Simulation of Synchronous Machines

This chapter covers:

(A) Sections 5.2-5.7: Determination of initial conditions

(B) Section 5.8: Determination of machine parameters from manufacturers' data

(C) Sections 5.9: Digital simulation of synchronous machines

We will only cover (A). This breaks down into:

- Section 5.2: Steady-state and phasor diagrams
- Section 5.3: Machine connected to an infinite bus through a line
- Section 5.4: Machine connected to an infinite bus with local load at machine terminal
- Section 5.5: Determining steady-state conditions
- Section 5.6: Examples
- Section 5.7: Initial conditions for a multimachine system

Of the above, we will concentrate on Sections 5.2, 5.5, and 5.7, but I encourage you to read all of these sections 5.2-5.7. I will briefly touch on Sections 5.8 and 5.9.

The basic problem is motivated by the following fact:

Simulation of the transient response of any dynamical system represented by state variables requires initial conditions for those state variables.

So what are our state variables?

- In general, it depends on the machine model.
- However, there are two state variables that are common to all machine models: δ, ω.

The initial condition for ω is easy: $\omega(t=0) = 1$.

But what about the initial condition for δ?

What is δ? See page 93, Section 4.2, which says: "At t=0 the phasor V is located at the axis of phase a, i.e., at the reference axis in Fig. 4.1. The q-axis is located at an angle δ, and the d-axis is located at $\theta=\delta+\pi/2$. At t>0, the reference axis is located at an angle $\omega_R t$ with respect to the axis of phase a. The d-axis of the rotor is therefore located at $\theta=\omega_R t+\delta+\pi/2$ where ω_R is the rated (synchronous) angular frequency in rad/sec and δ is the synchronous torque angle in electrical radians." (Note $\omega_R=\omega_{Re}$ here).

We have emphasized this point before in our class; see notes called "Torque Equation," pp. 9-11. You should go back and re-read that part.

The below picture, Fig. 1, illustrates the relation between θ and δ for t=0, i.e., at $\theta=\delta+\pi/2$

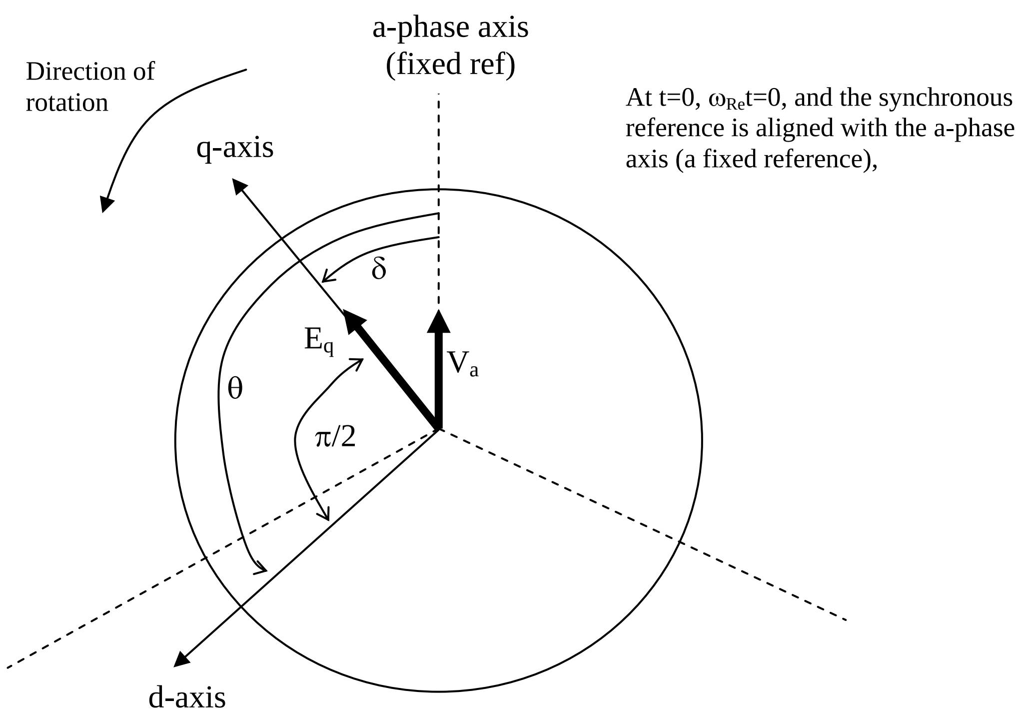


Fig. 1

So we see that δ is the angle between the *synchronous reference* and the q-axis of the machine.

So what is the reference? It is usually taken as the terminal bus voltage for one machine in the network. In the above picture, V_a identifies the reference.

So, the problem may be described by the following.

- We are about to perform a time domain simulation of a multi-machine system where each machine is represented using one of the Chapter 4 machine models. We will be simulating the electro-mechanical response of the power system to some identified disturbance.
- We have the corresponding power flow solved case to initialize the simulation. This power flow solution provides
    - V_a, the bus voltage (i.e., at the machine terminals) for all generator buses in the network, magnitude and angle, where the angle is given relative to the reference.
    - I_a, the bus current injection, magnitude and angle.

Since δ locates the q-axis for the machine, if we can find the angle of a quantity that lies along the q-axis, this angle will be δ.

What steady-state quantity lies along the q-axis?

This is the stator equivalent pu voltage corresponding to the field current i_F in pu. It is denoted by E in your text, but other books often denote it as E_q, to emphasize that it lies along the q-axis (and some books use E_I). It lies on the q-axis because it is entirely due to the field flux (see pp. 6-7 of "Simplified Models"). It is also equivalent to E_{FD} (see Ex 5.1, p. 175, eqts. (4.209 and 5.8).

➔<u>VERY IMPORTANT TO REMEMBER THAT E LIES ON THE q-AXIS!!!</u>

From Section 4.7.4, we recall that $\sqrt{3}E = \omega k M_F i_F$.

So our problem is now as follows:

Given V_a and I_a, find E.

Recall eq. 4.74 which was derived in the notes on per-unitization.

$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ \hline v_q \\ 0 \\ 0 \end{bmatrix} = -\left[\begin{array}{ccc|ccc} r & 0 & 0 & \omega L_q & \omega kM_G & \omega kM_Q \\ 0 & r_F & 0 & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 & 0 \\ \hline -\omega L_d & -\omega kM_F & -\omega kM_D & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{array}\right] \begin{bmatrix} i_d \\ i_F \\ i_D \\ \hline i_q \\ i_G \\ i_Q \end{bmatrix} - \left[\begin{array}{ccc|ccc} L_d & kM_F & kM_D & 0 & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & L_q & kM_G & kM_Q \\ 0 & 0 & 0 & kM_G & L_G & M_Y \\ 0 & 0 & 0 & kM_Q & M_Y & L_Q \end{array}\right] \begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_D \\ \hline \dot{i}_q \\ \dot{i}_G \\ \dot{i}_Q \end{bmatrix} \qquad (4.74)$$

Recall also that 4.74 is correct independent of whether units are MKS or per-unit. We will assume that we are in MKS.

We can obtain from 4.74 the steady-state relations between the d-q voltages and currents, by setting

- All derivatives to zero.
- $i_D=i_Q=i_G=0$

(because we are analyzing steady-state conditions).

The resulting equations are:

$$v_d = -ri_d - \omega L_q i_q = -ri_d - x_q i_q \qquad (*)$$

$$v_q = -ri_q + \omega L_d i_d + \omega kM_F i_F = -ri_q + x_d i_d + \sqrt{3}E \qquad (**)$$

Note the replacement of ωL_q and ωL_d with x_q and x_d, respectively.

From Park's relation $\underline{v}_{abc}=\underline{P}^{-1}\underline{v}_{0dq}$, with $v_0=0$, which is

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \cos\theta & \sin\theta \\ \frac{1}{\sqrt{2}} & \cos(\theta - 120) & \sin(\theta - 120) \\ \frac{1}{\sqrt{2}} & \cos(\theta + 120) & \sin(\theta + 120) \end{bmatrix} \begin{bmatrix} 0 \\ v_d \\ v_q \end{bmatrix}$$

This provides that

$$v_a = \sqrt{\frac{2}{3}} \left[v_d \cos\theta + v_q \sin\theta \right]$$

where θ is the angle of the D-axis given by $\theta = \omega_{\mathrm{Re}} + \delta + \pi/2$.

