Simplified Models of the Synchronous Machine

From CC Young's paper, "Equipment and System Modeling for Large-Scale Stability Studies," we read on pg. 1:

> The representation of synchronous machines in stability programs should be as simple as possible to minimize computer costs. But, at the same time, there are situations where an accurate representation is required. Furthermore, there may be uncertainty as to whether a simpler model will be adequate. For these reasons, models of the synchronous machine have been included which involve the representation of between one and three rotor-iron (or amortisseur) circuits. These models are discussed in Section II and their equations are given in Appendix I.

and on pg. 2:

> During any given system stability study, it is common to have synchronous machines in the system being represented by models of varying detail. For example, the machines near the disturbance may be represented by a very detailed model, whereas machines remote to the disturbance may be represented by the simplest model. Furthermore, the machines at intermediate locations may use still another model. Therefore, one requirement for a sophisticated stability program is that it have models available to the user which encompass the range of representation required.[7]

And VMAF write (p. 136), that

> In a stability study the response of a large number of synchronous machines to a given disturbance is investigated. The complete mathematical description of the system would therefore be very complicated unless some simplifications were used. Often only a few machines are modeled in detail, usually those nearest the disturbance, while others are described by simpler models. The simplifications adopted depend upon the location of the machine with respect to the disturbance causing the transient and upon the type of disturbance being investigated. Some of the more commonly used simplified models are given in this section. The underlying assumptions as well as the justifications for their use are briefly outlined. In general, they are presented in the order of their complexity.
>
> Some simplified models have already been presented. In Chapter 2 the classical representation was introduced. In this chapter, when the saturation is neglected as tacitly assumed in the current model, the model is also somewhat simplified. An excellent reference on simplified models is Young [24]…

Below is a summary of the different models that we will discuss:

Model #	Youngs ID	IEEE ID (d.q)	Description and relevant section in the text	G-cct?	Dmpr wdgs?	No. of states	States
1	~~Model III~~	2.2	Full model with G-cct; see 4.12.	Yes	Yes	8	$\lambda_d,\lambda_F,\lambda_D,\lambda_q,\lambda_G,\lambda_Q,\delta,\omega,$
2		2.1	Full model without G-cct.	No	Yes	7	$\lambda_d,\lambda_F,\lambda_D,\lambda_q,\lambda_Q,\delta,\omega$
3	~~Model II~~	1.1	Machine with solid round rotor; see 4.15.1.	Yes	No	6	$\lambda_d,\lambda_F,\lambda_q,\lambda_G,\delta,\omega$
4		1.0	E'q model: Same as 1.1 except for a salient pole machine; see 4.15.1.	No	No	5	$\Lambda_d,\Lambda_q,E'_q,\delta,\omega$
5	Model III	2.1	E" model: for a salient pole machine & $d\lambda_d/dt= d\lambda_q/dt=0$; see 4.15.2.	No	Yes	~~5~~ 6	$E''_d,\Lambda_D,\Lambda_Q,E'_q,\delta,\omega$
6	Model II	1.1	2-axis model: for a machine with solid round rotor & $d\lambda_d/dt= d\lambda_q/dt=0$; see 4.15.3.	Yes	No	4	E'_d,E'_q,δ,ω
7		1.0	One axis model: Same as 2-axis model (1.1) but without G-cct; see 4.15.4.	No	No	3	E'_q,δ,ω
8	Model I	0.0	Classical; see 4.15.5.	No	No	2	δ,ω

Observe:

- # of states decrease from 8 (mod#1) to 2 (mod#8)
- First 4 mods alternate between rnd-rot& sal-pole
- First 2 mods have dmprs; next 2 do not
- Mod#5-#8 assume $d\lambda_d/dt= d\lambda_q/dt=0$
- Mods#4, #7 have no q-axis ccts and are therefore "one-axis."

Note that the IEEE ID is useful: d.q, where d indicates how many direct-axis rotor circuits are modeled (F, D) and q indicates how many quadrature-axis rotor circuits are modeled (G, Q). Note that these numbers do not include the stator windings (d, q). You can get the number of electrical states from these numbers according to:

$$E=d+q+2-N$$

where "2" is for the d and q winding states and N is the number of states for which the derivative is assumed zero.

Then, the total number of states is E+M where M=2 is the number of mechanical states.

In the rest of these notes, we summarize the models indicated in Table 4.6, p. 136, of VMAF (p. 2 of these notes).

Model 1 (2.2) Full model with G-cct.

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \tag{4.126}$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \tag{4.128}$$

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD} \tag{4.129}$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \tag{4.130}$$

$$\dot{\lambda}_Q = -\frac{r_Q}{l_Q}\lambda_Q + \frac{r_Q}{l_Q}\lambda_{AQ} \tag{4.131a}$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ} \tag{4.131b}$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \tag{4.133}$$

$$\dot{\delta} = \omega - 1 \tag{4.102}$$

where

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F + \frac{L_{MD}}{l_D}\lambda_D \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q + \frac{L_{MQ}}{l_G}\lambda_G$$

and

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} + \frac{1}{l_G}\right]$$

This is the model we have developed in Chapter 4. In this model, we have E=d+q+2-N=2+2+2-0=6, and so the number of states is E+2=8, i.e., it is an 8-state model and is generally considered to be the model necessary for a round-rotor machine that is being studied in detail.

Model 2 (2.1): Full model without G-cct.

Same as model 1 except omit the state equation for λ_G (4.131a) & modify the auxiliary equations, resulting in:

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD} \qquad (4.129)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_Q = -\frac{r_Q}{l_Q}\lambda_Q + \frac{r_Q}{l_Q}\lambda_{AQ} \qquad (4.131)$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \qquad (4.133)$$

$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

where

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F + \frac{L_{MD}}{l_D}\lambda_D \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q$$

and

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q}\right]$$

In this model, we have E=d+q+2-N=2+1+2-0=5, and so the number of states is E+2=7, i.e., it is an 7-state model and is generally considered to be the model necessary for a salient-pole (hydro) machine that is being studied in detail.

