Windings and Axes

1.0 Introduction

In these notes, we will describe the different windings on a synchronous machine. We will confine our analysis to two-pole machines of the salient pole rotor construction. Results will be generalizable because

- A machine with p>2 poles will have the same phenomena, except p times/cycle.
- Round rotor machines can be well approximated using a salient pole model and proper designation of the machine parameters.

We will also define an important coordinate frame that we will use heavily in the future.

2.0 Defined axes

The magnetic circuit and all rotor winding circuits (which we will describe shortly) are symmetrical with respect to the polar and inter-polar (between-poles) axes. This proves convenient, so we give these axes special names:

- Polar axis: Direct, or d-axis
- Interpolar axis: Quadrature, or q-axis.

The q-axis is 90° from the d-axis, but which way?

- Ahead?
- Or behind?

Correct modeling can be achieved either way, and some books do it one way, and some another. We will remain consistent with your text and choose the q-axis to lag the d-axis by 90°, which is "consistent with the recommendation and rationale of [15][1]" (p. 92, VMAF).

Recommended Phasor Diagram for Synchronous Machines

IEEE COMMITTEE REPORT

The positive direction of the quadrature axis is chosen to lag the direct axis. That is, as the machine rotates, a point on the armature will be in the quadrature axis 90 electrical degrees of rotation after it is in the direct axis. The reasons for this choice are as follows.

1) The direction of the voltage generated by the flux due to field current is in the positive quadrature axis.

2) In mathematical analysis, the coefficient C_q in the following quadrature-axis transformation is a positive number:

$$i_q = C_q \left[i_A \sin\theta + i_B \sin\left(\theta - \frac{2\pi}{3}\right) + i_C \sin\left(\theta + \frac{2\pi}{3}\right) \right].$$

Thus both C_d and C_q are positive numbers if the positive reference directions of the direct and quadrature axes are chosen as recommended.

3) For a generator, the quadrature-axis component of armature current i_{aq} and the voltage due to field current e_{af} have positive values when referred to the reference direction of the quadrature axis. Since the product $I_{aq}E_{af}$[1] represents the electrical power transmitted from the rotor to the stator, it is quite reasonable to ask that both I_{aq} and E_{af} be positive numbers for generator operation.

4) It is common in system analysis to establish voltage as the phasor reference axis. Using this principle, the quadrature axis can be taken as the real axis. The direct axis then becomes the imaginary axis and the various electrical phasor equations can be written in terms of voltage and current phasors of the form $\boldsymbol{E} = E_q + j\boldsymbol{E}_d$ and $\boldsymbol{I} = I_q + jI_d$.

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[1] IEEE Committee Report, Harrington, D. (chair), "Recommended phasor diagram for synchronous machines, IEEE Trans. Power App. Syst. PAS-88: pp. 1593-1610, 1969.

Fig. 1 is from your text, and shows the q-axis lagging the d-axis, consistent with our assumption.

Fig. 2 is from Kundur, and shows the q-axis leading the d-axis, which we will NOT do.

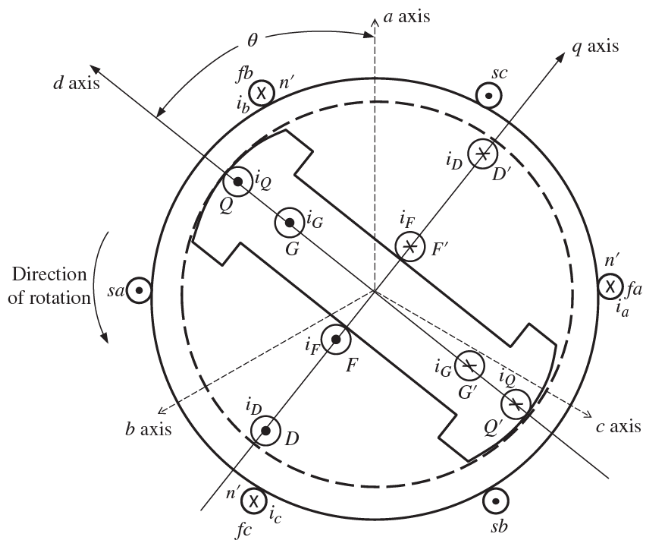

Figure 4.1a Pictorial representation of a synchronous machine.

