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POWER SYSTEM STEADY-STATE STABILITY AND THE LOAD-FLOW JACOBIAN

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Abstract

This paper presents the relationship between a detailed power system dynamic model and a standard load-flow model. The linearized dynamic model is examined to show how the load-flow Jacobian appears in the system dynamic state Jacobian for evaluating steady-state stability. Two special cases are given for the situation when singularity of the load-flow Jacobian implies singularity of the system dynamic state Jacobian.

Keywords

Steady-state stability, load-flow Jacobian, power system dynamics.

1. Introduction

The subjects of voltage collapse and voltage instability have created a renewed interest in load-flow Jacobian singularities and their relationship to steady-state stability [1-10]. While load-flow has been the primary method used to compute steady-state conditions, its role in evaluating stability has not been fully clarified. In 1975, V. A. Venikov et al published a paper which proposed that under certain conditions, there is a direct relationship between the singularity of the standard load-flow Jacobian and the singularity of the system dynamic state Jacobian [11]. This paper has been cited as the primary justification for studying the load-flow Jacobian matrix to determine critical load levels. In this paper, we clarify this result in the context of a fairly general dynamic model and show that the result should be considered optimistic for any type of steady-state stability analysis. The paper includes a tutorial on the role of load-flow in dynamic analysis.

2. A Detailed Dynamic Model Without Stator/Network Transients

This section presents a basic nonlinear multimachine dynamic model which includes the fundamental features of voltage and frequency control, but assumes that all stator/network transients have been eliminated. It has been shown that for pure R-L networks and loads, these transients are very fast compared to the slower mechanical or voltage type dynamics, and as such can be formally eliminated using singular perturbation and the concept of integral manifolds [12-14]. The elimination of the stator/network transients leads to algebraic equations which accompany the multimachine dynamic model as follows:

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \qquad i=1,\ldots,m \qquad (1)$$

$$M_i \frac{d\omega_i}{dt} = T_{Mi} - [E'_{qi} - X'_{di} I_{di}] I_{qi} - [E'_{di} + X'_{qi} I_{qi}] I_{di} - D_i(\omega_i - \omega_s) \qquad i=1,\ldots,m \qquad (2)$$

$$T'_{doi} \frac{dE'_{qi}}{dt} = -E'_{qi} - (X_{di} - X'_{di}) I_{di} + E_{fdi} \qquad i=1,\ldots,m \qquad (3)$$

$$T'_{qoi} \frac{dE'_{di}}{dt} = -E'_{di} + (X_{qi} - X'_{qi}) I_{qi} \qquad i=1,\ldots,m \qquad (4)$$

$$T_{Ei} \frac{dE_{fdi}}{dt} = -(K_{Ei} + S_{Ei}(E_{fdi})) E_{fdi} + V_{Ri} \qquad i=1,\ldots,m \qquad (5)$$

$$T_{Ai} \frac{dV_{Ri}}{dt} = -V_{Ri} + K_{Ai} R_{fi} - \frac{K_{Ai} K_{Fi}}{T_{Fi}} E_{fdi} + K_{Ai}(V_{refi} - V_i) \qquad i=1,\ldots,m \qquad (6)$$

$$T_{Fi} \frac{dR_{fi}}{dt} = -R_{fi} + \frac{K_{Fi}}{T_{Fi}} E_{fdi} \qquad i=1,\ldots,m \qquad (7)$$

$$T_{RHi} \frac{dT_{Mi}}{dt} = -T_{Mi} + \left(1 - \frac{K_{HPi} T_{RHi}}{T_{CHi}}\right) P_{CHi} + \frac{K_{HPi} T_{RHi}}{T_{CHi}} P_{SVi} \qquad i=1,\ldots,m \qquad (8)$$

$$T_{CHi} \frac{dP_{CHi}}{dt} = -P_{CHi} + P_{SVi} \qquad i=1,\ldots,m \qquad (9)$$

$$T_{SVi} \frac{dP_{SVi}}{dt} = -P_{SVi} + P_{Ci} - \frac{1}{R_i}\left(\frac{\omega_i}{\omega_s}\right) \qquad i=1,\ldots,m \qquad (10)$$

$$0 = V_i e^{j\theta_i} + (R_{si} + jX'_{di})(I_{di} + jI_{qi}) e^{j(\delta_i - \pi/2)} - [E'_{di} + (X'_{qi} - X'_{di}) I_{qi} + jE'_{qi}] e^{j(\delta_i - \pi/2)} \qquad i=1,\ldots,m \qquad (11)$$

$$0 = -P_i - jQ_i + V_i e^{j\theta_i}(I_{di} - jI_{qi}) e^{-j(\delta_i - \pi/2)} + P_{Li}(V_i) + jQ_{Li}(V_i) \qquad i=1,\ldots,m \qquad (12)$$

$$0 = -P_i - jQ_i + P_{Li}(V_i) + jQ_{Li}(V_i) \qquad i=m+1,\ldots,n \qquad (13)$$

$$0 = -P_i - jQ_i + \sum_{k=1}^{n} V_i V_k Y_{ik} e^{j(\theta_i - \theta_k - \alpha_{ik})} \qquad i=1,\ldots,n \qquad (14)$$

where the notation is standard for a machine with one damper winding plus field (two-axis model), IEEE type I excitation system, and simplified turbine/governor model [15].

