

**Instructions: Time: 75 minutes; calculators allowed; no communication devices;**  
**(i) Answers to questions 1-9 could be brought to the exam in advance and turned in with the exam;**  
**(ii) There are 11 technical papers that students may bring into the exam and use to help them answer questions;**  
**(iii) Except for the answers to questions 1-9, students should show all work on the exam pages.**

1. (4 pts) The 2004 “Terms & Definitions” paper says, “A stable equilibrium set thus has a finite region of attraction; the larger the region, the more robust the system with respect to large disturbances.” What does “region of attraction” mean in this context. In answering, consider the “stable equilibrium” studied in class.  
**Solution:** It is a collection of all points such that any trajectory starting at initial state at time will be attracted to the equilibrium point.
2. (4 pts) Section III-B of the 2004 “Terms & Definitions” paper classifies power system stability into three broad forms. (i) What are these forms? (ii) On which of these forms have we focused during the last several class discussions?  
**Solution:** (i) Rotor angle stability, frequency stability, and voltage stability; (ii) rotor angle stability.
3. (4 pts) The 2004 “Terms & Definitions” paper hardly mentions the notion of “hybrid systems,” but the 2021 “Terms & Definitions” paper devotes an entire subsection (III-D) to it. (i) What are hybrid systems? (ii) Why does the 2021 version put so much more emphasis on it in comparison to the 2004 paper?  
**Solution:** Hybrid systems are dynamical systems that exhibit both continuous and discrete dynamic behavior; (ii) In 2004, the amount of discrete dynamics in power systems was limited, but today and in the future, it is growing mainly due to the presence of converter interfaced resources.
4. (4 pts) The 2021 “Terms & Definitions” paper states in Section IV-B that “...as conventional synchronous generators are displaced by CIGs, the total inertia of the system will be reduced. This in turn has an impact on rotor angle stability...” We said in class that one effect will be that  $d\omega/dt$  will be larger for a given disturbance. How does the swing equation reflect this?  
**Solution:** The coefficient of  $d\omega/dt$  in the swing equation is inertia, therefore for a given amount of accelerating power, lower inertia systems such as those dominated by wind and solar will see higher rates of speed changes than systems dominated by synchronous machines.
5. (4 pts) The 1999 paper by Vittal which examined the effect of industry restructuring (deregulation) on transient stability analysis stated, “One aspect of the NERC planning standards that is directly impacted by deregulation is that the standards have now become more specific than in the past...” For example, the performance-disturbance table used to have categories A-D now has categories P0-P7. Why do you think this increased specificity happened?

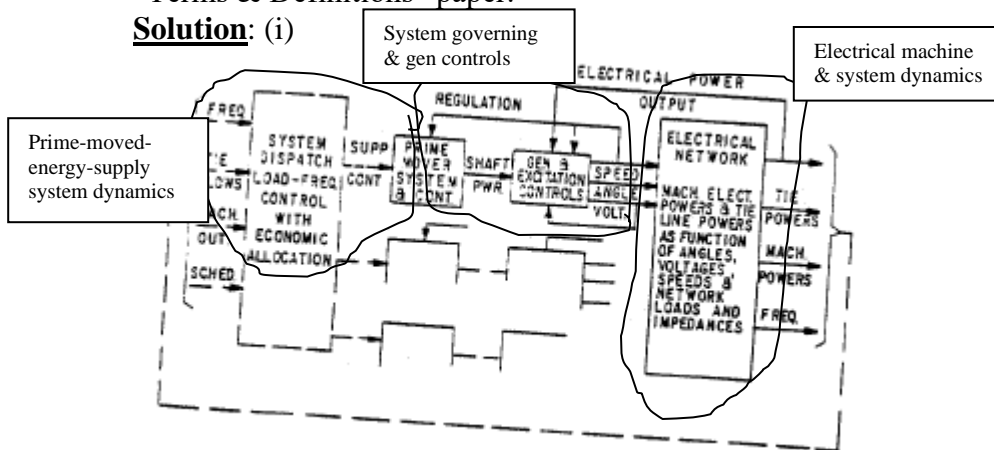
**Solution:** Deregulation brought greater diversity in terms of types of participants and in terms of their expertise, interests and motivations, therefore the rules had to be more explicitly defined.

6. (4 pts) The de Mello paper “Power system dynamics – overview” describes on pg. 2 the modeling of Figs. 3-10. Although this paper was written long before the growth of converter-interfaced generation (CIG), the pg. 2 description is relevant to it. Referring to the models by their figure number, (i) which model is the classical model? (ii) which model is most appropriate for high-fidelity transient stability analysis of a system without CIG? (iii) which model is most appropriate for high-fidelity transient stability analysis of a system with heavy CIG presence?

