Transactions on Power Systems, Vol. 7, No. 1, February 1992

THE CONTINUATION POWER FLOW: A TOOL FOR STEADY STATE VOLTAGE STABILITY ANALYSIS

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ABSTRACT

This paper presents a method of finding a continuum of power flow solutions starting at some base load and leading to the steady state voltage stability limit (critical point) of the system. A salient feature of the so-called continuation power flow is that it remains well-conditioned at and around the critical point. As a consequence, divergence due to ill-conditioning is not encountered at the critical point, even when single precision computation is used. Intermediate results of the process are used to develop a voltage stability index and identify areas of the system most prone to voltage collapse. Examples are given wherein the voltage stability of a system is analyzed using several different scenarios of load increase.

INTRODUCTION

In recent years, the increase in peak load demand and power transfers between utilities has elevated concerns about system voltage security. Voltage collapse has been deemed responsible for several major disturbances [1] and significant research efforts are under way in an effort to further understand voltage phenomena [2]. A large portion of this research is concentrated on the steady state aspects of voltage stability. Indeed, numerous authors have proposed voltage stability indexes based upon some type of power flow analysis [3-10]. A particular difficulty being encountered in such research is that the Jacobian of a Newton-Raphson power flow becomes singular at the steady state voltage stability limit. In fact, this stability limit, also called the critical point, is often defined as the point where the power flow Jacobian is singular. As a consequence, attempts at power flow solutions near the critical point are prone to divergence and error. For this reason, double precision computation and anti-divergence algorithms such as the one found in [12] have been used in attempts to overcome the numerical instability.

This paper demonstrates how singularity in the Jacobian can be avoided by slightly reformulating the power flow equations and applying a locally parameterized continuation technique. During the resulting "continuation power flow", the reformulated set of equations remains well-conditioned so that divergence and error due to a singular Jacobian are not encountered. As a result, single precision computations can be used to obtain power flow solutions at and near the critical point.

The continuation algorithm used in this work is from a well documented class of techniques used to find a path of equilibrium solutions of a set of nonlinear equations [13-16]. One particular application of these algorithms has been in civil engineering where the equilibrium solutions of the equations describing a structure have been been studied under a change in a load intensity parameter [16]. The method used in this paper is, however, different from the homotopy type of continuation used for the optimal power flow in [17] since singularity of the Jacobian remains a problem in the homotopy approach.

From its conception, the purpose of the continuation power flow was to find a continuum of power flow solutions for a given load change scenario. An early success was the ability to find a set of solutions from a base case up to the critical point in but a single program run [18]. Since then, however, certain intermediate results of the continuation process have been recognized to provide valuable insight into the voltage stability of the system and the areas prone to voltage collapse. Along these lines, a voltage stability index based upon results of the algorithm will be presented later in the paper.

The general principle behind the continuation power flow is rather simple. It employs a predictor-corrector scheme to find a solution path of a set of power flow equations that have been reformulated to include a load parameter. As shown in Fig. 1, it starts from a known solution and uses a tangent predictor to estimate a subsequent solution corresponding to a different value of the load parameter. This estimate is then corrected using the same Newton-Raphson technique employed by a conventional power flow. The local parameterization mentioned earlier provides a means of identifying each point along the solution path and plays an integral part in avoiding singularity in the Jacobian.

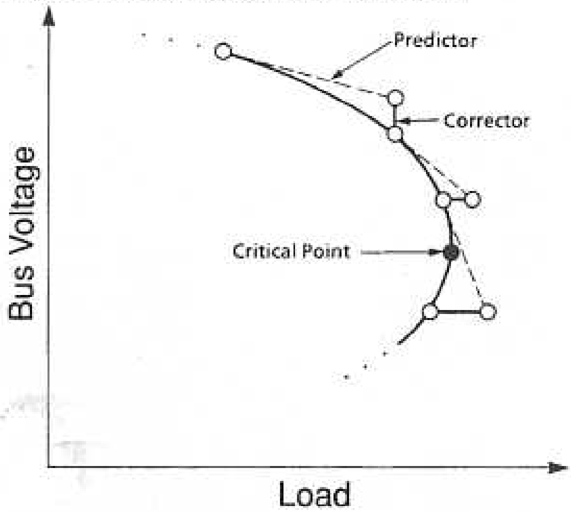

Fig. 1. An illustration of the predictor-corrector scheme used in the continuation power flow

In the sections that follow, each facet of the continuation power flow will be described and numerical results will be presented to demonstrate the usefulness of this technique in voltage stability analysis.