Substituting v_d, v_q given as in (*) and (**), we obtain:

$$v_a = \sqrt{\frac{2}{3}} \left[\left(-ri_d - x_q i_q \right) \cos\left(\omega_{\mathrm{Re}} t + \delta + \pi/2 \right) + \left(-ri_q + x_d i_d + \sqrt{3}E \right) \sin\left(\omega_{\mathrm{Re}} t + \delta + \pi/2 \right) \right]$$

Noting that the sin term in the above equation can be written as: $\sin\left(\omega_{\mathrm{Re}} t + \delta + \pi/2 \right) = \cos\left(\omega_{\mathrm{Re}} t + \delta \right)$, we have that:

$$v_a = \sqrt{\frac{2}{3}} \left[\left(-ri_d - x_q i_q \right) \cos\left(\omega_{\mathrm{Re}} t + \delta + \pi/2 \right) + \left(-ri_q + x_d i_d + \sqrt{3}E \right) \cos\left(\omega_{\mathrm{Re}} t + \delta \right) \right]$$

Now the above expression is the instantaneous expression, so that its magnitude is a peak quantity. To obtain RMS quantities, we need to divide by √2, resulting in:

$$V_a = \frac{1}{\sqrt{3}} \left[\left(-ri_d - x_q i_q \right) \cos\left(\omega_{\mathrm{Re}} t + \delta + \pi/2 \right) + \left(-ri_q + x_d i_d + \sqrt{3}E \right) \cos\left(\omega_{\mathrm{Re}} t + \delta \right) \right]$$

Converting to phasor notation, we have:

$$\bar{V}_a = \frac{\left(-ri_d - x_q i_q \right)}{\sqrt{3}} \angle\left(\delta + \pi/2 \right) + \frac{-ri_q + x_d i_d}{\sqrt{3}} \angle\left(\delta \right) + E\angle\left(\delta \right)$$

Combining terms in r yields:

$$\overline{V}_a = -r\left[\frac{i_d}{\sqrt{3}}\angle(\delta+\pi/2)+\frac{i_q}{\sqrt{3}}\angle(\delta)\right]-x_q\frac{i_q}{\sqrt{3}}\angle(\delta+\pi/2)+x_d\frac{i_d}{\sqrt{3}}\angle(\delta)+E\angle(\delta) \quad (5.9)$$

Recognizing that $\angle(\delta+\pi/2)= j\angle\delta$, and using the RMS equivalent d- and q-axis currents reflected to the stator as:

$$I_d = \frac{i_d}{\sqrt{3}} \qquad I_q = \frac{i_q}{\sqrt{3}}$$

See "SimplifiedModels" pp. 9-10 for this, where we wrote, "we conclude that the pu value of any d or q axis quantity is numerically equal to $\sqrt{3}$ times the pu quantity on the stator side."

we have that

$$\overline{V}_a = -r\left[jI_d\angle\delta + I_q\angle\delta\right] - jx_qI_q\angle\delta + x_dI_d\angle\delta + E\angle(\delta) \qquad (\#)$$

The quantity $jI_d\angle\delta + I_q\angle\delta$ is the stator current phasor decomposed into the d- and q-axes, i.e.,

$$\bar{I}_a = jI_d\angle\delta + I_q\angle\delta = \bar{I}_d + \bar{I}_q \qquad (\#\#)$$

where the j in front of the I_d term provides the necessary 90 degree rotation ahead of the q-axis for the d-axis component of the current.

Thus using (##) in equation (#), the a-phase voltage phasor, becomes:

$$\overline{V}_a = -r\bar{I}_a - jx_qI_q\angle\delta + x_dI_d\angle\delta + E\angle\delta$$

Solving for $E\angle\delta$, we have:

$$\overline{E} = E\angle\delta = \overline{V}_a + r\bar{I}_a + jx_qI_q\angle\delta - x_dI_d\angle\delta \qquad (5.13)$$

Error in text in (5.13) in that, in text, the second "+" sign is erroneously an "=" sign.

Now let's focus on the last two terms of the above equation.

Clearly, $I_q\angle\delta = \bar{I}_q$. But what about $\bar{I}_d$?

Recall (##): $\bar{I}_a = jI_d\angle\delta + I_q\angle\delta = \bar{I}_d + \bar{I}_q$ (##)

From (##), we see that $\bar{I}_d = jI_d\angle\delta$

$\rightarrow \frac{1}{j}\bar{I}_d = I_d\angle\delta \rightarrow -j\bar{I}_d = I_d\angle\delta \rightarrow j\bar{I}_d = -I_d\angle\delta$

Therefore (5.13) becomes:

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q I_q\angle\delta - x_d I_d\angle\delta \qquad (5.13)$$

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_q + jx_d\bar{I}_d \qquad (5.14)$$

Error in VMAF in (5.14) in that, in VMAF, the first "+" sign is erroneously an "=" sign.

Now, what has all of this work bought us?

If we have, from the power flow solution, $\bar{V}_a$ and $\bar{I}_a$, we can compute the first part of (5.14).

However, we do not yet know $\bar{I}_d$ and $\bar{I}_q$, which are required by the second part of (5.14)…because we do not know the location of the q-axis! That is, we do not yet know the angle δ.

What to do?

Here is a trick... Add and subtract $jx_q\bar{I}_d$ to (5.14) to obtain:

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_q \underbrace{+ jx_q\bar{I}_d}_{Added} \underbrace{- jx_q\bar{I}_d}_{subtracted} + jx_d\bar{I}_d$$

Collect terms in (jx_q) and in (jI_d) to yield:

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right) + j\bar{I}_d\left(x_d - x_q\right) \qquad (*)$$

To see the significance of eqt. (*), let's do two exercises in drawing phasor diagrams.

These exercises will use eqs. (5.14) and (*) as "instruction manuals" for drawing the phasor diagrams.

In both exercises, we will use two facts:

1. We know the angle of $\bar{V}_a$ so that it can be our reference angle, and we can assume that this reference is 0 degrees.
2. The stator-side voltage $\bar{E} = E\angle\delta$ must lie on the q-axis (see bottom of p. 3 of these notes), which says:

➔ <u>VERY IMPORTANT TO REMEMBER THAT E LIES ON THE q-AXIS!!!</u>

Exercise 1: Use eq. (5.14). Let’s assume that we know the phasors $\bar{I}_d$ and $\bar{I}_q$ (an important assumption!!!).

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_q + jx_d\bar{I}_d \qquad (5.14)$$

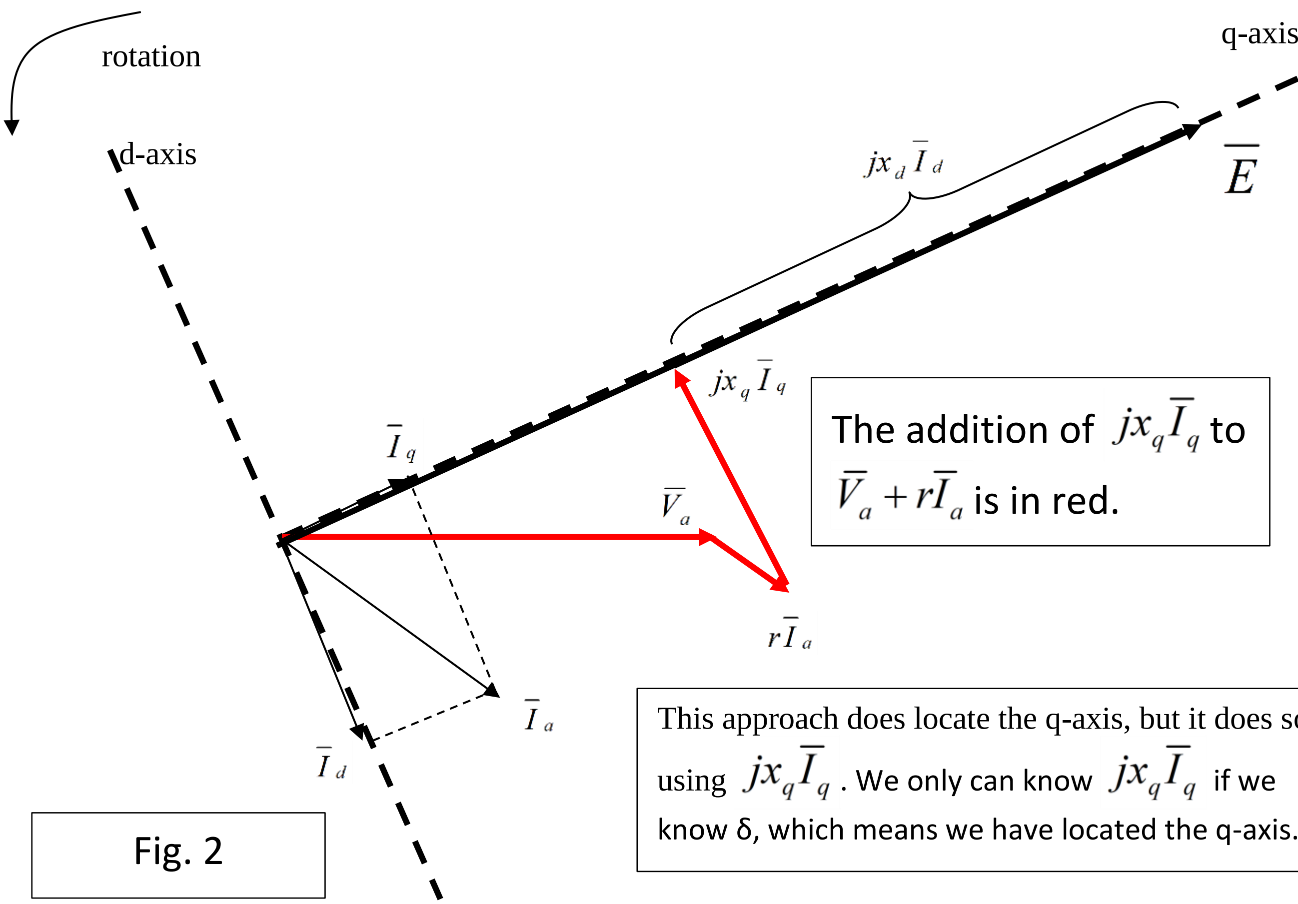
Error in text in (5.14) in that, in text, the first “+” sign is erroneously an “=” sign.