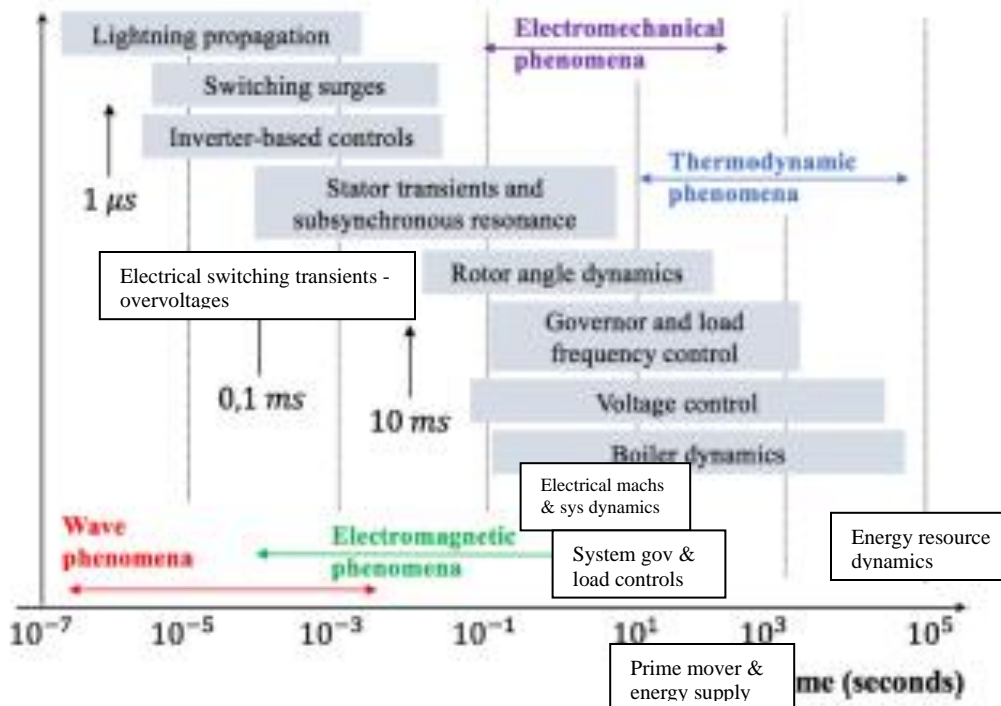
**Solution:** (i) Fig. 10 is the classical model. (ii) Fig. 8, to which the paper states, “This is the degree of detail generally used in stability studies where machine rotor flux behavior must be accounted for both for their effect on synchronizing and damping torques.” (iii) Fig. 6, to which the paper states, “Figure 6 represents the maximum of detail in both machine and network. Such effects as subsynchronous resonance, harmonics generated by saturation can be studied with this representation.”

7. (4 pts) The de Mello paper “Process dynamics in electric utility systems” identifies at the bottom of p. 1 three “areas” of “prime interest to instrumentation & control engineers,” that are in reference to Fig. 1 (Fig. 1 is on p. 6), and then describes them in the next three paragraphs. A “Fig. 2” (also on p. 6) is mentioned. (i) Cut and paste Fig. 1 and circle/label the part of the figure corresponding to each of the three areas listed on p. 1; (ii) Label each of the four time-frames of Fig. 2 on Fig. 1 of the 2021 “Terms & Definitions” paper.

**Solution:** (i)



(ii)



8. (4 pts) In the Fouad paper on DSA, R. Vierra of PG&E at the top right-hand column of p. 1318 defines “remedial actions” as “pre-planned actions ... which take place in response to specific disturbances and which are used to avoid uncontrolled loss of firm load following a major disturbance.” To what extent is the description of Section A under his “DSA Decisions” an example of a remedial action? Would you consider “fast valving” and “transient excitation boosting” (described in four other website papers) also types of remedial actions?

**Solution:** This example consists of “pre-planned actions...” and actually is the most complex remedial action ever. Yes, fast valving and TEB are examples of remedial actions.

9. (4 pts) The Byerly & Kimbark paper shows in Fig. 6 the post-fault power angle. (i) where is the fault-on curve? What angle corresponds to (ii) stable equilibrium; (iii) clearing; (iv) unstable equilibrium?

**Solution:**

- (i) The fault-on curve is the horizontal axis.
- (ii)  $\delta_{a1}$  corresponds to the stable equilibrium.
- (iii)  $\delta_{aT}$  corresponds to the clearing angle.
- (iv)  $\delta_{a2}$  corresponds to the unstable equilibrium.

10. (9 pts) Speed and frequency:

- a. What is the steady-state nominal frequency of any bus in the North American power grid, in electrical rad/sec?

**Solution:**  $\omega_{Re}=377$  rad/sec.

- b. What is the synchronous speed in mechanical rad/sec of a 6-pole machine that operates in the North American power grid?