Model 3 (1.1): Machine with solid round rotor. (See page 137) (No D or Q-axis damper windings).

For this model, VMAF (below left, p. 137) and A&F (below right, p. 127) reference Kimbark's Vol. III, saying:

"In this model, designated Model 1.1, the G-winding of a solid round rotor acts as a q axis damper winding, even with the D- and Q-windings omitted. The mathematical model for this type of machine will be the same as given in Sections 4.10 and 4.12 with i_D or λ_D omitted and i_Q or λ_Q omitted. For example in (4.103) and (4.138), the third and sixth rows and columns are omitted."	"The solid round rotor acts as a q axis damper winding, even with the d axis damper winding omitted. The mathematical model for this type of machine will be the same as given in Sections 4.10 and 4.12 with i_D or λ_D omitted. For example, in (4.103) and (4.138) the third row and column are omitted.

On checking Kimbark Vol. III, pg. 73, one finds state equations for both λ_Q and λ_D have been dropped, but the G-winding is still there[1].

So this model is the same as model 1 except we omit the state equations for λ_D (4.129) and λ_Q (4.131b), and modify the auxiliary equations, resulting in:

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ} \qquad (4.131a)$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \qquad (4.133)$$

$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

where

[1] In the 2nd edition of A&F, the text was assuming no G-winding, whereas Kimbark includes the G-winding. That is, whereas Kimbark drops both Q and D windings (but retains G), A&F 2nd edition drop only the D (and retain Q). Thus, A&F 2nd edition implicitly allow the Q-winding to substitute for the G-winding. In both cases, the model is d.q=1.1, where both have the F-winding which justifies the first "1"; A&F have the Q-winding (but not G) and Kimbark has the G-winding (but not Q) which justifies (in each case, respectively) the second "1". VMAF have reconciled this rather confusing issue by assuming the full model of Sections 4.0 and 4.12 include the G-winding, and the text in VMAF (as quoted in the left-box above) has been adjusted (relative to A&F 2nd edition) accordingly.

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_G}\lambda_G$$

and

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F}\right] \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_G}\right]$$

In this model, we have E=d+q+2-N=1+1+2-0=4, and so the number of states is E+2=6, i.e., it is an 6-state model and is generally considered to be a reasonable one (but not the best since it omits damper windings) for a for a round-rotor machine.

Model 4 (1.0): E'q model: Same as #3 except for a salient pole machine. (See Sec. 4.15.1, p. 137-142) (no D- or Q-axis dampers, & because it is salient pole, omit G-winding➔a "1-axis" model) From VMAF, p. 137,

> This is Model 1.0, where the G-winding and both amortisseur windings are omitted. Omitting these windings rests on the assumption that the effect of the G-windings and damper windings on the transient under study is small enough to be negligible. This is particularly true in system studies where the damping between closely coupled machines is not of interest. In this case the effect of the amortisseur windings may be included in the damping torque, i.e., by increasing the damping coefficient D in the torque equation. This model is obtained by omitting i_G, i_D, and i_Q in (4.103) or λ_G, λ_D, and λ_Q in (4.138).

One version of this model can be obtained from model 3 (which has no D- or Q-axis dampers) by omitting the state equation for λ_G (4.131a) and modifying Q-axis auxiliary equations appropriately.

But we may also describe this model in terms of some new "stator-side" states, E'_q, Λ_d, and Λ_q, which are just scaled versions of three corresponding rotor quantities. We will do this for the E'_q model, noting from the p. 2 table that models 5-7 also use these variables. It has been common in the literature to express machine models in terms of stator-side quantities because it provides for the ability to efficiently think about (and draw phasor diagrams for) the magnitudes of the various quantities. VMAF state (p. 107) "The basis for converting a field quantity to an equivalent stator EMF is that at open circuit a field current i_F A corresponds to an EMF of $i_F\omega_{Re}M_F$ V peak" (see p. 17 of perunitization notes).

The three new stator-side quantities are developed below:

- E'_q is the pu value of the stator equivalent EMF corresponding to the field flux linkage λ_F, in phase with the q-axis, given by:

$$E'_q = \lambda_F \frac{L_{AD}}{\sqrt{3}L_F} \qquad (4.203)$$

This comes about as follows. We call it E'_q because it is in phase with the q-axis. This is the case because it is a voltage due entirely to the field flux, and the field flux is generated along the d-axis. Because the corresponding induced stator winding voltage is proportional to dλ/dt, the induced voltage must be 90 degrees out of phase, meaning it must be along the q-axis.

Its magnitude can be deduced as follows. (This expands on the discussion in VMAF text, pp. 107-108). Recall that the mutual inductance between field and a-phase winding (before Park's transformation!) is given by

$$L_{aF} = M_F \cos\theta = M_F \cos\omega_{\text{Re}} t \tag{1}$$

We know that the mutual flux linking the a-phase winding is

$$\lambda_{aF} = L_{aF} i_F \tag{2}$$

and that the time derivative of this flux linkage gives the induced voltage in the a-phase winding, i.e.,

$$\frac{d\lambda_{aF}}{dt} = \frac{d(L_{aF} i_F)}{dt} \tag{3}$$

Assuming i_F is constant, (3) becomes

$$\frac{d\lambda_{aF}}{dt} = i_F \frac{d(L_{aF})}{dt} \tag{4}$$

Substitution of (1) into (4) yields

$$\frac{d\lambda_{aF}}{dt} = i_F \frac{d(M_F \cos\omega_{\text{Re}} t)}{dt} = -i_F M_F \omega_{\text{Re}} \sin\omega_{\text{Re}} t \tag{5}$$