<u>Observe</u>:

(1) We wrote on the top of p. 7 (from (##) on p. 6) that $-j\bar{I}_d = I_d\angle\delta$, which implies that $\bar{I}_d = -jI_d\angle\delta$. Since $I_d\angle\delta$ is necessarily on the q-axis, then $\bar{I}_d$ must be 90° behind it, i.e., on the negative d-axis.

(2) The addition of $jx_q\bar{I}_q$ to $\bar{V}_a + r\bar{I}_a$ must locate to the q-axis, since $\bar{E} = E\angle\delta$ must be on the q-axis and $jx_d\bar{I}_d$ is already on the q-axis and therefore its addition can offer no “directional correction.”

Summary of these 2 points:
1. $\bar{I}_d = -jI_d\angle\delta$ is on the d-axis.
2. addition of $jx_q\bar{I}_q$ to $\bar{V}_a + r\bar{I}_a$ must locate to the q-axis.



The addition of $jx_q\bar{I}_q$ to $\bar{V}_a + r\bar{I}_a$ is in red.

This approach does locate the q-axis, but it does so using $jx_q\bar{I}_q$. We only can know $jx_q\bar{I}_q$ if we know δ, which means we have located the q-axis.

Fig. 2

Exercise 2: Use eq. (*). Again, assume that we know the phasors $\overline{I}_d$ and $\overline{I}_q$.

$$\overline{E} = E\angle\delta = \overline{V}_a + r\overline{I}_a + jx_q\left(\overline{I}_q + \overline{I}_d\right) + j\overline{I}_d\left(x_d - x_q\right) \qquad (*)$$

Observe from (##), p. 6, that $\overline{I}_a = \overline{I}_d + \overline{I}_q$ and so the term $jx_q\left(\overline{I}_q + \overline{I}_d\right)$ must be rotated 90° from $\overline{I}_a$, and (similar to reasoning of ex 1), the addition of $jx_q\left(\overline{I}_q + \overline{I}_d\right)$ to $\overline{V}_a + r\overline{I}_a$ must locate to the q-axis, since $\overline{E} = E\angle\delta$ must be on the q-axis and $j\overline{I}_d\left(x_d - x_q\right)$ is already on the q-axis, and so its addition can offer no "directional correction."

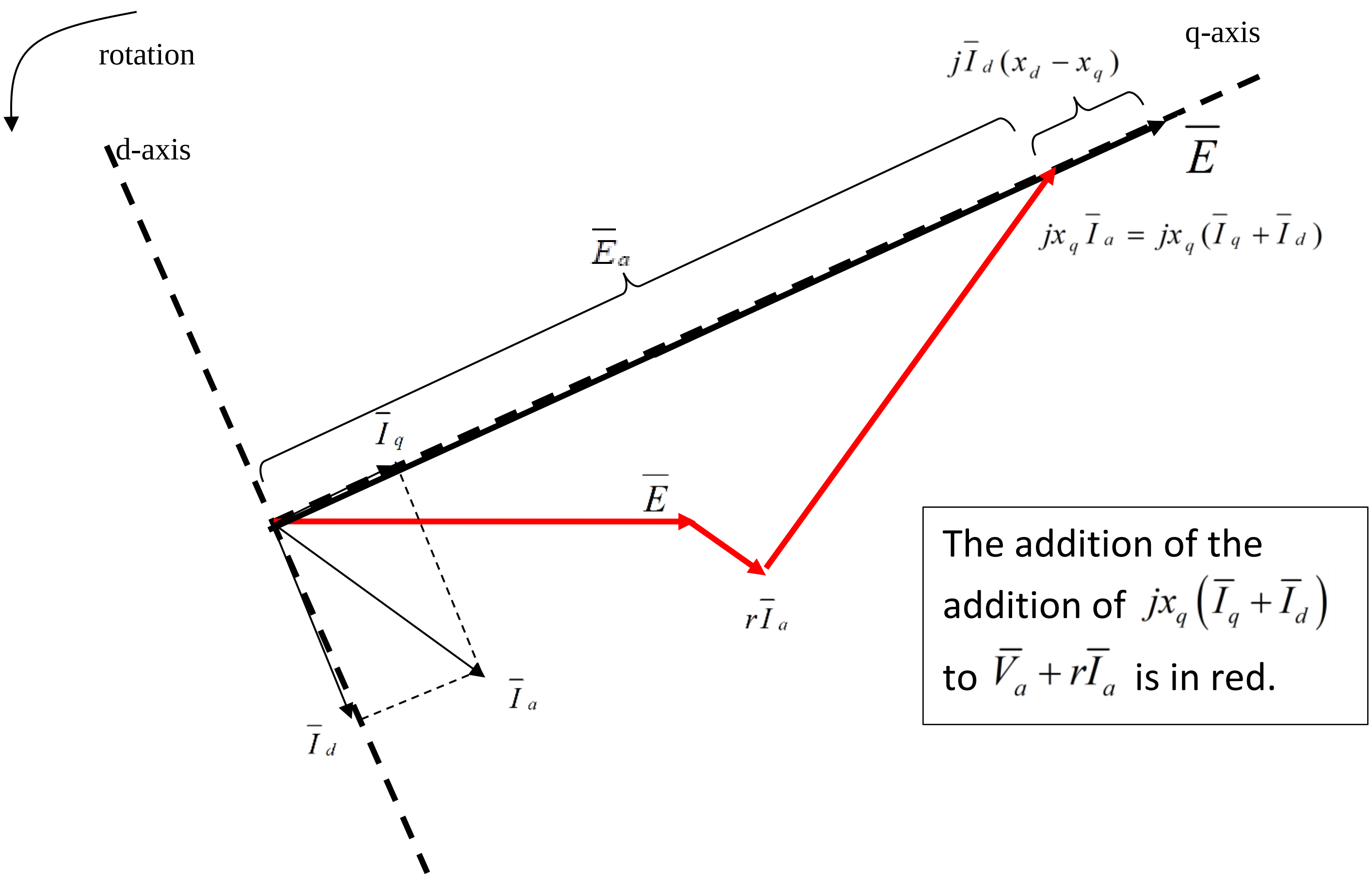


Fig. 3

Locating δ

Note that in exercise 2, we can express eq. (*) as

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right) + j\bar{I}_d\left(x_d - x_q\right) \qquad (*)$$

➔

$$\bar{E} = E\angle\delta = \underbrace{\bar{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right)}_{\bar{E}_a} + j\bar{I}_d\left(x_d - x_q\right) = \bar{E}_a + j\bar{I}_d\left(x_d - x_q\right)$$

where the first part of eq. (*) is given by:

$$\bar{E}_a = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_a$$

where $\bar{I}_a = \bar{I}_d + \bar{I}_q$.

If $\bar{E}$ is on the q-axis (and we have already proven that it is), then $\bar{E}_a$ must also be on the q-axis because the only difference between them is $j\bar{I}_d\left(x_d - x_q\right)$ which is a component along the q-axis (if a vector on the q-axis is added to another vector on the q-axis, the resultant vector must also be on the q-axis).