**Solution:**  $\omega_{Rm}=(2/p)\omega_{Re}=(2/6)(377)=125.667$  rad/sec

- c. What is the synchronous speed in revolutions per minute (rpm) of a 6-pole machine that operates in the North American power grid?

**Solution:**  $n_R = 125.667 \text{ rad/sec} * 60 \text{ sec/min} * 1 \text{ rev}/(2\pi \text{ rad}) = 1200 \text{ rpm}$

11. (10 pts) Answer the below questions:

- a. (x pts) Assume a reasonable value for the H-constant of a 3600 rpm turbine-generator set of a standard steam-fired power plant, given on the machine rating; write down this value and give correct units.

**Solution:**  $H = 5 \text{ sec}$ . (any value between 1 and 10 would be accepted).

- b. (x pts) The synchronous machine of (2a) is rated at 500 MVA. Based on your assumed value of H,

- i. compute the MW-sec of this machine.

**Solution:** Assume  $H = 5 \text{ sec} \rightarrow \text{MW-sec} = 5 * 500 = 2500 \text{ MW-sec}$ .

- ii. compute the H on a 100MVA system base:

**Solution:**  $H_{100} = H_{500} (500/100) = 5 * (500/100) = 25 \text{ sec}$ .

- c. (x pts) Compute the moment of inertia in  $\text{kg-m}^2$  for the synchronous machine of (a).

**Solution:**  $W_k = 0.5(J)\omega_R^2 \rightarrow J = 2W_k/\omega_R^2$

$W_k = H * 500 * 10^6 \text{ joules}$ ;

$\omega_R = 3600 \text{ rev/min} * 2\pi \text{ rad/rev} * 1 \text{ min}/60 \text{ sec} = 377 \text{ rad/sec}$

$\rightarrow J = 2 * H * 500 * 10^6 / (377^2) = 7035.9H$

With  $H = 5$ ,  $J = 35,179 \text{ kg-m}^2$ .

12. (9 pts) A 3-phase, 60Hz, 500 MVA, 15 kV, 32 pole hydroelectric generating unit has an H constant of 2.0 sec. It is initially operating at  $P_{mu} = P_{eu} = 1.0 \text{ pu}$ ,  $\omega_e = \omega_{Re} = 377 \text{ rad/sec}$  when a 3phase fault at the generator terminals causes  $P_{eu}$  to drop to 0 for  $t \geq 0$ .

- a. Give the per-unit swing equation for this unit corresponding to the pre-fault condition. Express accelerating power  $P_a$  numerically in this equation.

**Solution:**

$$\frac{2H}{\omega_{Re}} \ddot{\delta} = \frac{2(2)}{377} \ddot{\delta} = P_m - P_e \Rightarrow 0.0106 \ddot{\delta} = P_m - P_e = P_a = 0$$

- b. Give the per-unit swing equation for this unit corresponding to the fault-on period. Express accelerating power  $P_a$  numerically in this equation.

**Solution:**

$$0.0106 \ddot{\delta} = P_m - P_e = P_a = 1.0$$

- c. Determine the power angle 3 cycles after the fault commences. Assume  $P_{mu}$  remains constant at 1.0pu, and the initial angle is 0 degrees.

**Solution:**

$$0.0106 \ddot{\delta} = 1.0 \text{ or } 0.0106 \frac{d\omega}{dt} = 1.0 \text{ and so } 0.0106 d\omega = dt .$$

Now recall that the initial speed must be zero since the machine is initially at steady-state. Therefore we may integrate the above as follows:

$$\int_0^{\omega(t)} 0.0106 d\omega = \int_0^t dt \quad \Rightarrow 0.0106\omega(t) = t$$

which can be written as

$$0.0106 \frac{d\delta}{dt} = t \quad \Rightarrow 0.0106 d\delta = dt$$

Now integrate again:

$$\int_{\delta_0}^{\delta(t)} 0.0106 d\delta = \int_0^t t dt \quad \Rightarrow 0.0106[\delta(t) - \delta_0] = \frac{t^2}{2}$$

But  $\delta_0=0$ , and so

$$0.0106\delta(t) = \frac{t^2}{2}$$

$$\Rightarrow \delta(t) = \frac{1}{2(0.0106)} t^2 = 47.17t^2$$

At 3 cycles,  $t=3/60=0.05$ sec, and so

$$\Rightarrow \delta(t=0.05) = 47.17(0.05)^2 = 0.118$$

The above is in radians. In degrees, we obtain

$$\Rightarrow \delta(t=0.05) = 6.76^\circ$$

13. (12 pts) The equation for computing critical clearing angle is given as equation (\*) below:

$$\cos \delta_{cr} = \frac{\frac{P_m}{P_{M1}} (\delta_m - \delta_1) + r_2 \cos \delta_m - r_1 \cos \delta_1}{r_2 - r_1} \quad (*)$$

Assume the machine is generating 0.8pu power with an initial equilibrium angle  $21.09^\circ$  on a pre-fault power angle curve having amplitude of 2.223 pu, when a fault is applied at the machine terminals; the fault is cleared without removing any network elements.