A REFORMULATION OF THE POWER FLOW EQUATIONS

In order to apply a locally parameterized continuation technique to the power flow problem, a load parameter must be inserted into the equations. While there are many ways this could be done, only a simple example using a constant power load model will be considered here.

First let λ represent the load parameter such that

$$0 \leqslant \lambda \leqslant \lambda_{critical}$$

where $\lambda = 0$ corresponds to the base load and $\lambda = \lambda_{critical}$ corresponds to the critical load. We desire to incorporate λ into

$$0 = P_{Gi} - P_{Li} - P_{Ti}, \quad P_{Ti} = \sum_{j=1}^{n} V_i V_j \, y_{ij} \cos(\delta_i - \delta_j - \nu_{ij})$$

$$0 = Q_{Gi} - Q_{Li} - Q_{Ti}, \quad Q_{Ti} = \sum_{j=1}^{n} V_i V_j \, y_{ij} \sin(\delta_i - \delta_j - \nu_{ij})$$

for each bus i of an n bus system, where the subscripts $L, G,$ and T denote bus load, generation, and injection respectively. The voltages at buses i and j are $V_i \angle \delta_i$ and $V_j \angle \delta_j$ respectively and $y_{ij} \angle \nu_{ij}$ is the $(i,j)^{th}$ element of Y_{BUS}.

To simulate a load change, the P_{Li} and Q_{Li} terms must be modified. This can be done by breaking each term into two components. One component will correspond to the original load at bus i and the other component will represent a load change brought about by a change in the load parameter λ.

Thus,

$$P_{Li} = P_{Lio} + \lambda(k_{Li} S_{\Delta base} \cos\psi_i)$$

$$Q_{Li} = Q_{Lio} + \lambda(k_{Li} S_{\Delta base} \sin\psi_i)$$

where the following definitions are made;

- P_{Lio}, Q_{Lio} - original load at bus i, active and reactive respectively
- k_{Li} - multiplier to designate the rate of load change at bus i as λ changes
- ψ_i - power factor angle of load change at bus i
- $S_{\Delta base}$ - a given quantity of apparent power which is chosen to provide appropriate scaling of λ

In addition, the active power generation term can be modified to

$$P_{Gi} = P_{Gio}(1 + \lambda k_{Gi})$$

where P_{Gio} is the active generation at bus i in the base case and k_{Gi} is a constant used to specify the rate of change in generation as λ varies.

If these new expressions are substituted into the power flow equations, the result is

$$0 = P_{Gio}(1 + \lambda k_{Gi}) - P_{Lio} - \lambda(k_{Li} S_{\Delta base} \cos\psi_i) - P_{Ti}$$

$$0 = Q_{Gio} - Q_{Lio} - \lambda(k_{Li} S_{\Delta base} \sin\psi_i) - Q_{Ti}$$

Notice that values of k_{Li}, k_{Gi}, and ψ_i can be uniquely specified for every bus in the system. This allows for a very specific variation of load and generation as λ changes.

THE APPLICATION OF A CONTINUATION ALGORITHM

In the preceding discussion, the power flow equations for a particular bus i were reformulated to contain a load parameter λ. The next step is to apply a continuation algorithm to the system of reformulated power flow equations. If $\underline{F}$ is used to denote the whole set of equations, the problem can be expressed as

$$\underline{F}(\underline{\delta}, \underline{V}, \lambda) = \underline{0}, \qquad 0 \leqslant \lambda \leqslant \lambda_{critical}$$

where $\underline{\delta}$ represents the vector of bus voltage angles and $\underline{V}$ represents the vector of bus voltage magnitudes. As mentioned, the base case solution $(\underline{\delta}_o, \underline{V}_o, \lambda_o)$ is known via a conventional power flow and the solution path is being sought over a range of λ. In general, the dimension of $\underline{F}$ will be $2n_1 + n_2$, where n_1 and n_2 are the number of P-Q and P-V buses respectively.

To solve the problem, the continuation algorithm starts from a known solution and uses a predictor-corrector scheme to find subsequent solutions at different load levels. While the corrector is nothing more than a slightly modified Newton-Raphson power flow, the predictor is quite unique from anything found in a conventional power flow and deserves detailed attention.