The important point here is that $\bar{E}_a$ requires only $\bar{V}_a$ and $\bar{I}_a$ to compute it, which are known from the power flow solution! So we may locate the q-axis using equation (*). Note that, in exercise 1, we were required to first know $\bar{I}_d$ and $\bar{I}_q$ individually (which cannot be known without knowing δ).

Computing $\bar{I}_d$

In using eq. (*), we need $\bar{V}_a$ and $\bar{I}_a$ to obtain $\bar{E}_a$ and thus δ. Then we need $\bar{I}_d$ to compute $j\bar{I}_d\left(x_d - x_q\right)$, which is done as follows….

Define the familiar power factor angle as ϕ, the angle by which $\bar{I}_a$ lags $\bar{V}_a$ (see p. 172 in VMAF), or the angle by which $\bar{V}_a$ leads $\bar{I}_a$. The power factor angle is greater than zero for lagging current.

Let's also define β as the angle of $\overline{V}_a$, relative to the reference (necessary in a multi-machine system). Then it is the case that

$$\angle \bar{I}_a = \beta - \phi$$

The phasor diagram below illustrates the situation (see Fig. 5.1, p. 167, in VMAF):

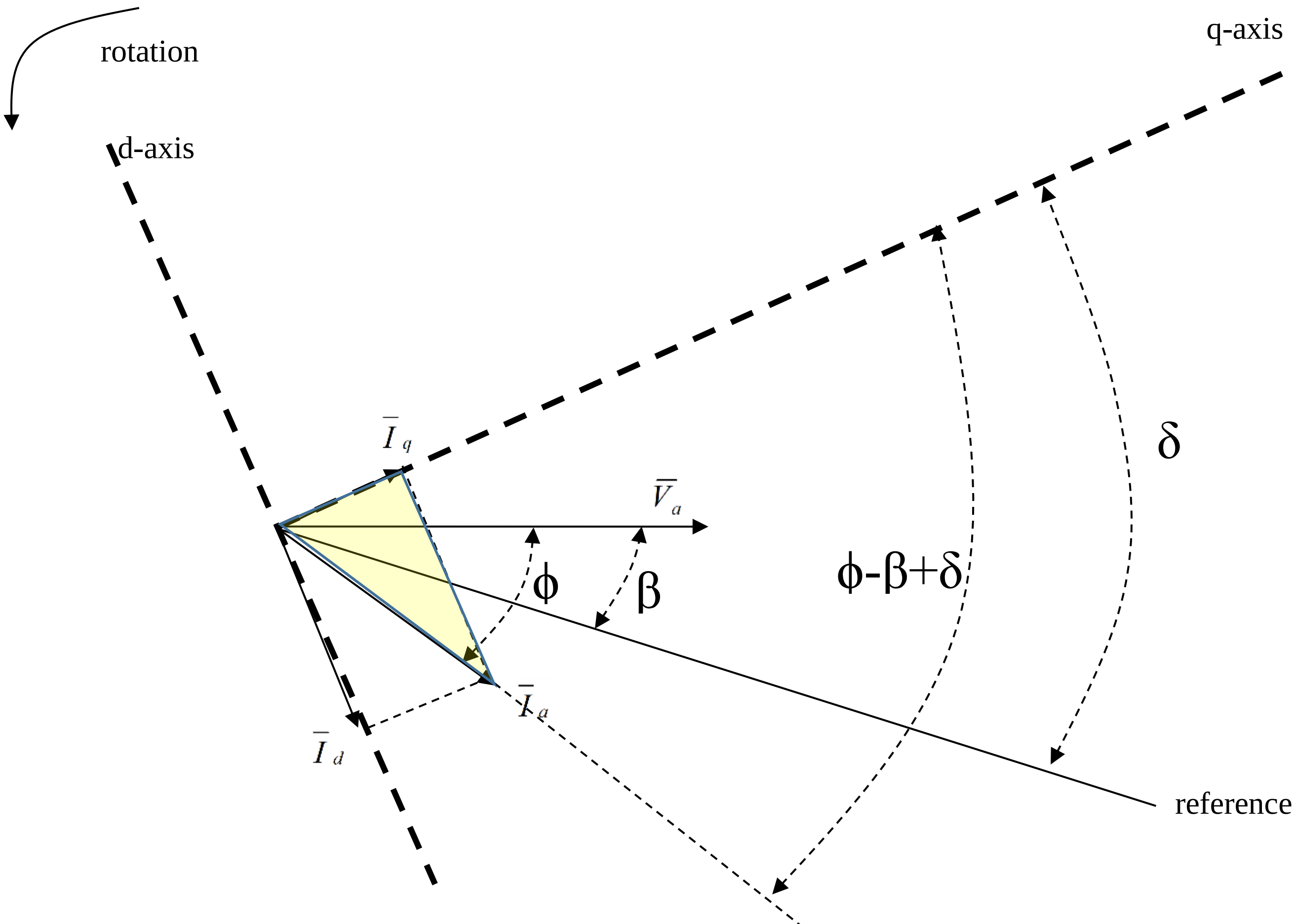


Fig. 4

From the phasor diagram, we can observe that

$$\angle \bar{I}_d = \delta - 90°$$

$$\left|\bar{I}_d\right| = \left|\bar{I}_a\right| \sin\left(\phi - \beta + \delta\right) \quad \text{(yellow triangle)}$$

The above relations provide us with $\bar{I}_d$, from which we may compute $\bar{E} = E\angle\delta$ from

$$\bar{E} = E\angle\delta = \bar{E}_a + j\bar{I}_d\left(x_d - x_q\right)$$

Some remarks on this….

<u>Remark 1</u>: $I_d = -\left|\bar{I}_d\right|$

Note that eq. 5.44 in your text indicates that

$$I_d = -I_a \sin(\phi - \beta + \delta)$$

which is different than the expression given above (p. 12) for $\left|\bar{I}_d\right|$, $\left|\bar{I}_d\right| = \left|\bar{I}_a\right|\sin(\varphi - \beta + \delta)$, as the text is assigning a sign to the magnitude of $\bar{I}_d$. Why is this?

We have said that $\bar{I}_a = \bar{I}_d + \bar{I}_q$ where:

$$\bar{I}_d = \left|\bar{I}_d\right|\angle\delta - 90$$

Which is right?

$$\bar{I}_q = \left|\bar{I}_q\right|\angle\delta$$

$$I_d = \frac{i_d}{\sqrt{3}} \qquad\qquad I_q = \frac{i_q}{\sqrt{3}}$$

Note that the text indicates, in eq. 5.12, that $\bar{I}_a = \left(I_q + jI_d\right)e^{j\delta}$, which we can write as

$$\bar{I}_a = \left(I_q + jI_d\right)e^{j\delta} = I_q\angle\delta + jI_d\angle\delta = I_q\angle\delta + I_d\angle\delta + 90 = \bar{I}_q + \bar{I}_d$$

But we have said that $\bar{I}_d = |I_d| \angle \delta - 90$. The implication is that $I_d = -|\bar{I}_d|$, which, if true, proves the equivalence of $I_d \angle \delta + 90$ and $|\bar{I}_d| \angle \delta - 90$, as follows:

$$\bar{I}_d = I_d \angle \delta + 90 = -|\bar{I}_d| \angle \delta + 90$$
$$= |\bar{I}_d| \angle -180 \angle \delta + 90 = |\bar{I}_d| \angle \delta + 90 - 180 = |\bar{I}_d| \angle \delta - 90$$

And so $\bar{I}_d$ can either be written as $I_d \angle \delta + 90$ or $|\bar{I}_d| \angle \delta - 90$ where $I_d = -|\bar{I}_d|$

Remark 2: Phasors

$\bar{I}_a$ is a *phasor* getting its rotation from the sinusoidal variation of the alternating currents.
On the other hand, I_d and I_q are equivalent values of i_d and i_q, respectively, and i_d and i_q are direct currents. So what are $\bar{I}_d$ and $\bar{I}_q$?

They are *phasors*, but their rotation comes from the rotor motion, not from the current variation.

Remark 3: Saliency

Recall eq. (*), where we found that

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right) + j\bar{I}_d\left(x_d - x_q\right) \qquad (*)$$

and with $\bar{I}_a = \bar{I}_d + \bar{I}_q$, we have that:

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_a\right) + j\bar{I}_d\left(x_d - x_q\right)$$

An equivalent circuit for this appears in Fig. 5.