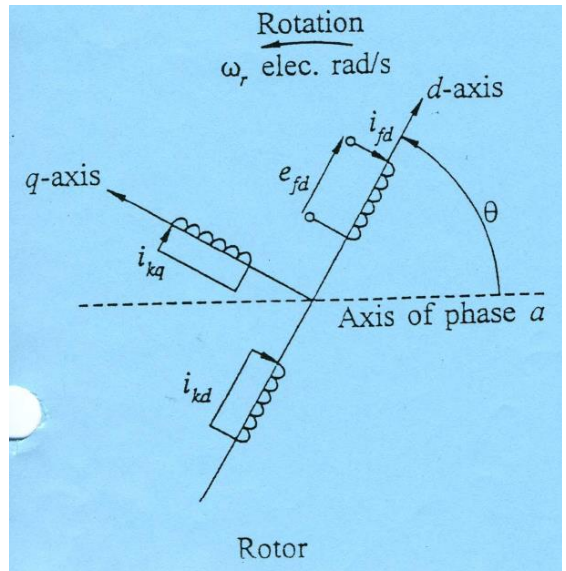

Fig. 1 Fig. 2

Example 5.1 in Chapter 5 is worked with the q-axis lagging the d-axis; Example 5.2 reworks the same problem with the q-axis leading the d-axis. Example 5.2 states that '"The objective of this example is to illustrate, though the calculations differ slightly, that the results are identical; thus, the choice of which coordinate reference frame is assumed is arbitrary, as long as calculations are consistent with the approach."

3.0 Physical windings

There are typically 5 physical windings on a synchronous machine:

- 3 stator windings (a-phase, b-phase, and c-phase)
- 1 main field winding
- Amortissuer windings on the pole-faces

The stator windings and the field winding are familiar to you based on the previous notes. The amortissuer winding might not be, so we briefly describe it.

Amortissuer is a French word for "shock absorber." These windings provide positive sequence damping under transient conditions and so are good for stability[2]. Under steady state, they are dead, i.e., they have no influence.

Amortissuer windings are not usually used on smooth-rotor machines, because the solid steel rotor cores of such machines provide paths for eddy currents and thus produce the same effects as amortissuer windings[3].

Amortissuer windings are often used in salient-pole machines, but even when not, eddy currents in pole

[2] E. Kimbark, "Power system stability, Vol. III: Synchronous Machines," IEEE Press, 1995 (orig pub 1956), p. 219.
[3] Ibid, p. 215.

faces contribute the same effect, although greatly diminished.

Amortissuers have a number of other good effects, as articulated by Kimbark in his Volume III book on synchronous machines[4] (he lists 10 such effects). For our purposes, they affect machine subtransient data[5].

Amortissuer windings are embedded in the pole-face (or "shoe" of the pole) and consist of copper or brass rods connected to end rings. They are similar in construction to the squirrel cage of an induction motor.

Figures 3 (from Sarma) and 4 (from Kundur) illustrate amortissuer windings. Note that they may be continuous (Fig. 3a and Fig. 4) or noncontinuous (Fig. 3b).

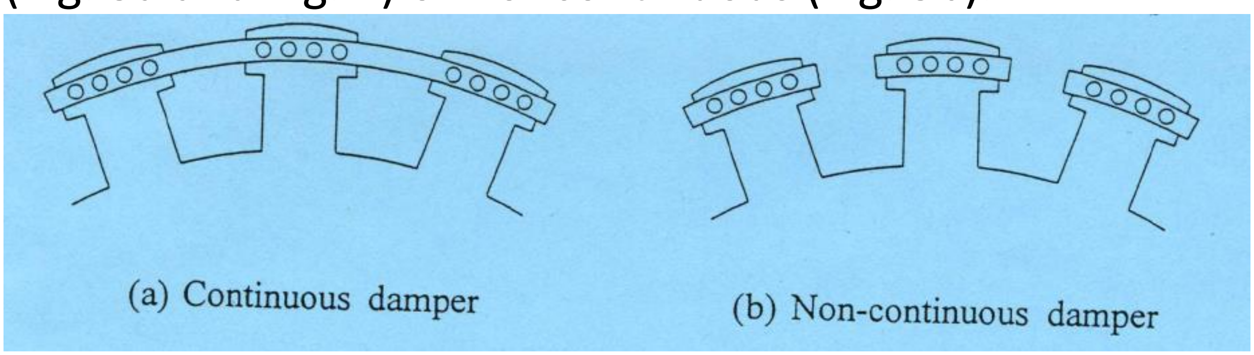

Fig. 3

[4] Ibid, p. 216.
[5] Ibid, p. 217-219.

