Power Angle & Time Domain Plots

Initial comment: Figs. 2.15, 2.16 in your VMAF text are to be compared; Fig. 2.15 is correct, but Fig. 2.16 is wrong. I have pasted both figures below to facilitate the comparison; for Fig. 2.16, I pasted incorrect (left) & correct (right) versions. The correct version is from an earlier edition of your text.

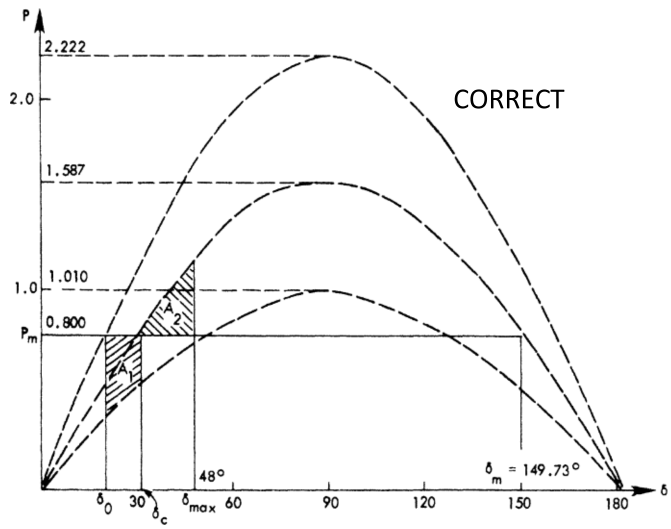


Fig. 2.15: Application of the equal area criterion to a stable system

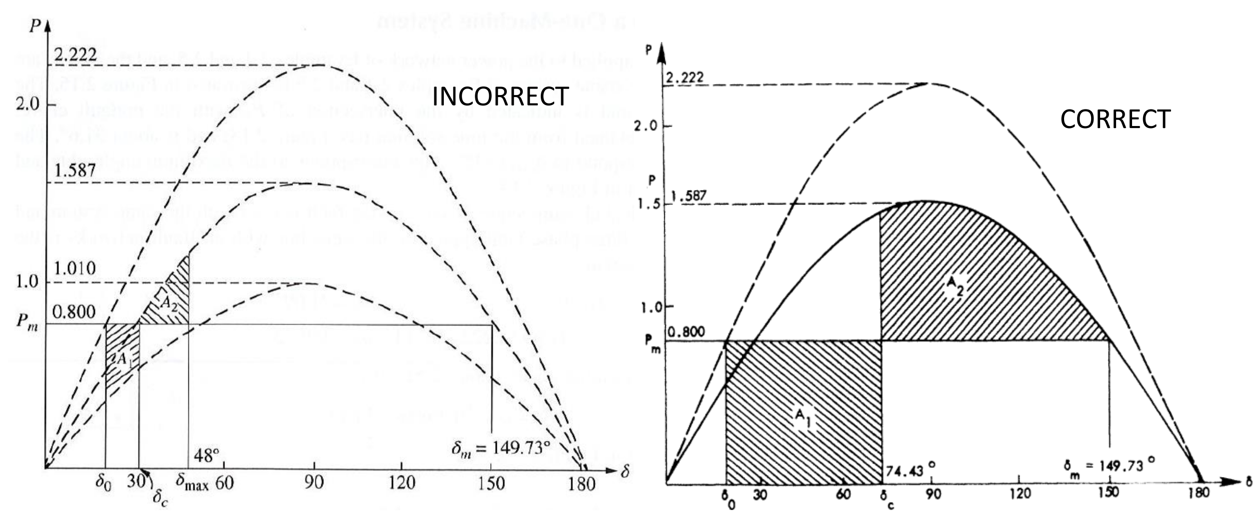


Fig. 2.16: Application of equal area criterion to critically cleared system: left - incorrect; right - correct

1.0 The power-angle curves

Consider a generator connected to an infinite bus through a network, as shown in Fig. 1 below (all in pu).

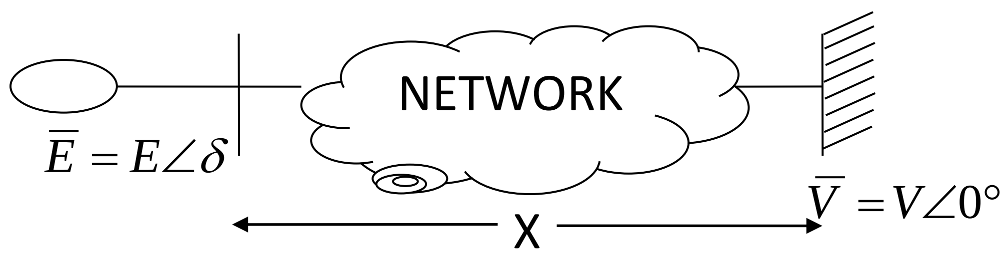


Fig. 1

We assume the network is lossless (all R=0), there is no station load, the machine is modeled classically, and that X includes network, transformer, and machine reactance. We consider 3 situations, consistent with previous notes.

1. Steady-state operating conditions (pre-fault):

$$P_{e1} = P_{M1}\sin\delta \tag{1}$$

2. Fault-on conditions:

$$P_{e2} = P_{M2}\sin\delta \tag{2}$$

3. Post-fault conditions:

$$P_{e3} = P_{M3}\sin\delta \tag{3}$$

Inspection of (1), (2), and (3) indicates that the electrical power out of a machine is a sinusoidal function of angle δ. We can and will explore plots of these three power-angle curves, but before doing that, let's observe that in general, we will have:

$$P_{M2} < P_{M3} \le P_{M1} \tag{4}$$

$$P_{Mi} = \frac{|E||V|}{X_i} \tag{5}$$

where we assume that $|E|$ and $|V|$ are constant throughout the sequence, and X_i is the impedance between the internal machine voltage and the infinite bus voltage under conditions i=1, 2, 3.

Some comments about X_i:

- For a fault at the machine terminals, $X_2=\infty$, because $X_2=(\mathbf{E}-\mathbf{V})/\mathbf{I}$ where **I** is the current flowing out of the generator into the network [or out of the network into the infinite bus with all other network sources and sinks idled (set to 0)]. When the machine terminals are short-circuited, **I**=0, so $X_2=\infty$. Having $X_2=\infty$ makes $P_{M2}=0$.

- In general, faults occur away from the terminals, in the network, and so $P_{M2}>0$.
- It is possible for $P_{M3}=P_{M1}$. How?

Answer: $P_{M3}=P_{M1}$ if the fault is only temporary, in which case the post-fault network is the same as the pre-fault network.

We draw the power-angle curves, Fig. 2, which show the dependence of P_{ei} on δ. It also shows the mechanical power, P_m, the pre-fault stable equilibrium angle δ_1 ($P_{e1}=P_m$), the post-fault stable equilibrium angle δ_3 ($P_{e3}=P_m$), & the post-fault unstable equilibrium angle δ_m. (We use δ_m here to be consistent with the nomenclature in Section 2.8 of the text; however, this δ_m should not be confused with the use of δ_m in previous notes as the angle of the rotor in mechanical radians.)

Recall that δ is relative to a moving, synchronous reference; here, we define ω to be the *relative speed* so that it is zero if $\omega_m=\omega_R$.