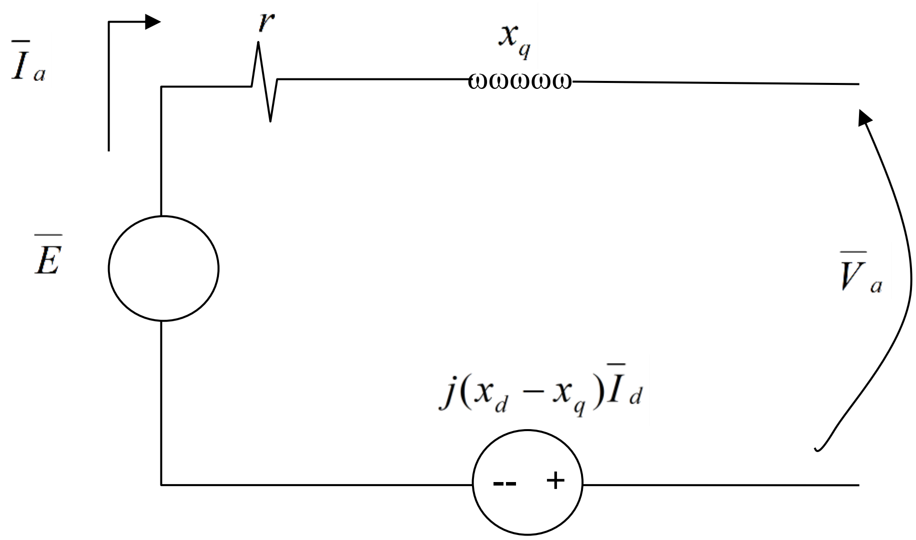


Fig. 5

Here, x_d and x_q are the synchronous machine reactances in the d- and q- axes. For a salient-pole machine, $x_d >> x_q$, and the lower voltage source is significant. For a round-rotor machine, $x_d \approx x_q$, and the lower voltage source is insignificant. We sometimes call the lower voltage the "**voltage due to saliency**."

Recall that for round rotor machines, the equivalent circuit for steady-state analysis is as in Fig. 6.

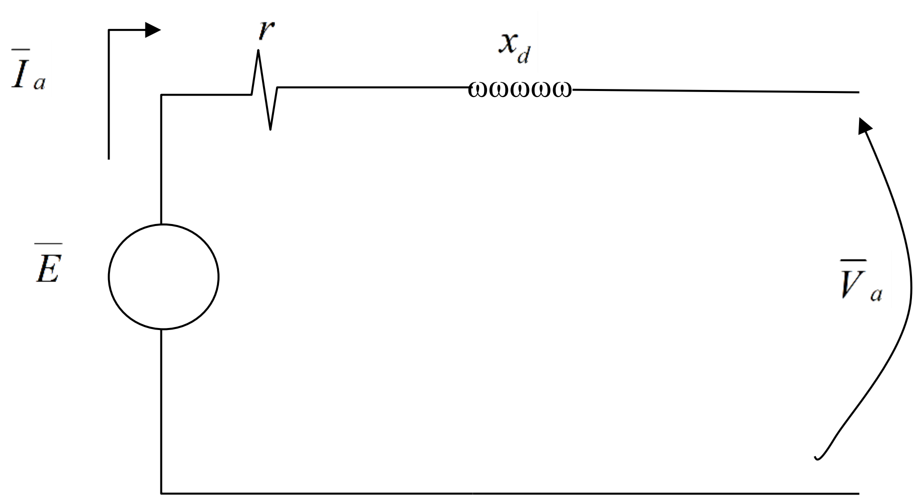


Fig. 6

The above circuit is likely quite familiar based on an undergraduate course in electromechanical energy conversion.

When r=0, we may derive from the circuit for the round-rotor machine the two familiar per-unit expressions (see appendix A):

$$P_{out} = \frac{|\bar{E}||\bar{V}_a|}{x_d}\sin(\delta - \beta)$$

$$Q_{out} = \frac{|\bar{E}||\bar{V}_a|}{x_d}\cos(\delta - \beta) - \frac{|\bar{V}_a|^2}{x_d}$$

If $\bar{V}_a$ is the reference, then β=0 in the above relations.
But what about the case of the salient-pole machine? The voltage due to saliency should change these expressions. Let's find out….
Let r=0 as in the round-rotor case, and return to eq. (5.14), which was eq. (*) before we performed the "add and subtract" trick. This equation was:

$$\bar{E} = E\angle\delta = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_q + jx_d\bar{I}_d \qquad (5.14)$$

To simplify the development, let

$$E = |E|\angle 0^\circ \qquad \bar{V}_a = |V_a|\angle -\delta$$

Thus we can write that:

$$\bar{V}_a = |V_a|e^{-j\delta} = |V_a|\cos\delta - j|V_a|\sin\delta$$

We want $S_{out} = \bar{V}_a\bar{I}_a^{\,*} = \bar{V}_a\left(\bar{I}_d + \bar{I}_q\right)^*$

We can obtain $\bar{I}_d$ and $\bar{I}_q$ from inspecting the phasor diagram resulting from eq. (5.14) (use 5.14 as "instruction manual" with $\bar{E}$ as the reference and r_a=0):

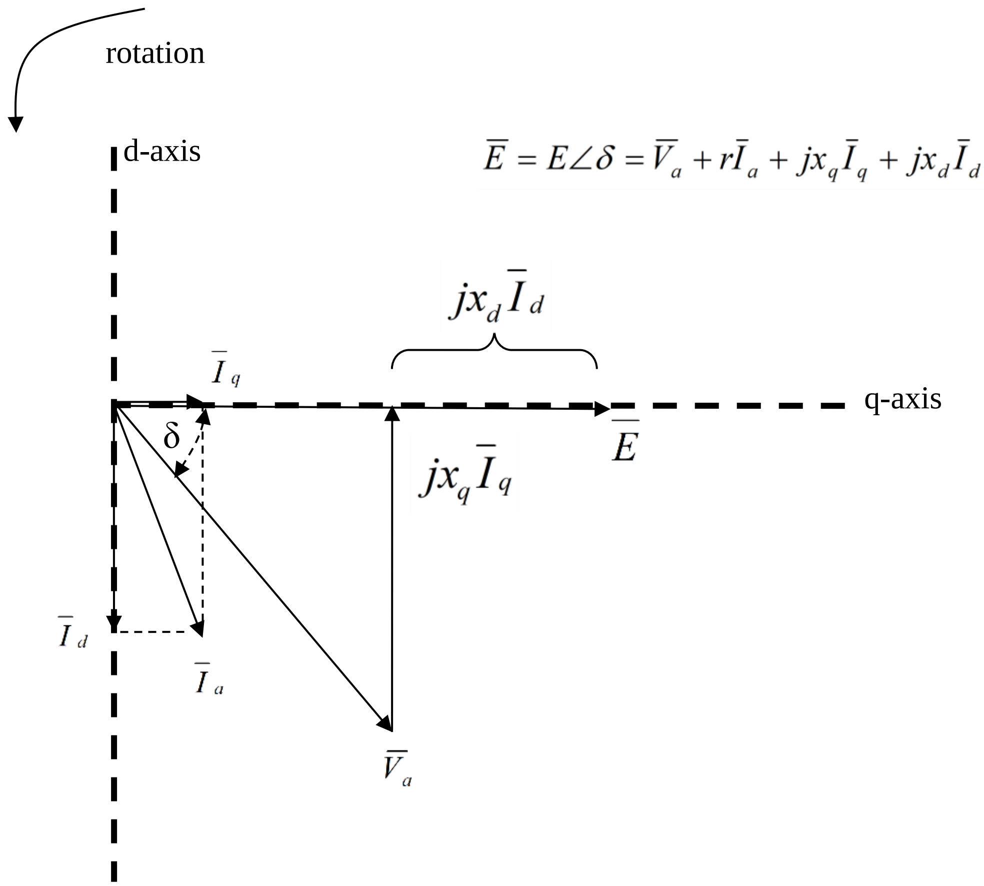


Fig. 7

From the above, we can see that

$$\bar{E} = \left|\bar{E}\right|\angle 0^\circ = \left|V_a\right|\cos\delta + jx_d\bar{I}_d \Rightarrow\Rightarrow \bar{I}_d = \frac{\left|\bar{E}\right| - \left|V_a\right|\cos\delta}{jx_d}$$

$$0 = -j\left|V_a\right|\sin\delta + jx_q\bar{I}_q \Rightarrow\Rightarrow \bar{I}_q = \frac{\left|V_a\right|\sin\delta}{x_q}$$