and we see that the peak value of the induced a-phase voltage is

$$E_{peak} = i_F M_F \omega_{\text{Re}} \tag{6}$$

The RMS value of this voltage would be V_{peak}/sqrt(2), i.e.,

$$E_{rms} = \frac{1}{\sqrt{2}} i_F M_F \omega_{\text{Re}} \tag{7}$$

If we multiple both sides by sqrt(3), we get

$$\sqrt{3} E_{rms} = \sqrt{\frac{3}{2}} i_F M_F \omega_{\text{Re}} \tag{8}$$

But recall our familiar k=sqrt(3/2), therefore

$$\sqrt{3} E_{rms} = i_F k M_F \omega_{\text{Re}} \tag{9}$$

To be consistent with the text, we will use E for E_{rms}, so that (9) becomes

$$\sqrt{3}E = i_F kM_F \omega_{\mathrm{Re}} \tag{10}$$

Equation (10) is given in your text at the bottom of page 107. Note that this is the voltage contribution to the a-phase from a certain value of field current.

VMAF text now (p. 108) considers the case of identifying the a-phase voltage that results from a certain amount of flux linkage seen by the field winding λ_F under steady-state (i_Q=i_D=0) and open circuit (i_d=i_q=0) conditions. Under these conditions, the only flux seen by the field winding is its own flux, and

$$\lambda_F = L_F i_F \Rightarrow i_F = \frac{\lambda_F}{L_F} \tag{11}$$

Substitution of (11) into (10) results in

$$\sqrt{3}E = \frac{\lambda_F}{L_F} kM_F \omega_{\mathrm{Re}} \tag{12}$$

The corresponding voltage is what VMAF call E'$_q$, to remind us that it is a voltage in phase with the q-axis, i.e.,

$$\sqrt{3}E'_q = \frac{\lambda_F}{L_F} kM_F \omega_{\mathrm{Re}} \qquad (4.58,\ 4.202)$$

We would like to per-unitize the above relation. To do so, recall the per-unit relations:

$$E'_q = E'_{qu} V_B, \qquad kM_F = kM_{Fu} M_{FB}, \qquad L_F = L_{Fu} L_{FB}, \qquad \lambda_F = \lambda_{Fu}\lambda_{FB} = \lambda_{Fu} L_{FB} I_{FB}$$

Substituting into (4.59), we have that

$$\sqrt{3}E'_{qu} V_B = \frac{\lambda_{Fu} L_{FB} I_{FB}}{L_{Fu} L_{FB}} kM_{Fu} M_{FB} \omega_{\mathrm{Re}}$$ (just before (4.203), p. 138 VMAF)

Bring over V_B to the right-hand-side & cancel the L_{FB}'s, to obtain

$$\sqrt{3}E'_{qu} = \frac{\lambda_{Fu} I_{FB}}{V_B L_{Fu}} kM_{Fu} M_{FB} \omega_{\mathrm{Re}}$$

Rearranging the right-hand-side

$$\sqrt{3}E'_{qu} = \frac{\lambda_{Fu}}{L_{Fu}} kM_{Fu} M_{FB} \frac{I_{FB}\omega_{\mathrm{Re}}}{V_B}$$

Noting that

$$\frac{I_{FB}}{V_B t_B} = \frac{1}{M_{FB}}$$

we may substitute to obtain

$$\sqrt{3}E'_{qu} = \frac{\lambda_{Fu}}{L_{Fu}} kM_{Fu} M_{FB} \frac{1}{M_{FB}} = \frac{\lambda_{Fu}}{L_{Fu}} kM_{Fu}$$

But in pu, $kM_{Fu}=L_{AD}$, and dropping the pu notation results in

$$\sqrt{3}E'_q = \frac{\lambda_F}{L_F} L_{AD} \qquad (4.203)$$

- We also need to define a "stator-side" quantity corresponding to the field voltage v_F, as v_F is our forcing function (and so we need to obtain the forcing function on the stator-side). This can be obtained by recognizing that in steady-state, $i_F=v_F/r_F$. Using (10), repeated here for convenience,

$$\sqrt{3}E = i_F kM_F \omega_{\mathrm{Re}} \qquad (10)$$

and denoting the stator-side emf as E_{FD}, we have

$$\sqrt{3}E_{FD} = \frac{v_F}{r_F} kM_F \omega_{\mathrm{Re}} \qquad (4.59)$$

Going through a similar process as in previous bullet to convert to per-unit, we get

$$E_{FD} = \frac{L_{AD} v_F}{\sqrt{3} r_F} \qquad (4.209)$$

- Finally, we need to obtain stator-side quantities of
  - Λ_d, the pu value of the stator equivalent flux linkage corresponding to the d-winding flux linkage λ_d,
  - Λ_q, the pu value of the stator equivalent flux linkage corresponding to the q-winding flux linkage λ_q
  - The d- and q- winding voltages v_d and v_q.

To obtain these, we recall that when we applied Park's transformation to a set of balanced a-b-c (stator-side) voltages, we got:

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3}V \sin\alpha \\ \sqrt{3}V \cos\alpha \end{bmatrix} \qquad (4.43)$$

From (4.203), (4.209), and the (4.43), we conclude that the pu value of any d or q axis quantity is numerically equal to √3 times the pu quantity on the stator side. Therefore, the stator-side per-unit equivalents of rotor side quantities are the rotor side quantity divided by √3. And so we have:

$$\Lambda_d = \frac{\lambda_d}{\sqrt{3}} \quad \Lambda_q = \frac{\lambda_q}{\sqrt{3}} \qquad V_d = \frac{v_d}{\sqrt{3}} \qquad V_q = \frac{v_d}{\sqrt{3}} \tag{4.212}$$

It is important to realize that E'_q (4.203), Λ_d, Λ_q (4.212), and E_{FD} (4.209) are given in pu.

