, 2008, p. 62.

[2] https://www.skm.com/Synchronous_Generator_Input_Data.html

Selected Model

Engine: 3512	Generator Frame: 824	Genset Rating (kW): 1500.0	Line Voltage: 480
Fuel: Diesel	Generator Arrangement: 2628102	Genset Rating (kVA): 1875.0	Phase Voltage: 277
Frequency: 60	Excitation Type: Permanent Magnet	Pwr. Factor: 0.8	Rated Current: 2255.3
Duty: STANDBY	Connection: SERIES STAR	Application: EPG	Status: Current

Version: 40400 /40268 /38580 /6517

Spec Information

Generator Specification

Frame: 824 Type: SR4B	No. of Bearings: 1
Winding Type: RANDOM WOUND	Flywheel: 21.0
Connection: SERIES STAR	Housing: 00
Phases: 3	No. of Leads: 6
Poles: 4	Wires per Lead: 8
Sync Speed: 1800	Generator Pitch: 0.6667

Generator Efficiency

Per Unit Load	kW	Efficiency %
0.25	375.0	91.7
0.5	750.0	94.4
0.75	1125.0	95.0
1.0	1500.0	94.9

Reactances	Per Unit	Ohms
SUBTRANSIENT - DIRECT AXIS X''_d	0.1831	0.0225
SUBTRANSIENT - QUADRATURE AXIS X''_q	0.1676	0.0206
TRANSIENT - SATURATED X'_d	0.2694	0.0331
SYNCHRONOUS - DIRECT AXIS X_d	3.3008	0.4056
SYNCHRONOUS - QUADRATURE AXIS X_q	1.5763	0.1937
NEGATIVE SEQUENCE X_2	0.1750	0.0215
ZERO SEQUENCE X_0	0.0098	0.0012

Time Constants	Seconds
OPEN CIRCUIT TRANSIENT - DIRECT AXIS T'_{d0}	5.8770
SHORT CIRCUIT TRANSIENT - DIRECT AXIS T'_d	0.4800
OPEN CIRCUIT SUBSTRANSIENT - DIRECT AXIS T''_{d0}	0.0086
SHORT CIRCUIT SUBSTRANSIENT - DIRECT AXIS T''_d	0.0074
OPEN CIRCUIT SUBSTRANSIENT - QUADRATURE AXIS T''_{q0}	0.0065
SHORT CIRCUIT SUBSTRANSIENT - QUADRATURE AXIS T''_q	0.0057
EXCITER TIME CONSTANT T_e	0.2225
ARMATURE SHORT CIRCUIT T_a	0.0497

Short Circuit Ratio: 0.35	Stator Resistance = 0.0023 Ohms	Field Resistance = 0.9324 Ohms

Voltage Regulation

Voltage level adustment: +/-	5.0%
Voltage regulation, steady state: +/-	0.5%
Voltage regulation with 3% speed change: +/-	0.5%
Waveform deviation line - line, no load: less than	3.0%
Telephone influence factor: less than	50

Generator Excitation

	No Load	Full Load, (rated) pf Series	Parallel
Excitation voltage:	6.73 Volts	32.63 Volts	Volts
Excitation current	1.77 Amps	7.06 Amps	Amps

The choice made by VMAF satisfies the above criteria; in addition, the VMAF choice ensures that the numerical values of the per-unit impedances are the same as those provided by manufacturers using their system of normalization.

In most undergraduate power system analyses courses, we learn that per-unitization requires *selection* of two base quantities out of the following four: V, I, Z, and S, and then the base quantities for the other two are *computed*. The situation is the same here, except that we also must deal with speed (or frequency). This necessitates that we must also *select* a base for either frequency (ω or f) or time, t.

In addition, we will also have need to compute base quantities associated with flux linkage (λ) and inductance (L or M).

Our approach will be to obtain the bases for the stator side and then the bases for the rotor side.

One may note two excellent references on the subject of per-unitizing synchronous machine models:

1. A. Rankin, "Per-unit impedance of synchronous machines," AIEE Transactions, 64, Aug., 1945.
2. M. Harris, P. Lawrenson, and J. Stephenson, "Per-unit systems with special reference to electrical machines," Cambridge University press, Cambridge, England, 1970.

Other references that address this subject, besides VMAF, include those by

- Sauer and Pai
- Concordia
- Padiyar
- Kundur

and also course notes from de Mello.

1.1 Stator side per-unitization:

We *select* our stator-side bases as:

- V_B: the stator rated line-neutral voltage, rms.
- S_B: the stator rated per-phase power, volt-amps
- ω_B: the generator rated speed, in electrical rad/sec ($=\omega_{Re}=377$)

We select per-phase power and line-neutral voltage bases for the voltage equations because it enables us develop the voltage equations in terms of per-phase circuits.

Then we may *compute* bases for the following 5 quantities:

- current: $I_B = \dfrac{S_B}{V_B}$
- impedance: $X_B = R_B = Z_B = \dfrac{V_B}{I_B} = \dfrac{V_B^2}{S_B}$
- time: $t_B = \dfrac{1}{\omega_B}$ (We could have used $t_B = \dfrac{2\pi}{\omega_B}$ but this would simply provide a different scaling and is therefore arbitrary. This choice of t_B is the time required for the rotor to move one electrical radian.)
- Flux linkage: $\lambda_B = V_B t_B$ (This comes from the fact that $v = \dfrac{d\lambda}{dt} \approx \dfrac{\Delta\lambda}{\Delta t} \Rightarrow \Delta\lambda = V\Delta t$)
- Inductance: $L_B = \dfrac{\lambda_B}{I_B} = \dfrac{V_B t_B}{I_B} = \dfrac{V_B}{\omega_B I_B} = \dfrac{X_B}{\omega_B}$

Question: How does our choice of stator base quantities affect the per-unit values of the d- and q-axis quantities? Note: although d- and q-axis quantities associate with fictitious rotor windings, we view them to be stator quantities.

To answer this question, let *V* and *I* be the rms magnitudes of the a-phase line-neutral voltage $V\angle\alpha$ and a-phase line current $I\angle\gamma$, respectively. Then the per-unit phasors are

$$\bar{V}_u = \frac{V}{V_B}\angle\alpha = V_u\angle\alpha \qquad \bar{I}_u = \frac{I}{I_B}\angle\gamma = I_u\angle\gamma$$

Now let's investigate the 0dq quantities.
To begin, recall that Peak=sqrt(2)*RMS; then the expressions for instantaneous voltages and currents for each phase are:

$$v_a = \sqrt{2}V\sin(\theta+\alpha) \qquad i_a = \sqrt{2}I\sin(\theta+\gamma)$$
$$v_b = \sqrt{2}V\sin(\theta+\alpha-120°) \quad i_b = \sqrt{2}I\sin(\theta+\gamma-120°)$$
$$v_c = \sqrt{2}V\sin(\theta+\alpha+120°) \quad i_c = \sqrt{2}I\sin(\theta+\gamma+120°)$$

Multiply the above by the Park's transformation matrix *P*. Recalling *P* as:

$$\underline{P} = \sqrt{\frac{2}{3}}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta & \cos(\theta-120) & \cos(\theta+120) \\ \sin\theta & \sin(\theta-120) & \sin(\theta+120) \end{bmatrix}$$

and performing the necessary trigonometry, we obtain:

$$v_{0dq} = \begin{bmatrix} 0 \\ \sqrt{3}V\sin\alpha \\ \sqrt{3}V\cos\alpha \end{bmatrix} \qquad i_{0dq} = \begin{bmatrix} 0 \\ \sqrt{3}I\sin\gamma \\ \sqrt{3}I\cos\gamma \end{bmatrix}$$

(This confirms our conclusion at the end of the last set of notes, "macheqts," that, for balanced conditions, the 0dq quantities are constants, i.e., DC.)

Now, per-unitize by dividing by V_B and I_B:

$$\underset{u}{v_{0dq}} = \begin{bmatrix} 0 \\ \frac{\sqrt{3}V}{V_B}\sin\alpha \\ \frac{\sqrt{3}V}{V_B}\cos\alpha \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3}V_u\sin\alpha \\ \sqrt{3}V_u\cos\alpha \end{bmatrix} \qquad \underset{u}{i_{0dq}} = \begin{bmatrix} 0 \\ \frac{\sqrt{3}I}{I_B}\sin\gamma \\ \frac{\sqrt{3}I}{I_B}\cos\gamma \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3}I_u\sin\gamma \\ \sqrt{3}I_u\cos\gamma \end{bmatrix}$$

Observe about the above that

1. The per-unit d and q voltages are equal to the per-unit a-phase voltage scaled by $\sqrt{3}\sin\alpha$ and $\sqrt{3}\cos\alpha$, respectively.
2. The per-unit d and q currents are equal to the per-unit line current scaled by $\sqrt{3}\sin\gamma$ and $\sqrt{3}\cos\gamma$, respectively.

1.2 Rotor-side per-unitization:

Recall that in system per-unitization, we must select a single power base for the entire system, independent of the fact that some sections of the system are magnetically coupled through transformers, i.e., we do NOT choose different power bases for different sides of a transformer.

The same restriction applies here, where the rotor circuit is magnetically coupled to the stator circuit, i.e., the power base selected for the stator side must also be the power base used on the rotor side. This is S_B.

In addition, we are required to select the same time (or frequency) base for both the stator side and the rotor side. This is t_B (or ω_B).

On the rotor side, we have one base left to choose (or compute). For transformers, we typically choose the one remaining base as the voltage base (or current base) according to the turns ratio. Here, however, we do not know a "turns ratio," and therefore we are left with problem of what, and how, to choose. (One text treats the problem under the assumption that a "turns ratio" is known between stator and rotor circuits - see the text by Padiyar, "Power System Dynamics," pp. 73-77.)

In making this choice, a problem results from the fact that stator power levels are typically several times the rotor power levels. VMAF give an interesting comparison (see pg 103) of a typical stator-side per-phase power rating of 100 MVA and field winding ratings of 250v, 1000A (250kw), a power ratio of 400:1. What are our choices of the one remaining rotor-side base quantity in this case?

- Choose voltage base=rated voltage=250v, but then the current base is I_B=100E6/250=400000 amps, and per-unit values of field currents will be very small.
- Choose current base=rated current=1000A, but then voltage base is V_B=100E6/1000=100000 volts, and per-unit values of field voltages will be very small.

Analogy to transformers:

Consider an illustration adapted from Krause, pp. 4-5. Fig. 1 shows magnetizing and leakage flux for a transformer configuration.

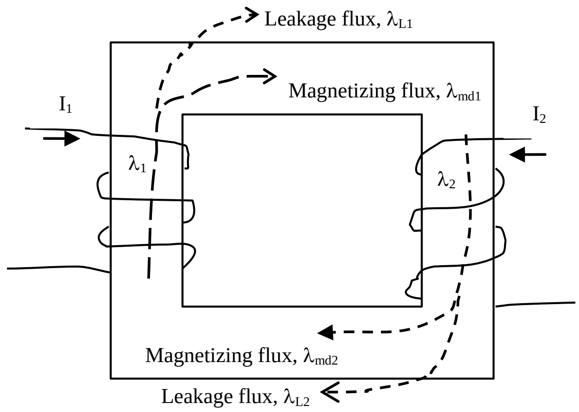


Fig. 1: Magnetizing and Leakage flux

Here we can write that

$$\lambda_1 = \underbrace{\frac{N_1^2 I_1}{R_{l1}} + \frac{N_1^2 I_1}{R_m}}_{\text{From } I_1} + \underbrace{\frac{N_1 N_2 I_2}{R_m}}_{\text{From } I_2} \tag{1}$$

where the first two terms represent flux linkage with coil 1 from I_1 and the third term flux linkage with coil 2 from I_2. We observe that of the flux linkage from I_1 (first two terms), the first term represents leakage flux and the second represents magnetizing flux. The first two terms comprise the flux linkage attributable to the self-inductance and the third term the flux linkage attributable to the mutual inductance. The first two terms attributable to self-inductance may be further expressed as

$$\lambda_1 = \left(\underbrace{\frac{N_1^2}{R_{l1}}}_{l_1} + \underbrace{\frac{N_1^2}{R_m}}_{L_{md1}} \right) I_1 + \frac{N_1 N_2}{R_m} I_2 = \left(\underbrace{l_1 + L_{md1}}_{L_1} \right) I_1 + M I_2 \tag{2}$$

where l_1 is the leakage inductance, L_{md1} is the magnetizing inductance, and M is the mutual inductance. Observe that the magnetizing inductance is not the same as the mutual inductance, i.e., $L_{md1} \neq M$, if $N_1 \neq N_2$. However, the magnetizing flux $\lambda_{md1}=L_{md1}I_1$ is the same as the mutual flux MI_2 since $N_1I_1=N_2I_2$ which we know to be true for transformers. **We want to impose a similar relationship on the synchronous machine mutual fluxes**.

Representing the self inductance as L_1, we have

$$L_1 = l_1 + L_{md1}$$

which indicates that the self inductance is comprised of the leakage inductance plus the magnetizing inductance.

Similar analysis results in $L_2 = l_2 + L_{md2}$.

<u>Back to synchronous machines</u>:

Here, we will select the base quantities according to the following criteria:

We select the base currents for the four rotor-side windings F, D (G, Q) to produce the same mutual flux in the air gap as produced by the stator-side base current I_B flowing in the corresponding fictitious d-axis (q-axis) coil.

The reason this is beneficial is that it will enable us to develop a relatively simple circuit to represent direct-axis pu quantities and another one to represent quadrature axis pu quantities. The form of this cct will be "tee". (see App C, top of p. 689) and pg. 34 of these notes.

We will begin by applying this idea to obtain the base current for the main field winding.