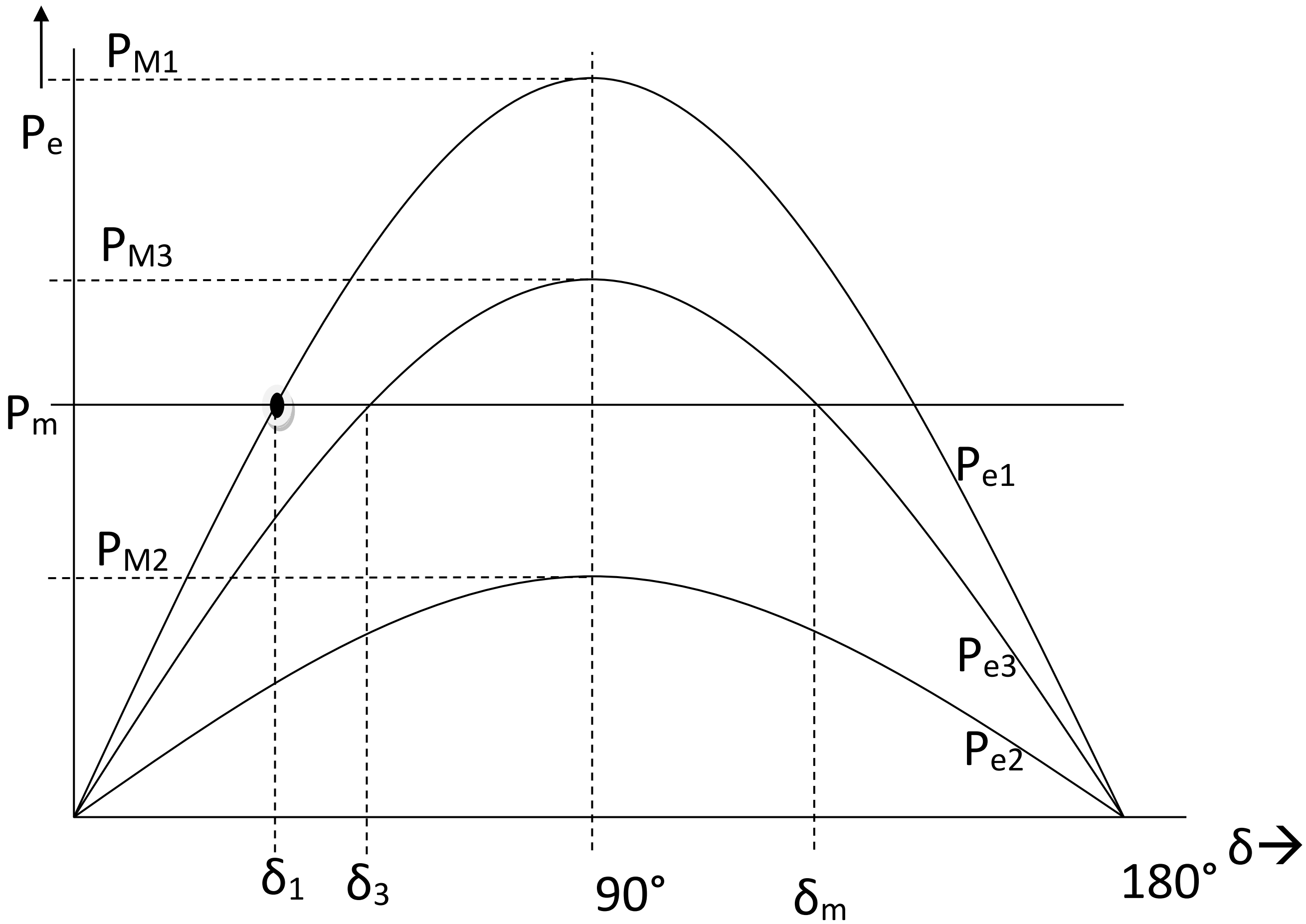


Fig. 2

Now we move through the sequence of events associated with occurrence of a fault, illustrating each change on the power-angle curves of Fig. 3. We focus on

- the sign of $P_a=P_m-P_e$, which determines whether ω is increasing or decreasing (note: ω may be increasing or decreasing while being either positive or negative), and
- the sign of ω, which determines whether δ is increasing or decreasing.

0. At δ_1, under the pre-fault condition, P_m-P_{e1}=0 ➔ P_a=0.

1. At t=0^+, fault occurs, and P_e=P_{e2}. The angle remains δ_1.

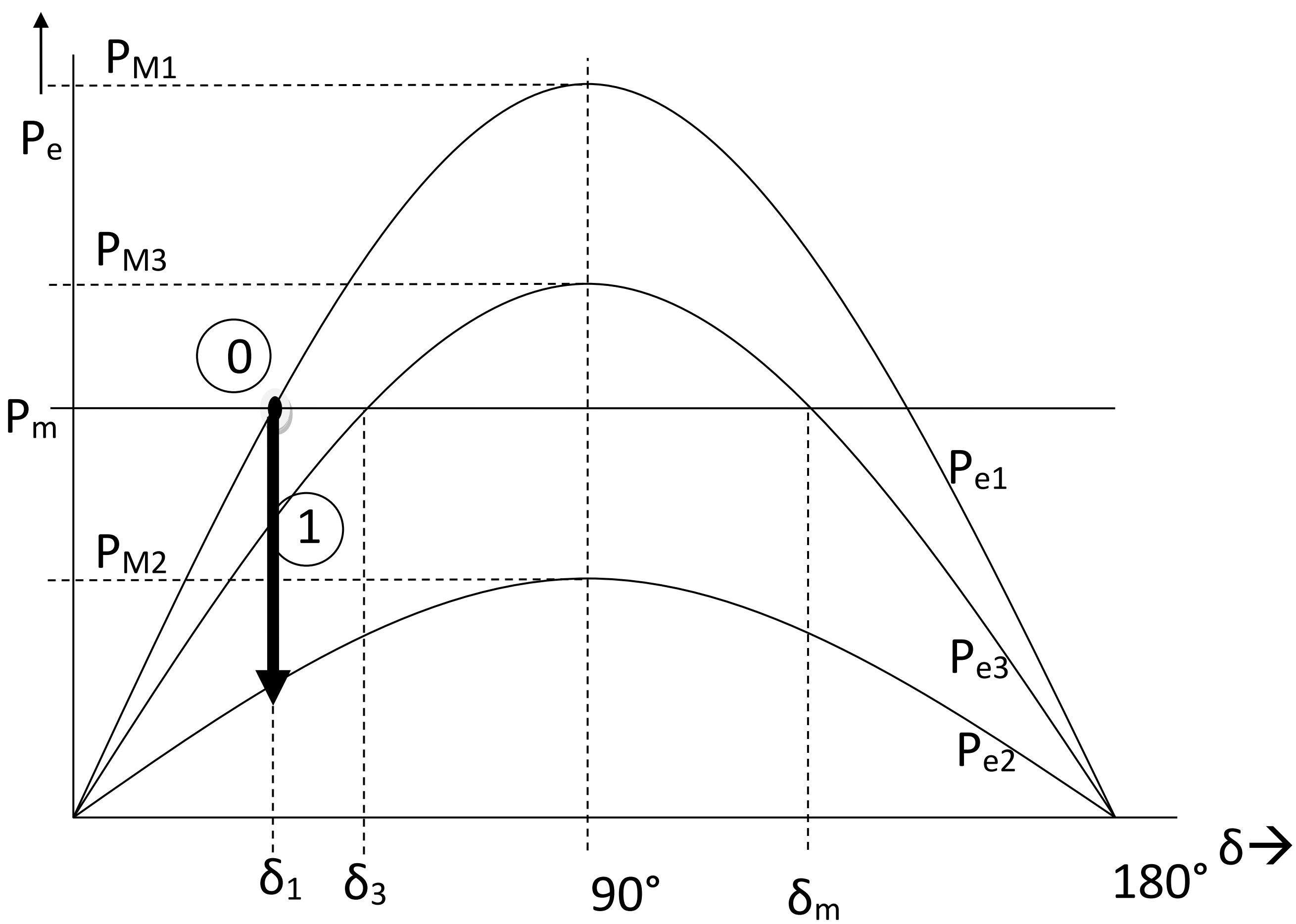


Fig. 3

2. From $t=0^+$ to clearing time t_c (corresponding to clearing angle δ_c), $P_m - P_{e2} = P_{a2} > 0$. So the machine accelerates, ω increases and is necessarily positive (since it was previously 0), and δ increases along the P_{e2} curve.