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The notation for an m-machine, n-bus system is,

$$V_i e^{j\theta_i} = \text{voltage at bus } i \qquad i=1,\ldots,n \tag{15}$$

$$V_i e^{j\theta_i} = (V_{di}+jV_{qi})e^{j(\delta_i-\pi/2)} \qquad i=1,\ldots,m \tag{16}$$

$$I_{Gi} e^{j\gamma_i} = (I_{di}+jI_{qi})e^{j(\delta_i-\pi/2)} \qquad i=1,\ldots,m \tag{17}$$

$$Y_{ik} e^{j\alpha_{ik}} = \text{standard load-flow bus admittance matrix entry} \qquad i,k=1,\ldots,n \tag{18}$$

$$P_i+jQ_i = \text{net injected power into bus } i \text{ from a path not included in the bus admittance matrix} \qquad i=1,\ldots,n \tag{19}$$

The algebraic variables P_i and Q_i are introduced so that the standard load-flow equations (14) are preserved for any dynamic load model (12) and (13). As written, this full dynamic model contains 10m dynamic states (δ,ω,E'_q, $E'_d,E_{fd},V_R,R_f,T_M,P_{CH},P_{SV}$) and 2m + 4n real algebraic states (I_d,I_q,P,Q,θ,V), and 2m inputs (V_{ref},P_C). Equation (11) is the stator algebraic equation which is normally expressed as a phasor diagram in the literature. We prefer the circuit form shown in Figure 1.

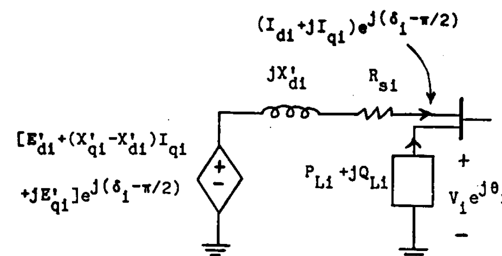

Figure 1 - Synchronous machine dynamic circuit

Note that $P_{Li}(V_i)$ and $Q_{Li}(V_i)$ are assumed to be given functions which describe the loads at all n buses as a function of their respective voltages. This representation allows most common load models including constant impedance. There are m+2n complex algebraic equations which should in principle be solved for the 2m+4n real algebraic states as functions of the 10m dynamic states. The machine currents I_d, I_q can easily be eliminated by solving (11) and substituting into (2)-(12). The P and Q algebraic states can easily be eliminated by substituting (12) and (13) into (14), leaving only n complex algebraic equations (14) to be solved for the 2n real algebraic states θ and V. These remaining algebraic equations cannot normally be solved explicitly. In the special case of constant impedance loads, it is customary to use an internal generator bus model and include all loads and the machine impedance $R_s+jX'_d$ in the bus admittance matrix (enlarged to m+n buses). With the additional assumption of $X'_q = X'_d$, all algebraic states can be explicitly eliminated with a reduced (mxm) admittance matrix. For nonlinear load models, the algebraic equations must be retained. This paper does not introduce internal machine buses.

3. Standard Load-Flow

Standard load-flow has been the traditional mechanism for computing a proposed steady-state operating point. For this paper, we define standard load-flow as the following algorithm [16-18]:

(a) Specify bus voltage magnitudes numbered 1 to m.
(b) Specify bus voltage angle number 1 (slack bus).
(c) Specify net injected real power P_i at buses numbered 2 to m.
(d) Specify load powers P_{Li} and Q_{Li} at all buses numbered 1 to n.

Solve the following equations ((13) and (14) rewritten) for $\theta_2,\ldots,\theta_n$, $V_{m+1},\ldots,V_n$,

$$0 = -P_i + \sum_{k=1}^{n} V_i V_k Y_{ik}\cos(\theta_i-\theta_k-\alpha_{ik}) \qquad i=2,\ldots,m \text{ (PV buses)} \tag{20}$$

$$0 = -P_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik}\cos(\theta_i-\theta_k-\alpha_{ik}) \qquad i=m+1,\ldots,n \text{ (PQ buses)} \tag{21}$$

$$0 = -Q_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik}\sin(\theta_i-\theta_k-\alpha_{ik}) \qquad i=m+1,\ldots,n \text{ (PQ buses)} \tag{22}$$

where $P_i(i=2,\ldots,m)$, $V_i(i=1,\ldots,m)$, $P_{Li}(i=m+1,\ldots,n)$, $Q_{Li}(i=m+1,\ldots,n)$, and θ_1 are specified numbers. The standard load-flow Jacobian matrix is the linearization of (20)-(22) with respect to $\theta_2,\ldots,\theta_n$, $V_{m+1},\ldots,V_n$. After this solution, compute

$$P_1+jQ_1 = \sum_{k=1}^{n} V_1 V_k Y_{1k} e^{j(\theta_1-\theta_k-\alpha_{1k})} \tag{23}$$

$$Q_i = \sum_{k=1}^{n} V_i V_k Y_{ik}\sin(\theta_i-\theta_k-\alpha_{ik}) \qquad i=2,\ldots,m \tag{24}$$

This standard load-flow has many variations including the addition of other devices such as tap changing under load (TCUL) transformers, switching var sources and HVDC converters. It can also include inequality constraints on quantities such as Q_i, and be revised to distribute the slack power between all generators.

We would like to make one important point about load-flow. Load-flow is normally used to evaluate operation at a specific load level (specified by a given set of powers). For a specified load and generation schedule, the solution is independent of the actual load model. That is, it is certainly possible to evaluate the voltage at a constant impedance load for a specific case where that impedance load consumes a specific amount of power. Thus the use of "constant power" in load-flow analysis does not require or even imply that the load is truly a constant power device. It merely gives the voltage at the buses when the loads (any type) consume a specific amount of power. The load characteristic is important when the analyst wants to study the system in response to a change such as contingency analysis or dynamic analysis. For these purposes, standard load-flow usually provides the "initial conditions."