With the above relations, we may substitute them into the model 3 state equations and then perform a considerable amount of algebra to obtain the state equations for the E'_q model, given as follows:

$$\dot{\Lambda}_d = -\left(r/L'_d\right)\Lambda_d + \left(r/L'_d\right)E'_q - \omega\Lambda_q - V_d \quad \text{pu} \tag{4.213}$$

$$\dot{\Lambda}_q = \omega\Lambda_d - \left(r/L_q\right)\Lambda_q - V_q \quad \text{pu} \tag{4.214}$$

$$\dot{E}'_q = -\frac{L_d}{L'_d\,\tau'_{d0}}E'_q + \frac{L_d - L'_d}{\tau'_{d0}\,L'_d}\Lambda_d + \frac{1}{\tau'_{d0}}E_{FD} \quad \text{pu} \tag{4.215}$$

$$\tau_j\dot{\omega} = T_m - T_e - D\omega \quad \text{pu} \tag{4.219}$$

$$\dot{\delta} = \omega - 1 \quad \text{pu} \tag{4.220}$$

In matrix form, equations (4.213-4.215) and (4.219-4.220) become:

$$\begin{bmatrix} \dot{\Lambda}_d \\ \dot{\Lambda}_q \\ \dot{E}'_q \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} \frac{-r}{L'_d} & -\omega & \frac{r}{L'_d} & 0 & 0 \\ \omega & \frac{-r}{L_q} & 0 & 0 & 0 \\ \frac{L_d - L'_d}{\tau'_{d0}L'_d} & 0 & \frac{-L_d}{\tau'_{d0}L'_d} & 0 & 0 \\ \frac{1}{\tau_j}\Lambda_q\left(\frac{1}{L'_d} - \frac{1}{L_q}\right) & \frac{-E'_q}{\tau_j L'_d} & 0 & \frac{-D}{\tau_j} & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Lambda_d \\ \Lambda_q \\ E'_q \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} -\mathrm{V}_d \\ -\mathrm{V}_q \\ \frac{1}{\tau'_{d0}}E_{FD} \\ \frac{1}{\tau_j}T_m \\ -1 \end{bmatrix}$$

We may also generate a block diagram for this model by taking the LaPlace transform of all equations. The resulting block diagram is shown in Figures 4.9 and 4.10 of your text, provided below.

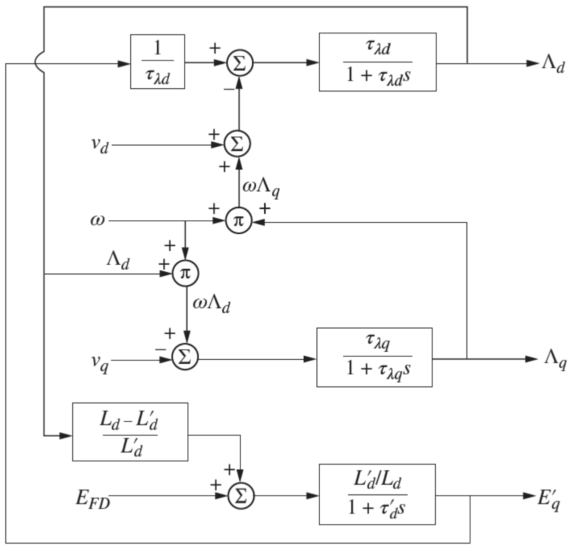


Figure 4.9 Block diagram representation of the E'_q model.

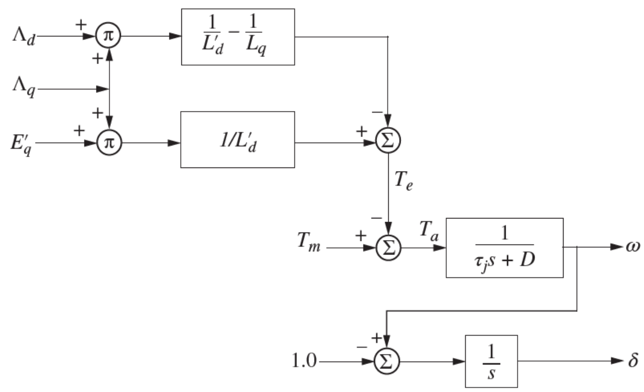


Figure 4.10 Block diagram representation of (4.218)–(4.220).

A final comment with respect to this model is that the driving functions are the inputs E_{FD} (stator-side field voltage which is regulated by the excitation system), T_m (mechanical power which is regulated by the turbine-governor system), and the voltages V_d and V_q which are functions of the external network.

In this model, we have E=d+q+2-N=1+0+2-0=3, and so the number of states is E+2=5.

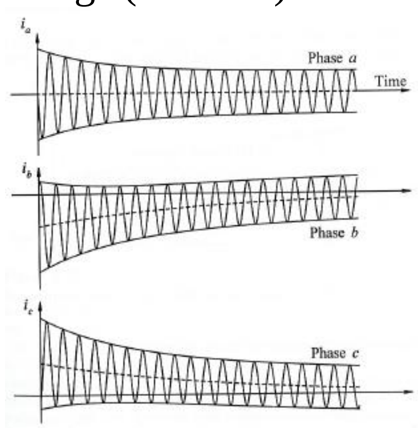
Comment on $d\lambda_d/dt=d\lambda_q/dt=0$.

