Introduction

Stability considerations have been recognized as an essential part of power system planning for a very long time. Fundamental papers on the subject, a group of which are listed in Table I, were published in the Transactions of the AIEE nearly 50 years ago. Within a few years, stability studies of new generation and transmission facilities had become routine. Over most of the history of power system growth in the United States and Canada, acceptable stability performance has been achieved without great difficulty. Approximately within the last decade, however, changes have occurred which require power system engineers to devote much more of their effort to this problem.

Depending on individual experience and viewpoint, there are no doubt various explanations for the increased concern with stability. The underlying cause, however, would appear to be the very extensive interconnection of power systems with greater dependence on firm power flow over ties. This magnifies the undesirable consequences of instability and complicates the analytical processes through which acceptable system behavior is assured.

By the late 1960's, most of the generators in the United States and Canada, with the exception of certain isolated systems, were connected to one of two large transmission networks, one in the east and one in the west. The primary motivation for interconnection is to obtain the economic benefits of large-scale generation and transmission facilities.

Interconnection can also improve reliability through mutual support in emergencies. However, improved reliability is realized only if the bulk transmission system remains intact through such emergencies. If ties between systems are not strong enough to maintain synchronism in the face of disturbances, the interconnected system breaks up and the ability to provide support to areas which become isolated vanishes.

The possible consequences of instability in an interconnected system were dramatized by the Northeast Power Failure [1]-[3] of 1965. Changes in design and operation since

\[\text{TABLE I}
\]

| Some Early Synchronous Stability Papers from the Transactions of the AIEE |
| Vol. XLIII, 1924. |
| Vol. XLIV, 1925. |
| Vol. XLV, 1926. |
| Discussions, pp. 80-94. |
| Vol. XLI, 1927. |

that event virtually assure that such widespread failure will not happen again, even when system separation does occur. Even so, integrity of a bulk transmission system is of primary importance.

Stability behavior of a large power system is difficult to predict. Size is in itself a complication. In the Northeast Power Failure, for example, the systems most directly affected included hundreds of generators, with hundreds more in neighboring systems less affected by the instability. Such large systems are subjected to a wide range of operating conditions, depending on load level and equipment availability, and may

\[1\] Ties between east and west were first closed in 1967. These have a very limited capacity in terms of the systems they interconnect and do not provide firm power. They are subject to frequent tripping. \[2\]
undergo a limitless variety of disturbances involving faults, loss of generation, loss of transmission facilities, and loss of load. The inherently nonlinear nature of ac power transmission limits the ability to extrapolate from one situation to another with confidence. Thus, the minimum effort required to determine the stability characteristics of any large system is substantial, and choosing between design alternatives or establishing operating guides becomes a time-consuming and costly undertaking.

The difficulties of interconnected system stability are compounded by the fact that certain systems exhibit tendencies toward dynamic instability. Although power system oscillations are at best only lightly damped, the assumption of positive damping has in the past been used to simplify studies and has been validated by operating experience. In the eastern United States, there is still no general evidence of dynamic instability. However, in the west, in parts of the midwest, and in parts of Canada, dynamic instability is a problem. Power oscillations may be initiated by some disturbance or occur spontaneously, and may on occasion increase in magnitude to the point of causing system separation. The common feature of systems displaying such behavior is long-distance transmission. Load centers tend to be widely separated and partially supplied by remote generation with large angular displacements between remote generators and those near load centers. Ties between load areas are also long, and interchange of power requires relatively large angle differences, frequently resulting in still larger angle separations between some remote generator groups and load center machines with which they must synchronize. The overall result is reduced capacity for synchronizing power flow between machine groups and low natural oscillation frequencies that are poorly damped.

It is the intent of this book to present, by means of reprints, the stability aspects of the recent growth of large power systems in the United States and Canada. The book is intended to provide a reference for practicing engineers. It should also be useful to students and teachers in the identification of an industry problem, and may help achieve perspective through organization and editorial comment.