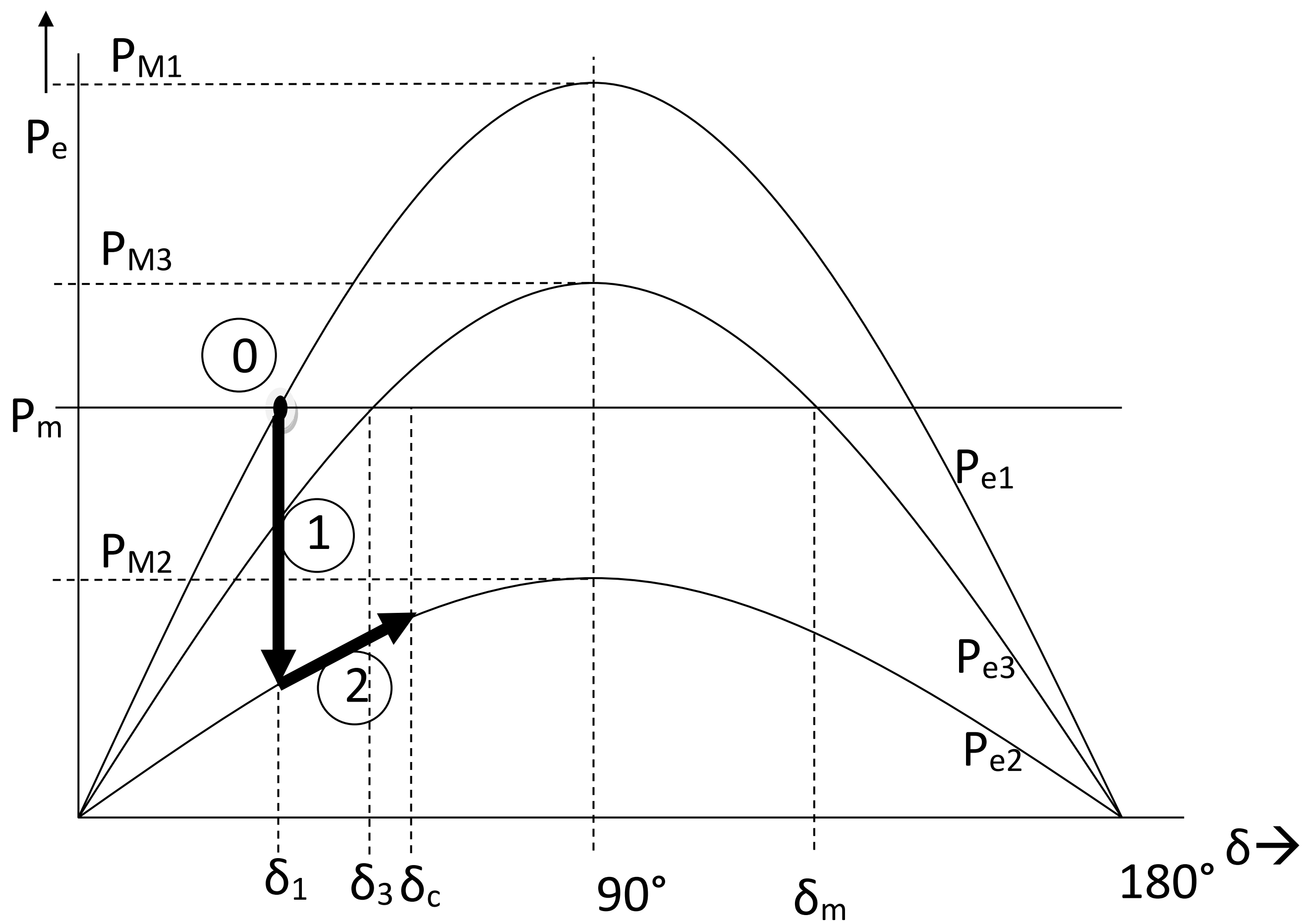


Fig. 4

3. At clearing time $t=t_c$ (corresponding to clearing angle δ_c), the breakers operate and remove the faulted line. Electric power P_e immediately changes (increases) to $P_{e3}(\delta_c)$, but δ does not change.

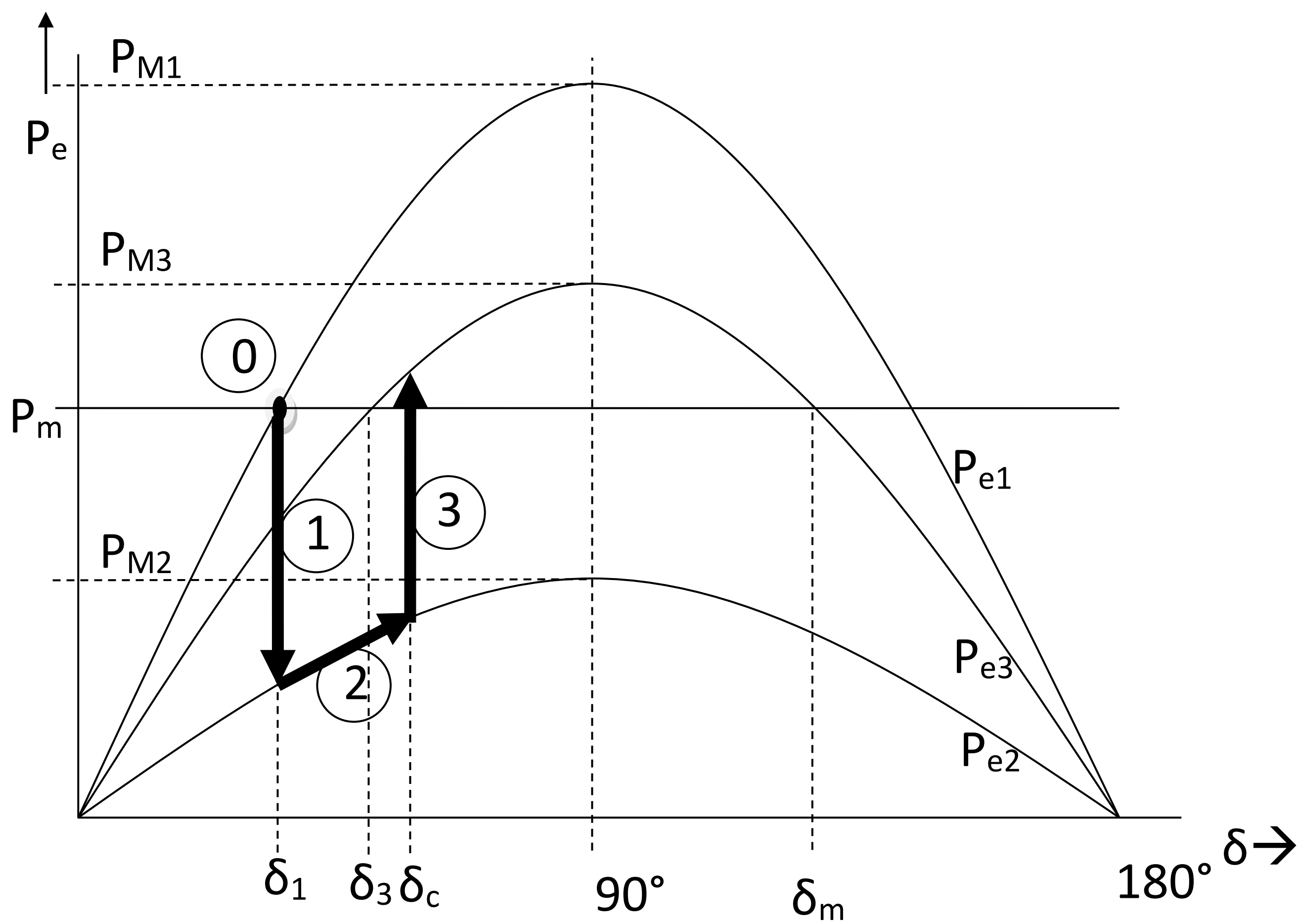


Fig. 5

4. For t just after clearing time t_c, P_m-P_{e3}=P_{a3}<0. The fact that accelerating power is negative means that the machine decelerates, i.e., its speed decreases (as long as $\delta<\delta_m$, implying a stable swing). However, ω is still positive, i.e., the angle δ is increasing, and it continues to increase until the speed reaches 0, at, say, δ_r. If the swing is stable, then at that angle δ_r, the speed reaches 0 and the angle δ begins to decrease.