Reference to the above table indicates that the E'' model, the 2-axis model, the one-axis model, and the classical model all have $d\lambda_d/dt=d\lambda_q/dt=0$. This issue is addressed in [1, 2], Kundur's book Sections 3.7 and 5.1.1 [3], and Krause's book Chapter 8 [4]. Some comments about this issue follow:

1. Analysis of a voltage source $e(t)=E_m\sin(\omega t+\alpha)$ connected to an RL circuit will show that the time-domain response of the current to a short circuit consists of two terms: a transient unidirectional (DC offset) component and a steady-state alternating component, i.e.,

$$i(t)=K_1e^{-Rt/L}+K_2\sin(\omega t+\alpha-\phi) \qquad (*)$$

2. Each of the three phases of a synchronous machine behave similarly under the conditions of a bolted three-phase fault applied to the machine terminals at t=0. Kundur [3, p. 107] illustrates this with the below figure. Here, we observe
   a. The DC offset (the dotted line) component decaying exponentially to zero within about 20 cycles, as represented in the first term of (*).
   b. The fundamental frequency component, as represented in the second term of (*), except here there is a difference in that this component has amplitude that also decays with time to a steady-state value, with
      i. the initial rapid decay due to decay of flux linking the subtransient windings (D and Q) and
      ii. the slower decay due to decay of flux linking the transient windings (F and G).



3. Kundur [3], p. 173, indicates that the effect on power system stability of the torque corresponding to the DC offset currents is to produce a DC braking torque which reduces rotor acceleration following a disturbance. Neglecting it is therefore conservative; in addition, its effects can be approximated via another torque term on the right-hand-side of the swing equation [5], [6, pg. 233-234], [7]. Of most importance, neglecting it has some major benefits, as follows:
   a. There is a similar DC offset effect in the network (it is an RLC circuit!), but including that effect increases the system size significantly, contributes high frequency components

This is an interesting issue for those of you considering how to handle IBRs in DSA... you might consider reviewing the refs in this section of my notes!

(requiring small time steps for numerical integration), and inhibits use of phasor representation for the network solution part of time-domain simulation. Section 7.3.1 of VMAF addresses this last point, which is further characterized by the following statement from [2]:

VMAF Section 7.3.1: "Network in the transient state".

> "In stability studies it has been found adequate to represent the network as a collection of lumped resistances, inductances, and capacitances, and to neglect the short-lived electrical transients in the transmission system.[8],[5],[9],[10] As a consequence of this fact, the terminal constraints imposed by the network appear as a set of algebraic equations which may be conveniently solved by matrix methods."

b. Neglecting synchronous machine DC offsets and in the network means that both are being treated consistently. ➔

Although treating them consistently is good, it is not essential in DSA studies; stator transients are fast but not as fast as network transients. Is this correct?

4. The above figure shows abc (phase) currents. The corresponding quantities following Park's transformation are i_d and i_q, where
   a. The fundamental frequency components in the phase currents are reflected as unidirectional components in i_d and i_q. We have encountered this idea before when we recognized that balanced steady-state phase (abc) currents transform to DC quantities.
   b. The DC offsets in the phase currents are reflected as fundamental frequency components in i_d and i_q. Neglecting phase current DC offsets is equivalent to setting $di_d/dt=di_q/dt=0$, and since our transformed inductance matrix is constant, setting $di_d/dt=di_q/dt=0$ is equivalent to setting $d\lambda_d/dt=d\lambda_q/dt=0$.
5. Setting $d\lambda_d/dt=d\lambda_q/dt=0$ is referred to in the literature as "neglecting stator transients" (is this the same as "neglecting network transients" per section 7.3.1?)
6. Other ways this assumption is expressed include:
   a. "transformer voltages are neglected,"
   b. "transformer voltages are assumed small compared to speed voltages" or

$$\left|\dot{\lambda}_d\right| << \left|\omega\lambda_q\right|$$

$$\left|\dot{\lambda}_q\right| << \left|\omega\lambda_d\right|$$

   c. The stator equations become algebraic.

7. In this case, the state-space equations become, from p. 11 in "flux linkage equations," become (changes made to 4.126 and 4.129).

$$\dot{\lambda}_d = 0 = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_D = 0 = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD} \qquad (4.129)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ} \qquad (4.131a)$$

$$\dot{\lambda}_Q = -\frac{r_Q}{l_Q}\lambda_Q + \frac{r_Q}{l_Q}\lambda_{AQ} \qquad (4.131b)$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \qquad (4.133)$$

$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

Model 5 (2.1): E'' model, for a salient pole machine & $d\lambda_d/dt= d\lambda_q/dt=0$. See section 4.15.2, p. 142-150.

The intent of this model is to develop a simple model but one that does account for the effects of both damper windings. So this model includes both transient and subtransient effects. However, we do not represent the G-winding, therefore it should only be used for a salient pole machine.

Recall from our fundamental voltage equations (4.36) that:

$$v_d = -ri_d - \dot{\lambda}_d - \omega\lambda_q$$

$$v_q = -ri_q - \dot{\lambda}_q + \omega\lambda_d$$

(Note the above are in MKS, not per-unit)
An important simplifying assumption for this model is that

$$\left|\dot{\lambda}_d\right| << \left|\omega\lambda_q\right|$$

$$\left|\dot{\lambda}_q\right| << \left|\omega\lambda_d\right|$$

(see "**Comment on $d\lambda_d/dt=d\lambda_q/dt=0$.** on p. 11 of these notes).

We will use E'_q as a state, as defined for model 4. But we will also define one new state:

- The d-axis component of the EMF produced by subtransient flux voltages:

$$e''_d = \omega\lambda_q$$

(There is a corresponding q-axis component, defined by $e''_q=-\omega\lambda_q$, but it will not be used as a state, since we have E'_q.)