4. Initial Conditions for Dynamic Analysis

For any dynamic analysis using (1)-(14), it is necessary to compute the initial values of all dynamic states and to specify the fixed inputs (V_{ref} and P_C).

in power system dynamic analysis, the fixed inputs and initial conditions are normally found from a base case load-flow solution. That is, the values of V_{ref} are computed such that the m generator voltages are as specified in the load-flow. The values of P_C are computed such that the m generator real power outputs are as specified and computed in the load-flow for rated speed (ω_s). To see how this is done, we assume that a load-flow solution (as defined in section 3) has been found. The first step in computing initial conditions is normally the calculation of generator currents from (12) and (17) as,

$$I_{Gi}e^{j\gamma_i} = ((P_i - P_{Li}(V_i)) - j(Q_i - Q_{Li}(V_i))/(V_i e^{-j\theta_i}) \qquad i=1,\ldots,m \qquad (25)$$

and machine relative rotor angles from manipulation of (11) and the algebraic equation from (4),

$$\delta_i = \text{angle of } (V_i e^{j\theta_i} + (R_{si} + jX_{qi})I_{Gi}e^{j\gamma_i}) \qquad i=1,\ldots,m \qquad (26)$$

With these quantities, the remaining dynamic and algebraic states can be found by,

$$I_{di} + jI_{qi} = I_{Gi}e^{j(\gamma_i - \delta_i + \pi/2)} \qquad i=1,\ldots,m \qquad (27)$$

$$V_{di} + jV_{qi} = V_i e^{j(\theta_i - \delta_i + \pi/2)} \qquad i=1,\ldots,m \qquad (28)$$

followed by E_{fd} from (3), (4), (11) and (16)

$$E_{fdi} = X_{di}I_{di} + V_{qi} + R_{si}I_{qi} \qquad i=1,\ldots,m \qquad (29)$$

With this field voltage, R_{fi}, V_{Ri} and V_{refi} can be found from (5)-(7) as,

$$R_{fi} = \frac{K_{Fi}}{T_{Fi}} E_{fdi} \qquad i=1,\ldots,m \qquad (30)$$

$$V_{Ri} = (K_{Ei} + S_{Ei}(E_{fdi}))E_{fdi} \qquad i=1,\ldots,m \qquad (31)$$

$$V_{refi} = V_i + (V_{Ri}/K_{Ai}) \qquad i=1,\ldots,m \qquad (32)$$

The initial values of E'_{qi} and E'_{di} are then found from (3) and (4),

$$E'_{qi} = -(X_{di} - X'_{di})I_{di} + E_{fdi} \qquad i=1,\ldots,m \qquad (33)$$

$$E'_{di} = (X_{qi} - X'_{qi})I_{qi} \qquad i=1,\ldots,m \qquad (34)$$

Note that if the machine saturation is included, this calculation for E'_{qi} and E'_{di} may be iterative. The mechanical states and P_{Ci} are found from (1), (2) and (8)-(10) as,

$$\omega_i = \omega_s \qquad i=1,\ldots,m \qquad (35)$$

$$T_{Mi} = (E'_{qi} - X'_{di}I_{di})I_{qi} + (E'_{di} + X'_{qi}I_{qi})I_{di} \qquad i=1,\ldots,m \qquad (36)$$

$$P_{CHi} = T_{Mi} \qquad i=1,\ldots,m \qquad (37)$$

$$P_{SVi} = P_{CHi} \qquad i=1,\ldots,m \qquad (38)$$

$$P_{Ci} = P_{SVi} + (1/R_i) \qquad i=1,\ldots,m \qquad (39)$$

This completes the computation of all dynamic state initial conditions and fixed inputs.

For a given disturbance, the inputs remain fixed throughout the simulation. If the disturbance occurs in the algebraic equations, the algebraic states must change instantaneously to satisfy the initial condition of the dynamic states and the new algebraic equations. Thus it may be necessary to re-solve the algebraic equations with the dynamic states specified at their initial conditions to determine the new initial values of the algebraic states.

From the above description it is clear that once a standard load-flow solution is found, the remaining dynamic states and inputs can be found in a straightforward way. The machine relative angles δ_i can always be found provided,

$$V_i e^{j\theta_i} + (R_{si} + jX_{qi})I_{Gi}e^{j\gamma_i} \neq 0 \qquad i=1,\ldots,m \qquad (40)$$

If control limits are enforced, a solution satisfying these limits may not exist. In this case, the state which is limited would need to be fixed at its limiting value and a corresponding new steady-state solution would have to be found. This would require a new load-flow specifying either different values of generator voltages, different generator real powers, or possibly specifying generator reactive power injections, thus allowing generator voltage to be a part of the load-flow solution. In fact, the use of reactive power limits in load-flow can usually be traced back to an attempt to consider excitation system limits or generator capability limits.

5. Angle Reference

In any rotational system, the reference for angles is arbitrary. Examination of (1)-(14) clearly shows that the order of this dynamic system can be reduced from 10m to 10m-1 by introducing the new relative angles (arbitrarily selecting δ_1 as reference)

$$\delta'_i = \delta_i - \delta_1 \qquad i=1,\ldots,m \qquad (41)$$

$$\theta'_i = \theta_i - \delta_1 \qquad i=1,\ldots,n \qquad (42)$$

The full system remains exactly the same as (1)-(14) with each δ_i replaced by δ'_i, each θ_i replaced by θ'_i and ω_s replaced by ω_1 in (1). During a transient, the angle δ_1 still changes from its initial condition (as found in the last section) as ω_1 changes, so that each original δ_i and θ_i can be easily recovered if needed. The angle δ'_1 remains at zero for all time. Thus for dynamic simulation, the differential equation for δ_1 is normally replaced by the algebraic equation which simply states $\delta'_1 = 0$. Notice that θ_1 is normally arbitrarily selected as zero for the load flow analysis. This means that the initial value of δ_1 is normally not zero. During a transient, θ'_1 and θ_1 change as all angles except δ'_1 change. If the inertia of machine 1 is set to infinity, ω_1 and δ_1 remain constant for all time.