- a. Compute  $r_1$  and  $r_2$ .

**Solution:**

Since the fault is at the machine terminals,  $P_{M2}=0$ , so  $r_1=P_{M2}/P_{M1}=0$ .

Since no elements are removed,  $P_{M3}=P_{M1}$ , so  $r_2=P_{M3}/P_{M1}=1$ .

- b. Using the values of  $r_1$  and  $r_2$  computed in part (a), provide a simplified version of equation (\*).

**Solution:**

The radian equivalent of

$$\cos \delta_{cr} = \frac{\frac{P_m}{P_{M1}}(\delta_m - \delta_1) + (1)\cos \delta_m - (0)\cos \delta_1}{(1) - (0)} = \frac{\frac{P_m}{P_{M1}}(\delta_m - \delta_1) + \cos \delta_m}{(1) - (0)}$$

- c. Compute the maximum angle for stability,  $\delta_m$  (also called the angle of the post-fault unstable equilibrium).

**Solution:**

Since no elements are removed, the pre-and post-fault power angle curves are the same, and in this case,  $\delta_m=180- \delta_1=180-21.09=158.91^\circ$ , or 2.77 rad

- d. Compute the critical clearing angle for this case.

**Solution:**

The angle of  $21.09^\circ$  is 0.3681 rad.

$$\cos \delta_{cr} = \frac{P_m}{P_{M1}}(\delta_m - \delta_1) + \cos \delta_m = \frac{0.8}{2.223}(2.77 - .3681) + \cos(2.77) = -.0674$$

$$\Rightarrow \delta_{cr} = \text{acos}(-0.0674) = 1.6382\text{rad} = 93.863^\circ.$$

14. (24 pts) True false:

- a. NERC's disturbance-performance table is based on the planning philosophy that a lower level of performance is allowed for disturbances having a higher frequency of occurrence.
- b. The "Dy Liacco" operational state diagram presented in class shows that an effective way to transition from the alert state to the normal state is to shed load while performing off-economic dispatch.
- c. Transient instability, a nonlinear problem, is typically assessed using time-domain simulations tools modeling faults, fault clearing and associated removal of one or more elements, and 5-20 seconds post-fault system response.
- d. Today's transient stability time-domain simulators use phasor representation of network voltages and currents, neglecting the faster electromagnetic transients associated with line inductance and capacitance.
- e. Following a generation trip in the network, a remaining machine with a 5% regulation constant will see that the magnitude of the per-unit steady-state frequency deviation will be 5% of the magnitude of the per-unit steady-state power change.
- f. During the post-fault condition, a synchronous machine may oscillate about its unstable equilibrium point.
- g. The equal area criterion states that, for a synchronous machine to exhibit stable performance following a fault, there must be sufficient decelerating energy after fault clearing to counter the accelerating energy associated with the fault-on period.
- h. Two ways to improve a synchronous machine's stability performance for a given fault/fault-clearing/post-fault sequence are to (i) lower the pre-fault power generated by the machine; and (ii) reduce the pre-fault internal voltage magnitude of the machine.

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|----|---|
| a. | F |
| b. | F |
| c. | T |
| d. | T |
| e. | T |
| f. | F |
| g. | T |
| h. | F |
| i. | T |
| j. | F |
| k. | T |
| l. | F |

- \_\_\_\_\_ i. A certain synchronous machine is connected to the network via a single transformer that is connected on the high-side to two 500 kV lines. Line 1 has a reactance of  $X_1$ , and line 2 has a reactance of  $X_2=2X_1$ . For a fault at the machine terminals followed by fault clearing and loss of one of the lines, we expect stability performance to be worse (less stable) for loss of line 1.
- \_\_\_\_\_ j. The paper “Turbine Fast Valving to Aid System Stability...” indicates that “tripping units at the plant would maintain stable operation for the postulated dual contingency...” This benefit occurs entirely because it increases voltage at the high-side of the plants step-up transformer.
- \_\_\_\_\_ k. The paper “Turbine Fast Valving to Aid System Stability...” indicates that “the single phase switching feature will enhance the stability performance of the Rockport units...” This benefit occurs because, for the most common fault type (line-to-ground), single pole switching clears only one phase while leaving the remaining two phases in operation, thereby enhancing post-fault system strength.
- \_\_\_\_\_ l. The paper “Transient excitation boosting at Grand Coulee Third Power Plant...” refers to post-fault insertion of series capacitors into transmission lines, e.g., column 2 of p. 1292, columns 2 and 3 of p. 1295, and column 2 of p. 1298. This “special control” enhances stability because it increases the effective reactance of the lines in the transmission system.