PREDICTING THE NEXT SOLUTION

Once a base solution has been found ($\lambda = 0$), a prediction of the next solution can be made by taking an appropriately sized step in a direction tangent to the solution path. Thus, the first task in the predictor process is to calculate the tangent vector. This tangent calculation is derived by first taking the derivative of both sides of the power flow equations.

$$d[\underline{F}(\underline{\delta}, \underline{V}, \lambda)] = \underline{F}_{\delta} \, d\underline{\delta} + \underline{F}_{V} \, d\underline{V} + \underline{F}_{\lambda} \, d\lambda = \underline{0}$$

Factorizing;

$$\begin{bmatrix} \underline{F}_{\delta} & \underline{F}_{V} & \underline{F}_{\lambda} \end{bmatrix} \begin{bmatrix} d\underline{\delta} \\ d\underline{V} \\ d\lambda \end{bmatrix} = \underline{0}$$

On the left side of this equation is a matrix of partial derivatives multiplied by a vector of differentials. The former is the conventional load flow Jacobian augmented by one column $(\underline{F}_{\lambda})$, while the latter is the tangent vector being sought. There is, however, an important barrier to overcome before a unique solution can be found for the tangent vector. The problem arises from the fact that one additional unknown was added when λ was inserted into the power flow equations, but the number of equations remained unchanged. Thus, one more equation is needed.

This problem can be solved by choosing a non-zero magnitude (say one) for one of the components of the tangent vector. In other words, if $\underline{t}$ is used to denote the tangent vector;

$$\underline{t} = \begin{bmatrix} d\underline{\delta} \\ d\underline{V} \\ d\lambda \end{bmatrix}, \quad t_k = \pm 1$$

This results in

$$\begin{bmatrix} \underline{F}_{\delta} & \underline{F}_{V} & \underline{F}_{\lambda} \\ & \underline{e}_k & \end{bmatrix} \begin{bmatrix} \underline{t} \end{bmatrix} = \begin{bmatrix} \underline{0} \\ \pm 1 \end{bmatrix} \tag{1}$$

where $\underline{e}_k$ is an appropriately dimensioned row vector with all elements equal to zero except the k^{th}, which equals one. If the index k is chosen correctly, letting $t_k = \pm 1$ imposes a non-zero norm on the tangent vector and guarantees that the augmented Jacobian will be nonsingular at the critical point [15]. Whether +1 or -1 is used depends on how the k^{th} state variable is changing as the solution path is being traced. If it is increasing a +1 should be used and if it is decreasing a -1 should be used. A method for choosing k and the sign of t_k will be presented later in the paper.

Once the tangent vector has been found by solving (1), the prediction can be made as follows:

$$\begin{bmatrix} \underline{\delta}^* \\ \underline{V}^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} \underline{\delta} \\ \underline{V} \\ \lambda \end{bmatrix} + \sigma \begin{bmatrix} d\underline{\delta} \\ d\underline{V} \\ d\lambda \end{bmatrix} \tag{2}$$

where "*" denotes the predicted solution for a subsequent value of λ (loading) and σ is a scalar that designates the step size. The step size should be chosen so that the predicted solution is within the radius of convergence of the corrector. While a constant magnitude of σ can be used throughout the continuation process, more elaborate methods of choosing the step size are described in [14] and [15].

PARAMETERIZATION AND THE CORRECTOR

Now that a prediction has been made, a method of correcting the approximate solution is needed. Actually, the best way to present this corrector is to expand on parameterization, which is vital to the process.

Every continuation technique has a particular parameterization scheme. The parameterization provides a method of identifying each solution along the path being traced. The scheme used in this paper is referred to as local parameterization.

In local parameterization the original set of equations is augmented by one equation that specifies the value of one of the state variables. In the case of the reformulated power flow equations, this means specifying either a bus voltage magnitude, a bus voltage angle, or the load parameter λ. In equation form this can be expressed as follows:

let

$$\underline{x} = \begin{bmatrix} \underline{\delta} \\ \underline{V} \\ \lambda \end{bmatrix}, \qquad \underline{x} \in R^{2n_1+n_2+1}$$

and let

$$x_k = \eta$$

where η is an appropriate value for the k^{th} element of $\underline{x}$.

Then the new set of equations would be

$$\begin{bmatrix} \underline{F}(\underline{x}) \\ x_k - \eta \end{bmatrix} = \begin{bmatrix} \underline{0} \end{bmatrix} \tag{3}$$

Now, once a suitable index k and value of η are chosen, a slightly modified Newton-Raphson power flow method (altered only in that one additional equation and one additional state variable are involved) can be used to solve the set of equations. This provides the corrector needed to modify the predicted solution found in the previous section.

Actually, the index k used in the corrector is the same as that used in the predictor and η will be equal to x_k^*, the predicted value of x_k. Thus, the state variable x_k is called the continuation parameter. In the predictor it is made to have a non-zero differential change ($dx_k = t_k = \pm 1$) and in the corrector its value is specified so that the values of other state variables can be found. How then does one know which state variable should be used as the continuation parameter?