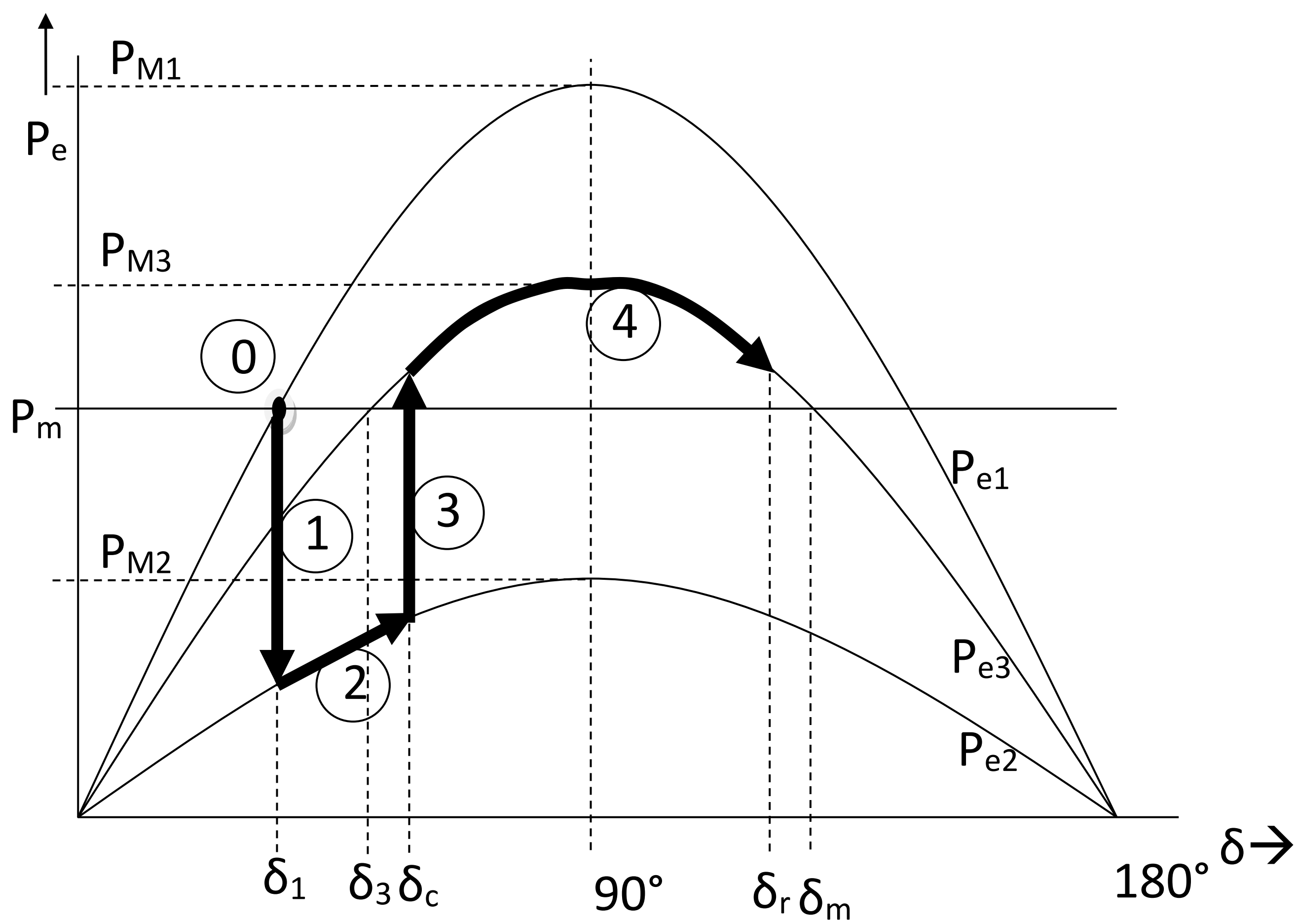


Fig. 6

5. The angle decreases from δ_r because speed is negative. Speed continues to be negative (actually becomes more negative) because $P_a<0$. This continues until $\delta=\delta_3$.

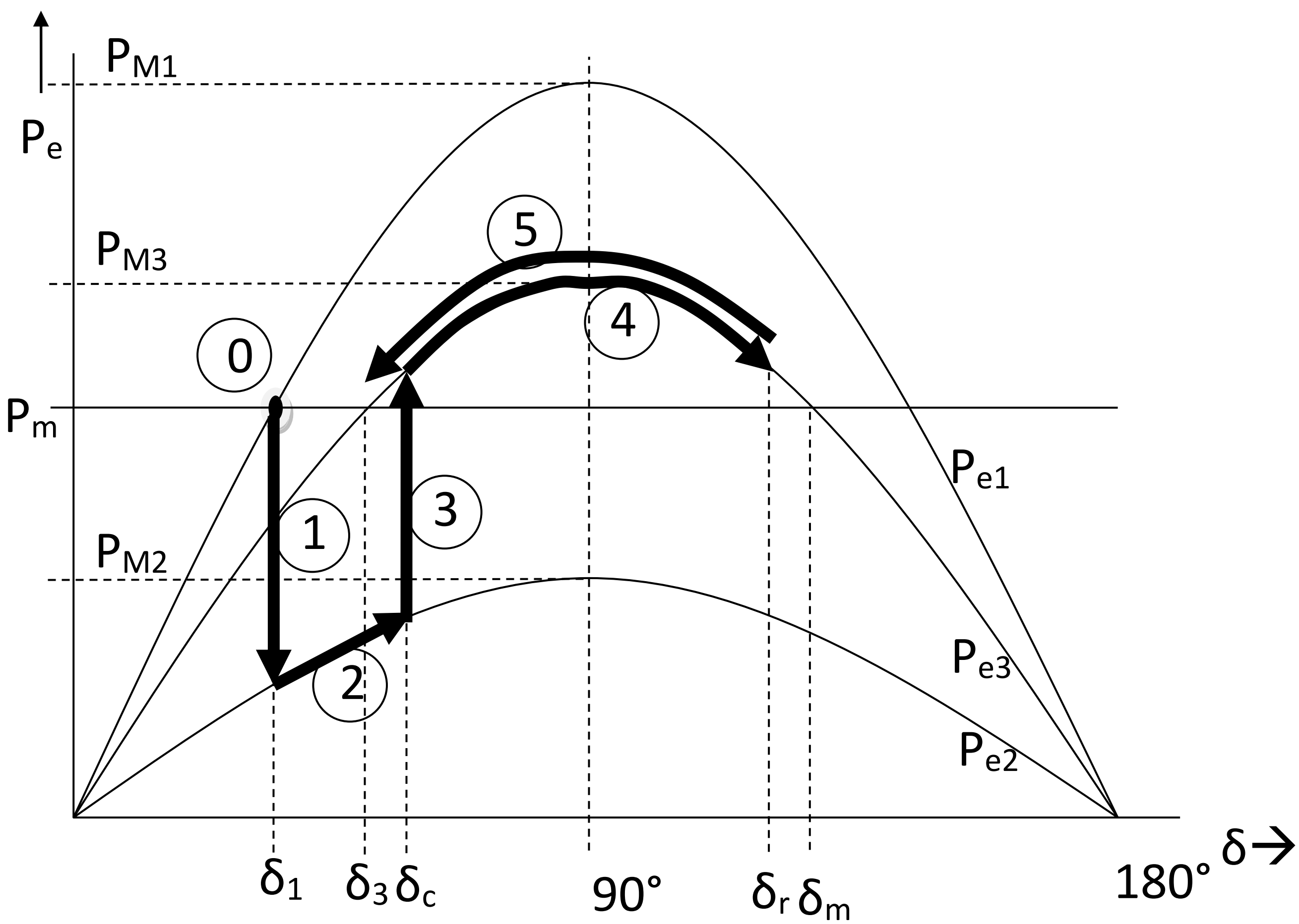


Fig. 7

6. As the rotor swings through the point $\delta=\delta_3$, accelerating power goes positive, i.e., $P_m-P_{e3}=P_{a3}>0$, and speed begins to decrease (becomes less negative). The rotor will swing to some minimum angle, where its speed will reach 0, it will turn around, and come back.