Selection of Papers

It is interesting to note some statistics regarding stability papers. An IEEE bibliography [4] published in 1965 lists just 35 AIEE and IEEE TRANSACTIONS papers over the 30-year period from 1935 through 1964. Then, abruptly, the number jumps from 2 in 1964 to 18 in 1965, if one includes PICA conference papers, and has remained at a high level to the present time. Even allowing for the general increase in the number of published papers in all fields, there is no doubt that power system engineers have been much more occupied with stability considerations in the past decade than they were previously.

The reader should be aware of the fact that the papers in this book are selected from a much larger set. The set includes all of the papers in the IEEE TRANSACTIONS ON POWER APPARATUS AND SYSTEMS for the years 1966 to the present. It includes the AIEE and IEEE TRANSACTIONS papers in the IEEE report “Power System Stability Bibliography, 1935–1965.” The set also includes all of the papers in the PICA conference records, and a very large number of conference papers from the general meetings of the Power Engineering Society of recent years. This entire set was examined in the selection process, and the papers chosen are each intended to document in some way the impact of the stability problem on the power systems of the United States and Canada. To provide depth and diversity of opinion, most topics are represented by several papers. Some papers from sources outside the set described above are in the book to fill specific needs, but there has been no exhaustive search of other published literature beyond that already mentioned.

The limitations of size do not permit reprinting all of the applicable papers from the sources searched. There are many excellent papers omitted for this reason alone, and the Editors can only offer sincere regrets to their authors. It should also be observed that there is a large body of IEEE literature from recent years outside the scope of this book. This generally consists of research papers which, however excellent, have not had a noticeable influence on the design of apparatus, the design or operation of systems, or the analytical methods employed by the industry. Representative bibliographies are included for two categories of papers, namely, those dealing with optimal generator control and those dealing with the so-called direct methods of determining stability.

Background Reading

The reprints in this book generally require the reader to be familiar with power system stability. There are a number of English language textbooks devoted entirely or in part to the subject, some of which are listed in Table II. The following is a brief presentation of the elementary concepts of synchronous stability.

Synchronous Stability—Concepts and Definitions

Power system generators connected through a transmission network must run in synchronism, that is, at the same average speed. Automatic load and frequency control systems and individual machine speed-governing systems tend to keep generator speeds, and consequently speed differences, within narrow bounds, but it is the effect of variational power flow through the network which forces speed differences to be zero on the average. If any generator runs faster than another, the angular position of its rotor relative to that of the slower machine will continue to advance as long as the speed difference exists, and its generated voltage will likewise advance in phase relative to the voltage of the slower machine. The resultant phase difference, within limits, shifts a load from the slow machine to the fast one, tending to reduce the speed difference.

The shift in load between generators is a nonlinear function of the difference in rotor angles, and above a certain angle difference, nominally 90°, the incremental load shift due to

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incremental angle change reverses, and the forces which tended to reduce speed differences become forces tending to increase speed differences. This, in essence, is the loss-of-synchronism phenomena. Because levels of kinetic energy change, recovery of synchronous operation is uncommon. Moreover, as machines lose synchronism, currents and voltages vary over wide ranges, and protective apparatus should operate, and normally will, to trip affected generators and lines. Loss of synchronism can occur between one machine and the rest of the system or between groups of machines, with synchronism perhaps maintained within those groups.

Power system stability is primarily concerned with variations in speeds, rotor positions, and generator loads. It focuses attention on the transmission network, since it is the network, more than power plant or system controls, which provides for the power shifts between generators required to maintain synchronism. Generating plant controls do have some influence, however. Automatic generator excitation systems modify power flow through the network by their influence on generator magnetic flux levels, and turbine speed-governing systems affect the power delivered by prime movers to generators.

Power system engineers have found it useful to identify three types of unstable behavior, designated steady-state instability, dynamic instability, and transient instability. There are no universally accepted precise definitions of this terminology, and there is no intent to imply definitions in what follows. Instead, the purpose is simply to describe generally several ways in which instability occurs.