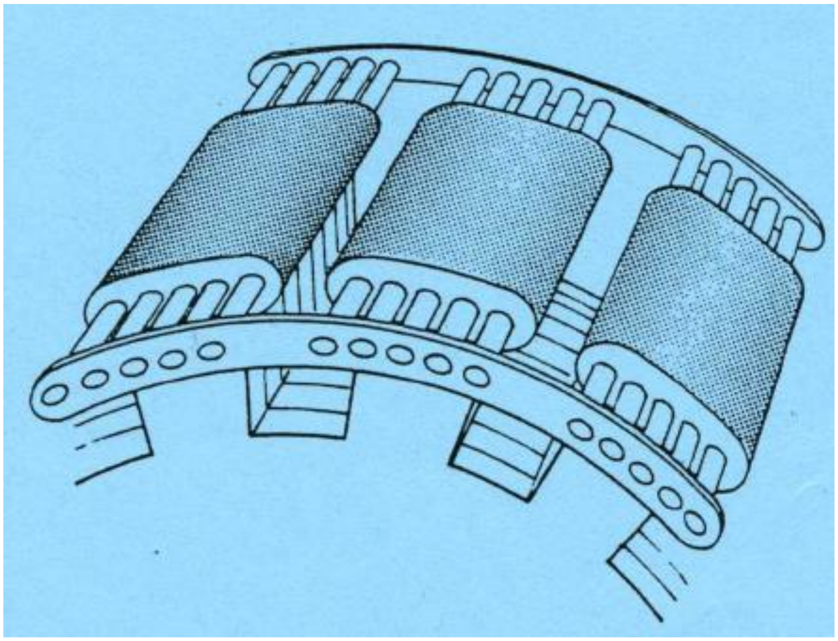

Fig. 4

4.0 Modeled windings and currents

Although there are typically 5 physical windings on a machine, we will model a total of 7, with associated currents as designated below.

- 3 stator windings: i_a, i_b, i_c
- Field windings: There are 2: one physical; one fictitious
  - Main field winding: carrying current i_F and producing flux along the d-axis.
  - G-winding: carrying current i_G and producing flux along the q-axis. This is the fictitious one, but it serves to improve the model accuracy of the round-rotor machine (by modeling the q-axis flux produced by the eddy-current effects in the rotor

during the transient period), and it can simply be omitted when modeling the salient pole machine (in salient pole machines, there is little q-axis flux produced by the eddy current effect in the rotor). The G-winding is like the F-winding of the main field, except it has no source voltage in its circuit. Kimbark suggests modeling it in his Vol. III, pg. 73.

- Amortissuer winding: This one represents a physical winding for salient-pole machines with dampers, and a fictitious winding otherwise. Because these produce flux along both the d-axis and the q-axis, we model two windings:
    - d-axis: amortissuer winding carrying current i_D
    - q-axis: amortissuer winding carrying current i_Q

It is important to understand the difference between the F and G windings and the D and Q windings, respectively, driven by the fact that D and Q windings have higher resistance than F and G windings. Therefore:

- Both the F and D produce flux along the D-axis, but D is “faster” (lower time constant or L/R ratio) than F.
- Both the G and Q produce flux along the Q-axis, but Q is “faster” (lower time constant or L/R ratio) than G.

5.0 Flux linkages and currents

So we have seven windings (circuits) in our synchronous machine. The flux linkage seen by any winding i will be a function of

- Currents in all of the windings and
- Magnetic coupling between winding i and winding j, as characterized by L_{ij}, where j=1,…,7.

That is

$$\lambda_i = \sum_{j=1}^{7} L_{ij} i_j \tag{1}$$

For example, the flux linking the main field winding is:

$$\lambda_F = L_{Fa} i_a + L_{Fb} i_b + L_{Fc} i_c + L_{FF} i_F + L_{FD} i_D + L_{FQ} i_Q + L_{FG} i_G \tag{2}$$

Repeating for all windings results in Equation (4.11) in VMAF. (Note in older editions the G-winding is not represented).

$$
\begin{array}{c} \text{stator}\left\{\begin{array}{c} \\ \\ \\ \end{array}\right. \\ \text{rotor}\left\{\begin{array}{c} \\ \\ \\ \\ \end{array}\right. \end{array}
\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_G \\ \lambda_D \\ \lambda_Q \end{bmatrix}
=
\left[\begin{array}{ccc:cccc}
L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aG} & L_{aD} & L_{aQ} \\
L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bG} & L_{bD} & L_{bQ} \\
L_{ca} & L_{bc} & L_{cc} & L_{cF} & L_{cG} & L_{cD} & L_{cQ} \\
\hdashline
L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FG} & L_{FD} & L_{FQ} \\
L_{Ga} & L_{Gb} & L_{Gc} & L_{GF} & L_{GG} & L_{GD} & L_{GQ} \\
L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DG} & L_{DD} & L_{DQ} \\
L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QG} & L_{QD} & L_{QQ}
\end{array}\right]
\begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix}
\qquad (3)
$$

Note the blocks of the above matrix correspond to

- Lower right-hand 4×4 are rotor-rotor terms.
- Upper-left-hand 3×3 are stator-stator terms;
- Upper right-hand 3×4 are stator-rotor terms;
- Lower left-hand 4×3 are rotor-stator terms;

VMAF summarize the expressions for each of these groups of terms on pp. 94-96. I expand on this summary in the next section.