<u>Base-current for main field winding, approach 1</u>:

One can visualize the above concept for the case of the relationship between the F-winding and the d-winding, in Fig. 2.

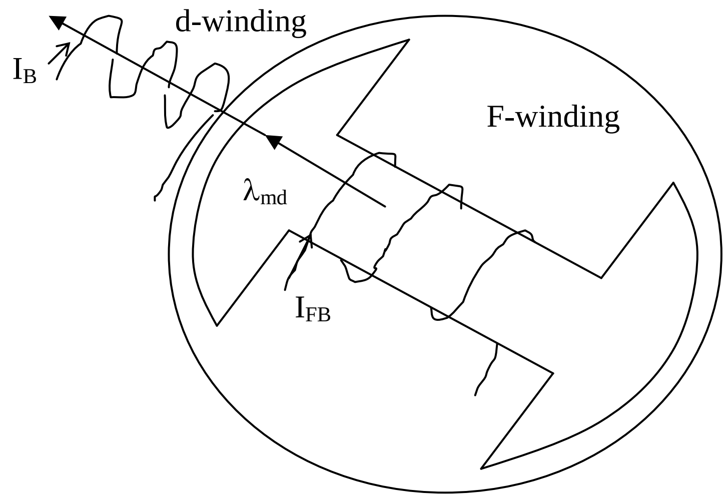


Fig. 2: Base currents in d and F windings

We see from Fig. 2 that we select I_{FB}, the field winding base current, as

> *that current when flowing in the F-winding will produce a mutual flux λ_{md} equal to the same mutual flux that is produced by a current I_B flowing in the d-winding.*

But how do we express (compute) I_{FB}?

From our previous set of notes (p. 29, "macheqts"), and also eq. 4.20 in VMAF, we derived

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_G \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & 0 & kM_D & 0 \\ 0 & 0 & L_q & 0 & kM_G & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & 0 & M_R & 0 \\ 0 & 0 & kM_G & 0 & L_G & 0 & M_Y \\ 0 & kM_D & 0 & M_R & 0 & L_D & 0 \\ 0 & 0 & kM_Q & 0 & M_Y & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} \quad \text{eq. (4.20)}$$

From the second equation in (4.20), we can see that

$$\lambda_d = L_d i_d + kM_F i_F + kM_D i_D \qquad \text{(eq. 3)}$$

where $k=\sqrt{(3/2)}$.

$L_d i_d$ is all of the flux produced by the d-winding, but only a part of this flux links with the F-winding. Call this flux from the d-winding that links with the F-winding λ_{md}, given by $\lambda_{md}=L_{md}i_d$, where L_{md} is the magnetizing inductance associated with this flux.

The difference between the total flux from the d-winding and the mutual flux is attributed to the leakage flux λ_{L}, so that,

$$l_d i_d = L_d i_d - L_{md} i_d \qquad \text{(eq. 4)}$$

Canceling the current i_d, we see that

$$l_d = L_d - L_{md} \Rightarrow L_d = l_d + L_{md} \qquad \text{(eq. 5)}$$

When I_B flows in the d-winding, so that $i_d=I_B$, the mutual flux is given by

$$\lambda_{md} = L_{md} I_B \qquad \text{(eq. 6)}$$

Looking back at eq (3), we see that the flux from the F-winding that links the d-axis winding is just $kM_F i_F$.
Our criteria for selecting I_{FB} says that when I_{FB} flows in the F-winding, the mutual flux linking the d-winding should equal the mutual flux from the d-winding linking the F-winding when it carries I_B. Thus, we write that

$$\lambda_{md}=L_{md}I_B=kM_F I_{FB} \tag{eq. 7}$$

And we see that

$$I_{FB} = \frac{L_{md}}{kM_F} I_B \tag{eq. 8}$$

M_F and L_{md} are generally provided in (or can be obtained from) manufacturer's data for a given machine[3].

- M_F can be computed as illustrated in Example 4.1 (which we review below), using the magnetization curve,
- $L_{md}=L_d-l_d$, where manufacturer's data sheets contain L_d and l_d.

Therefore, once I_B is selected, I_{FB} may be computed.

Base current for main field winding, approach 2:
One may also develop a relation for I_{FB} from the perspective of the flux linking the field winding, i.e., instead of using eq. (3) from (4.20'), use:

$$\lambda_F = kM_F i_d + L_F i_F + M_R i_D \tag{eq. 9}$$

Similar to eq. (5), the self inductance L_F is comprised of the leakage and the magnetizing, i.e.,

$$L_F=l_f+L_{mF} \tag{eq. 10}$$

Inspecting eq. (9), we see that the flux from the d-winding linking with the F-winding is $kM_F i_d$, so that when $i_d=I_B$ and $i_F=I_{FB}$, we have that

$$L_{mF}I_{FB}=kM_F I_B \tag{eq. 11}$$

and we see that

$$I_{FB} = \frac{kM_F}{L_{mF}} I_B \tag{eq. 12}$$

where, as before, M_F is obtained per Example 4.1 below, $L_{mF}=L_F-l_F$, and L_F, l_F are obtained from manufacturer's data sheet.

[3] Note: in MKS units (i.e., henries), L_{md} is not the same as kM_F , i.e., the reciprocal mutuals are not equal.

Base current for D-winding:

We select the D-winding base current, I_{DB} according to the following criteria:

> *We select I_{DB}, the D-winding base current, as that current when flowing in the D-winding will produce a mutual flux λ_{md} equal to the same mutual flux that is produced by a current I_B flowing in the d-winding.*

Similar analysis as for the F-winding results in

$$I_{DB} = \frac{L_{md}}{kM_D} I_B, \qquad I_{DB} = \frac{kM_D}{L_{mF}} I_B \qquad \text{(eq. 13)}$$

We may also utilize a similar procedure between D and F windings to obtain

$$I_{DB} = \frac{L_{mF}}{M_R} I_{FB} \qquad \text{(eq. 14)}$$

Base current for Q-winding:

We select Q-winding base current, I_{QB} according to the following criteria:

> *We select I_{QB}, the Q-winding base current, as that current when flowing in the Q-winding will produce a mutual flux λ_{mq} equal to the same mutual flux that is produced by a current I_B flowing in the q-winding.*

Similar analysis as for the F-winding results in

$$I_{QB} = \frac{L_{mq}}{kM_Q} I_B, \qquad I_{QB} = \frac{kM_Q}{L_{mQ}} I_B \qquad \text{(eq. 15)}$$

Base current for G-winding:

We select the G-winding base current, I_{GB} according to the following criteria:

> *We select I_{GB}, G-winding base current, as that current when flowing in the G-winding will produce a mutual flux λ_{mq} equal to the same mutual flux that is produced by a current I_B flowing in the q-winding.*

Similar analysis as for the F-winding results in

$$I_{GB} = \frac{L_{mq}}{kM_G} I_B, \qquad I_{GB} = \frac{kM_G}{L_{mG}} I_B \qquad \text{(eq. 16)}$$

We may also utilize a similar procedure between Q and G windings to obtain

$$I_{GB} = \frac{L_{mQ}}{M_Y} I_{QB} \qquad \text{(eq. 17)}$$

Summary:

Eqt. (8) together with eqts. (12-17) provide the ability to develop any of the equations given as (4.54) in VMAF. These equations are referred to as the "*fundamental constraints among base currents*" and are given by:

$$L_{md} I_B^2 = L_{mF} I_{FB}^2 = L_{mD} I_{DB}^2 = kM_F I_B I_{FB} = kM_D I_B I_{DB} = M_R I_{FB} I_{DB}$$

$$L_{mq} I_B^2 = kM_Q I_B I_{QB} = L_{mQ} I_{QB}^2 = M_Y I_{QB} I_{GB}$$

For example, recalling (8) is $I_{FB} = \frac{L_{md}}{kM_F} I_B$ and (12) is $I_{FB} = \frac{kM_F}{L_{mF}} I_B$, we can multiply the left-hand-sides together and the right-hand-sides together to obtain: $I_{FB}^2 = \frac{L_{md} kM_F}{kM_F L_{mF}} I_B^2 \Rightarrow L_{mF} I_{FB}^2 = L_{md} I_B^2$.

Now define the following k-factors:

$$k_F = \frac{I_B}{I_{FB}}, \qquad k_D = \frac{I_B}{I_{DB}}, \quad k_Q = \frac{I_B}{I_{QB}}, \quad k_G = \frac{I_B}{I_{GB}}$$

Because we have the same power base on all stator and rotor circuits, we obtain:

$$S_B = V_B I_B = V_{FB} I_{FB} = V_{DB} I_{DB} = V_{QB} I_{QB} = V_{GB} I_{GB}$$

Then the above k-factors may be expressed. For example,

$$V_B I_B = V_{FB} I_{FB} \Rightarrow \frac{V_{FB}}{V_B} = \frac{I_B}{I_{FB}} = k_F$$

In summary,

$$k_F = \frac{V_{FB}}{V_B}, \qquad k_D = \frac{V_{DB}}{V_B}, \quad k_Q = \frac{V_{QB}}{V_B}, \quad k_G = \frac{V_{GB}}{V_B}$$

Note that these k-factors may be considered to be effective turns ratios.

We may also derive expressions for the resistance and inductance bases. Our desire is to be able to compute rotor-side bases as a function of stator-side bases. The k-factors given above will be very handy here.

<u>Rotor-side resistance bases</u>:

$$R_{FB} \equiv \frac{V_{FB}}{I_{FB}} = \frac{V_{FB}}{V_B}\frac{I_B}{I_{FB}}\frac{V_B}{I_B} = k_F^2 R_B$$

Likewise,

$$R_{DB} = k_D^2 R_B, \qquad R_{QB} = k_Q^2 R_B, \qquad R_{GB} = k_G^2 R_B$$

<u>Rotor-side inductance bases</u>:

$$L_{FB} \equiv \frac{V_{FB} t_B}{I_{FB}} = \frac{V_{FB}}{V_B}\frac{I_B}{I_{FB}}\frac{V_B t_B}{I_B} = k_F^2 L_B$$

Likewise,

$$L_{DB} = k_D^2 L_B, \qquad L_{QB} = k_Q^2 L_B, \qquad L_{GB} = k_G^2 L_B$$

<u>Rotor-stator mutuals</u>:

Your text, pg. 103 refers to HW problem 4.18 which states that base mutuals must be the geometric mean of the base self-inductances, i.e.,

$$M_{12B} = \sqrt{L_{1B} L_{2B}}$$

Thus, we have that the base for the field winding to stator winding mutual terms is given by (see eq. 4.57 in VMAF):

$$M_{FB} = \sqrt{L_B L_{FB}} = \sqrt{L_B k_F^2 L_B} = k_F L_B$$

Note that M_{FB} is not the same as the base self inductance L_{FB} given above.
Likewise, we get (see eq. 4.57 in VMAF):

$$M_{DB} = k_D L_B, \qquad M_{QB} = k_Q L_B, \qquad M_{GB} = k_G L_B$$

Rotor-rotor mutuals:
There are just 2 of them (see eq. 4.57 in VMAF):

$$M_{RB} = \sqrt{L_{FB} L_{DB}} = \sqrt{k_F^2 L_B k_D^2 L_B} = k_F k_D L_B$$

Likewise,

$$M_{YB} = k_G k_Q L_B$$

2.3 Example 4.1, pg 105 of text

This is a good example that you should review carefully. Here is the first part of it (p. 106-107 continues with it).

Example 4.1. Find the pu values of the parameters of the synchronous machine for which the following data are given (values are for an actual machine with some quantities, denoted by an asterisk, being estimated for academic study):

←Why are some values "estimated for academic study"? (See pg. 20 below)

Rated MVA = 160 MVA	$L_Q = 1.423 \times 10^{-3}$ H*
Rated voltage = 15 kV Y connected	$\ell_d = \ell_q$(unsaturated) $= 0.5595 \times 10^{-3}$ H
Excitation voltage = 375 V	$kM_D = 5.782 \times 10^{-3}$ H*
Stator current = 6158.40 A	$kM_G = 5.0 \times 10^{-3}$ H*
Field current = 926 A	$kM_Q = 2.779 \times 10^{-3}$ H*
Power factor = 0.85	$r(125°\text{C}) = 1.542 \times 10^{-3}\ \Omega$
$L_d = 6.341 \times 10^{-3}$ H	$r_F(125°\text{C}) = 0.371\ \Omega$
$L_F = 2.189$ H	$r_D = 18.421 \times 10^{-3}\ \Omega$*
$L_D = 5.989 \times 10^{-3}$ H*	$r_G = 6.64 \times 10^{-3}\ \Omega$*
$L_q = 6.118 \times 10^{-3}$ H	$r_Q = 18.969 \times 10^{-3}\ \Omega$*
$L_G = 5.31 \times 10^{-3}$ H*	Inertia constant = 1.765 kW·s/hp

From the no-load magnetization curve, the value of field current corresponding to the rated voltage on *the air gap line* is 365 A.

←Note this statement; What is "air gap line"? See Fig. 3 and Fig. 4 below and related comments.

Solution

Stator base quantities:

$$S_B = 160/3 = 53.3333\,\mathrm{MVA/phase}$$
$$V_B = 15{,}000/\sqrt{3} = 8660.25\,\mathrm{V}$$
$$I_B = 6158.40\,\mathrm{A}$$
$$t_B = 2.6526 \times 10^{-3}\,\mathrm{s}$$
$$\lambda_B = 8660 \times 2.65 \times 10^{-3} = 22.972\,\mathrm{Wb\,turn/phase}$$
$$R_B = 8660.25/6158.40 = 1.406\,\Omega$$
$$L_B = 8660/(377 \times 6158) = 3.730 \times 10^{-3}\,\mathrm{H}$$
$$L_{md} = L_d - \ell_d = (6.341 - 0.5595)10^{-3} = 5.79 \times 10^{-3}\,\mathrm{H}$$
$$L_{mq} = L_q - \ell_q = (6.118 - 0.5595)10^{-3} = 5.56 \times 10^{-3}\,\mathrm{H}$$

4.11 relates abcFGDQ flux linkages to abcFGDQ currents, e.g., $\lambda_a = L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{af}i_F + L_{aG}i_G$ $L_{aD}i_D + L_{aQ}i_Q$. At open cct, this is $\lambda_a = L_{af}i_F$ which is, by 4.16, $\lambda_a = (M_F\cos\theta)i_F$.