6. Steady-State Stability

The steady-state stability of multimachine systems is usually evaluated by computing the eigenvalues of the system dynamic state Jacobian matrix which is the linearized version of (1)-(14) with all algebraic equations eliminated. This dynamic model has one zero eigenvalue corresponding to the angle reference discussed above. Elimination of δ_1 through the use of (41) and (42) would eliminate this zero eigenvalue. The system is linearized around a steady-state operating point found using standard load-flow. The Jacobian matrix for this standard load-flow appears as a sub-matrix in the lower right block and is denoted as J_{LF} below:

$$\begin{bmatrix} \frac{d\Delta y}{dt} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A & B & \\ C & D_{11} & D_{12} \\ & D_{21} & J_{LF} \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta z \\ \Delta v \end{bmatrix} \tag{43}$$

where v contains the load flow variables $\theta_2, \theta_3, \ldots, \theta_n$, $V_{m+1}, \ldots, V_n$. In order to evaluate the stability of the dynamic system, the algebraic equations must be eliminated. This requires the nonsingularity of the algebraic equation Jacobian (J_{AE}),

$$J_{AE} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & J_{LF} \end{bmatrix} \tag{44}$$

The stability of the steady-state equilibrium point is then determined by the system dynamic state Jacobian (J_{sys}),

$$J_{sys} = A - B\, J_{AE}^{-1}\, C \tag{45}$$

Special cases where these three Jacobians J_{sys}, J_{AE} and J_{LF} can be more explicitly related are given in the following section.

7. Special Cases

There are two special cases where the standard load flow Jacobian can be directly related to the system dynamic state Jacobian. We do not claim that these are necessarily realistic cases, only that they lead to cases where the three Jacobians can be related.

(a) The first special case makes the following assumptions:

(a1) Stator resistance of every machine is negligible ($R_{si} = 0$, $i=1,\ldots,m$).

(a2) Transient reactances of every machine are negligible ($X'_{di} = 0$, $X'_{qi} = 0$, $i=1,\ldots,m$).

(a3) Field and damper winding time constants for every machine are infinitely large (E'_{qi} = constant, E'_{di} = constant, $i=1,\ldots,m$).

(a4) Constant mechanical torque to the shaft of each generator (T_{Mi} = constant, $i=1,\ldots,m$).

(a5) Generator number one has infinite inertia. This together with (a1)-(a3) makes V_1 = constant, θ_1 = constant (infinite bus).

(a6) All loads are constant power ($P_{Li}(V_i)$ = constant, $Q_{Li}(V_i)$ = constant, $i=1,\ldots,n$).

With these assumptions, each δ_i is equal to its corresponding θ_i plus a constant, and each V_i is constant.

Choosing ω_1 as ω_s and θ_1 as zero. the dynamic model for this special case (after eliminating P_i and Q_i) is,

$$\frac{d\theta_i}{dt} = \omega_i - \omega_s \qquad i=2,\ldots,m \tag{46}$$

$$M_i \frac{d\omega_i}{dt} = T_{Mi} + P_{Li} - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) - D_i(\omega_i - \omega_s) \qquad i=2,\ldots,m \tag{47}$$

$$0 = -P_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \qquad i=m+1,\ldots,n \tag{48}$$

$$0 = -Q_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) \qquad i=m+1,\ldots,n \tag{49}$$

with

$$T_{Mi} = \text{constant} \qquad i=1,\ldots,m \tag{50}$$

$$V_i = \text{constant} \qquad i=1,\ldots,m \tag{51}$$

$$P_{Li} = \text{constant} \qquad i=1,\ldots,n \tag{52}$$

$$Q_{Li} = \text{constant} \qquad i=1,\ldots,n \tag{53}$$

$$\theta_1 = 0 \tag{54}$$

The linearized form of this model is,

$$\begin{bmatrix} \frac{d\Delta\theta_g}{dt} \\ M\frac{d\Delta\omega_g}{dt} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & I & 0 \\ K_1 & -D & K_2 \\ K_3 & 0 & K_4 \end{bmatrix} \begin{bmatrix} \Delta\theta_g \\ \Delta\omega_g \\ \Delta\theta_L V_L \end{bmatrix} \tag{55}$$

where $\Delta\theta_L V_L$ is the vector $[\Delta\theta_L \ \Delta V_L]^t$ of incremental load angles and voltages. For this case (a), the algebraic equation Jacobian ($J_{AE}^{(a)}$) is K_4. For nonsingular K_4, the system dynamic state Jacobian for this case (a) is,

$$J_{sys}^{(a)} = \begin{bmatrix} 0 & I \\ M^{-1}(K_1 - K_2 K_4^{-1} K_3) & -M^{-1}D \end{bmatrix} \tag{56}$$

The determinant of $J_{sys}^{(a)}$ is (see appendix A),

$$\det J_{sys}^{(a)} = \frac{\det(K_1 - K_2 K_4^{-1} K_3)}{\det M} (-1)^{m-1} \tag{57}$$

The standard load flow Jacobian as previously defined can be written in terms of these submatrices as,

$$J_{LF} = \begin{bmatrix} -K_1 & -K_2 \\ K_3 & K_4 \end{bmatrix} \tag{58}$$

Again for nonsingular K_4, the determinant of the load flow Jacobian is (see appendix A):

$$\det J_{LF} = \det K_4 \det(K_1 - K_2 K_4^{-1} K_3)(-1)^{m-1} \quad (59)$$

For this case, a clear relationship between the determinant of the standard load-flow Jacobian and the determinant of the system dynamic state Jacobian exists as:

$$\det J_{sys}^{(a)} = \frac{\det J_{LF}}{\det K_4 \det M} \quad (60)$$

This means that under these assumptions, monitoring the load-flow Jacobian determinant can detect a possible dynamic instability. This is discussed in section 8. The basic structure of this case (a) is used frequently, but the assumptions (a1)-(a3) are slightly different. The same structure of (46)-(49) can be obtained by assuming a constant voltage behind transient reactance model with the terminal buses eliminated. This leads to a non standard load-flow Jacobian matrix which includes machine parameters in the bus admittance matrix.