There are three basic steps to the development of this model given in the book. I refer to these steps as Step A, Step B, and Step C. Each one has several sub-steps, as summarized in what follows:

<u>Step A: Derive the auxiliary equations.</u>

Step A-1: Derive auxiliary equations for e''_q and e''_d.

1. Substitute expressions for currents i_d and i_q (4.134) into the equations for λ''_d and λ''_q (4.230).
2. Use $\sqrt{3}E'_q=L_{AD}\lambda_F/L_F$ to write in terms of E'_q.
3. Express $e''_q=\omega\lambda_q$ and $e''_d=-\omega\lambda_d$ to get (4.243, 4.245).

Step A-2: Derive the auxiliary equation for E (4.248).
Step A-3: Derive the auxiliary equation for i_D.

<u>Step B: Derive the differential equations.</u>

For each of these, we begin from the voltage equation from the corresponding winding.

Step B-1: D-axis damper: Derive differential equation for λ_D.
Step B-2: Q-axis damper: Derive differential equation for e''_d (which is produced by $d\lambda_q/dt$).
Step B-3: Field winding: Derive differential equation for E'_q (which is produced by $d\lambda_F/dt$).

State equations are given by (4.263, 4.264, 4.265, 4.267, 4.268).

<u>Step C: Convert to a state-space form</u>:

Step C-1: Convert the state variables to stator-side equivalents by dividing by $\sqrt{3}$, and define the constants K_1-K_4.
Step C-2: Bring in the inertial equations. This results in the equations of (4.270) in the text, which are written in the LaPlace domain (with "s" indicating differentiation).
Step C-3: Write equations (4.270) in the time domain and express them in matrix form to get a state-space model (your book does not do this part, so you do it).
NOTE: The remainder of these notes are incomplete. Please read your text p. 142-155.

$$\begin{bmatrix} \dot{E}_d^{''} \\ \dot{\Lambda}_d \\ \dot{E}_q^{'} \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{bmatrix} E_d^{''} \\ \Lambda_d \\ E_q^{'} \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

Note: We have 5 states in this model:

$$\delta, \omega, E''_d, \Lambda_d, E'_q$$

but we are modeling the following windings:

d, q, F, D, Q

With the E'_q model (model 4), we only had 3 windings:

d, q, F

but we also had 5 states

$$\delta, \omega, \Lambda_d, \Lambda_q, E'_q$$

Why is it that we are modeling more windings in the E" model than in the E'_q model, but we have the same number of states????

Because in the E" model, we set $d\lambda_d/dt= d\lambda_q/dt=0$, thus eliminating two stator states.

In this model, we have E=d+q+2-N=2+1+2-2=3, and so the number of states is E+2=5.

Model 6 (1.1): "Two-axis model" for a machine with solid round rotor & dλ_d/dt= dλ_q/dt=0 (pg. 138-140), Section 4.15.3

Overview

This model accounts for the transient effects but not the subtransient effects. It includes only two rotor circuits, F and G. So we are neglecting the D- and Q- damper windings. A&F say "The transient effects are dominated by the rotor circuits which are the field circuit in the d-axis and an equivalent circuit in the q-axis formed by the solid rotor."

The "equivalent circuit in the q-axis formed by the solid rotor" is the G-circuit (although A&F do not call it that in their second edition).

A&F also write, "An additional assumption made in this model is that in the stator voltage equations the terms λ_d-dot and λ_q-dot are negligible compared to the speed voltage terms…" This means that we let dλ_d/dt= dλ_q/dt=0, as in the E'' model.

A&F derive two state equations for this model, which are:

$$\dot{E}'_d = \frac{1}{\tau'_{q0}}(-E'_d - (x_d - x'_q)I_q$$

$$\dot{E}'_q = \frac{1}{\tau'_{d0}}(E_{FD} - E) \qquad \text{(4.288), (4.290)}$$

In this model, we have E=d+q+2-N=1+1+2-2=2, and so the number of states is E+2=4.

I will derive the above equations in what follows.

Derivation of (4.288)

From (4.282) we have

$$\lambda_G = L_{AQ} i_q + L_G i_G \quad \text{pu} \qquad (4.282)$$

Differentiating, we obtain:

$$\dot{\lambda}_G = L_{AQ} \dot{i}_q + L_G \dot{i}_G \quad \text{pu} \qquad (4.282')$$

The G-cct voltage equation is

$$r_G i_G + \dot{\lambda}_G = 0$$

Substitute (4.282') in the G-cct voltage equation to obtain:

$$r_G i_G + L_{AQ}\dot{i}_q + L_G\dot{i}_G = 0 \qquad (*)$$

From (4.286) we have

$$\sqrt{3}E_d = e_d = -L_{AQ} i_G \text{ pu}$$

$$\Rightarrow i_G = \frac{-\sqrt{3}}{L_{AQ}} E_d \qquad (4.286')$$

$$\Rightarrow \dot{i}_G = \frac{-\sqrt{3}}{L_{AQ}} \dot{E}_d \qquad (4.286'')$$

We also have from (4.47) (and see (5.11)) that

$$\sqrt{3}I_q = i_q \Rightarrow \sqrt{3}\dot{I}_q = \dot{i}_q \qquad (4.286''')$$

Substitution of (4.286'), (4.286''), and (4.286''') into (*) results in

$$-r_G \frac{\sqrt{3}}{L_{AQ}} E_d + L_{AQ}\sqrt{3}\dot{I}_q - L_G \frac{\sqrt{3}}{L_{AQ}} \dot{E}_d = 0$$

Eliminating the square root of 3, multiplying by -1, and moving terms to the right-hand-side, results in

$$\frac{L_G}{L_{AQ}} \dot{E}_d = L_{AQ}\dot{I}_q - \frac{r_G}{L_{AQ}} E_d \qquad (\#)$$