To obtain M_F, we use (4.11), (4.16), and (4.23). At open circuit the mutual inductance L_{aF} an the flux linkage in phase a are given by

$$L_{aF} = M_F\cos\theta \qquad \lambda_a = i_F M_F \cos\theta$$

🡐 [Also, see detail in these notes below.]

The instantaneous open circuit voltage of phase a is $\boxed{v_a = i_F\omega_R M_F \sin\theta,}$ where ω_R is the rated synchronous speed. Thus the peak phase voltage V_{peak} corresponds to the product $i_F\omega_R M_F$ so that $\boxed{M_F = V_{\text{peak}}/(i_F\omega_R)}$. From the air gap line of the no-load saturation curve, the value of the field current at rated voltage is 365 A. Therefore,

🡐4.23 gives $v_a = -r_a i_a - d\lambda_a/dt$; for open cct, $i_a = 0$ and so v_a is negative of the derivative of λ_a.

🡐Note "peak phase voltage".

$V_{peak} = V_{rms} * \sqrt{2}$

$$M_F = 8660\sqrt{2}/(377 \times 365) = 89.006 \times 10^{-3}\,\mathrm{H}$$
$$\mathrm{k}M_F = \sqrt{3/2} \times 89.006 \times 10^{-3} = 109.01 \times 10^{-3}\,\mathrm{H}$$

Then $k_F = \mathrm{k}M_F/L_{md} = 18.854$.

Then we compute, from (4.55) to (4.57),

$$I_{FB} = 6158.4/18.854 = 326.64\,\mathrm{A}$$ From (4.55) in VMAF, and bottom p. 12, $I_{FB} = I_B/k_F$

$$M_{FB} = 18.854 \times 3.73 \times 10^{-3} = 70.329 \times 10^{-3}\,\mathrm{H}$$ From (4.57) in VMAF, and top of p. 14, $M_{FB} = k_F L_B$

$$V_{FB} = \left(53.33 \times 10^6\right)/326.64 = 163{,}280.68\,\mathrm{V}$$ From (4.55) in VMAF, and bottom of p. 12, $V_{FB} = V_B I_B/I_{FB} = S_B/I_{FB}$

$$R_{FB} = 163{,}280.68/326.64 = 499.89\,\Omega$$ From (4.??) in VMAF, and bottom of p. , $R_{FB} = V_{FB}/I_{FB}$

$$L_{FB} = (18.845)^2 \times 3.73 \times 10^{-3} = 1.326\,\mathrm{H}$$

The only thing that is perhaps not too clear is the computation of M_F. I will just review that part of it here.

Computation of M_F: VMAF make the statement (see above):

> *"From the no-load magnetization curve, the value of field current corresponding to the rated voltage on the air-gap line is 365 A."*

The “open-circuit characteristic” or “magnetization curve” plots

- Something proportional to exciting (field) current on horizontal axis
- Something proportional to the flux on the vertical axis.

under open-circuit conditions (phase windings are open). Figure 3 below illustrates.

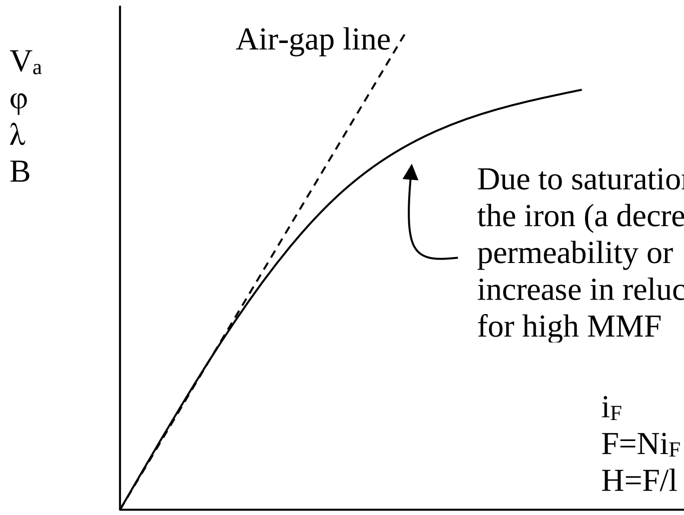


Fig. 3

The air-gap line is the V_a vs. i_F relation that results if the iron has constant permeability. The solid line that bends to the right is the actual characteristic that occurs, which shows that terminal voltage falls away from the air-gap line as the field current is raised beyond a certain point. This falling away is caused by saturation of the ferromagnetic material, resulting from the decrease in permeability under high flux conditions. Figure 4 illustrates a magnetization curve for a real 13.8 kV synchronous machine. The vertical axis is line-to-line voltage.

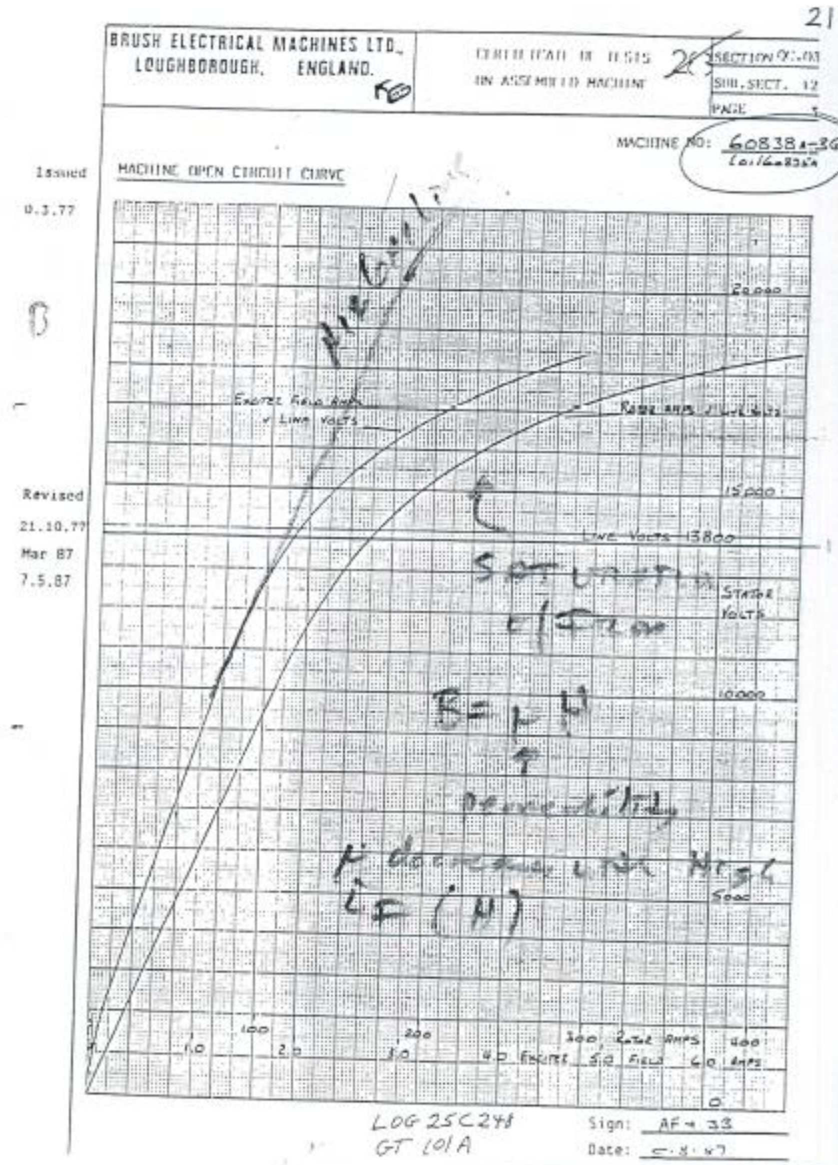
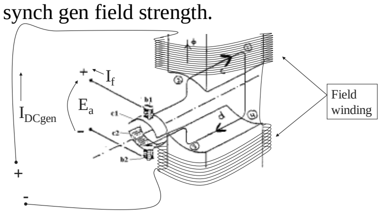


Fig. 4

This synch gen is separately excited, and so its field current I_f (here designated as "rotor amps") is supplied from the armature of a separate DC gen. The separate DC gen has field current I_{DCgen} (on DC gen stator) which creates field flux ϕ. The current I_f, which is the armature current of the DC gen and the field current of the synch gen, increases with DC gen armature voltage E_a, and $E_a=k\omega\phi$, where ϕ increases with I_{DCgen}. And so I_f and I_{DCgen} are both indicators of synch gen field strength.



The two magnetization curves to the left plot line-to-line open cct voltage of the synch gen against (a) I_{DCgen} and (b) I_f.

What is done in Ex. 4.1 (and what can be done in industry to obtain M_F), is that the field current is determined corresponding to steady-state rated open circuit terminal voltage. This voltage is $V_B=V_{LL\text{-rated}}/\text{sqrt}(3)$. For Ex. 4.1, this is V_B=15kV/sqrt(3)=8660 volts. This is the rms voltage, but VMAF indicate that we need the corresponding peak voltage: $V_{peak}=\sqrt{2}(8660)=12{,}247.1$ volts. But why do we need the peak voltage?

Let's consider this question.

From first page of previous notes titled "Machine Equations," or from eq. (4.11) in VMAF, we have

$$\lambda_a = L_{aa}i_a + L_{ab}i_b + L_{ac}i_c + L_{aF}i_F + L_{aD}i_D + L_{aQ}i_Q + L_{aQ}i_G$$

But $i_a=i_b=i_c=0$ under open circuit conditions.
And damper currents $i_D=i_Q=0$ under steady-state conditions. Therefore

$$\lambda_a = L_{aF}i_F + L_{aQ}i_G$$

Recall that the G-winding models the Q-axis flux produced by the eddy-current effects in the rotor during the transient period. But we are now considering only the steady-state condition, $i_G=0$. Therefore

$$\lambda_a = L_{aF}i_F \qquad (*)$$

Now recall from first page of previous notes titled "macheqts," or from eq. (4.16') in VMAF, that $L_{aF}=M_F\cos\theta$, and substitution into (*) yields

$$\lambda_a = M_F i_F \cos\theta \qquad (**)$$

Differentiating (**) results in

$$\frac{d\lambda_a}{dt} = -M_F i_F \sin\theta \frac{d\theta}{dt} = -\omega_{\mathrm{Re}} M_F i_F \sin\theta \qquad (***)$$

Now recall the voltage equation for the a-phase:

$$v_a = -i_a r_a - \dot{\lambda}_a + v_n \qquad (\#)$$

Substituting (***) into (#), we obtain

$$v_a = -i_a r_a + \omega_{\mathrm{Re}} M_F i_F \sin\theta + v_n$$

But under open circuit conditions, $i_a=0$, $i_n=0$ (implying $v_n=0$) and we have

$$v_a = \omega_{\mathrm{Re}} M_F i_F \sin\theta \qquad (\#*)$$

From (#*), we see that, under these conditions

$$V_{peak} = \omega_{\mathrm{Re}} M_F i_F \Rightarrow M_F = \frac{V_{peak}}{i_F \omega_{\mathrm{Re}}}$$

So we choose a point from the magnetization curve, for example, VMAF chose i_F=365A, V_{peak}=12,247.1volts (365A is the value of field current corresponding to the rated voltage on the air-gap line, and 12,247.1/sqrt(2)=8660volts is the rated RMS line-to-neutral voltage (corresponding to 8660sqrt(3)=15kV). Then

$$M_F = \frac{V_{peak}}{i_F \omega_{\mathrm{Re}}} = \frac{12{,}247.1}{(365)(377)} = 89.006 \times 10^{-3} \ \text{henries}$$

And from this we can compute

$$k_F = \frac{kM_F}{L_{md}} = \frac{kM_F}{L_d - l_d}$$

where the denominator is comprised of data provided by the manufacturer (see “Last comment” at the end of these notes).

The rest of Ex. 4.1 is just an application of our per-unitization formula.

There is an interesting paragraph in Appendix C, pg. 693 of your text, to which I want to draw your attention. It says,

> “*Note that a key element in determining the factor k_F, and hence all the rotor base quantities, is the value of M_F (in H). This is obtained from the air gap line of the magnetization curve provided by the manufacturer. Unfortunately, no such data is given for any of the amortisseur circuits. Thus, while the pu values of the various amortisseur elements can be determined, their corresponding MKS data are not known.*

I provide some comments on certain sentences in this paragraph:

- *“Note that a key element in determining the factor k_F, and hence all the rotor base quantities*,” refers to the fact that we use k_F to obtain L_{FB}, R_{FB} and M_{FB} from:

$$L_{FB} = k_F^2 L_B \qquad\qquad M_{FB} = k_F L_B$$

$$R_{FB} = k_F^2 R_B$$

- *“This is obtained from the air gap line of the magnetization curve provided by the manufacturer,”* as we have seen above by using

$$M_F = \frac{V_{peak}}{i_F \omega_{\mathrm{Re}}}$$

We are able to get M_F in this way because we can directly control the current i_F, with no other circuits energized (as a result of the open-circuit, steady-state conditions), and directly measure the induced voltage at the a-phase terminals.