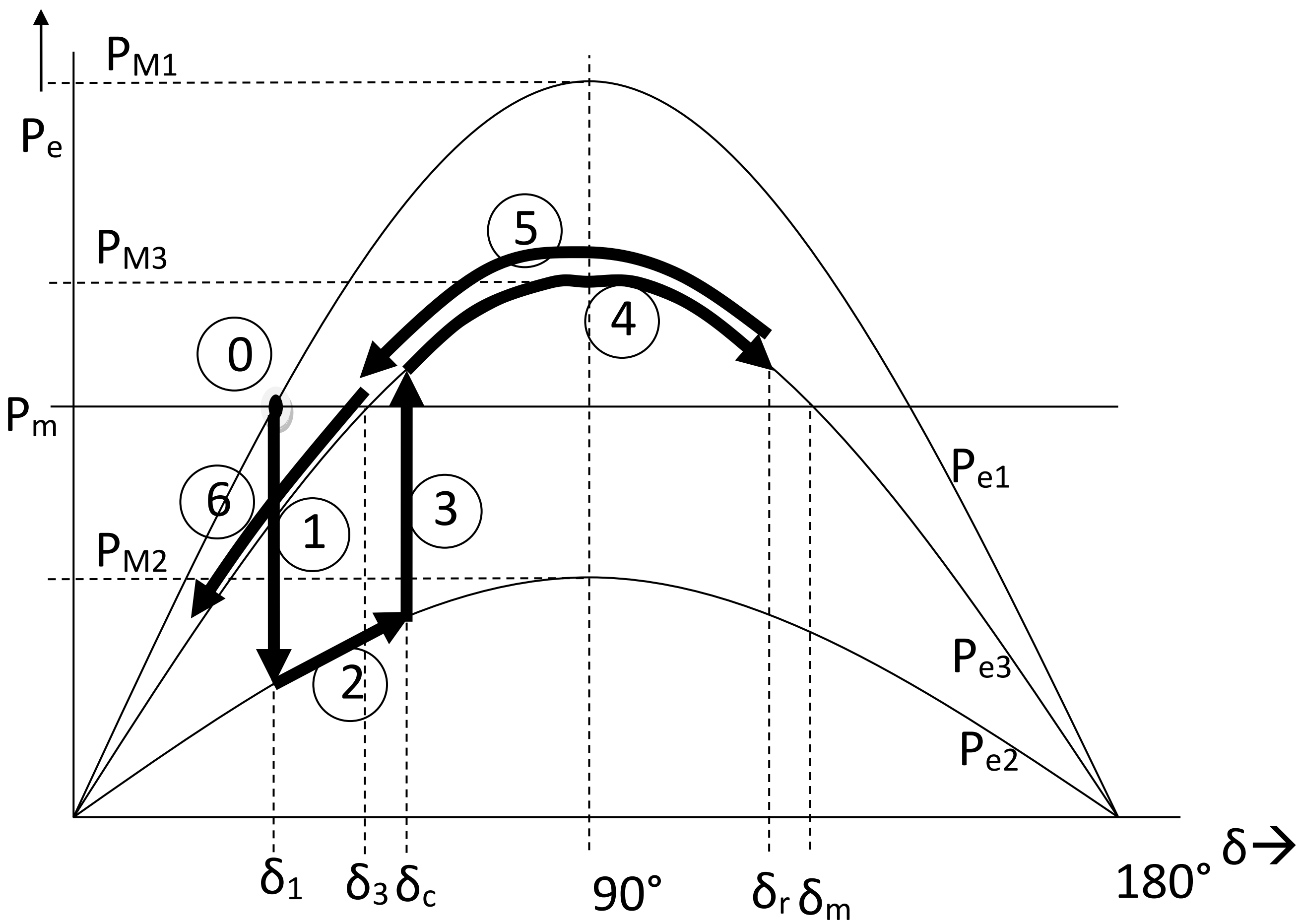


Fig. 8

7. The rotor will then oscillate back and forth about the post-fault stable equilibrium point δ_3. If there is no damping, the oscillations will continue forever. If there is damping, the oscillations will continuously decrease in amplitude, and after some time, the system will stabilize at the post-fault stable equilibrium point δ_3.

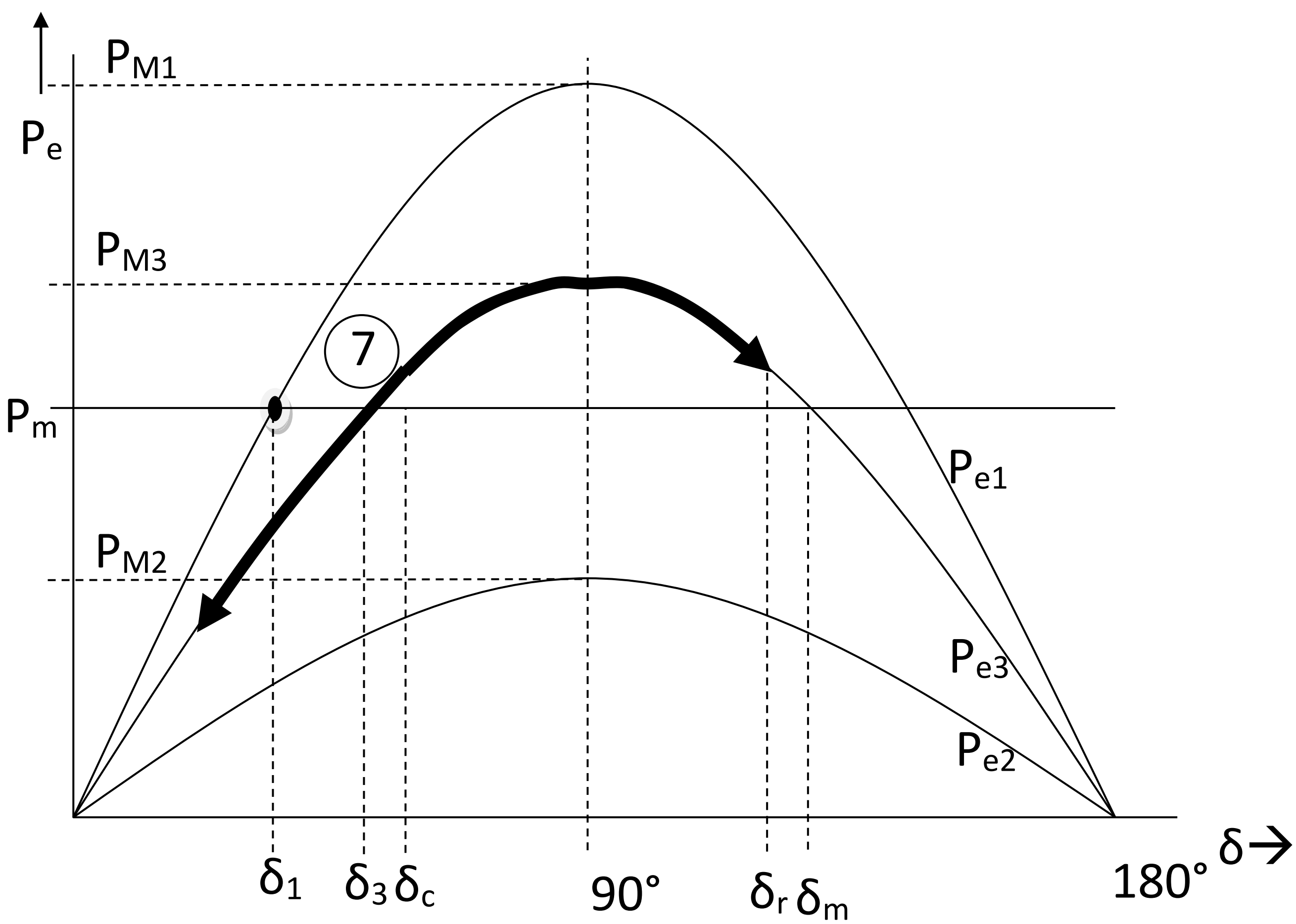


Fig. 9

2.0 Time-domain plots

We return to the example we worked on in the last set of notes where we had:

Pre-fault: $\ddot{\delta}(t) = 30.16 - 83.8071\sin\delta$

Fault-on: $\ddot{\delta}(t) = 30.16 - 38.0958\sin\delta$

Post-fault: $\ddot{\delta}(t) = 30.16 - 59.8638\sin\delta$

We can observe the oscillations described in the last section by plotting the two state variables against time. To do this, we need to convert the second-order equation to two first order equations. We will do this to the fault-on equation and to the post-fault equation. Recognizing that $\dot{\delta}(t) = \omega(t)$ and $\ddot{\delta}(t) = \dot{\omega}(t)$, we may write the fault-on swing equation as:

$$\begin{aligned} \dot{\delta}(t) &= \omega(t) \\ \dot{\omega}(t) &= 30.16 - 38.0958\sin\delta \end{aligned} \tag{29}$$

Similarly, the post-fault equation may be written as

$$\begin{aligned} \dot{\delta}(t) &= \omega(t) \\ \dot{\omega}(t) &= 30.16 - 59.8638\sin\delta \end{aligned} \tag{30}$$

Solution to these equations must be done numerically. We have not yet discussed numerical solutions of differential equations, but we will later in the course. For now, I simply provide you with the necessary Matlab commands.

Before doing this, however, it is necessary to realize our problem is an initial value problem, i.e., in order to solve it, we must know initial values. Since our variables are δ and ω, we observe that the initial values we must find are δ(0) and ω(0).

Of course, the initial value of ω, speed deviation, is very simple. Since the system is initially "at rest," the initial speed deviation is zero.

But what about the angle δ? To get this, we return to the pre-fault equation, and solve it for $P_a=P_m-P_{e1}=0$, i.e.,

$$\ddot{\delta}(t) = 30.16 - 83.8071\sin\delta = 0$$
$$\Rightarrow 83.8071\sin\delta = 30.16$$
$$\Rightarrow \delta = 0.3681 rad$$

And so we see that δ(0)=0.3681 radians (21.09°). This is what we have called δ_1 in all of our power-angle plots.

Now we are in a position to perform numerical solution. Note that Matlab offers a number of different solvers for ordinary differential equations. I choose ode45. The Matlab code is below (damping is zero).

```
%initial conditions have delta at 21.09 degrees and omega at 0.
delta0=21.09*pi/180;
X0=[delta0;0];
% Simulation time from 0 to 4 cycles.
TCLEAR=4/60;
TSPAN = [0  TCLEAR];
%call ODE solver to integrate from 0 to TCLEAR
[t, x]=ODE45('go1',TSPAN,X0);
%
% Now prepare to perform the post-fault integration.
%
% First, get initial conditions for post-fault
% integration. These are the final conditions
% for the fault-on integration.
last=length(t);
X0=[x(last,1); x(last,2)];
%Save solution to another matrix so it does
%not get overwritten. But do not save last point
%because last point will be first point in
%new solution.
xold=x(1:last-1,:);
told=t(1:last-1);
% Simulation time from 4 cycles to 5 seconds
TSPAN = [TCLEAR   5];
%call ODE solver to integrate from TCLEAR to 5 seconds.
[t, x]=ODE45('go2',TSPAN,X0);
% Now join xold with x.
delta=[xold(:,1)' x(:,1)']';
delta=delta*180/pi;
omega=[xold(:,2)' x(:,2)']';
ttot=[told' t']';
plot(ttot,delta)
grid
ylabel('Angle, delta (degrees)')
xlabel('Time (seconds)')
plot(ttot,omega)
grid
xlabel('Time (seconds)')
ylabel('Speed deviation, omega (rad/sec)')
```

The two functions “go1” and “go2” are given below.

```
% FUNCTION
function xp1 = go1(t,x)
%
% In:    t (time) is a scalar.
%        x is a 2-element vector whose elements are:
%           x(1)= delta(t)
%           x(2)= omega(t)
%
% Out:   xp1 is a 2-element vector whose elements are
%           xp1(1)=d delta(t)/dt
%           xp1(2)=d omega(t)/dt
%
xp1(1)=x(2);
xp1(2)=30.16-38.095*sin(x(1));
% ODE45 expects the function to return a column vector.
xp1=xp1';
```

```
% FUNCTION
function xp2 = go2(t,x)
%
% In:    t (time) is a scalar.
%        x is a 2-element vector whose elements are:
%           x(1)= delta(t)
%           x(2)= omega(t)
%
% Out:   xp2 is a 2-element vector whose elements are
%           xp2(1)=d delta(t)/dt
%           xp2(2)=d omega(t)/dt
%
xp2(1)=x(2);
xp2(2)=30.16-59.8638*sin(x(1));
% ODE45 expects the function to return a column vector.
xp2=xp2';
```

The resulting plots are shown in Figs. 10 and 11.

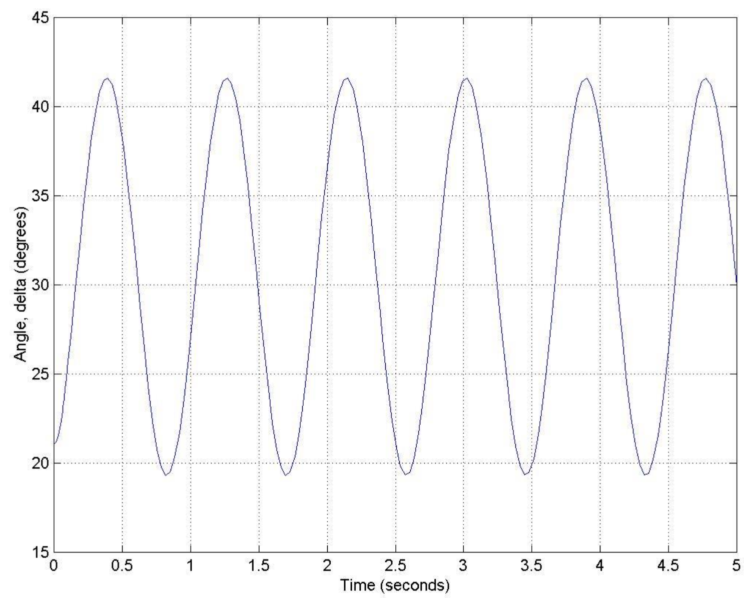


Fig.10: Angle

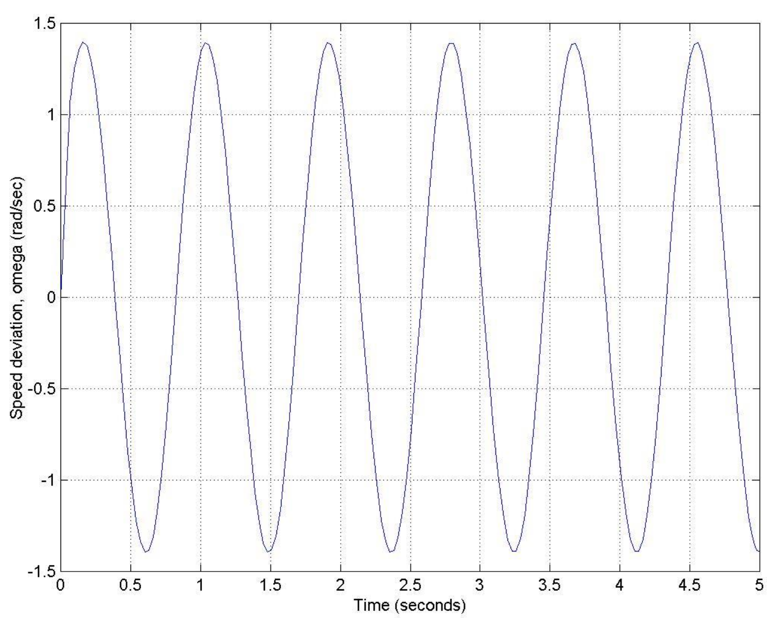


Fig. 11: Speed deviation

Observe in the plots the following:

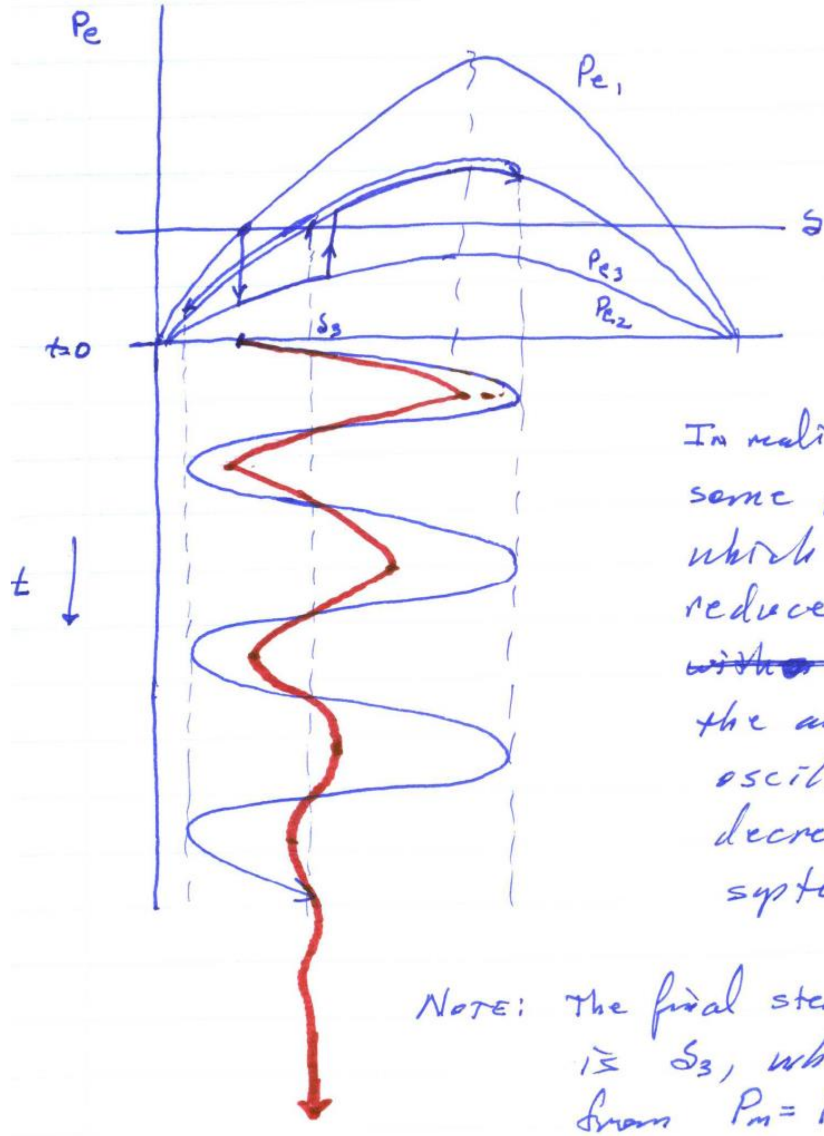
- Initial angle is 21.09°; initial speed deviation is 0.
- Oscillations are undamped.
- Angle oscillates about a value just above 30°. We can verify this by solving the post-fault equation for the equilibrium as follows:

$$\ddot{\delta}(t) = 30.16 - 59.8638 \sin \delta = 0$$
$$\Rightarrow 59.8638 \sin \delta = 30.16$$
$$\Rightarrow \delta = 0.528 rad$$

  This is 30.2524°.

- Speed deviation oscillates about a value of 0.

We have developed two different ways of understanding synchronous machine response to faults: the power angle curve and the time-domain simulation. You should understand how these two ways of viewing the situation are related. Fig. 12 on the next page illustrates.



In reality, we will have some positive damping which serves to always reduce the acceleration ~~with each~~ so that the amplitude of the oscillation always decreases for a stable system. See red curve.

NOTE: The final steady state angle is δ_3, which is found from $P_m = P_{e_3} = \frac{|E||V_\infty|}{X_3} \sin \delta_3$

Fig. 12

3.0 Additional observations & clarifications

We discuss two significant concepts in this section.

a. *Stability vs. instability*

It is possible for the machine to accelerate too much so that the angle does not "come back" after the first swing. What causes this to happen?

➔ It is too much acceleration, or, equivalently, not enough deceleration.

Notice that

- P_m-P_{e2} causes the acceleration, and this occurs between $t=0^+$ and $t=t_c$. During this time interval, our system will be "safer" if we decrease P_m (e.g., fast valving) and increase P_{M2} (e.g., transient excitation boosting) (recall P_{M2} is the maximum value of the fault-on-power-angle curve).
- P_{e3}-P_m causes the deceleration, and this just after $t=t_c$. During this time interval, our system will be "safer" if we increase P_{M3} and decrease P_m.

(Aside: Indeed, there are “special protection systems” (also known as “remedial action schemes”) which perform fast valving and transient excitation boosting to do just these things! See papers on website for these issues.)

Overall conclusion:

The system is “less stable” when

- P_m is high; then acceleration is larger and deceleration is smaller.

  ➔ “Worst-case” stability performance occurs when you are operating a machine at maximum output, and so this condition is what you study in design/planning.

- P_{e2} is low.

➔The worst case is when P_{e2}=0. This occurs when the fault is at the machine terminals. Fig. 13 illustrates.