Now consider the right-hand equation of (4.287) and its derivative:

$$E_d - x_q I_q = E'_d - x'_q I_q$$

$$\Rightarrow E_d = E'_d + (x_q - x'_q) I_q$$

$$\dot{E}_d = \dot{E}'_d + (x_q - x'_q)\dot{I}_q$$

Substituting into (#) results in

$$\frac{L_G}{L_{AQ}}\left(\dot{E}'_d + (x_q - x'_q)\dot{I}_q\right) = L_{AQ}\dot{I}_q - \frac{r_G}{L_{AQ}}\left(E'_d + (x_q - x'_q) I_q\right)$$

Now expand the terms and then multiply through by L_{AQ}/r_G to obtain

$$\frac{L_G}{r_G}\dot{E}'_d + \frac{L_G}{r_G}(x_q - x'_q)\dot{I}_q = \frac{L_{AQ}^2}{r_G}\dot{I}_q - \left(E'_d + (x_q - x'_q) I_q\right)$$

Now gather terms in $\dot{I}_q$

$$\frac{L_G}{r_G}\dot{E}'_d + \frac{1}{r_G}\left(L_G(x_q - x'_q) - L_{AQ}^2\right)\dot{I}_q = -\left(E'_d + (x_q - x'_q) I_q\right)$$

and because, in per-unit, $x_q = L_q$ and $x'_q = L'_q$, we can write the left-hand-side of the previous expression as

$$\frac{L_G}{r_G}\dot{E}'_d+\frac{1}{r_G}\left(L_G(L_q-L'_q)-L_{AQ}^2\right)\dot{I}_q=-\left(E'_d+(x_q-x'_q)I_q\right) \qquad (*\#)$$

Now we recall from (4.180) that

$$L'_q=L_q-\frac{L_{AQ}^2}{L_G} \qquad (4.180)$$

Substitution of (4.180) into the left-hand-side of (*#) results in

$$\frac{L_G}{r_G}\dot{E}'_d+\frac{1}{r_G}\left(L_G(L_q-L_q+\frac{L_{AQ}^2}{L_G})-L_{AQ}^2\right)\dot{I}_q=-\left(E'_d+(x_q-x'_q)I_q\right)$$

which becomes

$$\frac{L_G}{r_G}\dot{E}'_d+\frac{1}{r_G}\left(L_{AQ}^2-L_{AQ}^2\right)\dot{I}_q=-\left(E'_d+(x_q-x'_q)I_q\right)\Rightarrow\frac{L_G}{r_G}\dot{E}'_d=-E'_d-(x_q-x'_q)I_q$$

And using $\tau'_{q0}=L_G/r_G$ from (4.289) we obtain

$$\tau'_{q0}\dot{E}'_d=-E'_d-\left(x_q-x'_q\right)I_q$$

which is (4.288) in the A&F text.

QED

<u>Derivation of (4.290)</u>

From (4.280a)

$$\lambda_F=L_{AD}i_d+L_Fi_F \quad \text{pu}$$

Differentiating, we obtain:

$$\dot{\lambda}_F=L_{AD}\dot{i}_d+L_F\dot{i}_F \quad \text{pu}$$

Substitute the last equation into the voltage equation (4.127)

$$\upsilon_F=r_Fi_F+\dot{\lambda}_F\Rightarrow r_Fi_F+L_{AD}\dot{i}_d+L_F\dot{i}_F=v_F \qquad (\&)$$

From (4.286) we have

$$\sqrt{3}E=e_q=L_{AD}i_F\ \text{pu}\Rightarrow i_F=\frac{\sqrt{3}E}{L_{AD}}\Rightarrow\dot{i}_F=\frac{\sqrt{3}\dot{E}}{L_{AD}} \qquad (4.286)$$

$$\sqrt{3}I_d=i_d\ \text{pu}\ \Rightarrow\sqrt{3}\dot{I}_d=\dot{i}_d$$

And from (4.209),

$$\sqrt{3}E_{FD}=\frac{L_{AD}}{r_F}v_F\Rightarrow v_F=\frac{r_F}{L_{AD}}\sqrt{3}E_{FD} \qquad (4.209)$$

Substitution of these last two relations into (&) results in

$$r_F\frac{\sqrt{3}E}{L_{AD}}+L_{AD}\sqrt{3}\dot{I}_d+L_F\frac{\sqrt{3}\dot{E}}{L_{AD}}=\frac{r_F}{L_{AD}}\sqrt{3}E_{FD}$$

Divide through by the square root of 3:

$$r_F \frac{E}{L_{AD}} + L_{AD}\dot{I}_d + L_F \frac{\dot{E}}{L_{AD}} = \frac{r_F}{L_{AD}} E_{FD} \qquad \text{(\#0a)}$$

Now recall the left-hand-side of (4.287), and its derivative:

$$E + x_d I_d = E'_q + x'_d I_d$$
$$\Rightarrow E = E'_q + (x'_d - x_d) I_d \quad \dot{E} = \dot{E}'_q + (x'_d - x_d)\dot{I}_d \qquad \text{(\#0b)}$$

Substitution of the last two equations in (#0b) into (#0a) results in

$$\frac{r_F}{L_{AD}}\left(E'_q + (x'_d - x_d) I_d\right) + L_{AD}\dot{I}_d + \frac{L_F}{L_{AD}}\left(\dot{E}'_q + (x'_d - x_d)\dot{I}_d\right) = \frac{r_F}{L_{AD}} E_{FD} \qquad \text{(\#0c)}$$