6.0 Inductance blocks

<u>6.1a Rotor-rotor terms: self inductances</u>

Recall (see eq (15) in notes called "Preliminary Fundamentals) that the general expression for self-inductance is

$$L_{ii} = \frac{\lambda_i}{i_i} = \frac{N_i^2}{\mathscr{R}_i} \tag{4a}$$

where $\mathscr{R}_i$ is the reluctance of the path seen by λ_i, given by

$$\mathscr{R}_i = \frac{l}{\mu A} \tag{4b}$$

where l is the mean length of the path, μ is the permeability of the path's material, and A is the cross-sectional area of the path.

At any given moment, the stator and the rotor present a constant reluctance path to flux developed by a winding on the rotor, i.e., the reluctance path seen by any rotor winding is independent of the "position" angle θ. This is illustrated in Fig. 5 for the main field (F) winding.

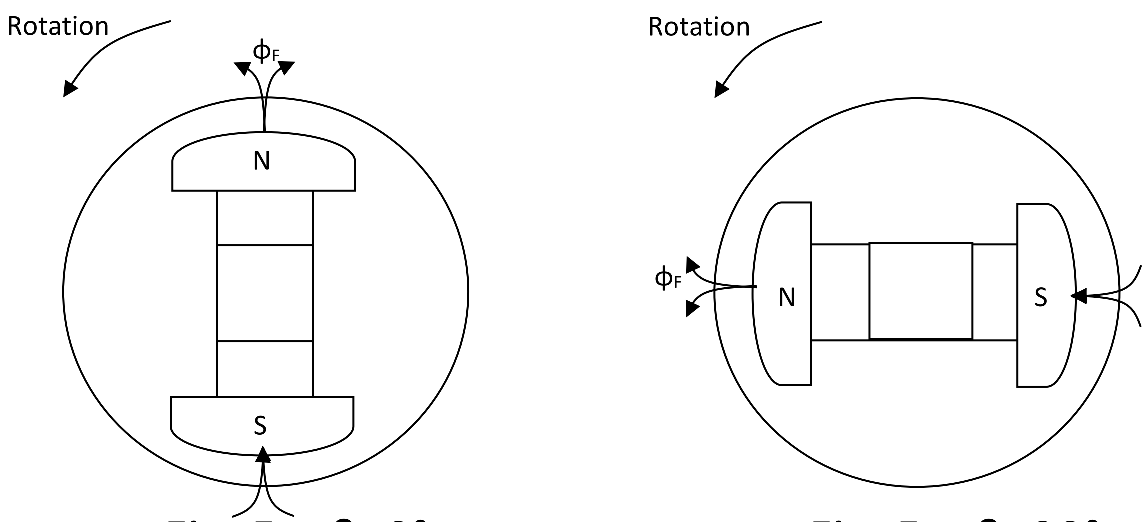

Fig. 5a: θ=0°

Fig. 5a: θ=90°

Fig. 5

Thus, since $L_{ii}=(N_i)^2/\mathcal{R}$, rotor winding self-inductances are constants, and we define the following nomenclature, consistent with eq. (4.13) in VMAF.

- d-axis field winding $L_{FF} = L_F$ (5)
- q-axis field winding $L_{GG} = L_G$ (6)
- d-axis amortissuer winding: $L_{DD} = L_D$ (7)
- q-axis amortissuer winding: $L_{QQ} = L_Q$ (8)

Note your text's convention of using only a single subscript for constant terms.

6.1b Rotor-rotor terms: mutual inductances

Recall (eq. (15) in "Preliminary Fundamentals") that:

$$L_{ij} = \frac{\lambda_i}{i_j} = \frac{N_i N_j}{\mathcal{R}_{ij}} \tag{9}$$

where $\mathcal{R}_{ij}$ is the reluctance of the path seen by λ_i in linking with coil j or the path seen by λ_j in linking with coil i (either way – it is the same path!). Again, by similar reasoning as in section 6.1a, these mutual terms are constants (i.e., independent of θ).

Therefore, we have the following:

- F (field)–D (amort): $L_{FD} = L_{DF} = M_R$ (10a)
- G (field)-Q (amort): $L_{GQ} = L_{QG} = M_Y$ (10b)

But we have four other pairs to address:

- F (field)-G (field): $L_{FG} = L_{GF} = 0$ (11a)
- F (field)-Q (amort): $L_{FQ} = L_{QF} = 0$ (11b)
- G (field)-D (amort): $L_{GD} = L_{DG} = 0$ (11c)
- D (amort)-Q (amort): $L_{DQ} = L_{QD} = 0$ (11d)

The last four pairs of windings are each in quadrature, so the flux from one winding does not link the coils of the other winding, as illustrated in Fig. 6. Therefore the above four terms are zero, as indicated in eqs (11a-11d).