(b) A second case where such a relationship can be firmly established was proposed in principle by Venikov in [11]. This special case makes the following assumptions:

- (b1) Stator resistance is negligible ($R_{si} = 0$, $i=1,\ldots,m$)
- (b2) No damper windings or speed damping ($T'_{qoi} = 0$, $D_i = 0$, $i=1,\ldots,m$)
- (b3) High gain and fast excitation systems so that all generator terminal voltages are constant (V_i = constant, $i=1,\ldots,m$)
- (b4) Constant mechanical torque to the shaft of each generator (T_{Mi} = constant, $i=1,\ldots,m$)
- (b5) Generator number one has infinite inertia and negligible reactances. This together with (b1)-(b3) makes V_1 = constant, θ_1 = constant (infinite bus)
- (b6) All loads are constant power ($P_{Li}(V_i)$ = constant, $Q_{Li}(V_i)$ = constant, $i=1,\ldots,n$)

With these assumptions, the special case dynamic model (after eliminating P_i and Q_i) is,

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \qquad i=2,\ldots,m \quad (61)$$

$$M_i \frac{d\omega_i}{dt} = T_{Mi} - [E'_{qi} + (X_{qi} - X'_{di}) I_{di}] I_{qi} \qquad i=2,\ldots,m \quad (62)$$

$$0 = V_i e^{j\theta_i} + jX'_{di}(I_{di} + jI_{qi}) e^{j(\delta_i - \pi/2)} - [(X_{qi} - X'_{di}) I_{qi} + jE'_{qi}] e^{j(\delta_i - \pi/2)} \qquad i=2,\ldots,m \quad (63)$$

$$0 = -\sum_{k=1}^{n} V_i V_k Y_{ik} e^{j(\theta_i - \theta_k - \alpha_{ik})} + V_i e^{j\theta_i}(I_{di} - jI_{qi}) e^{-j(\delta_i - \pi/2)} + P_{Li} + jQ_{Li} \qquad i=2,\ldots,m \quad (64)$$

$$0 = -\sum_{k=1}^{n} V_i V_k Y_{ik} e^{j(\theta_i - \theta_k - \alpha_{ik})} + P_{Li} + jQ_{Li} \qquad i=m+1,\ldots,n \quad (65)$$

with

$$T_{Mi} = \text{constant} \qquad i=1,\ldots,m \quad (66)$$

$$V_i = \text{constant} \qquad i=1,\ldots,m \quad (67)$$

$$P_{Li} = \text{constant} \qquad i=1,\ldots,n \quad (68)$$

$$Q_{Li} = \text{constant} \qquad i=1,\ldots,n \quad (69)$$

$$\theta_1 = 0 \quad (70)$$

The m+n-2 complex algebraic equations must be used to eliminate the 2m+2n-4 real algebraic variables E'_{qi}, I_{di}, I_{qi} ($i=2,\ldots,m$), θ_i ($i=2,\ldots,n$), V_i ($i=m+1,\ldots,n$). We begin by first noting that from (63) and (64),

$$[E'_{qi} + (X_{qi} - X'_{di}) I_{di}] I_{qi} = -P_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \qquad i=2,\ldots,m \quad (71)$$

This can be substituted into (62). Secondly, we note that (63) and (64) can be rewritten (using cos(A+B) = cosAcosB-sinAsinB and sin(A+B) = sinAcosB+cosAsinB),

$$X_{qi} I_{qi} = -V_i \sin(\theta_i - \delta_i) \qquad i=2,\ldots,m \quad (72)$$

$$E'_{qi} - X'_{di} I_{di} = V_i \cos(\theta_i - \delta_i) \qquad i=2,\ldots,m \quad (73)$$

$$V_i I_{di} = \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\delta_i - \theta_k - \alpha_{ik}) + P_{Li} \sin(\theta_i - \delta_i) - Q_{Li} \cos(\theta_i - \delta_i) \qquad i=2,\ldots,m \quad (74)$$

$$V_i I_{qi} = \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\delta_i - \theta_k - \alpha_{ik}) - P_{Li} \cos(\theta_i - \delta_i) - Q_{Li} \sin(\theta_i - \delta_i) \qquad i=2,\ldots,m \quad (75)$$

Eliminating E'_{qi}, I_{di}, I_{qi} (simply equating I_{qi} in (72) and (75)) gives

$$-V_i^2 \sin(\theta_i - \delta_i) = X_{qi} \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\delta_i - \theta_k - \alpha_{ik}) - X_{qi} P_{Li} \cos(\theta_i - \delta_i) - X_{qi} Q_{Li} \sin(\theta_i - \delta_i) \qquad i=2,\ldots,m \quad (76)$$

Using (71), (76) and (65) this special case dynamic model with E'_{qi}, I_{di}, I_{qi} ($i=2,\ldots,m$) eliminated is,