Steady-state instability is a possible but improbable event on large power systems. In the simplest hypothetical two-machin system, loss of synchronism will occur if an attempt is made to operate with an angle separation between machines greater than 90°. For real multimachine systems, large angle differences also tend toward steady-state instability, but it is not possible at this time to formulate steady-state stability limits for large systems in terms of angle differences. Between widely separated machines on a large system, angle differences may greatly exceed 90° with no threat of steady-state instability. On the other hand, depending on the location and the characteristics of system loads, steady-state instability may occur with angle differences less than 90°. In those systems where this kind of instability is a genuine hazard, operators are frequently able to recognize limiting conditions in terms of some gradual change, such as a sagging bus voltage, and alter operation in time to avoid instability. If limiting conditions are exceeded, rates of change increase enormously, and loss of synchronism with system breakup can occur within seconds.

Dynamic instability is more probable than steady-state instability, or at least is more common in existing North American power systems. Small speed deviations occur continuously in normal operation with corresponding variations in angle differences and generator loads. If the variations resulting from any initial change diminish with time, the system is said to be dynamically stable. Conversely, if these variations, in the form of oscillations, increase with time, the system is dynamically unstable. Due to the effects of nonlinearities, such oscillations may be limited at some magnitude, or they may increase to the point of loss of synchronism and system breakup unless some intervention occurs.

Transient instability may occur on any system subjected to a major disturbance. There is no limit to the kinds of disturbances which can occur, but a fault on a heavily loaded line which requires opening the line to clear the fault is usually of greatest concern. The tripping of a loaded generator or the abrupt dropping of a large load may also cause instability. Usually, disturbances alter the system at least temporarily so that the subsequent steady-state operation will be different from that prior to the disturbance. There is then the necessity for that altered system to be stable in its new steady state. There is also the possibility that the altered system will be dynamically unstable, and that oscillations subsequent to the disturbance will be sustained, or eventually increase in magnitude to the point of causing system breakup. Most often, however, instability following a disturbance is due to the relatively large transient deviations in speeds, rotor angle differences, and power shifts initiated by the disturbance. Loss of synchronism can occur quite rapidly, within 1 s for a generator close to a transmission line fault. In the Northeast Power Failure of 1965, transient instability was initiated by tripping a 230-kV transmission circuit, and within about 7 s, the Northeast had become isolated from the larger eastern network, and had subdivided into four major isolated subsystems.

**Elementary Analytical Viewpoints**

A basic understanding of synchronous stability can be acquired using simple analytical methods. The reader should be advised that these methods cannot be readily extended to large systems and are presented only to provide insight.

**Generator and Infinite Bus**

Consider a remote generator connected radially to a major substation of a very large system. Assume that the voltage at the substation is so strongly influenced by large nearby generation that it is independent of events at the remote generator. A single line diagram of this system is shown in Fig. 1. A sta-
bility model for this generator consisting of three equations will be described.

**Acceleration Equation**

The combined inertias of the generator and its prime mover will be accelerated by any unbalance in the applied torques. Neglecting certain losses, one may write

$$ J \frac{d\omega}{dt} = T_M - T_E. \tag{1} $$

Symbols are interpreted as follows:
- $T_M$: prime mover torque, megajoules/radian
- $T_E$: electrical torque, megajoules/radian
- $\omega$: speed, radians/second
- $J$: moment of inertia for generator and prime mover, megajoules·second$^2$/radian$^2$
- $H = \frac{1}{2} J \omega_R^2 / S_B. \tag{2}$

Substituting for $J$ in (1),

$$ \left( \frac{1}{\omega_R} \right) \frac{d\omega}{dt} = \left( \frac{1}{2H} \right) \left( \frac{\omega R}{\omega} \right) \left( \frac{\omega T_M - \omega T_E}{S_B} \right). \tag{3} $$