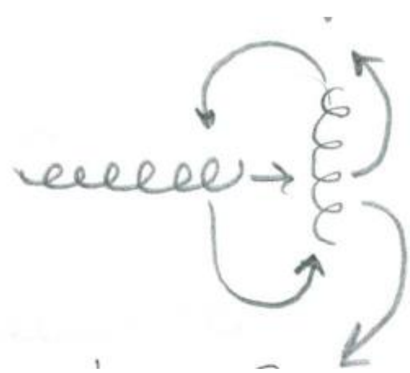

Fig. 6

6.2a Stator-stator terms: self inductances

We can derive these rigorously (see Kundur pp. 61-65) but the insight gained in this effort may not be great. Rather, we may be better served by gaining a conceptual understanding of ***four ideas***, as follows:

1. *Sinusoidal dependence of permeance on* θ: Due to saliency of the poles (and to field winding slots in a smooth rotor machine), the path reluctance seen by the stator windings depends on θ, as illustrated in Fig. 7.

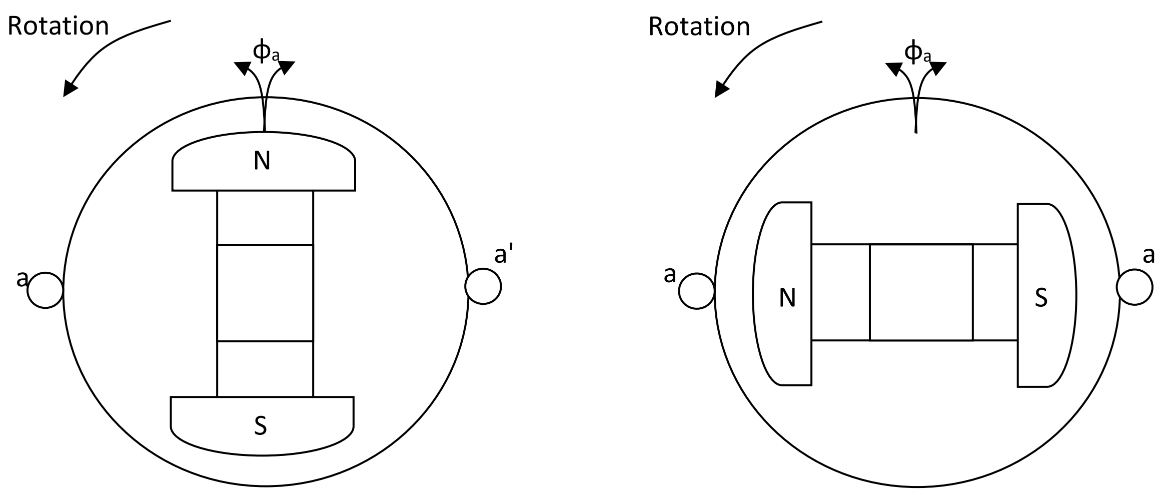

Fig. 7a: θ=0°   Fig. 7b: θ=90°

Fig. 7

From Fig. 7a, we observe that when θ=0°, the path of phase-a flux contains more iron than at any other angle 0→180°, and therefore the reluctance seen by the phase-a flux in this path is at a minimum, and permeance is at a maximum.

From Fig. 7b, we observe that when θ=90°, the path of phase-a flux contains more air that at any other angle 0→180°, and therefore the reluctance seen by the phase-a flux in this path is at a maximum, and permeance is at a minimum. This suggests a sinusoidal variation of permeance with θ.

2. *Constant permeance component*: There will be a constant permeance component due to the amount of permeance seen by the phase-a flux at any angle. This will include the iron in the middle part of the rotor (indicated by a box in Figs. 7a and 7b), the stator iron, and the air gap. Denote the corresponding component as $\mathscr{P}_s$.

3. *Double angle dependence*: Because the effects described in 1 and 2 above depend on permeance (or reluctance), and not on rotor polarity, the maximum permeance occurs twice each cycle, and not once.

Taking (1), (2), and (3) together, we may write that

$$\mathscr{P} = \mathscr{P}_s + \mathscr{P}_m \cos 2\theta \qquad (12)$$

4. *Inductance:* Because L=N^2/$\mathscr{R}$=N$^2\mathscr{P}$, the self inductance of the a-phase winding can be written as

$$L_{aa} = L_s + L_m \cos 2\theta \tag{13}$$

Likewise, we will obtain:

$$L_{bb} = L_s + L_m \cos 2(\theta - 120) \tag{14}$$

$$L_{cc} = L_s + L_m \cos 2(\theta - 240) \tag{15}$$

Equations (13), (14), (15) are denoted (4.12) in your text. Note that because θ is a function of *t*, then (13), (14), (15) imposes that stator self-inductances are functions of *t*! (Recall our discussion in "Preliminary Fundamentals".) This means in Faraday's law, e=d(Li)/dt, L is ***not*** constant.