CHOOSING THE CONTINUATION PARAMETER

There are several ways of explaining the proper choice of continuation parameter. Mathematically, it should correspond to the state variable that has the largest tangent vector component. More simply put, this would correspond to the state variable that has the greatest rate of change near the given solution. In the case of a power system, the load parameter λ is probably the best choice when starting from the base solution. This is especially true if the base case is characterized by normal or light loading. Under such conditions, the voltage magnitudes and angles remain fairly constant under load change. On the other hand, once the load has been increased by a number of continuation steps and the solution path approaches the critical point, voltage magnitudes and angles will likely experience significant change. At this point λ would be a poor choice of continuation parameter since it may change only a small amount in comparison to the other state variables. For this reason, the choice of continuation parameter should be re-evaluated at each step. Once the choice has been made for the first step, a good way to handle successive steps is to use

$$x_k : \left| t_k \right| = \max \left\{ \left| t_1 \right|, \left| t_2 \right|, \cdots \left| t_m \right| \right\} \tag{4}$$

where $\underline{t}$ is the tangent vector with a corresponding dimension $m = 2n_1 + n_2 + 1$ and index k corresponds to the component of the tangent vector that is maximal [13]. When the continuation parameter is chosen, the sign of its corresponding tangent component should be noted so that the proper value of +1 or -1 can be assigned to t_k in the subsequent tangent vector calculation.

SENSING THE CRITICAL POINT

The only thing left to do amid the predictor-corrector process is to check to see if the critical point has been passed. This is easily done if one keeps in mind that the critical point is where the loading (and therefore λ) reaches a maximum and starts to decrease. Because of this, the tangent component corresponding to λ (i.e., $d\lambda$) is zero at the critical point and is negative beyond the critical point. Thus, once the tangent vector has been calculated in the predictor step, a test of the sign of the $d\lambda$ component will reveal whether or not the critical point has been passed. If it was desired to find the exact location of the critical point, a direct method such as the one presented in [19] and [20] could be used.

SUMMARY OF THE PROCESS

Now that the continuation power flow has been described in some detail, a summary of the process may be helpful. Fig. 2 provides a brief summary in the form of a flow chart.

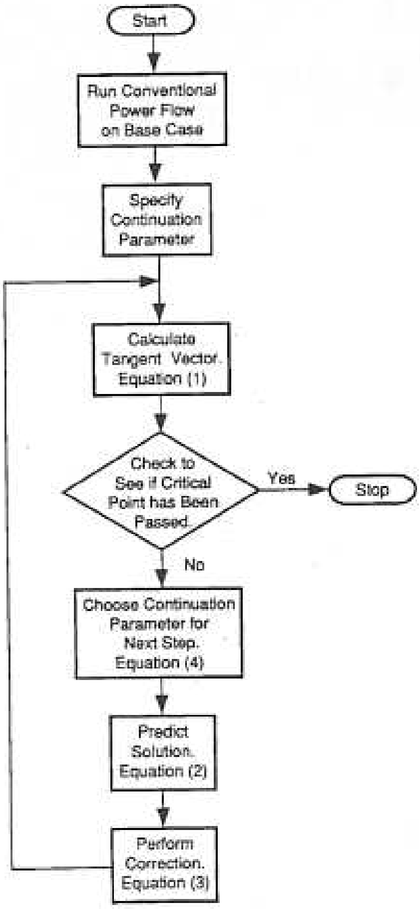

Fig. 2. A flow chart of the continuation power flow

SENSITIVITY INFORMATION FROM THE TANGENT VECTOR

Up until now, the discussion has been focused on finding a continuum of power flow solutions up to and just past the critical point. Although the accomplishment of this primary task is welcome, even more information is available from intermediate results. In fact, both a voltage stability index and an indicator of "weak" buses are available at almost no extra calculation cost by analyzing the tangent vector at each step.

In the continuation process, the tangent vector proves useful because it describes the direction of the solution path at a corrected solution point. A step in the tangent direction is used to estimate the next solution. However, if one looks at the elements of the tangent vector as differential changes in the state variables (dV_i or $d\delta_i$) in response to a differential change in system load ($Cd\lambda$, where C is some constant), the potential for a meaningful sensitivity analysis becomes apparent. Not so apparent, though, are the details of exactly how this analysis should be performed so that a stability index and identification of weak buses are obtained.