- *"Unfortunately, no such data is given for any of the amortisseur circuits."* It is not possible to directly control the currents i_D, i_Q, and i_G, since their corresponding circuits do not have sources. The only way to energize these circuits is via a transient condition, but there is no way to provide a transient condition that will also not energize other circuits, which would result in the measured terminal voltage being induced from the mutual inductance between itself and the other circuits as well.
- "*Thus, while the pu values of the various amortisseur elements can be determined, their corresponding MKS data are not known."* In example 4.1, the text puts an asterisk by some of the parameters (L_D, L_Q, kM_D, kM_Q, r_D, and r_Q), indicating they were "estimated for academic study"). This is because manufacturer's datasheets do not usually include the parameters for the amortisseur (and G-winding) circuits, simply because they are hard to measure (based on the comments of the previous bullet). However, if one can obtain any q-axis mutual inductance, then others are also identified because, as we shall see in Section 3.0 below, in per-unit, all direct-axis mutuals are equal and all quadrature-axis mutuals are equal! In other words:
    - D-axis mutuals:

      F-d winding mutual, kM_F
      D-d winding mutual, kM_D
      F-D winding mutual, M_R (is called M_X in some texts)

      That is, we will show that in per-unit, $kM_{Fu} = kM_{Du} = M_{Ru}$
    - Q-axis mutuals:

      G-q winding mutual, kM_G
      Q-q winding mutual, kM_Q
      G-Q winding mutual, M_Y

      That is, we will show that in per-unit, $kM_{Qu} = kM_{Gu} = M_{Yu}$
      (See "Last Comment" at end of these notes.)

2.4 Applying the bases to voltage equations (section 4.8 in VMAF):

Recall our voltage equation as written in MKS units:

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \\ -v_F \\ 0 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r_a+3r_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_b & \omega L_q & 0 & \omega\sqrt{\frac{3}{2}}M_G & 0 & \omega\sqrt{\frac{3}{2}}M_Q \\ 0 & -\omega L_D & r_c & -\omega\sqrt{\frac{3}{2}}M_F & 0 & -\omega\sqrt{\frac{3}{2}}M_D & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix}$$

$$-\begin{bmatrix} L_0+3L_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & \sqrt{\frac{3}{2}}M_F & 0 & \sqrt{\frac{3}{2}}M_D & 0 \\ 0 & 0 & L_q & 0 & \sqrt{\frac{3}{2}}M_G & 0 & \sqrt{\frac{3}{2}}M_Q \\ 0 & \sqrt{\frac{3}{2}}M_F & 0 & L_F & 0 & M_R & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_G & 0 & L_G & 0 & M_Y \\ 0 & \sqrt{\frac{3}{2}}M_D & 0 & M_R & 0 & L_D & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_Q & 0 & M_Y & 0 & L_Q \end{bmatrix} \begin{bmatrix} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_G \\ \dot{i}_D \\ \dot{i}_Q \end{bmatrix}$$

Let's normalize them using our chosen bases to obtain the equations in per-unit. The per-unit equations should appear as above when done, except that everything must be in per-unit.

Step 1: Replace all MKS voltages on the left with

- the product of their per-unit value and their base value (use V_B for the first 3 equations and V_{FB}, V_{DB}, V_{QB}, V_{GB} for the last four equations),

and replace all currents on the right with

- the product of their per-unit value and their base value (use I_B for the first 3 equations and I_{FB}, I_{DB}, I_{QB}, I_{GB} for the last four equations).

This results in eq. 4.60 in the text, as follows….

$$
\begin{bmatrix} v_{0u}V_B \\ v_{du}V_B \\ v_{qu}V_B \\ -v_{Fu}V_{FB} \\ 0 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r+3r_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega L_q & 0 & \omega kM_G & 0 & \omega kM_Q \\ 0 & -\omega L_D & r & -\omega kM_F & 0 & -\omega kM_D & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_{0u}I_{\mathrm{B}} \\ i_{du}I_{\mathrm{B}} \\ i_{qu}I_{\mathrm{B}} \\ i_{Fu}I_{\mathrm{FB}} \\ i_{Gu}I_{\mathrm{GB}} \\ i_{Du}I_{\mathrm{DB}} \\ i_{Qu}I_{\mathrm{QB}} \end{bmatrix}
$$

$$
-\begin{bmatrix} L_0+3L_n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & 0 & kM_D & 0 \\ 0 & 0 & L_q & 0 & kM_G & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & 0 & M_R & 0 \\ 0 & 0 & kM_Q & 0 & L_G & 0 & M_Y \\ 0 & kM_D & 0 & M_R & 0 & L_D & 0 \\ 0 & 0 & kM_G & 0 & M_Y & 0 & L_Q \end{bmatrix} \begin{bmatrix} \dot{i}_{0u}I_{\mathrm{B}} \\ \dot{i}_{du}I_{\mathrm{B}} \\ \dot{i}_{qu}I_{\mathrm{B}} \\ \dot{i}_{Fu}I_{\mathrm{FB}} \\ \dot{i}_{Gu}I_{\mathrm{GB}} \\ \dot{i}_{Du}I_{\mathrm{DB}} \\ \dot{i}_{Qu}I_{\mathrm{QB}} \end{bmatrix} \qquad \text{(eq. 4.60)}
$$

Step2: For each of the equations in the above, we need to divide through by the voltage base. For those equations containing ω, we replace it with $\omega=\omega_u\omega_B$ ($\omega_B=\omega_{Re}$). Then we do some algebra on each equation to express the coefficients of each current and current derivative as per-unitized self or mutual inductances. As an example, the 2nd equation is done for you in VMAF (p.109); here, I do the 5th equation, corresponding to the G-winding.

$$
0(V_{GB}) = -r_G i_{Gu} I_{GB} - \sqrt{\frac{3}{2}} M_G \dot{i}_{qu} I_B - M_Y \dot{i}_{Qu} I_{QB} - L_G \dot{i}_{Gu} I_{GB}
$$

Step 2a: Divide through by V_{GB} to obtain:

$$
0 = \frac{-r_G}{V_{GB}/I_{GB}} i_{Gu} - \sqrt{\frac{3}{2}} \frac{M_G}{\dfrac{V_{GB}}{I_B}} \dot{i}_{qu} - \frac{M_Y}{\dfrac{V_{GB}}{I_{QB}}} \dot{i}_{Qu} - \frac{L_G}{\dfrac{V_{GB}}{I_{GB}}} \dot{i}_{Gu}
$$

The first term has a denominator of R_{GB}. The last 3 terms are not so obvious. We desire them to have denominators of M_{GB}, M_{YB}, and L_{GB}, respectively, where, from above (p. 14), we recall $M_{GB}=k_G L_B$, $M_{YB}=k_G k_Q L_B$, $L_{GB}=k_G^2 L_B$, where $k_G=\dfrac{V_{GB}}{V_B}=\dfrac{I_B}{I_{GB}}$, and $k_Q=\dfrac{I_B}{I_{QB}}$.

Step 2b: Let's multiply the denominator of the last three terms by V_B/V_B. This results in:

$$0 = \frac{-r_G}{R_{GB}} i_{Gu} - \sqrt{\frac{3}{2}} \frac{M_G}{\frac{V_{GB}}{V_B}\frac{V_B}{I_B}} \dot{i}_{qu} - \frac{M_Y}{\frac{V_{GB}}{V_B}\frac{V_B}{I_{QB}}} \dot{i}_{Qu} - \frac{L_G}{\frac{V_{GB}}{V_B}\frac{V_B}{I_{GB}}} \dot{i}_{Gu}$$

Step 2c: Let's multiply the denominator of the last two terms by I_B/I_B. This results in:

$$0 = \frac{-r_G}{R_{GB}} i_{Gu} - \sqrt{\frac{3}{2}} \frac{M_G}{\frac{V_{GB}}{V_B}\frac{V_B}{I_B}} \dot{i}_{qu} - \frac{M_Y}{\frac{V_{GB}}{V_B}\frac{V_B}{I_B}\frac{I_B}{I_{QB}}} \dot{i}_{Qu} - \frac{L_G}{\frac{V_{GB}}{V_B}\frac{V_B}{I_B}\frac{I_B}{I_{GB}}} \dot{i}_{Gu}$$

Step 2d: Recall k-factors (pg 104 of text): $k_G = \frac{V_{GB}}{V_B} = \frac{I_B}{I_{GB}}$, and $k_Q = \frac{I_B}{I_{QB}}$.

Substitution yields:

$$0 = \frac{-r_G}{R_{GB}} i_{Gu} - \sqrt{\frac{3}{2}} \frac{M_G}{k_G \frac{V_B}{I_B}} \dot{i}_{qu} - \frac{M_Y}{k_G k_Q \frac{V_B}{I_B}} \dot{i}_{Qu} - \frac{L_G}{k_G{}^2 \frac{V_B}{I_B}} \dot{i}_{Gu}$$

Step 2e: We are close now, as we need $M_{GB}=k_G L_B$, $M_{YB}=k_G k_Q L_B$, and $L_{GB}=(k_G)^2 L_B$, respectively, on the denominator of the last three terms. Recall that $L_B=V_B/(\omega_B I_B)$, so we need to divide top and bottom on the denominators of the last three terms by ω_B. Doing so yields:

$$0 = \frac{-r_G}{R_{GB}} i_{Gu} - \sqrt{\frac{3}{2}} \frac{M_G}{k_G \frac{V_B}{\omega_B I_B}\omega_B} \dot{i}_{qu} - \frac{M_Y}{k_G k_Q \frac{V_B}{\omega_B I_B}\omega_B} \dot{i}_{Qu} - \frac{L_G}{k_G{}^2 \frac{V_B}{\omega_B I_B}\omega_B} \dot{i}_{Gu}$$

Step 2f: And substituting in L_B results in:

$$0=\frac{-r_G}{R_{GB}}i_{Gu}-\sqrt{\frac{3}{2}}\frac{M_G}{k_G L_B\omega_B}\dot{i}_{qu}-\frac{M_Y}{k_G k_Q L_B\omega_B}\dot{i}_{Qu}-\frac{L_G}{k_G{}^2 L_B\omega_B}\dot{i}_{Gu}$$

Step 2g: Recalling that $M_{GB}=k_G L_B$, $M_{YB}=k_G k_Q L_B$, and $L_{GB}=(k_G)^2 L_B$, we may write:

$$0=\frac{-r_G}{R_{GB}}i_{Gu}-\sqrt{\frac{3}{2}}\frac{M_G}{M_{GB}\omega_B}\dot{i}_{qu}-\frac{M_Y}{M_{YB}\omega_B}\dot{i}_{Qu}-\frac{L_G}{L_{GB}\omega_B}\dot{i}_{Gu}$$

which results in

$$0=-r_{Gu}i_{Gu}-\sqrt{\frac{3}{2}}\frac{M_{Gu}}{\omega_B}\dot{i}_{qu}-\frac{M_{Yu}}{\omega_B}\dot{i}_{Qu}-\frac{L_{Gu}}{\omega_B}\dot{i}_{Gu}$$

Step 2h: However, we still have one problem. Recall that we want the equations to be identical in pu to their form in MKS units. But in the last equation, we still have ω_B, which does not appear in our MKS equation. We can take care of it, however, by recalling that $\omega_B=1/t_B$, so that:

$$\frac{1}{\omega_B}\dot{i}_{qu}=\frac{1}{1/t_B}\frac{di_{qu}}{dt}=\frac{di_{qu}}{d\left(t/t_B\right)}=\frac{di_{qu}}{d\tau};$$

$$\frac{1}{\omega_B}\dot{i}_{Qu}=\frac{1}{1/t_B}\frac{di_{Qu}}{dt}=\frac{di_{Qu}}{d\left(t/t_B\right)}=\frac{di_{Qu}}{d\tau};$$

$$\frac{1}{\omega_B}\dot{i}_{Gu}=\frac{1}{1/t_B}\frac{di_{Gu}}{dt}=\frac{di_{Gu}}{d\left(t/t_B\right)}=\frac{di_{Gu}}{d\tau}$$

where $\tau=t/t_B$ is the normalized time.

With this last change, we can write, finally, that

$$0=-r_G i_{Gu}-\sqrt{\frac{3}{2}}M_{Gu}\dot{i}_{qu}-M_{Yu}\dot{i}_{Qu}-L_{Gu}\dot{i}_{Gu}$$

which is the per-unitized form of the last equation in eq. (4.60).

Note that it is exactly the same form as the original equation in MKS units!