Each of the products of speed and torque is power in megawatts, so that division by $S_B$ yields power in per unit on that base. The symbols $P_M$ and $P_E$ will be used for these per unit variables. Speed divided by rated speed is per unit speed and will be designated $\bar{\omega}$. The ratio of rated speed to speed is always quite close to unity in practical cases, and that ratio on the right-hand side of (3) can be set to unity as a simplification. The acceleration equation then becomes

$$ \frac{d\bar{\omega}}{dt} = \frac{P_M - P_E}{2H}. \tag{4} $$

**Power Flow Equation**

It is convenient here to assume that $P_M$ is equal to the initial prime mover power through the time interval of analysis, This ignores speed-governing effects. Acceleration is then entirely due to variations in $P_E$. An equation for $P_E$ can be written beginning with the positive-sequence diagram of Fig. 2, in which all resistances are assumed to be negligible. The remote generator is represented by a complex voltage $E'$ behind an internal generator reactance $X_D$, the transformer by a reactance $X_T$, the transmission lines by a reactance $X_L$, and the infinite bus by a complex voltage $E$. All of these are per unit quantities.

Let the total reactance of the circuit be designated $X$:

$$ X = X_D + X_T + X_L. \tag{5} $$

Write the complex voltages in polar form using the following notation:

$$ E' = V'e^{j\delta'} \tag{6} $$
$$ E = V'e^{j\delta}. \tag{7} $$

Define the voltage angle difference

$$ \delta = \theta' - \theta. \tag{8} $$

With a little effort, it can be shown that

$$ P_E = \frac{VV' \sin \delta}{X}. \tag{9} $$

**Angle Difference Equation**

Ignoring variations in generator rotor flux, the speed induced voltage $V'$ may reasonably be assumed to have constant magnitude. Since the induced voltage is caused by flux which is fixed in relationship to the generator rotor, the relative phase of the voltage will be advanced for rotor speeds higher than synchronous, and retarded for speeds less than synchronous. Mathematically,

$$ \frac{d\delta'}{dt} = N(\omega - \omega_R). \tag{10} $$

where $N$ is the number of pole pairs on the generator rotor. Let the rated electrical frequency in radians/second be

$$ \omega_0 = N\omega_R. \tag{11} $$

Then (10) may be written

$$ \frac{d\delta'}{dt} = \omega_0(\bar{\omega} - 1). \tag{12} $$

Finally, since the angle $\theta$ of the infinite bus voltage is assumed to be constant,

$$ \frac{d\theta}{dt} = \omega_0(\bar{\omega} - 1). \tag{13} $$

**Summary of Basic Equations**

A stability model for the remote generator consists of the following three equations:

$$ \frac{d\omega}{dt} = \frac{P_M - P_E}{2H} \tag{4} $$
$$ P_E = \frac{VV' \sin \delta}{X} \tag{9} $$
$$ \frac{d\theta}{dt} = \omega_0(\bar{\omega} - 1). \tag{13} $$
As an alternative, a single equation can be obtained by differentiating (13) and using that result and (9) in (4):

\[
\frac{d^2 \delta}{dt^2} = \frac{\omega_0}{2H} \left[ P_M - \frac{V' \sin \delta}{X} \right].
\]

(14)

**Power-Angle Diagram and Steady-State Stability**

A graphical plot of (9) is called a power-angle diagram. The diagram can be plotted over any range of angles, but it is usually sufficient to show it over the interval \(0 \leq \delta \leq \pi\). Such a plot is shown in Fig. 3. This diagram serves a number of purposes, the first of which is to define stable and unstable equilibrium angles. A horizontal line is drawn on the diagram at a power level equal to \(P_M\), the prime mover power. This horizontal line also represents the electrical power at equilibrium, i.e., no acceleration. The remote generator voltage angle may lead the infinite bus voltage angle by either of the two values \(\delta_1\) or \(\delta_2\) at which \(P_M\) intersects \(P_E\). The angle \(\delta_1\) is a stable equilibrium angle because small variations in \(\delta\) about \(\delta_1\) produce torques on the generator which tend to restore the angle to \(\delta_1\). For example, if \(\delta\) slightly exceeds \(\delta_1\), the electrical power \(P_E\) given by the curve will exceed the prime mover power \(P_M\), and the generator will tend to slow down, reducing \(\delta\). A similar argument for restoration of equilibrium may be made for values of \(\delta\) slightly less than \(\delta_1\). On the other hand, for \(\delta\) slightly greater than \(\delta_2\), \(P_E\) is now less than \(P_M\), and the generator will tend to speed up. This further increases \(\delta\), so that \(\delta_2\) is not a stable equilibrium. If \(\delta\) is less than \(\delta_1\), the generator power exceeds the prime mover power so that the generator tends to slow down, and tends toward \(\delta_1\), the stable equilibrium.