First of all, in discussing weak buses, the term "weak" must be defined. In the context of this paper, the weakest bus is the one that is nearest to experiencing voltage collapse. If one were to think of this in terms of a received power versus bus voltage (P-V) curve, the weak bus would be the one that is closest to the turning point or "knee" of the curve. Equivalently, a weak bus is one that has a large ratio of differential change in voltage to differential change in load $\left(\frac{dV}{dP}\right)$. But here it is important to think about which load is changing and which load change will affect the stability of the bus. If bus i was affected only by its own load change, then the ratio dV_i/dP_{Li} would be a good indicator of relative weakness. However, one must concede that load changes at other buses in the system will also play a key role. For this reason, the best method of deciding which bus is nearest to its voltage stability limit is to find the bus with the largest dV_i/dP_{TOTAL}, where dP_{TOTAL} is the differential change in active load for the whole system.

The differential change in voltage at each bus for a given differential change in system load is available from the tangent vector. Using the reformulated power flow equations, the differential change in active system load is

$$dP_{TOTAL} = \sum_{n} dP_{Li} = \sum_{n}\left(k_{Li} S_{\Delta BASE} \cos\psi_i\right) d\lambda$$

$$= \left(S_{\Delta BASE} \sum_{n} k_{Li} \cos\psi_i\right) d\lambda = C d\lambda$$

Thus, in light of the previous discussion, the weakest bus would be

$$bus\ j: \left|\frac{dV_j}{dP_{TOTAL}}\right|$$

$$= \left|\frac{dV_j}{Cd\lambda}\right| = \max\left(\left|\frac{dV_1}{Cd\lambda}\right|, \left|\frac{dV_2}{Cd\lambda}\right|, \cdots \left|\frac{dV_n}{dC\lambda}\right|\right)$$

Since the value of $Cd\lambda$ is the same for each dV element in a given tangent vector, choosing the weakest bus is as easy as choosing the bus with the largest dV component. Note, though, that the location of the weakest bus may change as the load shifts in intensity, characteristic, and location.

When the weakest bus, j, reaches its steady state voltage stability limit, the ratio of $-dV_j/dP_{TOTAL}$ will become infinite, or equivalently, the ratio of $-dP_{TOTAL}/dV_j$ would be zero. This latter ratio, being easier to handle numerically, makes a good voltage stability index for the entire system. This index will be high when the weakest bus is far from instability but will be zero when the weakest bus experiences voltage collapse. (The negative sign is used so that the index will be positive before the critical point is encountered and negative afterwards.)

Alternatively, a ratio of $-dV_i/dQ_{TOTAL}$ could also be used to compare weak buses and $-dQ_{TOTAL}/dV_j$ would accompany this as a voltage stability index. In this case, dQ_{TOTAL} would be expressed as a constant multiplied by $d\lambda$ as was done for dP_{TOTAL}. The use of dQ_{TOTAL} is, of course, required if a scenario involves a change in reactive load only, as do some of those used in the following examples.

A DEMONSTRATION OF RESULTS

The continuation power flow just described was coded in FORTRAN (single precision) and tested using the 30 bus New England System. Since this system is popular for voltage stability research, various load increase scenarios have already been described in [3], [8], and [11]. In this paper, two of these scenarios will be used as test cases and two extra test cases will be presented because they help to further demonstrate the capabilities of the continuation power flow. Each case begins with the same base load shown in [3] and [11], and entails increasing various loads until voltage collapse occurs. A graph of the voltage stability index obtained via the continuation power flow will be shown for each case.

Case 1

Case 1, which has been used in [3], [8], and [11], is well suited for testing the basic performance of a voltage stability index. It involves starting with a given system load, and subjecting bus 11 to an increasing reactive load until voltage collapse occurs. Five cumulative contingencies accompany this basic scenario so that the trajectory of the $-dQ_{TOTAL}/dV_j$ index can be demonstrated for each of the following:

A. No contingencies

B. Remove Generator 6

C. Remove Generator 10

D. Increase generation at Buses 2, 25, and 29 by 200 MW each

E. Remove line (9,30)

F. Install a 500 MVAR capacitor bank at Bus 11

The results, shown in Fig. 3, are very interesting when compared to those submitted in [3] and [8]. The basic result (curve A) is very similar to the response of the VIPI shown in [3]. This includes a profound impact on the index when generator reactive power output limits are met. The lower portion of the graph, where the index is below approximately 180, corresponds to the operating range shown in [8] where an energy based method is demonstrated.