Substitution into the expression for S_{out} yields:

$$S_{out} = \left(\left|\bar{V}_a\right|\angle -\delta\right)\left(\frac{\left|\bar{E}\right| - \left|V_a\right|\cos\delta}{jx_d} + \frac{\left|V_a\right|\sin\delta}{x_q}\right)^* = \left(\left|\bar{V}_a\right|(\cos\delta - j\sin\delta)\right)\left(\frac{\left|V_a\right|\sin\delta}{x_q} - j\frac{\left|\bar{E}\right| - \left|V_a\right|\cos\delta}{x_d}\right)^*$$

Now taking care of the conjugation yields:

$$\Rightarrow\Rightarrow S_{out} = \left(\left|\bar{V}_a\right|(\cos\delta - j\sin\delta)\right)\left(\frac{|V_a|\sin\delta}{x_q} + j\frac{\left|\bar{E}\right| - |V_a|\cos\delta}{x_d}\right)$$

Taking the real part to find P_{out}:

$$P_{out} = \left|\bar{V}_a\right|\left[\frac{\left|\bar{V}_a\right|\cos\delta\sin\delta}{x_q} + \frac{\left|\bar{E}\right|\sin\delta - \left|\bar{V}_a\right|\cos\delta\sin\delta}{x_d}\right]$$

Multiplying through by $\left|\bar{V}_a\right|$ and rearranging the order of the terms yields:

$$P_{out} = \frac{\left|\bar{E}\right|\left|\bar{V}_a\right|\sin\delta}{x_d} + \left|\bar{V}_a\right|^2\cos\delta\sin\delta\left[\frac{1}{x_q} - \frac{1}{x_d}\right]$$

Recalling the trigonometric identity sin2x=2 cosx sinx, we have:

$$P_{out} = \frac{\left|\bar{E}\right|\left|\bar{V}_a\right|\sin\delta}{x_d} + \frac{\left|\bar{V}_a\right|^2}{2}\left[\frac{1}{x_q} - \frac{1}{x_d}\right]\sin 2\delta$$

Similarly, we may derive from S_{out} the expression for reactive power out of a salient-pole machine, as:

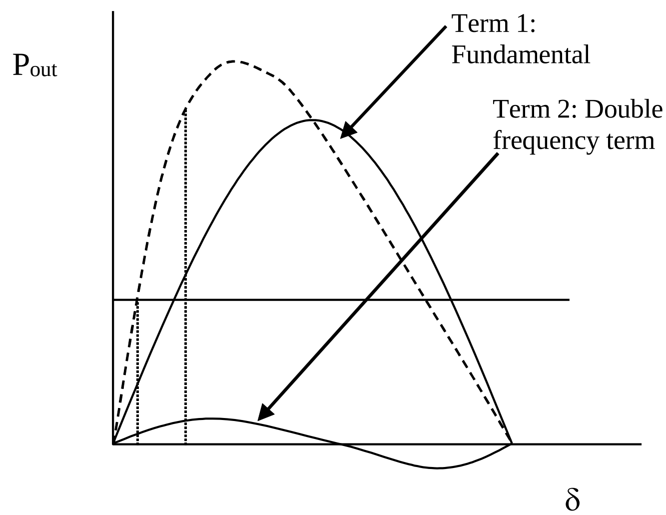
$$Q_{out} = \frac{\left|\bar{E}\right|\left|\bar{V}_a\right|\cos\delta}{x_d} - \frac{\left|\bar{V}_a\right|^2}{2x_d x_q}\left[\left(x_d + x_q\right) - \left(x_d - x_q\right)\cos 2\delta\right]$$

Note that both P_{out} and Q_{out} collapse to round-rotor equations if $x_d = x_q$.

Question: What does saliency do to stability?

Refer back to the expression for P_{out} and call the first term "term 1" and the second term "term 2."

$$P_{out} = \underbrace{\frac{|\overline{E}||\overline{V}_a|\sin\delta}{x_d}}_{Term1} + \underbrace{\frac{|\overline{V}_a|^2}{2}\left[\frac{1}{x_q} - \frac{1}{x_d}\right]\sin 2\delta}_{Term2}$$



From the above figure, we observe that P_{max} is greater for a salient-pole machine relative to a round-rotor machine. This fact means that, for a given power output level, a salient-pole machine will typically have more decelerating energy available than a corresponding round-rotor machine, with all other things being equal. ➔ Saliency tends to improve stability (but the effect is not large).

See pp. 80-89 of Kimbark Vol. III – it provides sample calculations regarding the above conclusion.

<u>Initial conditions for a multi-machine system (Section 5.7)</u>:

Assume that the power flow solution give us $\overline{V}_a$ and $\bar{I}_a$ for every generator such that

$\overline{V}_a = |\overline{V}_a| \angle\beta \qquad \bar{I}_a = |\bar{I}_a| \angle\beta - \phi$

Then, for each generator, we need to perform the following procedure in order to obtain the initial conditions:

1. Compute $\bar{E}_a = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_a$ (and this gives us δ)
2. Compute $\bar{I}_d$ and $\bar{I}_q$ from:

   $$\left|\bar{I}_d\right| = \left|\bar{I}_a\right|\sin(\phi - \beta + \delta) \qquad \left|\bar{I}_q\right| = \left|\bar{I}_a\right|\cos(\phi - \beta + \delta)$$

   where $\delta = \angle\bar{E}_a$, $\phi = \beta - \angle\bar{I}_a$, $\bar{I}_d = \left|\bar{I}_d\right|\angle\delta - 90$, and $\bar{I}_q = \left|\bar{I}_q\right|\angle\delta$
3. Compute $\bar{E} = E\angle\delta = \bar{E}_a + j\bar{I}_d\left(x_d - x_q\right)$
4. Compute: $I_d = -\left|\bar{I}_d\right|$, and $I_q = \left|\bar{I}_q\right|$
5. Compute $i_d = \sqrt{3}I_d$, $i_q = \sqrt{3}I_q$, and $i_F = \dfrac{\sqrt{3}\left|E\right|}{L_{AD}}$

The relation for i_F is obtained from $i_F\omega_R kM_F = \sqrt{3}\,E$ (section 4.7.4), where, in pu, ω_R=1 and $L_{AD}=kM_F$, as expressed in 4.226.

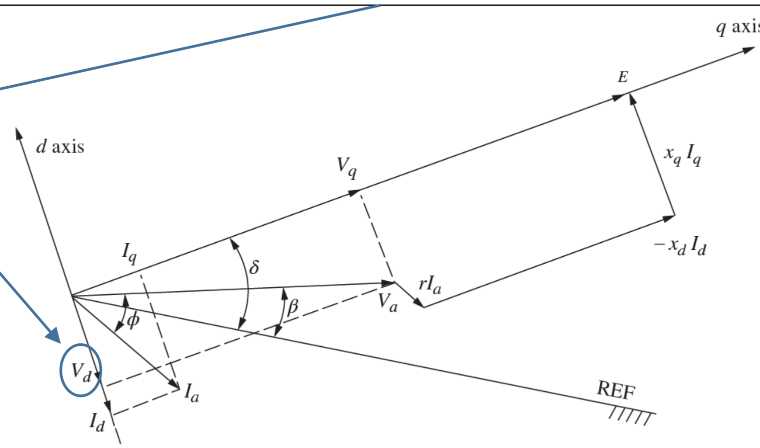
6. Now compute v_d and v_q. From below phasor diagram (Fig. 5.1), we can decompose $\bar{V}_a$ into its component in phase with the d-axis and its component in phase with the q-axis. This results in:

Note V_d, like I_d, is a negative number. This is consistent with its assumed location, see below (Fig. 5.1 in VMAF).

$$\left|\bar{V}_q\right| = \left|\bar{V}_a\right|\cos(\delta - \beta) \qquad \left|\bar{V}_d\right| = \left|\bar{V}_a\right|\sin(\delta - \beta)$$

$$V_q = \left|\bar{V}_q\right| \qquad V_d = -\left|\bar{V}_d\right|$$

$$v_q = \sqrt{3}V_q \qquad v_d = \sqrt{3}V_d$$



7. Compute $v_F = i_F r_F$

All of the above steps are "generic;" they apply to all of the machines. The remaining steps, however, depend on the particular model being used for the generator at this bus.

Let's assume we are using the E'$_q$ model (model 1.0). In this model, we neglect the G-winding and both D- and Q-damper windings, so that the only rotor winding accounted for is the main field winding.