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \qquad i=2,\ldots,m \quad (77)$$

$$M_i \frac{d\omega_i}{dt} = T_{Mi} + P_{Li} - \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \qquad i=2,\ldots,m \quad (78)$$

$$0 = V_i^2 \sin(\theta_i - \delta_i) - X_{qi} Q_{Li} \sin(\theta_i - \delta_i) - X_{qi} P_{Li} \cos(\theta_i - \delta_i) + X_{qi} \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\delta_i - \theta_k - \alpha_{ik}) \qquad i=2,\ldots,m \quad (79)$$

$$0 = -P_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \qquad i=m+1,\ldots,n \qquad (80)$$

$$0 = -Q_{Li} + \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) \qquad i=m+1,\ldots,n \qquad (81)$$

with

$$T_{Mi} = \text{constant} \qquad i=1,\ldots,m \qquad (82)$$

$$V_i = \text{constant} \qquad i=1,\ldots,m \qquad (83)$$

$$P_{Li} = \text{constant} \qquad i=1,\ldots,n \qquad (84)$$

$$Q_{Li} = \text{constant} \qquad i=1,\ldots,n \qquad (85)$$

The linearized form of this dynamic model is,

$$\begin{bmatrix} \frac{d\Delta\delta_g}{dt} \\ M\frac{d\Delta\omega_g}{dt} \\ \hline 0 \\ 0 \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & I & 0 & 0 \\ 0 & 0 & K_1 & K_2 \\ \hline K_5 & 0 & K_6 & K_7 \\ 0 & 0 & K_3 & K_4 \end{array}\right] \begin{bmatrix} \Delta\delta_g \\ \Delta\omega_g \\ \hline \Delta\theta_g \\ \Delta\theta_L V_L \end{bmatrix} \qquad (86)$$

For this case (b), the algebraic equation Jacobian ($J_{AE}^{(b)}$) is,

$$J_{AE}^{(b)} = \begin{bmatrix} K_6 & K_7 \\ K_3 & K_4 \end{bmatrix} \qquad (87)$$

Defining B' and C' as,

$$B' = [K_1 \quad K_2] \quad , \quad C' = \begin{bmatrix} K_5 & 0 \\ 0 & 0 \end{bmatrix} \qquad (88)$$

for nonsingular $J_{AE}^{(b)}$, the system dynamic state Jacobian for this case (b) is,

$$J_{sys}^{(b)} = \begin{bmatrix} 0 & I \\ M^{-1}(-B'J_{AE}^{(b)-1}C') & 0 \end{bmatrix} \qquad (89)$$

and (see appendix A)

$$\det J_{sys}^{(b)} = \frac{\det(-B'J_{AE}^{(b)-1}C')}{\det M} (-1)^{m-1} \qquad (90)$$

Note that the eigenvalues of $J_{sys}^{(b)}$ will all be either pure imaginary or will include one or more which is positive real. We will consider the dynamic system case (b) to be stable as long as no eigenvalues are positive real. By rearranging rows and columns of the matrix in (86)(see appendix A),

$$\det J_{AE}^{(b)} \det(-B'J_{AE}^{(b)-1}C') = \det K_5 \det J_{LF} \qquad (91)$$

where J_{LF} is as in (58). This gives the following relationship between the determinants of the standard load-flow Jacobian, the algebraic equation Jacobian, and the system dynamic state Jacobian,

$$\det J_{sys}^{(b)} = \frac{\det K_5 \det J_{LF}}{\det M \det J_{AE}^{(b)}} \qquad (92)$$

This means that under these assumptions, monitoring the load-flow Jacobian determinant can detect a possible dynamic instability. This is discussed in the following section.

8. Instability and Maximum Loadability

When studying a proposed load or interchange level, a load-flow solution is required before steady-state stability can be analyzed. If a load-flow solution cannot be found, then it is normally assumed that the proposed loading exceeded the "maximum power transfer" capability of the system. This maximum power transfer point is normally assumed to coincide with a zero determinant for the standard load-flow Jacobian. Using this as a criteria, any load level which produces a zero determinant for the standard load-flow Jacobian is an upper bound and hence an optimistic value of the maximum loadability. This upper bound has been analyzed in the past, and is regaining interest as voltage collapse is associated with this point [1-11,19-21]. It is also important to note that non-convergence of load-flows is also a matter of solution technique. Cases have been cited where Gauss-Seidel routines converge when Newton-Raphson routines do not.

If a standard load-flow solution and associated dynamic system equilibrium point are found (as described in sections 3 and 4) the stability of the point must be determined. In order to do this, the algebraic equation Jacobian must be nonsingular. This matrix is given by (44) in general, by K_4 for case (a) and by (87) for case (b). Assuming these algebraic equation Jacobians are nonsingular for a given case, steady-state stability must be evaluated from the eigenvalues of the system dynamic state Jacobian. This matrix is given by (45) in general, by (56) for case (a), and by (89) for case (b). A system is at a critical point when the real part of one of its eigenvalues is zero. If a real eigenvalue is zero then the determinant is zero. In the general case of (45), the zero eigenvalue due to the angle reference can easily be removed by using a dynamic model reduced in order by 1 (see section 5). Clearly many cases can be found where an equilibrium point can be critically unstable (at least one eigenvalue has a zero real part) and the load-flow Jacobian is nonsingular.