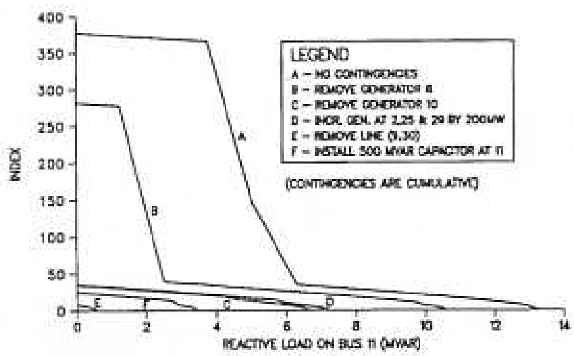

Fig. 3. Trace of the $-dQ_{TOTAL}/dV_j$ index as reactive power load is added to bus 11

Case 2

In contrast to Case 1, Case 2 involves increasing load on several buses as opposed to just one. In this case, any active and reactive load that existed in the base case is simply multiplied. Similarly, active power generation is increased by the same multiple as load, but a relaxed constraint is imposed on the reactive power output of each generator. Instead of using a constant limit as is usually done in power flow studies, the reactive power output is allowed to go as high as .8 of the constantly increasing active power output. While this case may not be realistic, the relaxed constraint was used in [3] because it mitigates the instability encountered in using a conventional Newton-Raphson power flow for calculation near the critical point. A more realistic constraint will be used in the next case.

The $-dQ_{TOTAL}/dV_j$ index for Case 2 is shown in Fig. 4. Again, the result is very similar to, although a little less curved than that of the VIPI in [3].

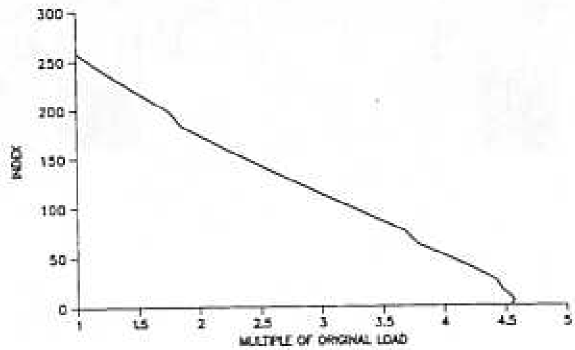

Fig. 4. $-dQ_{TOTAL}/dV_j$ index for Case 2

Case 3

The robustness of the continuation power flow can be demonstrated by slightly modifying Case 2. Here the relaxed reactive power output constraints will be removed so that the usual constant constraints are imposed.

As Fig. 5 shows, this more realistic constraint had no visible impact on the numerical stability of the continuation power flow. The implications of these constraints are manifested in the index as it takes noticeable dives when the reactive power output limits of various generators are reached. In addition, the stability limit for this case occurred at less than half of the load of Case 2.

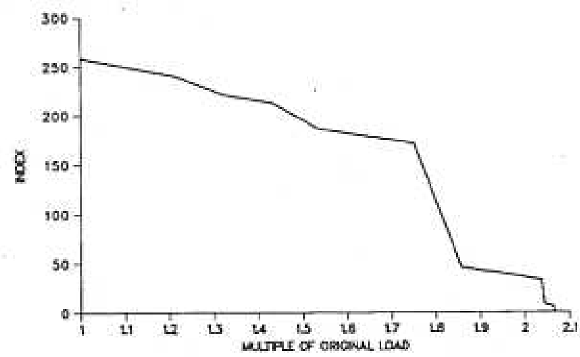

Fig. 5. $-dQ_{TOTAL}/dV_j$ index for Case 3

Since this case is fairly realistic, a meaningful demonstration of the weakest buses identified via the tangent vector can be made. Fig. 6 shows the five weakest buses identified at five different load levels. In each case, diamond shapes are used to denote weak buses and the circle representing a generator is filled in when the generator reaches its reactive power output limits.

When the loads are 1.31 times their original value, only generator 2 has reached its limit and the weakest buses are concentrated in the top center of the diagram. Next, the weakest buses begin to move toward the lower left-hand corner of the system when generator 6 reaches its limit. This move is completed with the change in generator 10 when the load multiple is 1.83. When the load is 2.01 times its original value and yet another two generators reach their limits, the location of the five weakest buses remains unchanged. But this situation is altered drastically as generators 20 and 23 can no longer supply unlimited reactive power. The stability limit is reached at 2.05 times the original load with the weakest buses found on the right-hand side of the diagram.