6.2b Stator-stator terms: mutual inductances

We will identify 3 important concepts for understanding mutual terms of stator-stator inductances.

1. *Sign*:

First, we need to remind ourselves of a preliminary fact:

➔ For any circuits i and j, L_{ij} is positive if positive currents in the two circuits produce fluxes in the same direction.

With this fact, we can state important concept 1:

➔As a result of defined stator current directions, the stator-stator mutual inductance is always negative.

To see this, we can observe that the flux produced by positive currents of a and b phases are in opposite directions, as indicated in Fig. 8.

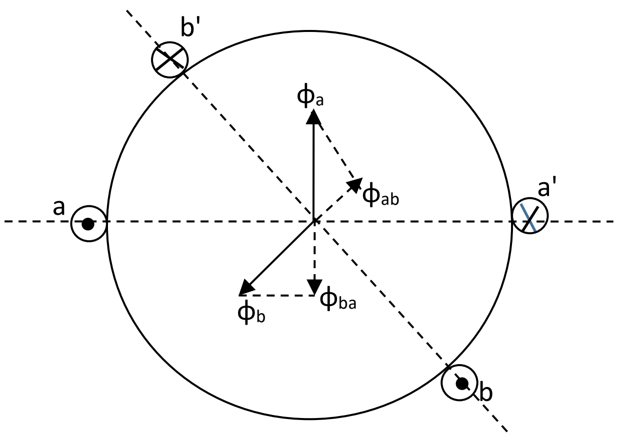

Fig. 8

"X" shows current into the plane; "●" shows current out of the plane. RHR gives flux direction.

Observe that physical location of the b-phase will cause its voltage to lag the a-phase voltage by 120°, as, for counter-clockwise (CCW) rotation, the "leading edge" of the CCW-rotating mag field is seen first by the "a" pole of the a-phase winding and then, 120° later, by the "b" pole of the b-phase winding.

Observe the following in Fig. 8:

- The component of flux from winding-a that links with winding-b, ϕ_{ab}, is 180° from ϕ_b.
- The component of flux from winding-b that links with winding-a, ϕ_{ba}, is 180° from ϕ_a.

The implication of the above 2 observations are that

➔Mutual inductance is negative.

➔This implies that mutually induced voltages are negative relative to self induced voltages.

2. *Function of position*:

2a. Maximum Permeance for Mutual Flux:

Recall conditions where, for self-flux, the amount of iron in the path yields a maximum permeance (minimum reluctance) condition (remember θ specifies rotor location).

This condition for phase-a self-flux is θ=0°.

This condition for phase-b self-flux is θ=-60°.

Therefore the condition for maximum permeance for the mutual flux between phases a and b (which maximizes flux produced from one winding that links with the other winding) is halfway between these two at θ=-30°[6].

2b. Periodicity of Permeance for Mutual Flux:

Starting at the maximum permeance condition, a rotation by 90° to θ=60° gives minimum permeance. See Fig. 9.

[6] The thinking here is that if θ=0° results in max flux (min reluctance path) from a-current seen by a-winding, and if θ=-60° results in max flux (min reluctance path) from b-current seen by b-winding, then halfway between the two will result in max flux (min reluctance path) from a-(b-) current seen by b-(a-) winding.

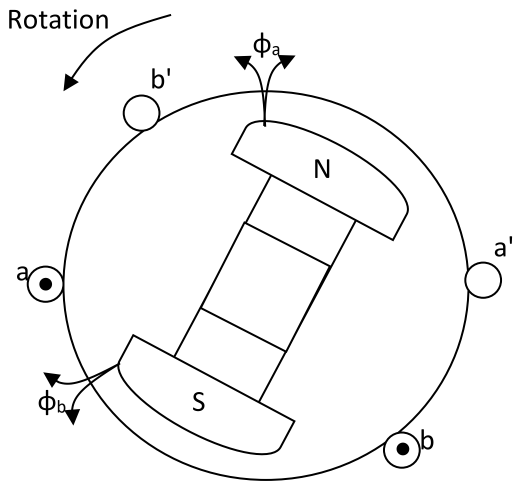

Fig. 9a: θ=-30°

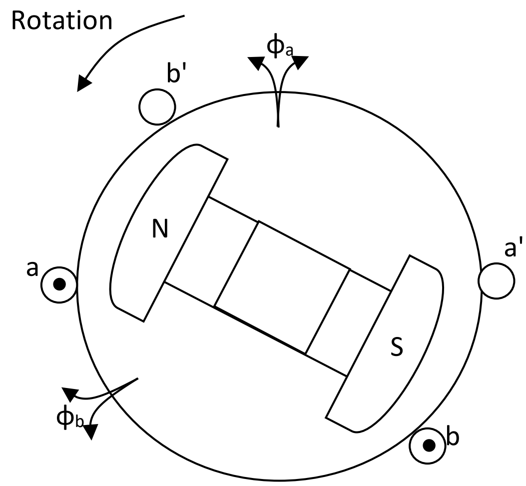

Fig. 9b: θ=60°

Fig. 9

Starting at the maximum permeance condition, a rotation by 180° to θ=150° gives maximum permeance again.