In cases (a) and (b), all detailed model dynamic states have been eliminated by making rather drastic assumptions. In special case (a), as long as det M and det K_4 are non zero and bounded, a dynamic equilibrium point exists and has a system dynamic state Jacobian which is singular if and only if the load-flow Jacobian is singular. In special case (b), we need to look at the matrix K_5. Examination of (79) shows that K_5 is diagonal with the i^{th} diagonal entry equal to:

$$K_{5i} = -V_i^2 \cos(\theta_i-\delta_i)+X_{qi}Q_{Li}\cos(\theta_i-\delta_i) - X_{qi}P_{Li}\sin(\theta_i-\delta_i)-X_{qi}\sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\delta_i-\theta_k-\alpha_{ik}) \qquad i=2,\ldots,m \qquad (93)$$

From (74),

$$K_{5i} = - X_{qi}V_iI_{di} - V_i^2 \cos(\theta_i - \delta_i) \qquad i=2,\dots,m \qquad (94)$$

and from (73)

$$K_{5i} = - V_i(E'_{qi} + (X_{qi} - X'_{di})I_{di}) \qquad i=2,\dots,m \qquad (95)$$

In steady-state, (4) and (11) give (with $R_{si}=0$)

$$0 = V_i e^{j\theta_i} + jX_{qi}(I_{di} + jI_{qi})e^{j(\delta_i - \pi/2)} - j[E'_{qi} + (X_{qi} - X'_{di})I_{di}]e^{j(\delta_i - \pi/2)} \qquad i=2,\dots,m \qquad (96)$$

This means that K_{5i} can only be zero (for nonzero V_i) if (see (17))

$$V_i e^{j\theta_i} + jX_{qi}I_{Gi}e^{j\gamma_i} \neq 0 \qquad i=2,\dots,m \qquad (97)$$

This also shows that K_{5i} is proportional to the magnitude of the voltage behind X_{qi} in steady-state. This was discussed in section 4 as a condition for the existence of a dynamic equilibrium from a load-flow solution. Thus, if a dynamic equilibrium point exists (equation (40) is satisfied), then K_5 cannot be singular. Thus if det M and det $J_{AE}^{(b)}$ are non zero and bounded, then the system dynamic state Jacobian of case (b) is singular if and only if the load-flow Jacobian is singular.

Since $J_{sys}^{(b)}$ must have all pure imaginary eigenvalues to be stable, det $J_{sys}^{(b)}$ must be positive to be stable. Reference [11] originally proposed the monitoring of the load-flow Jacobian determinant during load-flow iterations to see if it changed sign between the initial guess and the converged solution. The implication was that if it did, then the case (b) dynamic model would be unstable at that solution, and if it did not then the case (b) dynamic model would be stable (all pure imaginary eigenvalues). Our interpretation indicates that they did not account for possible values of det K_5 and det $J_{AE}^{(b)}$. A change in sign of either of these would affect stability issues. We have shown that det K_5 would probably never change sign, but whether the det $J_{AE}^{(b)}$ changes sign or not remains an open question.

9. Conclusions

Standard load-flow is used to find system voltages for a specified level of loading or interchange (regardless of the dynamic load model). It is also the starting point for determining the initial conditions for dynamic analysis. The standard load-flow Jacobian can provide information about the existence of a steady-state equilibrium point for a specified level of loading or interchange. There are two very special cases when the determinant of the standard load-flow Jacobian implies something about the steady-state stability of a dynamic model. Both of these cases involve very drastic assumptions about the synchronous machines and their control systems. The load level which produces a singular load-flow Jacobian should be considered an optimistic upper bound on maximum loadability. The actual upper bound would be either the same or lower since it requires both the existence of a solution and stable dynamics.

For voltage collapse and voltage instability analysis, any conclusions based on the singularity of the standard load-flow Jacobian would apply only to the phenomena of voltage behavior near maximum power transfer. Such analysis would not detect any voltage instabilities associated with synchronous machine characteristics or their controls.

10. Acknowledgements

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APPENDIX A

The following facts of determinants can be found in most linear algebra texts.

1. If a square matrix J is block upper triangular then the determinant of J is equal to the product of determinants of the diagonal blocks.

   Example

$$J = \begin{bmatrix} A & B \\ 0 & D \end{bmatrix} \text{ (A and D square)} \tag{A.1}$$

$$\det J = \det A \det D \tag{A.2}$$

2. If a square matrix J is partitioned with A and D square as,

$$J = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{A.3}$$

   and if det D≠0, then

$$\det J = \det D \det(A-BD^{-1}C) \tag{A.4}$$

   This is often called Schur's formula.

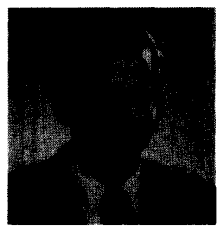
3. If A and B are sqaure of the same dimension then

$$\det AB = \det A \det B \tag{A.5}$$

4. If any two rows (or two columns) of a square matrix are interchanged, then the determinant of the resulting matrix is the negative of the determinant of the original matrix.

5. The determinant of a square matrix is equal to the product of its eigenvalues.

6. If each element of a row (or column) of a square matrix is multipled by -1, the determinant of the matrix changes sign.



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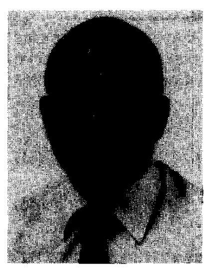

M. A. Pai was born in Mangalore, India on October 5, 1931. After receiving his B.E. in Electrical Engineering at Madras University in 1953, he worked in Bombay Electric Supply and Transport undertaking till 1957. He obtained his M.S. and Ph.D. degrees from the University of California, Berkeley in 1958 and 1961 respectively. After teaching for a year year each at the University of California Berkeley and U.C.L.A., he joined I.I.T. Kanpur in 1963 where he was on the faculty of Electrical Engineering till 1981. He was visiting Professor at Iowa State University from 1979-1981 and since 1981 he is Professor of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign.