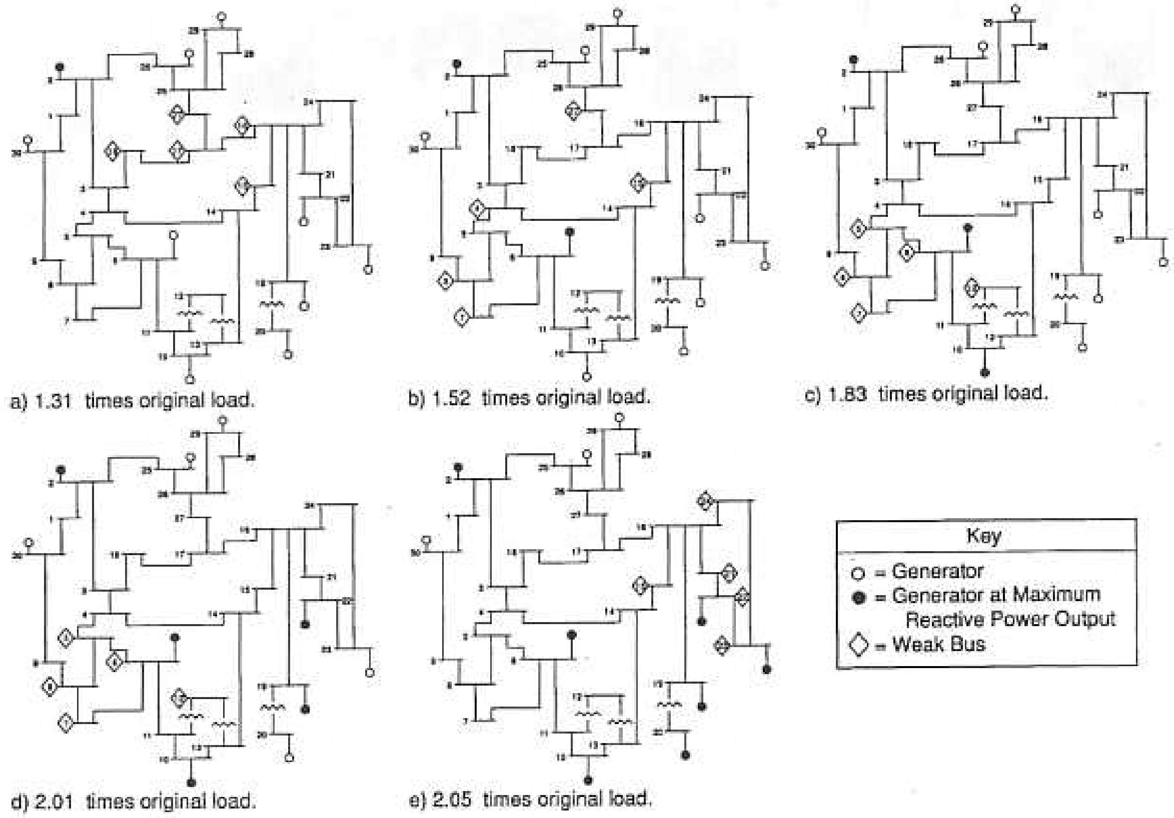

Fig. 6: Weakest buses identified via the tangent vector as the load is increased in Case 3

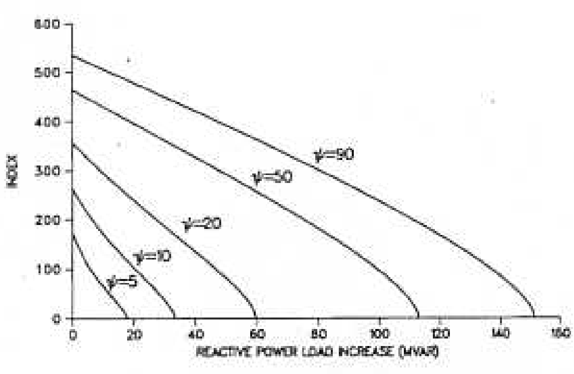

Fig. 7. The $-dQ_{TOTAL}/dV_j$ index for Case 4 when generator reactive power output limits are not imposed (ψ in degrees)

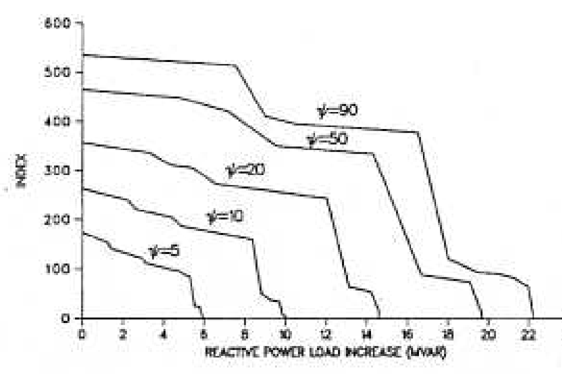

Fig. 8. The $-dQ_{TOTAL}/dV_j$ index for Case 4 when generator reactive power output limits are imposed (ψ in degrees)

Case 4

Case 4 is a slightly different scenario than Case 3. Although the same base load is used, the load multiplication constants (k_{Li}'s) are now set proportional to the original active power load at each bus. In addition, the load added to each bus during continuation is made to have the same power factor. In terms of the reformulated power flow equations, this latter item is accomplished by using the same value of ψ_i at each bus. Although this type of load change would not be seen in an actual power system, the scenario is useful for observing the response of the index when various power factors are used.