The implication of these observations are that permeance, and therefore inductance, is a sinusoidal function of 2(θ+30°).

3. *Constant term*:

There is an amount of permeance that is constant, independent of rotor position. Like before, this is composed of the stator iron, the air gap, and the inner part of the rotor. We will denote the corresponding inductance as M_S.

From above 1, 2, and 3, we express mutual inductance between the a- and b-phases as

$$L_{ab} = -\left[M_s + L'_{ab}\cos 2(\theta + 30°)\right] \quad (16)$$

One last comment: The amplitude of the permeance variation for the mutual flux is the same as the amplitude of the permeance variation for the self-flux, therefore $L'_{ab}=L_m$.

And so the three mutual expressions we need are

$$L_{ab} = -\left[M_s + L_m\cos 2(\theta + 30°)\right] \quad (17)$$

$$L_{bc} = -\left[M_s + L_m\cos 2(\theta - 90°)\right] \quad (18)$$

$$L_{ca} = -\left[M_s + L_m\cos 2(\theta + 150°)\right] \quad (19)$$

We see that stator-stator mutuals, like stator self inductances, are functions of time!

6.3 Stator-rotor terms

These are all mutual inductances. There are four windings on the rotor (F, G, D, and Q) and three windings on the stator (a, b, c phases). Therefore there are 12 mutual terms in all.

Central idea: Recall that for stator-stator mutuals,

- windings are locationally fixed, and…
- the path of mutual flux is fixed, but…

- the rotor moves within the path of mutual flux and causes the iron in the path to vary, and for this reason, the path permeance varies.

Now, in this case, for stator-rotor terms (all mutuals),

- the rotor winding locations vary, the stator winding locations are fixed, and so…
- the path of mutual flux varies, and so…
- the iron in the path of mutual flux varies, and for this reason, the path permeance varies.

To illustrate, consider the permeance between the a-phase winding and the main field winding (F).

- When the main field winding and the stator winding are aligned, as in Fig. 10a, the permeance is maximum, and therefore inductance is maximum.

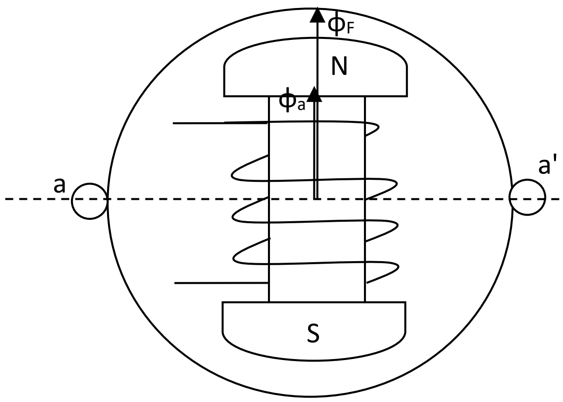

Fig. 10a

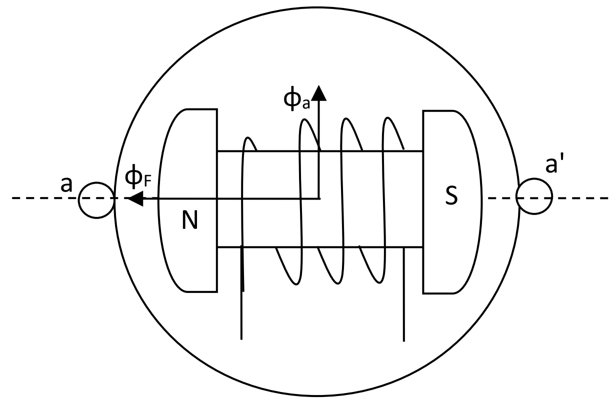

Fig. 10b

- When the main field winding and the a-phase stator winding are 90° apart, as in Fig. 10b, there is no linkage at all, and inductance is zero.
- When the rotor winding and the a-phase stator winding are 180° apart, as in Fig. 11, the permeance is again maximum, but now polarity is reversed.