8. From 4.104, we obtain λ_d, λ_q, and λ_F from:

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_F \end{bmatrix} = \begin{bmatrix} L_d & 0 & kM_F \\ 0 & L_q & 0 \\ kM_F & 0 & L_F \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_F \end{bmatrix}$$

9. We also need E_{FD} as an input. (E_{FD} is the RMS stator value corresponding to the field current of v_F/r_F; it differs from E'_q, which is the RMS stator value corresponding to the field flux linkage λ_F, as explained on p. 108 of VMAF). We obtain it from

$$E_{FD} = \frac{1}{\sqrt{3}} \frac{L_{AD}}{r_F} v_F$$

10. Get the initial conditions on the other states:

$$E'_q = \frac{1}{\sqrt{3}} \frac{kM_F}{L_F} \lambda_F \qquad \Lambda_d = \frac{\lambda_d}{\sqrt{3}} \qquad \Lambda_q = \frac{\lambda_q}{\sqrt{3}}$$

These, along with δ (see step 2) & ω=1 comprise initial conditions.

This "additional comment" which summarizes Sec 5.5 in VMAF is not significant since we will always have β for every bus, from the power flow solution.

Additional comment on step 2 above (Section 5.5 in VMAF):

If the angle β (angle of $\bar{V}_a$) is not explicitly given, then the calculation can still be made except it is necessary to think a bit more about how to make it (see p. 172).

Consider decomposing the current $\bar{I}_a$ into components I_r in phase and I_x in quadrature with the terminal voltage V_a so that

$$\bar{I}_a = I_r + jI_x$$

With ϕ as the power factor angle (the angle by which $\bar{I}_a$ lags $\bar{V}_a$, positive for lagging power factor), then

$$I_r = |I_a| \cos\phi \qquad I_x = -|I_a| \sin\phi$$

The minus sign on the expression for I_x is to account for the fact that when ϕ is positive, current is lagging the voltage so that the x-component should be negative in this case.

Now recall our $\bar{E}_a$ vector is

$$\bar{E}_a = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_a.$$

Substituting for I_a, we have:

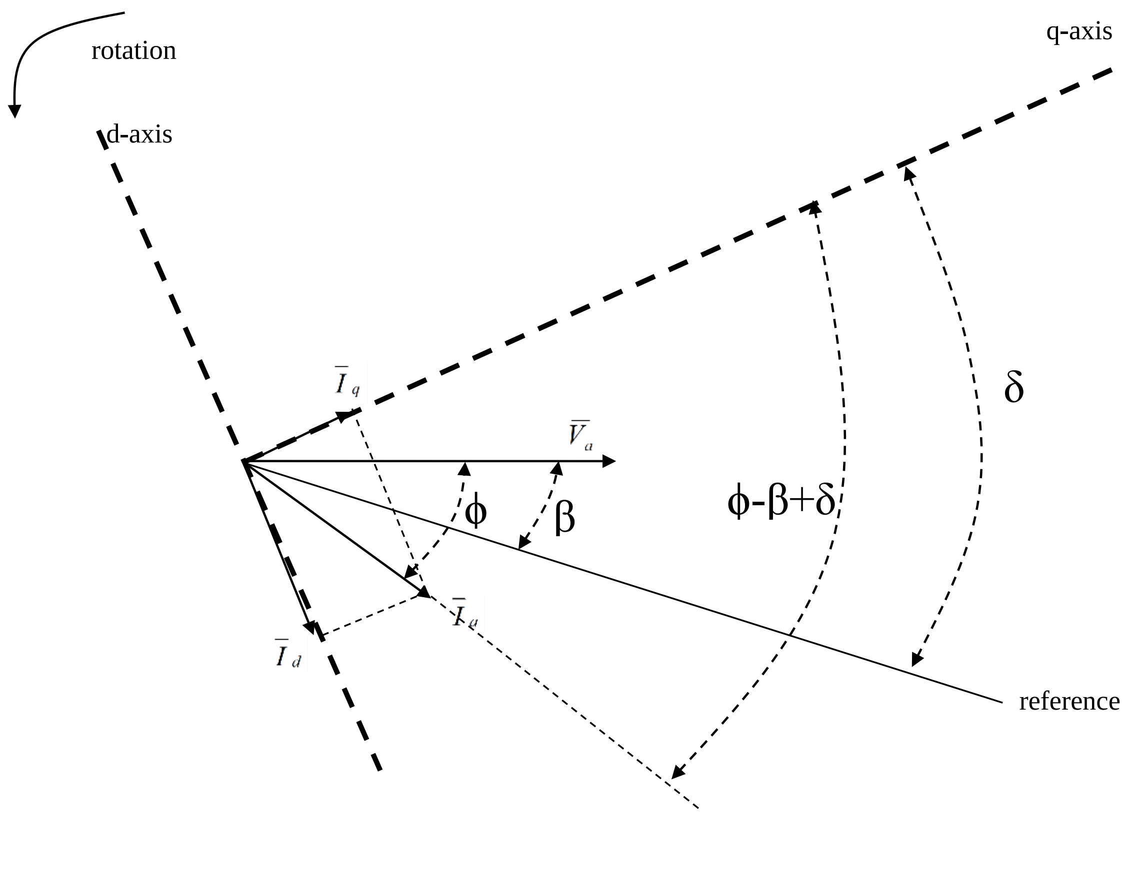
$$\bar{E}_a = \bar{V}_a + r(I_r + jI_x) + jx_q(I_r + jI_x)$$

Collecting real and imaginary parts, we have:

$$\bar{E}_a = \bar{V}_a + (rI_r - x_qI_x) + j(rI_x + x_qI_r)$$

With $\bar{V}_a$ having an angle of β, the above calculation results in an $\bar{E}_a$ with an angle of δ.

But let's rotate $\bar{V}_a$ by -β (see Fig. 4) so that it has an angle of β-β=0, i.e., so that $\bar{V}_a$ is aligned with the reference. I have repeated Fig. 4 below.



In this case, the computed quantity on the left-hand-side, $\overline{E}_a$, will have an angle of δ-β (without the rotation of -β, it has an angle of δ), and we may rewrite the above expression for $\overline{E}_a$ so that $\overline{V}_a$ is expressed as an entirely real number, i.e., with zero imaginary part (since it has angle of 0). Thus $\overline{V}_a = V_a$ and

$$\overline{E}_a = \left(V_a + rI_r - x_q I_x\right) + j\left(rI_x + x_q I_r\right)$$

Thus, we have that

$$\delta - \beta = \tan^{-1}\left(\frac{rI_x + x_q I_r}{V_a + rI_r - x_q I_x}\right)$$

which locates $\overline{E}_a$ and thus the q-axis.

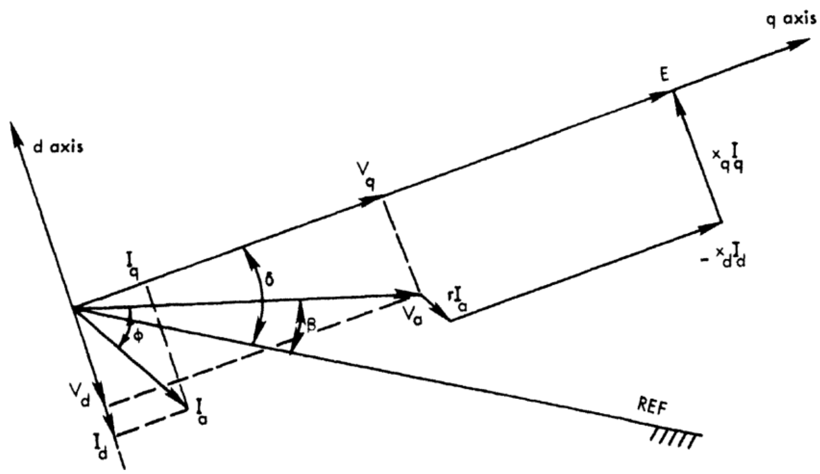
There are a series of examples in the text which deserve some study, but our time does not allow us to treat them in class. These examples use the machine data from Examples 4.1-4.3 in Chap. 4. Here are comments on main points to draw from these examples.

Example 5.1: Steps 1-6 are illustrated here. Subsequent steps assume the full flux linkage model, i.e., model 2.2 and initial conditions are computed for the corresponding states λ_d, λ_q, λ_F, λ_D, λ_G, λ_Q. Per-unit electrical torque on a per-phase base is then computed from $T_{e\varphi} = i_q \lambda_d - i_d \lambda_Q$, and we divide by 3 to get it on a 3-phase base. We then compute the infinite bus voltage.