Similar work can be done for the other equations (and you should try to do one of the others yourself), resulting in equation 4.74 in your text:

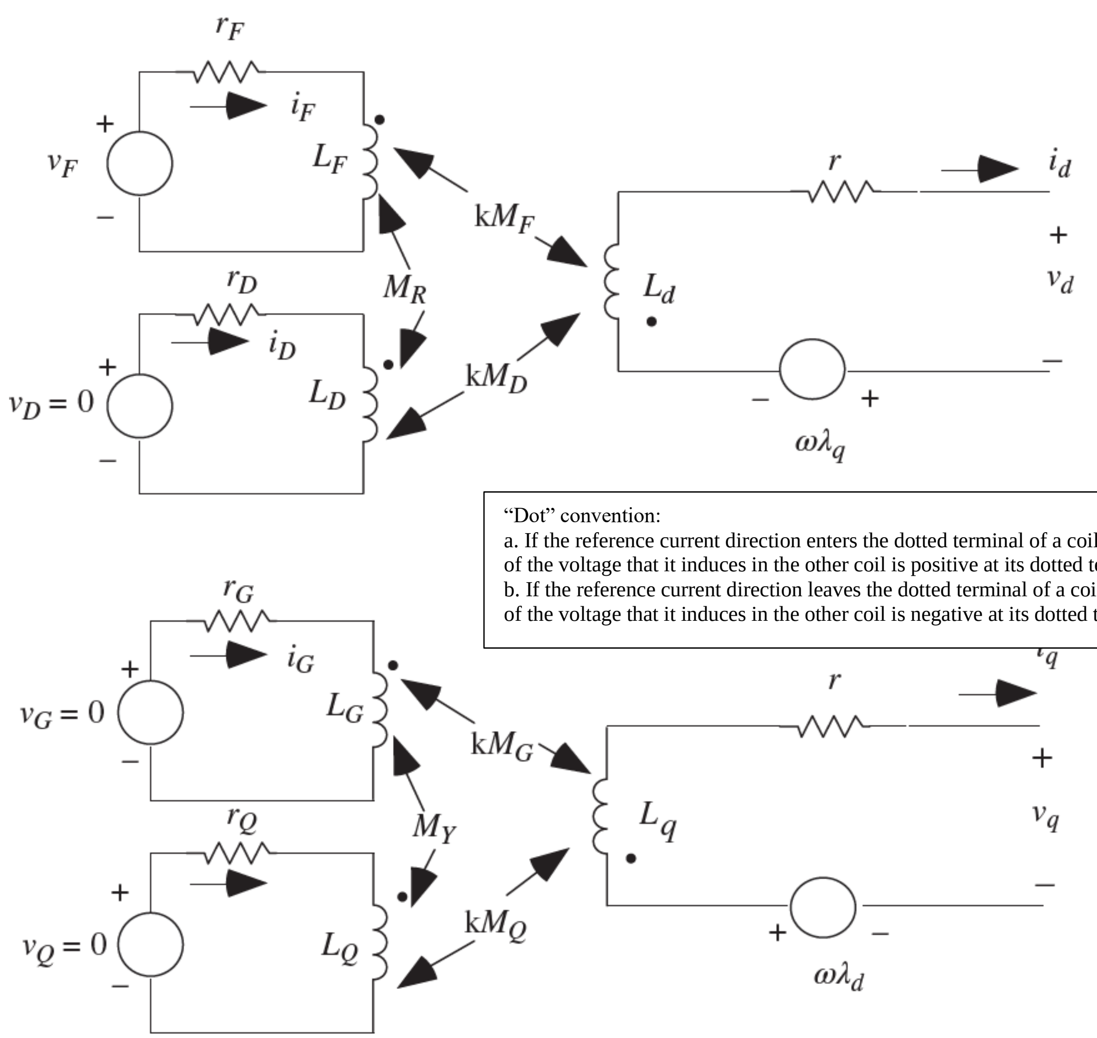
$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ \hline v_q \\ 0 \\ 0 \end{bmatrix} = -\left[\begin{array}{ccc|ccc} r & 0 & 0 & \omega L_q & \omega kM_G & \omega kM_Q \\ 0 & r_F & 0 & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 & 0 \\ \hline -\omega L_d & -\omega kM_F & -\omega kM_D & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{array}\right]\begin{bmatrix} i_d \\ i_F \\ i_D \\ \hline i_q \\ i_G \\ i_Q \end{bmatrix} - \left[\begin{array}{ccc|ccc} L_d & kM_F & kM_D & 0 & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & L_q & kM_G & kM_Q \\ 0 & 0 & 0 & kM_G & L_G & M_Y \\ 0 & 0 & 0 & kM_Q & M_Y & L_Q \end{array}\right]\begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_D \\ \hline \dot{i}_q \\ \dot{i}_G \\ \dot{i}_Q \end{bmatrix} \qquad \text{(eq. 4.74)}$$

Note that in the above equation,

- The "u" subscript is dropped; however, all parameters are in pu.
- We have dropped the zero-sequence voltage equation since we will be interested in balanced conditions for stability studies. (A system having a three-phase fault, considered to be, usually, the most severe, is still a balanced system. This does not mean that we cannot analyze unbalanced faults using stability programs. It is possible to analyze the effects of unbalanced faults on the positive sequence network represented in stability programs – see

Kimbark Vol I, pp. 220-221). Otherwise, eq. (4.74) is precisely the same as eqt. 4.39 in VMAF (see the same equation as 4.39, in the notes called "macheqts").

- The equations are rearranged from stator then rotor equations to d-axis then q-axis equations to better display coupling and decoupling between various circuits. This coupling is well illustrated by Fig. 4.3 in VMAF, given below. Note that coupling between F and D windings is captured by M_R. Some works call this mutual inductance M_X. We use M_R to remain consistent with VMAF. These are physically-realizable circuits for which KVL in each of the 6 circuits results in the 6 equations of 4.74 above.



"Dot" convention:
a. If the reference current direction enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.
b. If the reference current direction leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal.

Fig 4.3

Now let's make some definitions:

$$\underline{R} = \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r_F & 0 & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix}; \qquad \underline{N} = \begin{bmatrix} 0 & 0 & 0 & L_q & kM_G & kM_Q \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -L_d & -kM_F & -kM_D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} L_d & kM_F & kM_D & 0 & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_G & kM_Q \\ 0 & 0 & 0 & kM_G & L_G & M_Y \\ 0 & 0 & 0 & kM_Q & M_Y & L_Q \end{bmatrix}$$

$$\underline{v} = \begin{bmatrix} v_d \\ -v_F \\ 0 \\ v_q \\ 0 \\ 0 \end{bmatrix}; \qquad \underline{i} = \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_G \\ i_Q \end{bmatrix}$$

With these definitions, we rewrite eqt. (4.74) in compact notation:

$$\underline{v} = -(\underline{R} + \omega \underline{N})\underline{i} - \underline{L}\dot{\underline{i}} \qquad \text{(eq. 4.75)}$$

We may solve eq. (4.75) for di/dt so that it is in state-space form:

$$\dot{\underline{i}} = -\underline{L}^{-1}(\underline{R} + \omega \underline{N})\underline{i} - \underline{L}^{-1}\underline{v} \qquad \text{(eq. 4.76)}$$

It is useful, once again, to recall the power of Park: the above (although just for 1 machine and no network) is a set of ODEs with constant coefficients!!!

3.0 Per-unit mutuals (See Section 4.11)

<u>A useful observation regarding per-unit values of M_F, M_D, and M_R</u>:

Recall our definitions of the D-axis k-factors:

$$k_F = \frac{I_B}{I_{FB}} = \frac{V_{FB}}{V_B}$$

$$k_D = \frac{I_B}{I_{DB}} = \frac{V_{DB}}{V_B}$$

and the *fundamental constraints among base currents* that we developed (see eqs. 4.54, pg 12 of these notes):

$$\underset{1}{L_{md} I_B^2} = \underset{2}{L_{mF} I_{FB}^2} = \underset{3}{L_{mD} I_{DB}^2} = \underset{4}{kM_F I_B I_{FB}} = \underset{5}{kM_D I_B I_{DB}} = \underset{6}{M_R I_{FB} I_{DB}} \quad (18)$$

From the first and fourth expression in eq (18), we have:

$$L_{md} I_B^2 = kM_F I_B I_{FB}$$

1.

Thus,

$$\frac{I_B}{I_{FB}} = \frac{kM_F}{L_{md}} \quad (19)$$

Likewise, from the first and fifth, and from the fourth and sixth expressions in eq (18), we have:

$$L_{md} I_B^2 = kM_D I_B I_{DB} \qquad kM_F I_B I_{FB} = M_R I_{FB} I_{DB}$$

Thus,

$$\frac{I_B}{I_{DB}} = \frac{kM_D}{L_{md}} \quad \text{and} \quad \frac{I_B}{I_{DB}} = \frac{M_R}{kM_F} \quad (20)$$

From the definitions of the k-factors (top of this page), and eqs (19) and (20), we have:

$$k_F = \frac{kM_F}{L_{md}} \quad \text{and} \quad k_D = \frac{kM_D}{L_{md}} = \frac{M_R}{kM_F} \quad (21)$$

And from eq. 4.57 in text (also see p. 14 of these notes), we find

2.

$$M_{FB} = k_F L_B, \; M_{DB} = k_D L_B, \; M_{RB} = k_F k_D L_B \quad (22)$$

which we obtained by using the fact that base mutuals must be the geometric mean of the base self-inductances (see prob 4.18).

Now, recall the elements in the per-unitized voltage equations as given by eq. 4.74 (see page 25 of these notes).

$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ \hline v_q \\ 0 \\ 0 \end{bmatrix} = -\left[\begin{array}{ccc|ccc} r & 0 & 0 & \omega L_q & \omega kM_G & \omega kM_Q \\ 0 & r_F & 0 & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 & 0 \\ \hline -\omega L_d & -\omega kM_F & -\omega kM_D & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{array}\right]\begin{bmatrix} i_d \\ i_F \\ i_D \\ \hline i_q \\ i_G \\ i_Q \end{bmatrix}$$

$$-\left[\begin{array}{ccc|ccc} L_d & kM_F & kM_D & 0 & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & L_q & kM_G & kM_Q \\ 0 & 0 & 0 & kM_G & L_G & M_Y \\ 0 & 0 & 0 & kM_Q & M_Y & L_Q \end{array}\right]\begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_D \\ \hline \dot{i}_q \\ \dot{i}_G \\ \dot{i}_Q \end{bmatrix} \qquad 4.74$$

3.

In particular, consider the mutual terms in the last matrix for the direct axis. These are in the upper left-hand 3x3 block, in the blue box. These terms, in pu, are by definition the ratio of the term in MKS to the appropriate base. Therefore:

- <u>Stator-field mutual</u>: $kM_{Fu} = \dfrac{kM_F}{M_{FB}}$. **3.**

  2. **1.**

  Substituting for M_{FB} from eq. (22) and then k_F from eq. (21) results in:

$$kM_{Fu} = \frac{kM_F}{M_{FB}} = \frac{kM_F}{k_F L_B} = \frac{kM_F L_{md}}{kM_F L_B} = \frac{L_{md}}{L_B} \equiv L_{mdu}$$

So this comes from (i) k_F definition; (ii)fundamental constraints among base currents (eq. 4.54), & (iii)definition of pu mutual kM_{Fu}

- <u>Stator-D-winding damper mutual</u>: $kM_{Du} = \dfrac{kM_D}{M_{DB}}$. **3.**

  2. **1.**

  Substituting for M_{DB} from eq. (22) and then k_D from eq. (21) results in:

$$kM_{Du} = \frac{kM_D}{M_{DB}} = \frac{kM_D}{k_D L_B} = \frac{kM_D L_{md}}{kM_D L_B} = \frac{L_{md}}{L_B} \equiv L_{mdu}$$

So this comes from (i) k_D definition; (ii)fundamental constraints among base currents (eq. 4.54), & (iii)definition of pu mutual kM_{Du}

- Field-D-winding damper mutual: $M_{Ru} = \frac{M_R}{M_{RB}}$ **3.**

2.

Substituting for M_{RB} from eq. (22) and then k_F and k_D from eq. (21) (using the 2nd expression for k_D in eq. (4)) results in: **1.**

$$M_{Ru} = \frac{M_R}{M_{RB}} = \frac{M_R}{k_F k_D L_B} = \frac{M_R L_{md} kM_F}{kM_F M_R L_B} = \frac{L_{md}}{L_B} \equiv L_{mdu}$$

So this comes from (i) k_F, k_D definitions; (ii)fundamental constraints among base currents (eq. 4.54), & (iii)definition of pu mutual kM_{Ru}

Important fact: *In per-unit, all d-axis mutuals are numerically equal to L_{mdu} (per p. 9, L_{md} is the magnetizing inductance, i.e., $L_d=L_{md}+l_d$).* We will define a new term for them, L_{AD}, as the per-unit value of any d-axis mutual inductance, so that:

$$L_{AD} \equiv L_{mdu} = kM_{Fu} = kM_{Du} = M_{Ru}$$

Also note that the mutual is the difference between the self and the leakage, so that

$$L_{du}-l_{du}=L_{Du}-l_{Du}=L_{Fu}-l_{Fu}=L_{AD}$$

The above relations are given in eqs. 4.107 and 4.108 in VMAF.

We can go through a similar process for the q-axis mutuals (from 4.74, we see that these are the terms in the lower right-hand block of the matrix, kM_Q, kM_G, and M_Y). I will leave this for you to do. The result is:

$$L_{AQ} \equiv L_{mqu} = kM_{Qu} = kM_{Gu} = M_{Yu}$$

$$L_{qu}-l_{qu}=L_{Qu}-l_{Qu}=L_{Gu}-l_{Gu}=L_{AQ}$$

The above relations are given by eq. 4.109 in your text.

L_{AD} and L_{AQ} are very important for drawing the equivalent circuits.

They are also important in dealing with saturation because they provide for the definition of the per-unit mutual flux (we will see this in our development of the flux-linkage state-space model).

4.0 Equivalent Circuits (See Section 4.11)

Let's return to the voltage equations that we had before we folded in the speed voltage terms. They were:

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \\ -v_F \\ 0 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r_a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_b & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix}\begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} - \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & \sqrt{\frac{3}{2}}M_F & 0 & \sqrt{\frac{3}{2}}M_D & 0 \\ 0 & 0 & L_q & 0 & \sqrt{\frac{3}{2}}M_G & 0 & \sqrt{\frac{3}{2}}M_Q \\ 0 & \sqrt{\frac{3}{2}}M_F & 0 & L_F & 0 & M_R & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_G & 0 & L_G & 0 & M_Y \\ 0 & \sqrt{\frac{3}{2}}M_D & 0 & M_R & 0 & L_D & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_Q & 0 & M_Y & 0 & L_Q \end{bmatrix}\begin{bmatrix} \dot{i}_0 \\ \dot{i}_d \\ \dot{i}_q \\ \dot{i}_F \\ \dot{i}_G \\ \dot{i}_D \\ \dot{i}_Q \end{bmatrix}$$

$$+\begin{bmatrix} 0 \\ -\omega\lambda_q \\ \omega\lambda_d \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 3r_n i_0 - 3L_n \dot{i}_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Assume all of the above is in per-unit (but we have dropped the u-subscript).