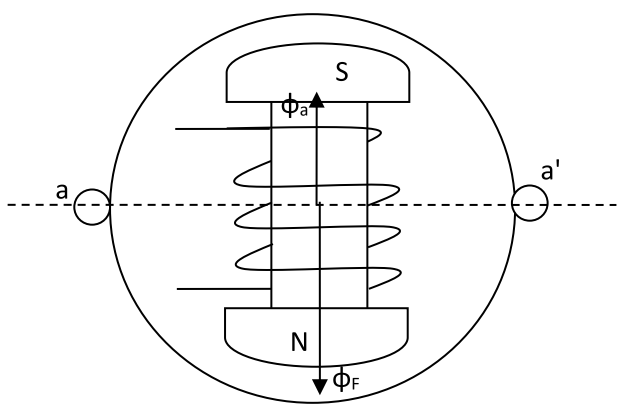

Fig. 11

This discussion results in a conclusion that the mutual inductance between a-phase winding and the main field winding should have the form:

$$L_{aF} = M_F \cos\theta \tag{20a}$$

The d-axis damper (amortissuer) winding is positioned concentric with the main field winding, both producing flux along the d-axis. Therefore, the reasoning about the mutual inductance between the a-phase winding and the d-axis damper winding will be similar to the reasoning about the mutual inductance between the a-phase winding and the main field (F) winding, leading to

$$L_{aD} = M_D \cos\theta \tag{21a}$$

Now consider the mutuals between the a-phase winding and the windings on the q-axis, i.e., the G-winding and the Q damper (amortissuer) winding.

The only difference in reasoning about these mutuals and the mutuals between the a-phase winding and the windings on the d-axis (the F-winding and the D damper winding) is that the windings on the q-axis are 90° behind the windings on the d-axis. Therefore, whereas the a-phase/d-axis mutuals were cosine functions, these mutuals will be sine functions, i.e.,

$$L_{aQ} = M_Q \sin\theta \tag{22a}$$

$$L_{aG} = M_G \sin\theta \tag{23a}$$

Summarizing stator-rotor terms for all three phases, we obtain the equations on the next page.

$$L_{aF} = M_F \cos\theta \quad (20a)$$

$$L_{bF} = M_F \cos(\theta - 120^\circ) \quad (20b)$$

$$L_{cF} = M_F \cos(\theta - 240^\circ) \quad (20c)$$

$$L_{aD} = M_D \cos\theta \quad (21a)$$

$$L_{bD} = M_D \cos(\theta - 120^\circ) \quad (21b)$$

$$L_{cD} = M_D \cos(\theta - 240^\circ) \quad (21c)$$

$$L_{aQ} = M_Q \sin\theta \quad (22a)$$

$$L_{bQ} = M_Q \sin(\theta - 120^\circ) \quad (22b)$$

$$L_{bQ} = M_Q \sin(\theta - 240^\circ) \quad (22c)$$

$$L_{aG} = M_G \sin\theta \quad (23a)$$

$$L_{bG} = M_G \sin(\theta - 120^\circ) \quad (23b)$$

$$L_{bG} = M_G \sin(\theta - 240^\circ) \quad (23c)$$

7.0 Summary

Summarizing all of our needed expressions:

Rotor-rotor self terms: 5, 6, 7, 8

Rotor-rotor mutuals: 10a, 10b, 11a, 11b, 11c, 11d

Stator-stator self terms: 13, 14, 15

Stator-stator mutuals: 17, 18, 19

Rotor-stator mutuals: 20a, 20b, 20c, 21a, 21b, 21c, 22a, 22b, 22c, 23a, 23b, 23c

Counting the above expressions, we see that we have 28. But let's look back at our original flux linkage relation (3):

$$\begin{array}{l} \text{stator}\left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. \\ \text{rotor}\left\{ \begin{array}{l} \\ \\ \\ \\ \end{array} \right. \end{array} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_G \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \left[\begin{array}{ccc:cccc} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aG} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bG} & L_{bD} & L_{bQ} \\ L_{ca} & L_{bc} & L_{cc} & L_{cF} & L_{cG} & L_{cD} & L_{cQ} \\ \hdashline L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FG} & L_{FD} & L_{FQ} \\ L_{Ga} & L_{Gb} & L_{Gc} & L_{GF} & L_{GG} & L_{GD} & L_{GQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DG} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QG} & L_{QD} & L_{QQ} \end{array} \right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} \quad (3)$$

We have 49 terms! Where are the other 21 expressions?

Note because $L_{ij}=L_{ji}$, the inductance matrix will be symmetric. Of the 49 terms, 7 are diagonal. The other 42 terms are off-diagonal and are repeated twice. So we are "missing" the 21 expressions corresponding to the off-diagonal elements for which we did not provide expressions. But we do not need to, since those "missing" equations for the off-diagonal elements L_{ij} are exactly the same as the expressions for the off-diagonal elements L_{ji}.

We will look closely at this matrix in the next set of notes.