An important relation that is used is

$$E = V_q + rI_q - x_d I_d$$

We have not developed this relation in our notes, but it comes from noticing that it is computing magnitude only (i.e., it is not a phasor), and then just observing that this magnitude is the sum of the corresponding terms in Fig 5.1, repeated below for convenience.



Example 5.2: Example 5.1 is repeated, except here it is assumed that the q-axis leads the d-axis. Calculations are the same except we get a sign change for I_d, in which case the above diagram indicates we must use

$$E = V_q + rI_q + x_d I_d$$

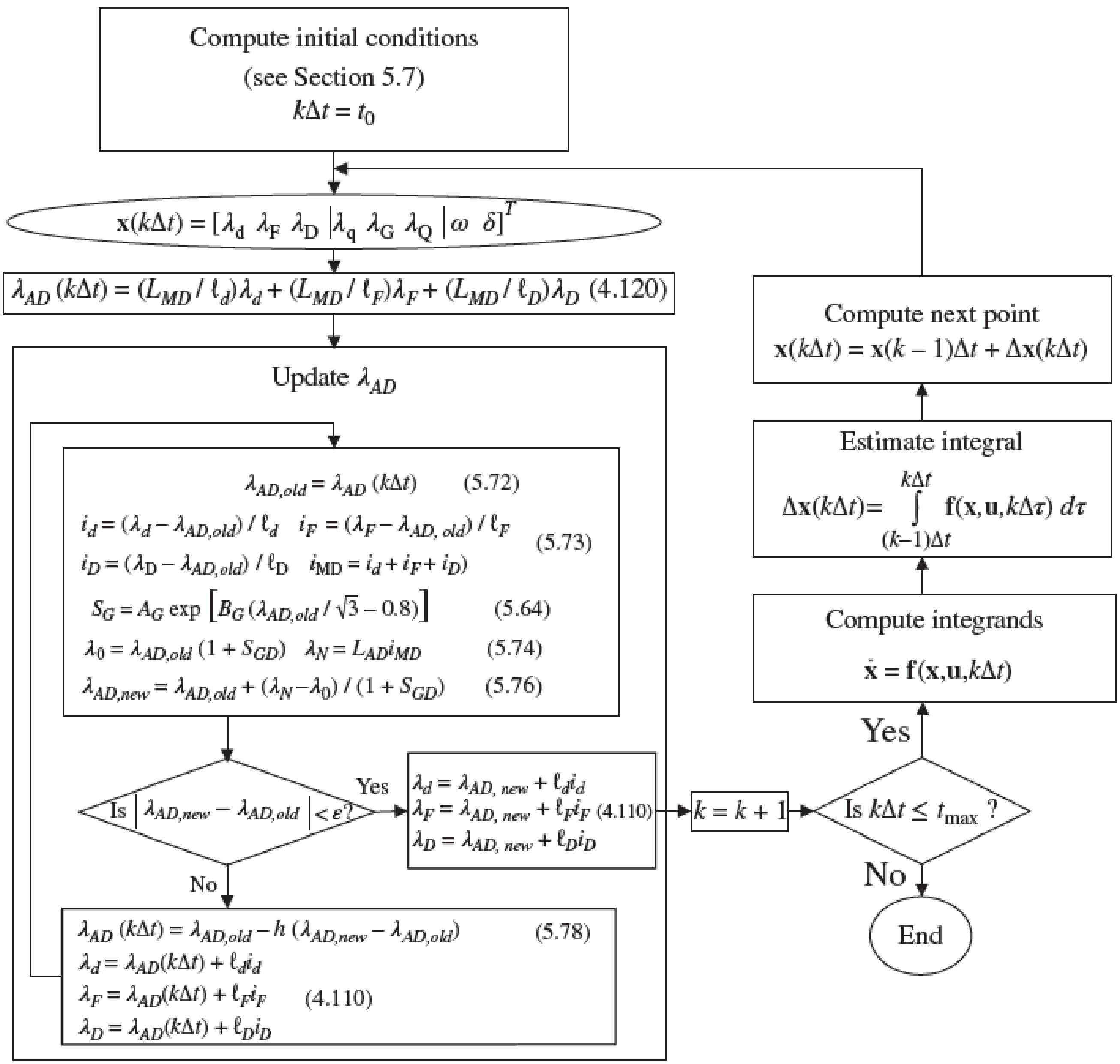
Example 5.3: This repeats Example 5.1 except instead of computing the infinite bus voltage (given the terminal voltage), we are given the infinite bus voltage. This is no different than Ex. 5.1 in terms of finding initial conditions, except for the fact that the point of known voltage is electrically further away from the internal voltage.

Example 5.4: This repeats Example 5.3 (where infinite bus voltage is given) except the "external network" is a *little* more complex in that it has load at the machine terminals (so-called "station load").

Example 5.5: Given the initial conditions of Example 5.1, this example models station load (a different station load than the one in Example 5.4) and computes the infinite bus voltage.

Section 5.8: This is useful material related to Sec.4.16 on parameter determination for gen dynamic models. In Sec.4.16, we learned most measurement methods provide so-called standard parameters. These parameters are used with simplified models (Sec. 4.15), but they cannot be used for model 2.2 (flux or current), which requires so-called fund. parameters. This section, 5.8, shows how to compute fund. parameters from standard parameters.

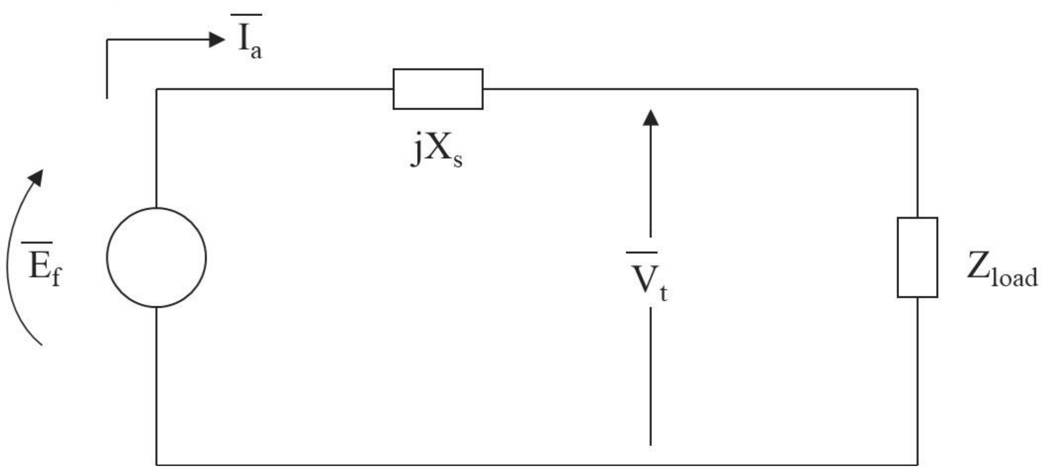
Section 5.9: This section, titled "Digital simulation of synchronous machines," briefly touches on the topic of Chapter 7, except here we remain focused on just a single synchronous machine without incorporating the network as we will do in Chapter 7. The main focus in Section 5.9 is the incorporation of saturation in the simulation, as indicated in Fig. 5.13, a flowchart provided below.



Appendix A: Derivation of Steady-State Power Relations for Round Rotor Machine from Equivalent Circuit

Power relationships

Recall the power angle, δ, as the angle at which the excitation voltage, $\bar{E}_f = E_f \angle \delta$, leads the terminal voltage, $\bar{V}_t = V_t \angle 0°$. Therefore, from the circuit….



Power relationships

$$\bar{I}_a = \frac{E_f \angle \delta - V_t \angle 0°}{jX_s} = \frac{E_f \cos\delta + jE_f \sin\delta - V_t}{jX_s}$$

$$= \frac{E_f \cos\delta - V_t}{jX_s} + \frac{jE_f \sin\delta}{jX_s}$$

$$= \frac{E_f \sin\delta}{X_s} - j\left[\frac{E_f \cos\delta - V_t}{X_s}\right] \qquad (1)$$

But $\quad \bar{I}_a = I_a \cos\theta - jI_a \sin\theta \qquad (2)$

Power relationships (cont'd)

Equating real and imaginary parts of eqs. 1 and 2 and multiplying both sides of the equations by $3V_t$:

$$P_{out} = 3V_t I_a \cos\theta = \frac{3V_t E_f \sin\delta}{X_s} \qquad (3)$$

$$Q_{out} = 3V_t I_a \sin\theta = \frac{3V_t E_f \cos\delta}{X_s} - \frac{3V_t^2}{X_s} \qquad (4)$$

Note: reactive power is positive when the machine is operated overexcited, i.e., when it is lagging