There is some advantage to re-writing these equations in terms of L_{AD} and L_{AQ}. For example, consider the d-axis equation. It is:

$$v_d = -ri_d - L_d \dot{i}_d - kM_F \dot{i}_F - kM_D \dot{i}_D - \omega\lambda_q$$

Recall that $L_d = L_{md} + l_d \rightarrow L_{md} = L_d - l_d$

Let's modify the d-axis voltage equation by adding and subtracting $l_d\,(di_d/dt)$:

$$v_d = -ri_d - L_d\dot{i}_d + l_d\dot{i}_d - l_d\dot{i}_d - kM_F\dot{i}_F - kM_D\dot{i}_D - \omega\lambda_q\,,$$

which can be written as:

$$v_d = -ri_d - l_d\dot{i}_d - [(L_d - l_d)\dot{i}_d + kM_F\dot{i}_F + kM_D\dot{i}_D] - \omega\lambda_q$$

The advantage to this is that, in per-unit, we recall that L_d-l_d=kM_F=kM_D≡L_{AD}. Therefore,

$$v_d = -ri_d - l_d\dot{i}_d - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D] - \omega\lambda_q$$

Let's repeat this for the G-axis equation, which is, from the matrix equation at the beginning of this section:

$$v_G = 0 = -r_G i_G - kM_G\dot{i}_q - M_Y\dot{i}_Q - L_G\dot{i}_G$$

Let's modify the G-winding voltage equation by adding and subtracting $l_G\,(di_G/dt)$:

$$v_G = 0 = -r_G i_G - kM_G\dot{i}_q - M_Y\dot{i}_Q - L_G\dot{i}_G + l_G\dot{i}_G - l_G\dot{i}_G$$

which can be written as:

$$v_G = 0 = -r_G i_G - kM_G\dot{i}_q - M_Y\dot{i}_Q - (L_G - l_G)\dot{i}_G - l_G\dot{i}_G$$

The advantage to this is that, in pu, we have L_G-l_G=kM_G=M_Y≡L_{AQ}.

Therefore

$$v_G = 0 = -r_G i_G - l_G\dot{i}_G - L_{AQ}(\dot{i}_q + \dot{i}_Q + \dot{i}_G)$$

Repeating this procedure for the F, D, and q equations, and then summarizing, we obtain:

<u>D-axis relations</u>:

$$v_d = -ri_d - l_d\dot{i}_d - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D] - \omega\lambda_q$$

$$-v_F = -r_F i_F - l_F\dot{i}_F - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D]$$

$$v_D = 0 = -r_D i_D - l_D\dot{i}_D - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D]$$

<u>Q-axis relations</u>:

$$v_q = -ri_q - l_q\dot{i}_q - L_{AQ}(\dot{i}_q + \dot{i}_Q + \dot{i}_G) + \omega\lambda_d$$
$$v_Q = 0 = -ri_Q - l_Q\dot{i}_Q - L_{AQ}(\dot{i}_q + \dot{i}_Q + \dot{i}_G)$$
$$v_G = 0 = -r_G i_G - l_G\dot{i}_G - L_{AQ}(\dot{i}_q + \dot{i}_Q + \dot{i}_G)$$

We desire to draw circuits that are characterized by these equations.

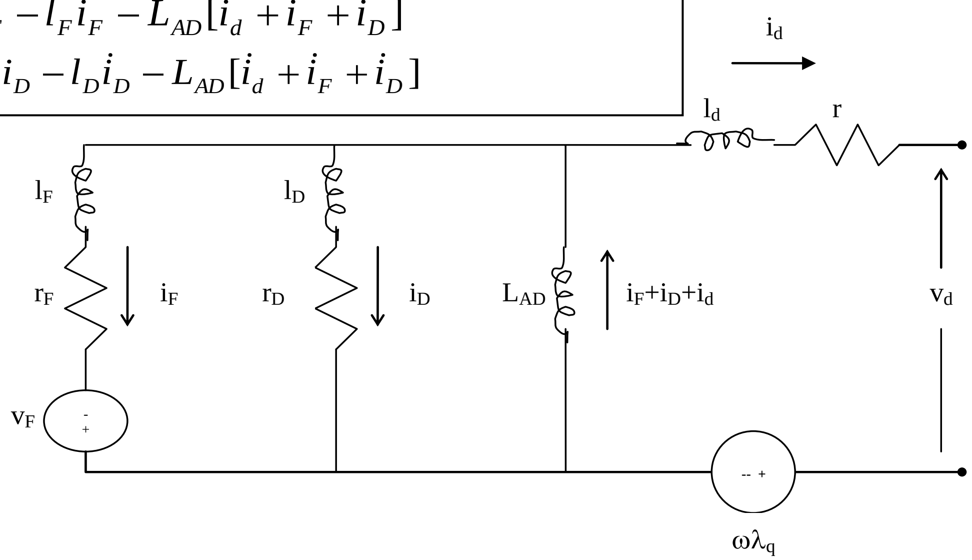
Note:

- The D-axis relations are coupled through the L_{AD} terms.
- This term, for each equation, may be represented by a single "center" branch.
- The other terms, for each equation, may be represented as single branches which feed the center branch.

This results in the circuit of Fig 4.5 in your text.

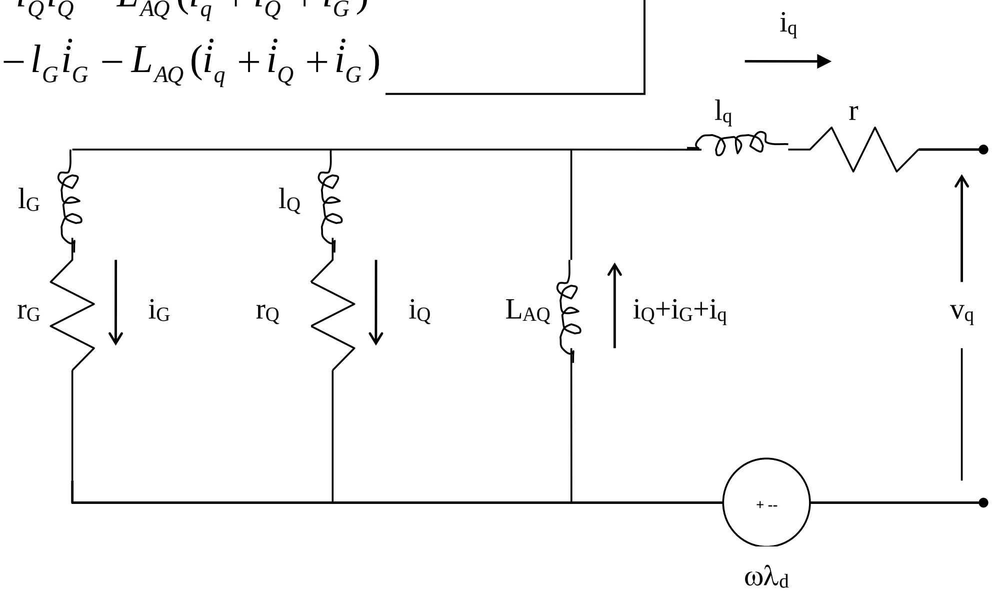
Similar reasoning results in the circuit of Fig. 4.6 in your text. We redraw these circuits below.

$$v_d = -ri_d - l_d\dot{i}_d - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D] - \omega\lambda_q$$
$$-v_F = -r_F i_F - l_F\dot{i}_F - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D]$$
$$v_D = 0 = -r_D i_D - l_D\dot{i}_D - L_{AD}[\dot{i}_d + \dot{i}_F + \dot{i}_D]$$



Direct-axis equivalent circuit:
The above is the same as Fig. 4.5 in your text

$$
\begin{aligned}
v_q &= -ri_q - l_q\dot{i}_q - L_{AQ}(\dot{i}_q + \dot{i}_Q + \dot{i}_G) + \omega\lambda_d \\
v_Q &= 0 = -ri_Q - l_Q\dot{i}_Q - L_{AQ}(\dot{i}_q + \dot{i}_Q + \dot{i}_G) \\
v_G &= 0 = -r_G i_G - l_G\dot{i}_G - L_{AQ}(\dot{i}_q + \dot{i}_Q + \dot{i}_G)
\end{aligned}
$$



Quadrature-axis equivalent circuit:
The above is the same as Fig. 4.6 in your text

The ability to draw these circuits is a direct result of the L_{AD} and L_{AQ} relations that occur only in per-unit. Therefore, it is important to be in the per-unit system when utilizing these circuits. And the "equal mutuals" effect came as a result of the fact that we chose our base currents according to the following criteria (see p. 8 of these notes):

> *We select the base currents for the four rotor-side windings F, D (Q, G) to produce the same mutual flux in the air gap as produced by the stator-side base current I_B flowing in the corresponding fictitious d-axis (q-axis) coil.*

See Appendix C of your text, at the top of p. 689, for another way to articulate this fact.

These equivalent circuits are useful for:

- Remembering the voltage relations.
- Gaining physical understanding of relations between d-q-F-D-Q-G quantities.
- Reading the literature, where you will see them often.

Last comments: There are two electrical parameter sets (inductances and resistances, or inductances and time constants) used to characterize synchronous machines.

- The one we are using up until this point is sometimes referred to as the fundamental or basic parameter set; Kundur on p. 139 of his book uses this terminology. This includes self inductances L_q, L_F, L_D, L_q, L_G, and L_Q, mutual inductances M_F, M_R, M_D, M_G, M_Y, and M_Q, and resistances r, r_F, r_D, r_G and r_Q.
- Another parameter set is referred to as the standard parameter set; both Kundur (p. 144) and VMAF (p. 156, top paragraph) use this terminology for this parameter set. It includes self-inductances L_d, L'_d, L''_d, L_q, L'_q, L''_q, open circuit time constants T'_{d0}, T''_{d0}, T'_{q0}, T''_{q0}, and short circuit time constants T'_d, T''_d, T'_q, T''_q. We study this parameter set next in notes called "Subtransient & Transient Inductances" and "TimeConstants".

The two parameter sets provide equivalent info; one set may be derived from another. An important distinction between the parameter sets is how testing may be used to obtain them; this is an issue extending from Example 4.1 (see p. 14 of these notes) where we see we cannot get q-axis quantities at all and need manufacturer's data to get d-axis quantities (see p. 19). There are 3 kinds of testing that can be done to determine synchronous machine parameters.

- Short circuit test: As the name implies, the machine is exposed to a sudden short-circuit across its terminals; parameters may be computed based on recorded currents and voltages. This approach is able to obtain d-axis parameters but not q-axis parameters;
- Decrement test: This test is also called a step-response test and a stator current interruption test. It is able to obtain both d-axis and q-axis parameters.
- Standstill frequency response (SSFR) test: Here, one applies an excitation to the stator terminals over a range of frequencies, measuring the resulting currents, with the ratio providing what are referred to as operational inductances.

Section 4.16 of VMAF summarizes these tests and provides excellent references for further reading on parameter determination. In reading the VMAF Section 4.16 (appended below), you will also see that it describes "operational inductances" that are directly related to results of the SSFR test. Kundur, on pp. 139-143, develops this approach nicely in a section he titles "Operational parameters," which is obtained directly from Park's original 1929 paper. Kundur describes operational parameters this way (p. 139-140):

> "*A convenient method of identifying the machine electrical characteristics is in terms of operational parameters relating the armature and field terminal quantities… where*
>
> - *G(s) is the stator to field transfer function*
> - *$L_d(s)$ is the d-axis operational inductance*
> - *$L_q(s)$ is the q-axis operational inductance*."

The above expressions, which are functions of the Laplace variable "s," can be expressed in terms of various time constants and take on various forms depending on the time frame of interest.

Krause, in his book (p. 283), has a chapter titled "Operational impedances and time constants of synchronous machines," which contains in its introduction a useful discussion of this issue, as follows:

> R. H. Park [1] in his original paper did not specify the number of rotor circuits. Instead, he expressed the stator flux linkages in terms of operational impedances and a transfer function relating stator flux linkages to field voltage. In other words, Park recognized that, in general, the rotor of a synchronous machine appears as a distributed parameter system when viewed from the stator. The fact that an accurate, equivalent lumped parameter circuit representation of the rotor of a synchronous machine might require two, three, or four damper windings was more or less of academic interest until large digital computers became available. Prior to the 1970s the damper windings were seldom considered in stability studies; however, as the size of computers increased, it became desirable to represent the machine in more detail.
>
> The standard short-circuit test, which involves monitoring the stator short-circuit currents, provides information from which the parameters of the field winding and one damper winding in the *d* axis can be determined. The parameters for the *q*-axis damper winding are calculated from design data. Due to the need for more accurate parameters, frequency-response data are now being used as means of measuring the operational impedances from which the parameters can be obtained for any number of rotor windings in both axes.

Another good reference on testing is Padiyar. I have cut our his descriptions of short circuit tests, decrement tests, and SSFR tests and appended them at the end of these notes.