Stable equilibrium requires that the slope of the power-angle curve be positive at the operating angle. The power \(P_{max}\) represents the stable limit of power transfer between the remote generator and the infinite bus. A practical operating limit is considerably less than \(P_{max}\) if stability is to be maintained under transient conditions.

**Faults and Transient Stability**

Attention will now be directed toward the behavior of this stability model during and subsequent to a three-phase fault at the remote generator transformer high-side bus. A one-line diagram showing the fault location is presented in Fig. 4. Let the instant at which the fault occurs be the reference for time, that is, \(t = 0\), with the fault cleared by tripping the breaker \(A\) when \(t = T\). This leaves the transmission system intact subsequent to fault clearing, and to that extent simplifies the analysis.

During the fault, the electrical power is zero, and (4) and (13) are easily integrated to obtain the speed and angle at the fault clearing time \(T\):

\[
\bar{\omega} = 1 + \frac{P_M T}{2H}
\]

(15)

\[
\delta_T = \delta_1 + \frac{\omega_0 P_M T^2}{4H}.
\]

(16)

Fig. 5 shows the effect of the fault in terms of the power-angle diagram. Initially, \(P_E\) is equal to \(P_M\) with an angle separation of \(\delta_1\). During the fault, the electrical power is zero while the angle \(\delta\) increases from \(\delta_1\) to \(\delta_T\). Simultaneously, the generator speed increases from unity to \(\bar{\omega}T\) with a corresponding increase in kinetic energy. When the fault is cleared, the electrical power is greater than the prime mover power as indicated by the curve at the angle \(\delta_T\), so that the generator experiences a decelerating torque and its speed begins to return to normal. At some peak angle \(\delta_\rho\), the generator speed will recover its normal value, but the net torque on the generator is in the direction of deceleration so that the machine will continue to slows down, with the speed now dropping below normal. This results in a diminishing angle. When the angle reaches \(\delta_1\), the net torque on the generator will again be zero, but the speed will be less than normal, so the angle will continue to diminish. Below \(\delta_1\), the torque on the machine reverses, and the resulting acceleration begins to increase speed toward its normal value. At some minimum angle \(\delta_N\), normal speed again occurs, but the net torque now causes the machine to speed up and the angle to increase. According to the stability model established for this generator, this oscillating motion will continue indefinitely since no provision for damping has been included. Assuming dynamic stability, such oscillations will damp out eventually, and operation at the equilibrium angle will be restored. For the model behavior described, no loss of synchronism occurs, and the system is stable.
Equal Areas on the Power-Angle Diagram

The power-angle diagram of Fig. 6 corresponds to the transient discussed in the preceding paragraph, and shows two areas, \( A_1 \) and \( A_2 \), which must be equal. To see the necessity for this equality, consider the following argument. Let \( P_A \) be the net accelerating power on the generator, so that (4) may be written

\[
2H \frac{d\omega}{dt} = P_M - P_E = P_A. \tag{17}
\]

Equations (17) and (13) imply a differential relationship between \( \omega \) and \( \delta \) which, eliminating the independent variable time, can be written

\[
2\omega H (\omega - 1) \frac{d\omega}{d\delta} = P_A \delta. \tag{18}
\]