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Discussion

Adam Semlyen (University of Toronto): I would like to congratulate the authors for their interesting and thought provoking paper. Certainly, many power system engineers have wondered whether the load flow Jacobian could provide information on the stability of the system, in addition to its loadability. It appears from the paper that the answer is almost completely negative since, if there is any relation between J_{LF} and J_{sys} (the dynamic state Jacobian of the system), it is only related to their singularity, i.e. one real eigenvalue becoming zero. Before this happens, as the system is more and more heavily loaded, complex eigenvalue pairs of J_{sys} may cross the imaginary axis into the unstable domain. Even the singularity of J_{LF} or of J_{sys} is not a definitive indication whether the system is stable or not since in that case the stability analysis can not be based solely on linearized models. As the authors correctly point out, the exact load flow solution can be obtained even for a turning point[A] (i.e. extreme loading condition). Even Newton's method (in a modified form) can be used for that purpose[A,B].

The fact that system stability is not solely related to the load flow equations is well known but the general formulation of a dynamic problem in the form

$$\dot{x}=f(x) \tag{1}$$

may be suggesting the opposite, since it leads directly to the equations of the steady state

$$f(x)=0 \tag{2}$$

There are no distinct Jacobians in this formulation. In reality, what we use for the load flow problem is not eqn.(2) but is an equation embedded in $f(x)$=0. This will be shown in more detail below. For this purpose we first rewrite (1) and (2) in the more general form

$$\dot{x}=\phi(x,y) \tag{3a}$$

$$\psi(x,y)=0 \tag{3b}$$

and

$$\phi(x,y)=0 \tag{4a}$$

$$\psi(x,y)=0 \tag{4b}$$

where y are auxiliary variables to be eliminated using the second set of equations.

In the particular case of power systems, the dynamics of the synchronous machines and of the induction machines at load buses is described by

$$M\dot{\omega}+D(x,\omega)(\omega-\omega_0)=p_{mech}(x,\omega)-p_{el}(v,\theta,\delta) \tag{5}$$

where D and p_{mech} are shown to depend, in addition to ω, on other state variables x as well (reflecting thus the effect of controls); p_{el} depend on the machine angles δ and the bus voltages v and their angles θ. Thus the system dynamic equations are

$$\dot{\omega}=M^{-1}\left[p_{mech}(x,\omega)-p_{el}(v,\theta,\delta)-D(x,\omega)(\omega-\omega_0)\right] \tag{6a}$$

$$\dot{\delta}=\omega-\omega_0 \tag{6b}$$

$$\dot{x}=g(x,\omega) \tag{6c}$$

$$q_{load}(v,\omega)-q_{el}(v,\theta)=0 \tag{6d}$$

The last equation refers to the reactive powers.

In steady state the right hand side of eqns.(6) equals zero. This yields

$$\delta=\delta_0 \tag{7a}$$

$$\omega=\omega_0 \tag{7b}$$

$$x=x_0 \tag{7c}$$

Substitution of (7) into (6a) and (6d) gives

$$p_{el}(v,\theta,\delta_0)-p_{mech}(x_0,\omega_0)=p(v,\theta)=0 \tag{8a}$$

$$q_{el}(v,\theta)-q_{load}(v,\omega_0)=q(v,\theta)=0 \tag{8b}$$

Equations (8) are the load flow equations. They have been obtained directly from the right hand side of the dynamic equations but part of the content of the latter has disappeared in the process. Clearly, the load flow equations are embedded in the dynamic equations as a kernel and are devoid of the information which is essential to the dynamics of the system. Most importantly, the damping term with $D(x,\omega)$ is missing and p_{mech} does not include the effects of the controls. These remarks apply also to the linearized forms of the paper, in particular as shown in eqns.(55) and (86).

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K. R. Padiyar (IIS Bangalore, India): The authors of this paper are to be complimented for providing a clear exposition of the relationships between the standard load-flow Jacobian and system dynamic state Jacobian. There is renewed interest in the analysis of steady-state stability due to the concerns regarding voltage instability problems. Although the system dynamics are usually neglected in the analysis of voltage instability, this is not always realistic.

The steady-state instability is associated with the crossing of a real eigenvalue of the system from LHP to RHP, and hence the condition for singularity of the system Jacobian is of interest. As power flow is increased, the load flow Jacobian and consequently the system Jacobian tends to be singular. I would appreciate if the authors can respond to the following queries:

1. Is it possbile to predict the critical load pattern which would lead to the singularity of the Jacobian? For lossless systems with PV buses it is possible to invoke the concept of minimal cutsets (see Ref. A).

2. Is it possible to relate the determinant of $J_{AE}^{(b)}$ with that of J_{LF}, at least in the simplified case of systems with PV buses?

Reference [A] A. Arapostathis, S. Sastry and P. Varaiya, "Analysis of Power Flow Equation", Memorandum No. UCB/ERL M80/35, 1980.

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P. W. SAUER and M. A. PAI: We thank Professors Semlyen and Padiyar for their interest in the paper and added discussion. Both discussers comment on the critical load level and the presence of algebraic equations. In regard to critical load level, the fundamental issue is normally how load is distributed between buses (both real and reactive). If the distribution is specified in a meaningful way such as some percentage of total load, a critical load level can be computed. If the distribution is not specified in a meaningful way, the critical load patterns can be physically unreasonable.

In the relation of Jacobians, one issue which has caused considerable discussion is the notion of "PV" buses. In load flow this means bus voltage magnitude and real power are fixed while bus voltage angle and reactive power vary as required. In dynamic analysis we would assume this to mean that the bus voltage was controlled by an automatic voltage regulator with some source of reactive power such as a synchronous machine. If all buses are synchronous machines in special case (b), then $J_{AE}^{(b)}$ is simply K_6 and J_{LF} is simply $-K_1$. We have not as yet found any relationship between the determinants of K_6 and K_1.

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