4.16 PARAMETER DETERMINATION FOR GENERATOR DYNAMIC MODELS

The synchronous machine models used in this chapter, which are in common use by power system engineers, are based on a standard machine with discrete physical windings on the stator and rotor. As mentioned in Section 4.14, the solid iron rotor used in large steam turbine generators provides multiple paths for circulating eddy currents that act as equivalent damper windings under dynamic conditions. The representation of these paths by one discrete circuit on each axis in Model 2.1, and even when the G-circuit is added in Model 2.2, has been questioned for some time. Another source of concern to the power engineer is that the values of the machine constants (i.e., inductances $L_d, L'_d, L''_d, L_q, L'_q$, and L''_q and time constants τ'_{d0}, τ''_{d0}, τ'_{q0}, and τ''_{q0}), sometimes called standard parameters, used in dynamic studies are derived from data intended to define fault current magnitudes and decrements. In some stability studies, discrepancies between computer simulation and field data have been observed. One reason for these discrepancies is the inadequate definition of machine inductances in the frequency ranges encountered in stability studies. It is of interest that the standard parameters, as used in Sections 4.14 and 4.15, differ from the so-called Park's variables used in the earlier sections, e.g., equations (4.74), (4.103), (4.154), and (4.163). As indicated in [27], the preference for standard parameters is largely driven by the ability to directly determine them via machine tests, something that cannot be done in Park's variables.[3]

There have been three main approaches used to obtain machine parameters via testing. The first and earliest approach is the short circuit test, described in [28]; although effective for obtaining d axis parameters, other procedures must be used to obtain q axis parameters. A second approach, initially suggested in [29] and identified in its discussion as a step response test, was further investigated in [30, 31]. It is referred to as a decrement test in [32]. It is advantageous in that it obtains both d and q axis parameters and, unlike short circuit tests, it may be conducted under low loading conditions; in [33], it is identified as a stator current interruption test. A third approach, frequency response testing, is most often performed offline as in [22, 23, 34–36] where it is referred to as a standstill frequency response (SSFR) test, but it may also be conducted online [37, 38].

The studies made in [22, 23, 34–36] ascertained the accuracy of available dynamic models and data for turbine generators. These studies showed that a detailed representation of the rotor circuits can be more accurately simulated by up to three discrete rotor circuits on the d axis and three on the q axis. Data for these circuits can be obtained from SSFR tests. To fit the "conventional" view of rotor circuits that influence the so-called subtransient and transient dynamic behavior of the machine, it is found that two rotor circuits (on each axis) are sometimes adequate but the inductances and time constants are not exactly the same as those obtained via short circuit tests, as recommended in early standards [18]. Reference [39] described general SSFR testing while [40] focused on application to salient pole machines. References [28, 33, 41] provide recent descriptions of both short circuit and SSFR tests to determine machine parameters for dynamic analysis.

As described in [28], SSFR testing measures electrical responses of synchronous machines to small perturbations of stator and rotor quantities about an operating point characterized by the following s-domain relations:

$$\lambda_d(s) = G(s)\Delta v_F(s) - L_d(s)\Delta i_d(s) \qquad \lambda_q(s) = -L_q(s)\Delta i_q(s) \tag{4.302a}$$

where $G(s)$ is the armature flux to field voltage transfer function; $L_d(s)$, called the direct axis operational inductance, is the Laplace transform of the ratio of the direct axis armature flux linkages to the direct axis current, with the field winding short-circuited; and $L_q(s)$, called the quadrature axis operational inductance, is the Laplace transform of the ratio of the quadrature axis armature flux linkages to the direct axis current. Consistent with the approach described in [1], these functions are developed according to the following procedure: (1) Substitute the flux linkage equations of (4.104) into the voltage equations of (4.36). (2) Transform the equations into the Laplace domain and then solve for the rotor currents in terms of the field voltage $v_F(s)$ and the armature currents $i_d(s)$ and $i_q(s)$. (3) Substitute the relations resulting from step (2) into the armature flux linkage relations for $\lambda_d(s)$ and $\lambda_q(s)$ from (4.104).

The procedure for determining the machine constants is to assume equivalent circuits on each axis made up of a number of circuits in parallel. The transfer function for $L_d(s)$ and $L_q(s)$ is called an operational inductance of the form

$$L(s) = [N(s)/D(s)]L \tag{4.302b}$$

where L is the synchronous reactance and $N(s)$ and $D(s)$ are polynomials in s. Thus, for the d axis, we express a third-order operational inductance as

$$L_d(s) = L_d \frac{(1+a_1 s)(1+b_1 s)(1+c_1 s)}{(1+a_2 s)(1+b_2 s)(1+c_2 s)} \tag{4.303}$$

and the constants L_d, a_1, a_2, b_1, b_2, c_1, and c_2 are determined from the frequency domain response.

If the operational inductance is to be approximated by quadratic polynomials, i.e., as second order, the constants can be identified approximately with the transient and subtransient parameters. Thus, for the d axis, $L_d(s)$ becomes

$$L_d(s) = L_d \frac{\left[1 + \left(L'_d/L_d\right)\tau'_{d0}s\right]\left[1 + \left(L''_d/L'_d\right)\tau''_{d0}s\right]}{\left(1 + \tau'_{d0}s\right)\left(1 + \tau''_{d0}s\right)} \tag{4.304}$$

The time constants in (4.304) are different from those associated with the exponential decay of d or q axis open circuit voltages, hence the discrepancy with those obtained via short circuit tests.

An example of the data obtained by SSFR tests is given in [35] and is reproduced in Figure 4.21. Both third-order and second-order polynomial representations are given. Machine data thus obtained differ from standard data previously obtained by the manufacturer from short circuit tests. Reference

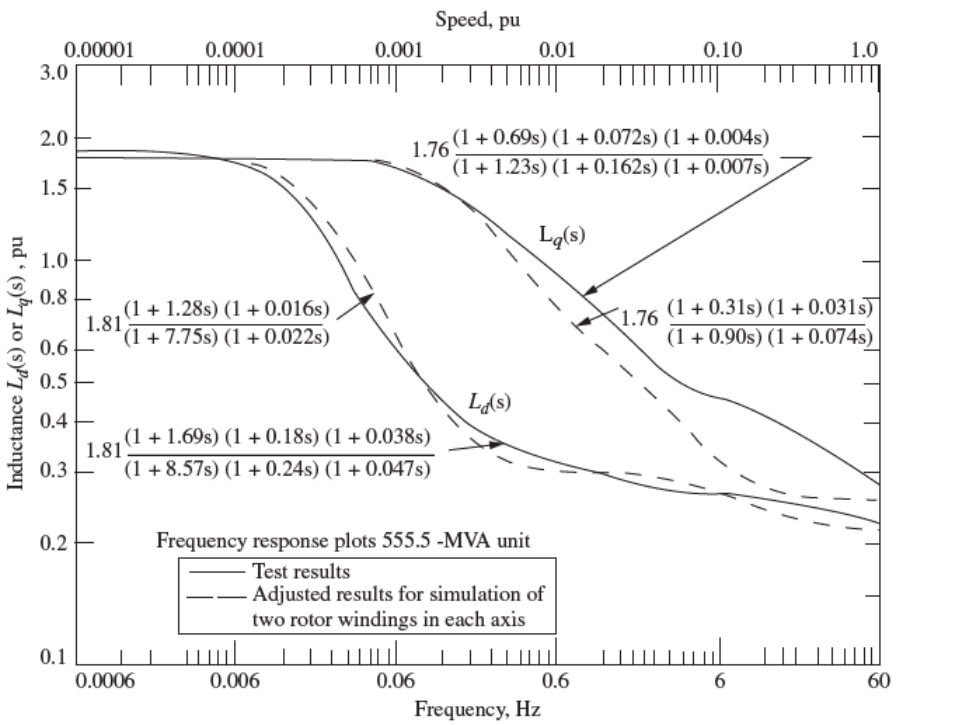


Figure 4.21 Frequency response plot for a 555-MVA turboalternator. (Source: © IEEE. Reprinted from [35].)

TABLE 4.7 Comparison of Standard Data with Data Obtained from Frequency Tests for a 555-MVA Turboalternator

Constants	Standard Data	Adjusted Data
L_d pu	1.97	1.81
L'_d pu	0.27	0.30
L''_d pu	0.175	0.217
L_q pu	1.867	1.76
L'_q pu	0.473	0.61
L''_q pu	0.213	0.254
L_ℓ pu	0.16	0.16
τ'_{d0} s	4.3	7.8
τ''_{d0} s	0.031	0.022
τ'_{q0} s	0.56	0.90
τ''_{q0} s	0.061	0.074

Source: © IEEE. Reprinted from [35].

[35] gives a comparison between the two sets of data for a 555-MVA turbogenerator. This comparison is given in Table 4.7.

The inductance versus frequency plot given in Figure 4.21 is nothing more than the amplitude portion of the familiar Bode plot with the amplitude given in pu rather than in decibels. The transfer functions plotted in Figure 4.21 can be approximated by the superposition of multiple first-order asymptotic approximations. If this is done, the break frequencies should give the constants of (4.304). The machine constants thus obtained are given in the third column of Table 4.6. If, however, the machine constants obtained from the standard data are used to obtain the breakpoints for the straight-line approximation of the amplitude–frequency plots, the approximated curve does not provide a good fit to the experimental data. For example, the d axis time constant τ'_{d0} of the machine, as obtained by standard methods, is 4.3 s. If this is used to obtain the first break frequency for $\log\left[1/\left(1+\tau'_{d0}s\right)\right]$, the computed break frequency is

$$1/\tau'_{d0} = 1/4.3 = 0.2326\,\text{rad/s} = 0.00062\,\text{pu} \tag{4.305}$$

The breakpoint that gives a better fit of the experimental data corresponds to a frequency of 0.1282 rad/s or 0.00034 pu. Since the amplitude at this frequency is the reciprocal of the d axis transient time constant, this corresponds to an adjusted value, denoted by τ'^{*}_{d0}, given by

$$\tau'^{*}_{d0} = 1/0.1282 = 7.8\,\text{s} \tag{4.306}$$

Reference [35] notes that the proper adjustment of τ'_{d0}, τ'_{q0}, and L'_q are all particularly important in stability studies.

A study conducted by the Northeast Power Coordinating Council (NPCC) [42] concludes that, in general, it is more important in stability studies to use accurate machine data than to use more elaborate machine models. Also, the accuracy of any dynamic machine model is greatly improved when the so-called standard machine data are modified to match the results of a frequency analysis of the solid iron rotor equivalent circuit. Indeed, as indicated in [33], frequency response tests is capable of verifying the model over the entire frequency range of interest, and it can be used to identify characteristics of individual subsystems of the excitation system, stabilizer, and governor. However, reference [28] indicates that SSFR must be conducted with the machine operating at nonstandard (overly low) magnetizing currents, and as a result, adjustment of SSFR-based parameters using short circuit data could be beneficial. It appears the most accurate characterization of machine parameters results from integrating data from both types of tests as done in, for example [43].

Finally, a comparison of these results and the machine models presented in this chapter is in order. Model 2.1 is one of the models investigated in the NPCC study [42] for solid-rotor machines. It was found to be inferior to the more elaborate model based on two windings in each axis, Model 2.2. This is not surprising since the added detail due to the extra q axis amortisseur should result in an improved simulation. Perhaps more surprising is the fact that the model developed with F-, D-, and Q-windings, Model 2.1, provided practically no improvement over a simpler model with only F- and Q-windings, Model 1.1. Furthermore, with the F–Q Model 1.1 based on time constants τ'_{d0} and τ'_{q0}, larger digital integration time steps are possible than with models that use the much shorter time constants τ''_{d0} and τ''_{q0}, as is the case for Models 2.1 and 2.2.

There has been a great deal of work over the past several decades to identify the best machine models and data for steady-state and dynamic stability studies and the proper means for testing or estimating these data. This work has culminated in three standards published by the North American Electric Reliability Corporation (NERC) focused on testing and model verification for power plants [44–47]; these standards are applicable for power plants composed of synchronous machines or inverter-based technologies (e.g., wind and solar PV plants). In particular, [45] addresses model verification, required every 10 years for each power plant exceeding the interconnection's capacity threshold (50 or 75 MVA in ERCOT, 75 MVA in the Western Interconnection, and 100 MVA in the Eastern Interconnection) to confirm that the dynamic performance of the generator, excitation system, power system stabilizer, and voltage compensator models accurately reflect response of the actual equipment installed in the field. Reference [46] does likewise for turbine governor models. Reference [33] provides insightful guidelines for use of these standards in relation to power plants composed of synchronous machines; reference [47] does the same in relation to power plants composed of inverter-based resources.

REFERENCES

1. Concordia, C. (1951). *Synchronous Machines*. New York: Wiley.
2. Kimbark, E.W. (1956). *Power System Stability*, vol. 3. New York: Wiley (Vol. 1, 1948).
3. Adkins, B. (1964). *The General Theory of Electrical Machines*. London: Chapman and Hall.
4. Crary, S.B. (1945). *Power System Stability*, vol. 1. New York: Wiley Vol. 2, 1947.
5. Lynn, T.W. and Walshaw, M.H. (1961). *Tensor Analysis of a Synchronous Two-Machine System*. IEE (British) Monograph. London: Cambridge University Press.