Let the right-hand side of (18) be integrated over the interval \( \delta_L \leq \delta \leq \delta_R \). The left-hand side vanishes for this interval since the upper and lower limits of integration are equal. That is to say, the initial per unit speed at the angle \( \delta_1 \) is unity, and the per unit speed at the peak angle swing \( \delta_R \) is also unity. Hence

\[
\int_{\delta_1}^{\delta_R} P_A \, d\delta = 0. \tag{19}
\]

This integral may be separated into two parts,

\[
\int_{\delta_1}^{\delta_R} P_A \, d\delta = \int_{\delta_1}^{\delta_T} P_M \, d\delta + \int_{\delta_T}^{\delta_R} (P_M - P_{\max} \sin \delta) \, d\delta \tag{20}
\]

so that

\[
\int_{\delta_1}^{\delta_T} P_M \, d\delta = \int_{\delta_T}^{\delta_R} (P_{\max} \sin \delta - P_M) \, d\delta. \tag{21}
\]

The left-hand side of (21) is the area \( A_1 \) of Fig. 6, and the right-hand side is \( A_2 \).

Critical Clearing Time and the Equal Area Criterion

From (16) it is clear that a fault of longer duration than that assumed would have resulted in a larger value for the angle at clearing time, \( \delta_T \). In Fig. 6, \( A_1 \) would have been larger, and since \( A_2 \) must equal \( A_1 \), \( \delta_R \) would have increased. Note, however, that the speed of the remote generator must be reduced to its normal value before the angle \( \delta \) reaches \( \delta_2 \) or synchronism with the infinite bus will be lost. To see this, assume that some speed greater than unity exists with \( \delta = \delta_2 \). This speed will shortly cause \( \delta \) to be greater than \( \delta_2 \), where the net accelerating power on the generator will be positive, further increasing the speed.

The angle \( \delta_2 \) is frequently referred to as the critical angle, and the fault clearing time which makes \( A_1 \) equal to \( A_2 \) in Fig. 9 is called the critical clearing time. For this relatively simple stability model, the critical clearing time can be calculated using (21) and (16). Critical clearing times for faults in real systems are much more difficult to obtain, involving a trial and error process with digital computer simulation. This can be quite expensive, so that critical clearing times may not often be determined with precision. Even so, the notion that a critical clearing time exists is important.

Factors Which Influence Transient Stability

The diagram of Fig. 7 depicts a more realistic fault location than that of Fig. 4 since it occurs on an operating transmission circuit. Clearing the fault requires removal of the line from service, and assuming equal reactance in each line, increases the post-fault reactance to

\[
X_{PF} = X_D + X_T + 2X_L. \tag{22}
\]

This reduces the power transmission capability and increases the risk of instability. Fig. 8 shows two power-angle diagrams corresponding to the pre-fault and post-fault transmission capacities. Note that there will be new post-fault operating and critical angles, \( \delta_1 \) and \( \delta_2 \), and more importantly, that the peak angle \( \delta_R \) moves closer to the critical angle \( \delta_2 \) than in the case previously discussed. This constitutes a reduction in the margin of stability, and leads to the consideration of other factors which tend to reduce the margin of stability. Symbols are to be interpreted in terms of Fig. 6.

1) Increasing the generator initial load \( P_M \) increases \( \delta_1 \) and \( \delta_2 \) also increased.

2) Increasing the fault duration increases \( A_1 \), hence \( \delta_R \).

3) Increasing the pre-fault impedance increases \( \delta_1 \), hence \( \delta_R \).

4) Reducing the generator and prime mover inertia increases \( \delta_T \), hence \( A_1 \) and \( \delta_R \).

It is important to note that the effects of these factors is non-linear, and that for small stability margins, the reduction in margin becomes extremely sensitive to unfavorable changes in
these important factors. The implication is that real systems must operate with sufficient margin to allow for variations in operating conditions and uncertainties in models and data if secure operations are to be achieved. Unfortunately, there are neither definitions nor standards for stability margins applicable to real systems.

REFERENCES