Since the generator reactive power output limits introduce extra nonlinearities into an already nonlinear system, this example is started by showing the response of the $-dQ_{TOTAL}/dV_j$ index for various power factors when the limits are not imposed. As can be seen from Fig. 7, the index is not linear in its response, but its basic shape is almost the same for the higher values of power factor angles. When the reactive power output limits are again imposed (Fig. 8), the trace of the index maintains a fairly similar shape over the range of power factor angles.

So far, each case has been viewed in terms of reactive power by using the $-dQ_{TOTAL}/dV_j$ index. As can be seen from Fig. 9 and Fig. 10, an active power perspective can also be used. These graphs show the response of the $-dP_{TOTAL}/dV_j$ index to Case 4. The same basic observations can be made of these two graphs as were made of the previous two except that the $-dP_{TOTAL}/dV_j$ index has a more uniform response at the lower power factor angles instead of at the higher ones seen with the $-dQ_{TOTAL}/dV_j$ index. However, now it is important to consider perspective. As seen from these results, a case that seems relatively severe in terms of active power ($\psi = 80°$, for example) would appear to be moderate in terms of reactive power. This is, of course, due to the fact that a large power factor angle signifies much larger changes in reactive load than in active load. Since this may precipitate confusion, one or the other index should be used when comparing cases, the choice depending upon which perspective is desired.

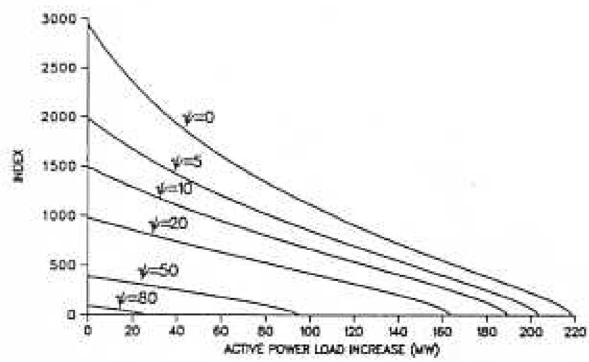

Fig. 9. The $-dP_{TOTAL}/dV_j$ index for Case 4 when generator reactive power output limits are not imposed (ψ in degrees)

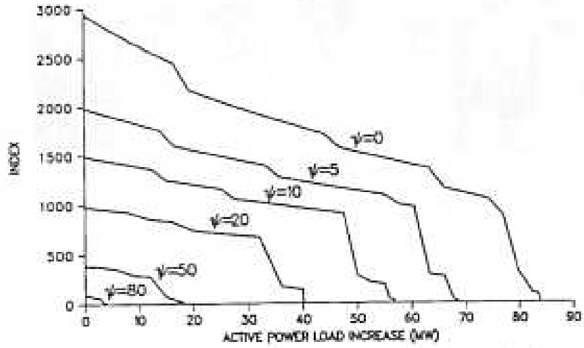

Fig. 10. The $-dP_{TOTAL}/dV_j$ index for Case 4 when generator reactive power output limits are imposed (ψ in degrees)

CONCLUSIONS

The examples presented in this paper have demonstrated both the capability and usefulness of the continuation power flow. Solution paths that went up to and beyond the critical point were found for various load change scenarios. In each case divergence was not a problem as it would have been in a conventional Newton-Raphson power flow. As an additional benefit, intermediate results from the tangent predictor of the continuation process proved useful in analyzing the voltage stability of the system. Both a stability index and a method of identifying the weakest buses in the system were readily obtained. Although the indexes used were effective and meaningful, there is great potential for developing more sophisticated indexes from the same tangent vector information.

As a next step in developing this method, nonlinear load models are being incorporated into the process so that a more accurate assessment of voltage stability performance can be made. In addition, it is recognized that the same algorithm employed by the continuation power flow can also be used in an optimal power flow (OPF). In fact, the locally parameterized continuation approach might overcome the ill-conditioning encountered in the OPF proposed in [17].

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