Now, according to the text, just after (4.280), it says: "By eliminating i_F and using (4.174) and (4.203),

$$\lambda_d - \sqrt{3} E'_q = L'_d i_d \text{ "} \qquad \text{(\#1)}$$

Differentiate (#1) to obtain

$$\dot{\lambda}_d - \sqrt{3}\dot{E}'_q = L'_d \dot{i}_d \qquad \text{(\#2)}$$

But according to the assumptions of this model, $\dot{\lambda}_d = 0$, i.e., the left-hand-side of (#2) is 0, therefore (#2) becomes

$$-\sqrt{3}\dot{E}'_q = L'_d \dot{i}_d \Rightarrow \dot{i}_d = \frac{-\sqrt{3}\dot{E}'_q}{L'_d} \qquad \text{(\#3)}$$

But recall from (4.286)

$$\sqrt{3} I_q = i_q \text{ pu} \quad \Rightarrow \sqrt{3}\dot{I}_q = \dot{i}_q$$

And so (#3) becomes

$$\dot{I}_d = \frac{-\dot{E}'_q}{L'_d} \qquad \text{(\#4)}$$

Substitution of (#4) into (#0c) results in

$$\frac{r_F}{L_{AD}}\left(E'_q + (x'_d - x_d) I_d\right) - L_{AD}\frac{\dot{E}'_q}{L'_d} + \frac{L_F}{L_{AD}}\left(\dot{E}'_q - (x'_d - x_d)\frac{\dot{E}'_q}{L'_d}\right) = \frac{r_F}{L_{AD}} E_{FD}$$

Distributing,

$$\frac{r_F}{L_{AD}} E'_q + \frac{r_F}{L_{AD}}(x'_d - x_d) I_d - L_{AD}\frac{\dot{E}'_q}{L'_d} + \frac{L_F}{L_{AD}}\dot{E}'_q - \frac{L_F}{L_{AD}}(x'_d - x_d)\frac{\dot{E}'_q}{L'_d} = \frac{r_F}{L_{AD}} E_{FD}$$

$$\frac{r_F}{L_{AD}} E'_q + \frac{r_F}{L_{AD}}(x'_d - x_d) I_d - \left(L_{AD} + \frac{L_F}{L_{AD}}(x'_d - x_d)\right)\frac{\dot{E}'_q}{L'_d} + \frac{L_F}{L_{AD}}\dot{E}'_q = \frac{r_F}{L_{AD}} E_{FD}$$

Bring the first term to the right-hand-side:

$$-\underbrace{\left(\mathrm{L}_{AD} + \frac{\mathrm{L}_F}{\mathrm{L}_{AD}}(\mathrm{x}'_d - \mathrm{x}_d)\right)}_{\text{TERM A}}\frac{\dot{E}'_q}{L'_d} + \frac{L_F}{L_{AD}}\dot{E}'_q = \frac{r_F}{L_{AD}} E_{FD} - \frac{r_F}{L_{AD}} E'_q - \frac{r_F}{L_{AD}}(x'_d - x_d) I_d \qquad \text{(\#5)}$$

Now consider TERM A of (#5), accounting for the fact that in per-unit, $x'_d = L'_d$ and $x_d = L_d$, and recalling that $L'_d = L_d - L_{AD}^2 / L_F$:

$$\mathrm{L_{AD}} + \frac{\mathrm{L_F}}{\mathrm{L_{AD}}}(\mathrm{x'_d}\text{-}\mathrm{x_d}) = \mathrm{L_{AD}} + \frac{\mathrm{L_F}}{\mathrm{L_{AD}}}(L'_d - L_d) = \mathrm{L_{AD}} + \frac{\mathrm{L_F}}{\mathrm{L_{AD}}}\left(L_d - \frac{L_{AD}^2}{L_F} - L_d\right) = \mathrm{L_{AD}} - \frac{\mathrm{L_F}}{\mathrm{L_{AD}}}\frac{L_{AD}^2}{L_F} = 0$$

Application of the above TERM A into (#5) results in

$$\frac{L_F}{L_{AD}}\dot{E}'_q = \frac{r_F}{L_{AD}}E_{FD} - \frac{r_F}{L_{AD}}E'_q - \frac{r_F}{L_{AD}}(x'_d - x_d)I_d \qquad (\#6)$$

Multiply through by L_{AD}/L_F to obtain

$$\dot{E}'_q = \frac{r_F}{L_F}E_{FD} - \frac{r_F}{L_F}E'_q - \frac{r_F}{L_F}(x'_d - x_d)I_d \qquad (\#7)$$

Factor out the r_F/L_F from the last two terms:

$$\dot{E}'_q = \frac{r_F}{L_F}E_{FD} - \frac{r_F}{L_F}\left(E'_q + (x'_d - x_d)I_d\right) \qquad (\#8)$$

Recall from the left-hand-side of (4.287) that

$$E = E'_q + (x'_d - x_d)I_d$$

which is recognized in the parentheses of the last term on the right-hand-side of (#8), so that we have

$$\dot{E}'_q = \frac{r_F}{L_F}E_{FD} - \frac{r_F}{L_F}E \qquad (\#9)$$

Recalling from (4.189) that $\tau'_{d0} = L_F/r_F$, (#9) becomes

$$\dot{E}'_q = \frac{1}{\tau'_{d0}}E_{FD} - \frac{1}{\tau'_{d0}}E \qquad (\#10)$$

which is equation (4.290) in the A&F text.

QED

Model 7 (1.0): One-axis model (4.15.4)

Here, we neglect the dynamics of E'_d.

In this model, we have E=d+q+2-N=1+0+2-2=1, and so the number of states is E+2=3.

Model 8 (1.0): Classical model.

Here, we assume constant E'_q.

In this model, we have E=d+q+2-N=0+0+2-2=0, and so the number of states is E+2=2.

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