6. Taylor, G.D. (1962). *Analysis of Synchronous Machines Connected to Power Network*. IEE (British) Monograph. London: Cambridge University Press.
7. Westinghouse Electric Corporation (1950). *Electrical Transmission and Distribution Reference Book*. Pittsburgh, PA: Westinghouse Electric Corporation.
8. Anderson, P.M. (1973). *Analysis of Faulted Power Systems*. Ames: Iowa State University Press.
9. Harris, M.R., Lawrenson, P.J., and Stephenson, J.M. (1970). *Per Unit Systems: With Special Reference to Electrical Machines*. IEE (British) Monograph. London: Cambridge University Press.
10. Krause, P., Wasynczuk, O., and Sudhoff, S. (1995). *Analysis of Electric Machinery*. IEEE Press.
11. Park, R.H. (1929). Two reaction theory of synchronous machines, Pt. 1. *AIEE Trans.* 48: 716–730.
12. Park, R.H. (1933). Two reaction theory of synchronous machines, Pt. 2. *AIEE Trans.* 52: 352–355.
13. Lewis, W.A. (1958). A basic analysis of synchronous machines, Pt. 1. *AIEE Trans.* PAS-77: 436–455.
14. Krause, P.C. and Thomas, C.H. (1965). Simulation of symmetrical induction machinery. *IEEE Trans. Power App. Syst.* PAS-84: 1038–1052.
15. IEEE Committee Report, Harrington, D. (chair) (1969). Recommended phasor diagram for synchronous machines. *IEEE Trans. Power App. Syst.* PAS-88: 1593–1610.
16. Prentice, B.R. (1929). Fundamental concepts of synchronous machine reactances. *AIEE Trans.* 56(Suppl. 1: 716–720.
17. Rankin, A.W. (1945). Per unit impedances of synchronous machines. *AIEE Trans.* 64: 569–572, 839–841.
18. IEEE (1965). *Test Procedures for Synchronous Machines*. Standard No. 115 (March). IEEE.
19. IEEE Standard 1110-2002 (R2007).20 (2003). *IEEE Guide for Synchronous Generator Modeling Practices and Applications in Power System Stability Analyses*. IEEE.
20. Prabhashankar, K. and Janischewskyj, W. (1968). Digital simulation of multimachine power systems for stability studies. *IEEE Trans. Power App. Syst.* PAS-87: 73–80.
21. Fouad, A. and Vittal, V. (1992). *Power System Transient Stability Analysis Using the Transient Energy Function Method*. Prentice-Hall.
22. Jackson, W.B. and Winchester, R.L. (1969). Direct and quadrature axis equivalent circuits for solid-rotor turbine generators. *IEEE Trans. Power App. Syst.* PAS-88: 1121–1136.
23. Schulz, R.P., Jones, W.D., and Ewart, D.N. (1973). Dynamic models of turbine generators derived from solid rotor equivalent circuits. *IEEE Trans. Power App. Syst*. PAS-92: 926–933.
24. Young, C.C. (1972). Equipment and system modeling for large-scale stability studies. *IEEE Trans. Power App. Syst*. PAS-91: 99–109.
25. Task Force on Definitions and Procedures and Working Group on Determination and Application of Synchronous Machine Models for Stability Studies, Dandeno, P. (chair) (March 1986). Current usage & suggested practices in power system stability simulations for synchronous machines. *IEEE Trans. Energy Convers.* 1 (1).
26. Schulz, R.P. (1975). *Synchronous machine modeling. Symposium on Adequacy and Philosophy of Modeling: System Dynamic Performance*. IEEE Publ. 75 CH O970-PWR. IEEE.
27. Olive, D.W. (1969). Digital Simulation of Synchronous Machines Transients. *IEEE Trans. Power App. Syst.* PAS-87 (8): 1669–1675.
28. IEEE Standard 115-2009 (2010). *IEEE Guide for Test Procedures for Synchronous Machines*. IEEE.
29. de Mello, F.P. and Ribeiro, J.R. (1977). Derivation of synchronous machine parameters from tests. *IEEE Trans. Power App. Syst.* PAS-96: 1211–1218.
30. Shackshaft, G. and Poray, A.T. (1977). Implementation of new approach to determination of synchronous-machine parameters from tests. *IEE Proc* 124 (12).
31. de Mello, F.P. and Hannet, L.H. (1981). Validation of synchronous machine models and derivation of model parameters from tests. *IEEE Trans. Power App. Syst*. PAS-100 (2): 662–672.
32. Kundur, P. (1994). *Power System Stability and Control*. McGraw-Hill.
33. NERC Reliability Guideline (2018). *Power Plant Model Verification and Testing for Synchronous Machines* (July 2018). North American Electric Reliability Corporation (NERC).
34. Watson, W. and Manchur, G. (1974). Synchronous machine operational impedances from low voltage measurements at the stator terminals. *IEEE Trans. Power App. Syst.* PAS-93: 777–784.
35. Kundur, P. and Dandeno, P.L. (1974). Stability performance of 555 MVA turboalternators – digital comparisons with system operating tests. *IEEE Trans. Power App. Syst.* PAS-93: 767–776.
36. Dandeno, P.L., Hauth, R.L., and Schulz, R.P. (1973). Effects of synchronous machine modeling in large-scale system studies. *IEEE Trans. Power App. Syst*. PAS-92: 574–582.
37. Dandeno, P.L., Kundur, P., Poray, A.T., and Coultes, M.E. (1981). Validation of turbogenerator stability models by comparisons with power system tests. *IEEE Trans. Power App. Syst.* PAS-100 (4): 1637–1645.
38. Dandeno, P.L., Kundur, P., Poray, A.T., and Zein El-Din, H.M. (1981). Adaptation and validation of turbogenerator model parameters through on-line frequency response measurements. *IEEE Trans. Power App. Syst.* PAS-100 (4): 1656–1664.
39. Task Force on Definitions and IEEE Joint Working Group on Determination of Synchronous Machine Stability Constants, Dandeno, P. (chair) (1980). Supplementary definitions & associated test methods for obtaining parameters for synchronous machine stability study simulation. *IEEE Trans. Power App. Syst.* PAS-99 (4): 1625–1633.
40. IEEE/PES Working Group #12 of the Synchronous Machinery Subcommittee of the Electric Machinery Committee, Dandeno, P. (chair) (1999). Experience with standstill frequency response (SSFR) testing and analysis of salient pole synchronous machines. *IEEE Trans. Energy Convers.* 14 (4): 1209–1217.

41. IEC 60034-4-1:2018, International Standard (2018). *Rotating Electrical Machines – Part 4-1: Methods for Determining Electrically Excited Synchronous Machine Quantities from Tests.* IEC.
42. Northeast Power Coordinating Council (1971). *Effects of Synchronous Machine Modeling in Large-Scale System Studies.* Final Report, NPCC-10, Task Force on System Studies, System Dynamic Simulation Techniques Working Group. NPCC.
43. Canay, I. (1993). Modeling of alternating-current machines having multiple rotor circuits. *IEEE Trans. Energy Convers.* 8 (2): 280–296.
44. NERC Standard MOD-025-2 (2014). *Verification and Data Reporting of Generator Real and Reactive Power Capability and Synchronous Condenser Reactive Power Capability* (Approved 2014). NERC.
45. NERC Standard MOD-026-1 (2014). *Verification of Models and Data for Generator Excitation Control System or Plant Volt/Var Control Function* (Approved 2014). NERC.
46. NERC Standard MOD-027-1 (2014). *Verification of Models and Data for Turbine/Governor and Load Control or Active Power/Frequency Control* (Approved 2014). NERC.
47. NERC Reliability Guideline (2018). *Power Plant Model Verification for Inverter-Based Resources* (June 2018). NERC.

From Padiyar, pp. 85-87:

Short Circuit Tests [26]

IEEE Standard No.115 [26] describes in detail the short circuit tests which were first proposed in 1948. The latest revision of the test code was done in 1983.

A typical test is a three phase short circuit applied to the terminals of a synchronous machine which is running at rated speed on open circuit. The open circuit voltage can be chosen at any value within specifications. For determination of the reactances, the tests are performed for several voltage levels in a range typically up to about 0.5 to 0.6 p.u. of rated terminal voltage. The oscillograms of the armature currents are obtained and the variations in the peak to peak current magnitudes with time are plotted on semi-logarithmic paper. Generally, two slopes in the current variation are identified. The projection of each slope to zero time (when the fault is applied) will determine the initial magnitude of the current, which, when divided into the voltage magnitude before the fault, gives a reactance. The initial, smaller value is the subtransient reactance (x_d'') and the second larger value is the transient resistance (x_d'). The slopes are also used to derive the time constants T_d' and T_d''. There is no procedure in IEEE Standard No.115 for a similar test to obtain quantities in the q-axis.

Decrement Tests: [23, 24, 27]

These tests involve sudden changes imposed on either stator or field windings. In the method described in [24], the machine armature currents are interrupted under two initial operating conditions (i) $i_d = 0$ and (ii)$i_q = 0$. The conditions can be achieved by under-exciting or over exciting the machine at some percentage of the full load. Achieving an exact loading condition for either $i_d = 0$ or $i_q = 0$ is unnecessary provided an accurate measurement of the rotor angle is available.

Decrement tests proposed by Shackshaft [23, 27] have been used at Central Electricity Generating Board in U.K. There are two types of tests

a) Stator decrement tests

b) Rotor decrement tests

In a stator decrement test, the machine under test is operated at zero load and is excited totally from the power system, i.e. its field current is zero. The generator is then suddenly disconnected from the system and the subsequent variation of stator voltage and the current in the field winding (if closed) can be used to determine the machine parameters.

In a rotor decrement test with the stator on open circuit, the machine is excited via its field winding and the excitation supply is then suddenly shorted out. From the decay of the field current and stator voltage, some of the parameters can be obtained.

Frequency Response Tests [25, 29-32]

An alternative approach to the determination of machine parameters is through frequency response testing. Both (a) standstill and (b) on-line frequency response tests are used.

Conceptually, this approach involves viewing the machine model as a two port network in the d-axis and one port network in the q-axis. (See Fig. 3.14.) This representation enables even the most detailed model (3.3) to be considered. Also, these networks can be viewed as linear R-C networks whose immittance functions have certain properties. For example the function $X_d(s)$ can be expressed as

$$X_d(s) = \frac{x_d(1+sT_1)(1+sT_2)(1+sT_3)}{(1+sT_4)(1+sT_5)(1+sT_6)} \tag{3.115}$$

with 3 rotor windings in d-axis. From the properties of the immittance function, we have

$$T_3 < T_6 < T_2 < T_5 < T_1 < T_4$$

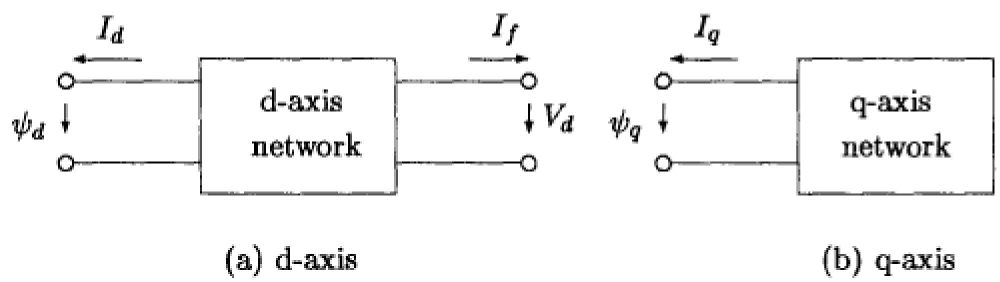


Figure 3.14: Representation of d-axis and q-axis models

For model (2.2), the above inequality can be expressed as

$$T_d'' < T_{do}'' < T_d' < T_{do}'$$

Similarly for the q-axis, we have

$$T_q'' < T_{qo}'' < T_q' < T_{qo}'$$

The standstill frequency response (SSFR) test is convenient to use and the details are given in [29]. In addition to the determination of the transfer functions $\frac{V_d}{I_d}(s)$, $\frac{V_q}{I_q}(s)$, with field winding shorted, Coultes and Watson [29] also recommend the measurement of two more transfer function $\frac{V_f}{I_d}(s)$ and $\frac{I_f}{I_d}(s)$, the former with the field winding open and the latter with the field shorted. The measurements also enable the computation of x_{rc} in the d-axis equivalent circuit of Fig. 3.7.

The direct axis operational impedance, $\frac{V_d}{I_d}(s)$ is measured using any one of the two possible connections shown in Fig. 3.15. In the first connection (a), the magnetic axes of phase a and field are aligned ($\theta = 0$). In the second connection (b), the magnetic axis of field winding is at 90^o to that of phase a ($\theta = 90^o$).

It can be shown for connection (a) that,

$$\frac{V_d(s)}{I_d(s)} = -\frac{2}{3}\frac{V}{I}(s) = -Z_d(s) \tag{3.116}$$

$$X_d(s) = \frac{\omega_B}{s}\ [Z_d(s) - R_a] \tag{3.117}$$

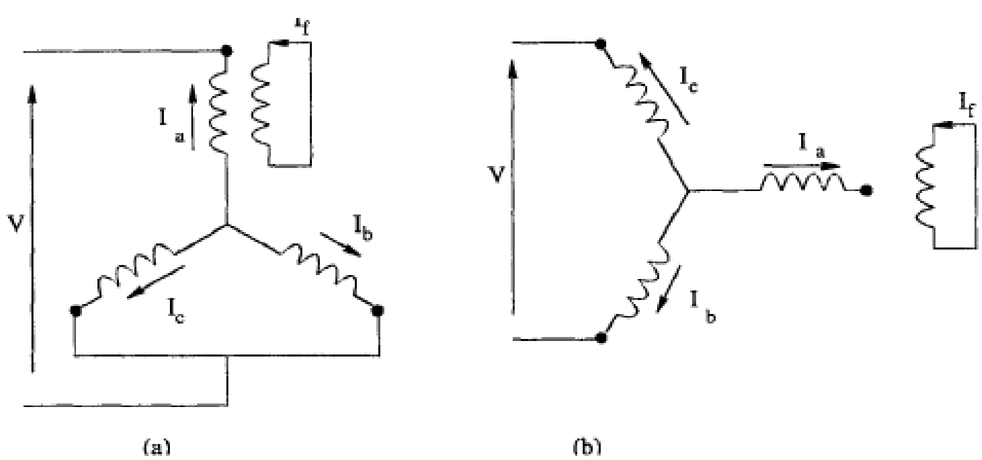


Figure 3.15: Two connections for measuring direct axis impedances