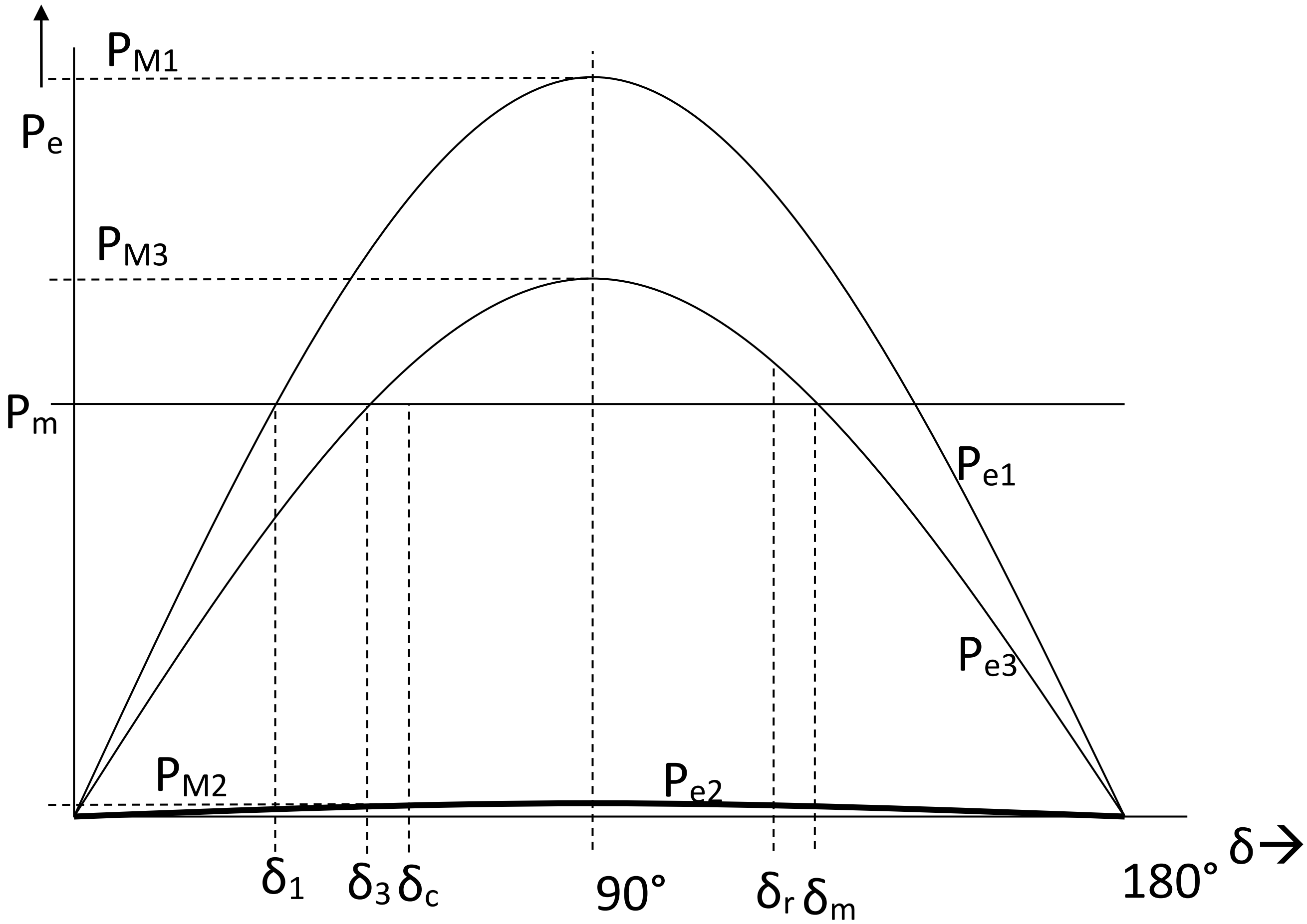


Fig. 13

- t_c is large; then we will see $P_m - P_{e2} = P_{a2} > 0$ for a longer time, & the machine has more time to accelerate.

➔So-called "stuck breaker operation" requires backup protection to clear the fault, and backup protection always takes longer time than primary protection.

- P_{e3} is low.

➔Losing "strong" lines is worse than losing "weak" lines.

b. ***Unstable equilibrium point***

What about the point δ_m?

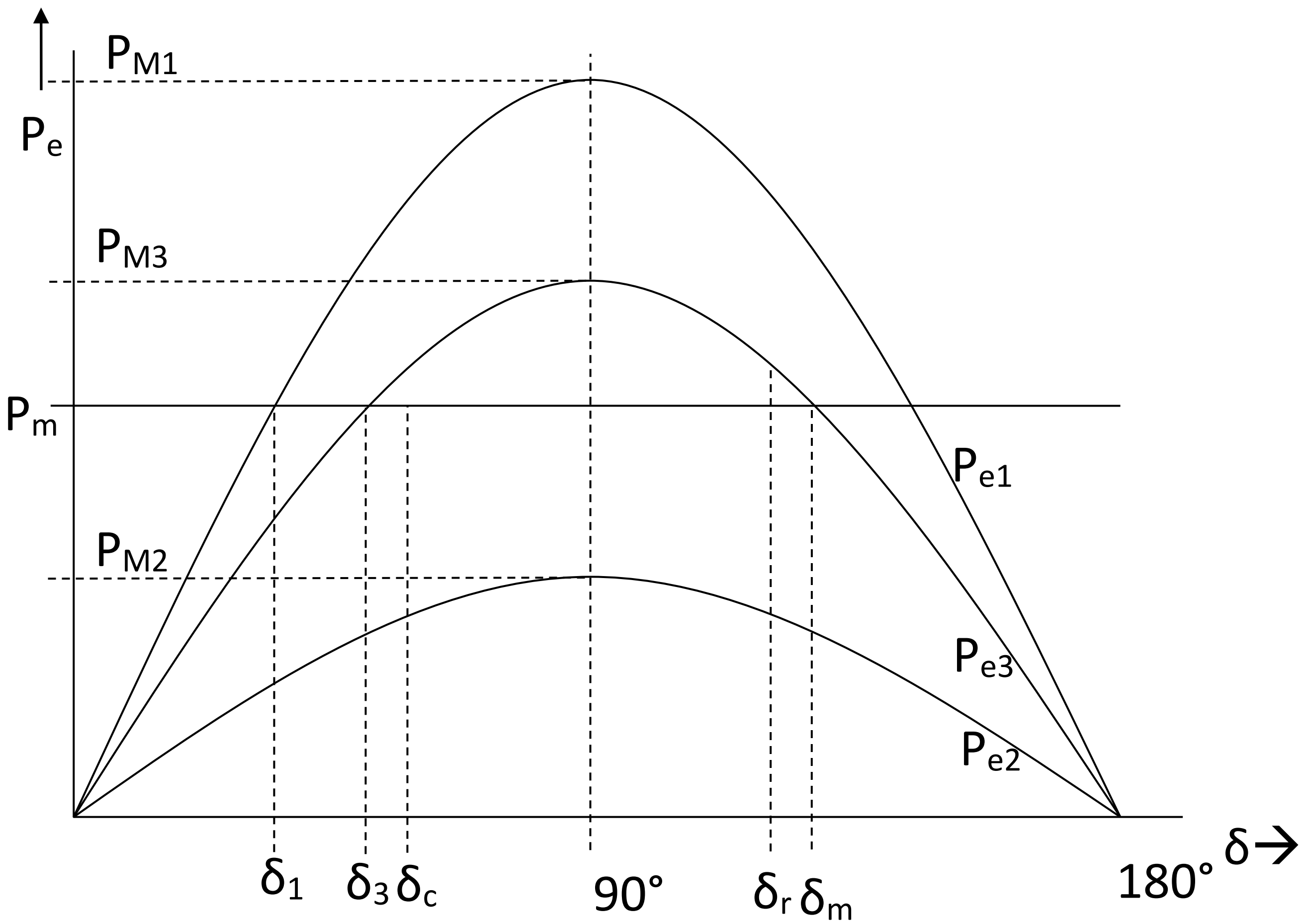


Fig. 14

- Notice that it can be computed as δ_m=180- δ_3. In the case of our example, this is 180-30.25=149.75°.
- This is an *unstable equilibrium point*. Why?

➔If you were ever able to maneuver a generator to this point, with ω=0 (and thus in equilibrium), then

- Any disturbance causing an angle decrease would immediately result in deceleration because $P_m<P_{e3}$. The deceleration would cause negative speed, and further angle decrease, until the machine reached δ_3. At that point, if further angle decrease occurred, then the machine would begin accelerating, tending to reduce (make smaller in absolute value) the negative speed, eventually reaching zero speed at which point the speed would become positive, and the angle would begin to move back towards the stable equilibrium point δ_3 about which it would oscillate; eventually, the machine would settle on the stable equilibrium point δ_3.
- Any disturbance causing an angle increase would immediately result in acceleration because $P_m>P_{e3}$. The acceleration would cause further angle increase, which would cause more acceleration,

and so on, with the angle increasing without bound. This is an unstable situation.

Note that this is similar to the case of the inverted pendulum, where the point at the top of the pendulum's apex is an unstable equilibrium point, in that movement in either direction causes the system to find a new equilibrium point, as shown in Fig. 15.

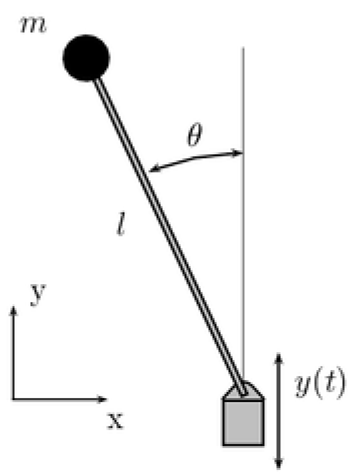


Fig. 15

- Is it possible to get the machine to operate at δ_m by using gradual changes to P_m? (Assume we are in post-fault state and so the electrical power out is characterized by P_{e3}).

➔ Answer: No! Because at δ=90°, any further increase in P_m causes a permanent state of acceleration, and the machine loses synchronism, as illustrated in Fig. 16.

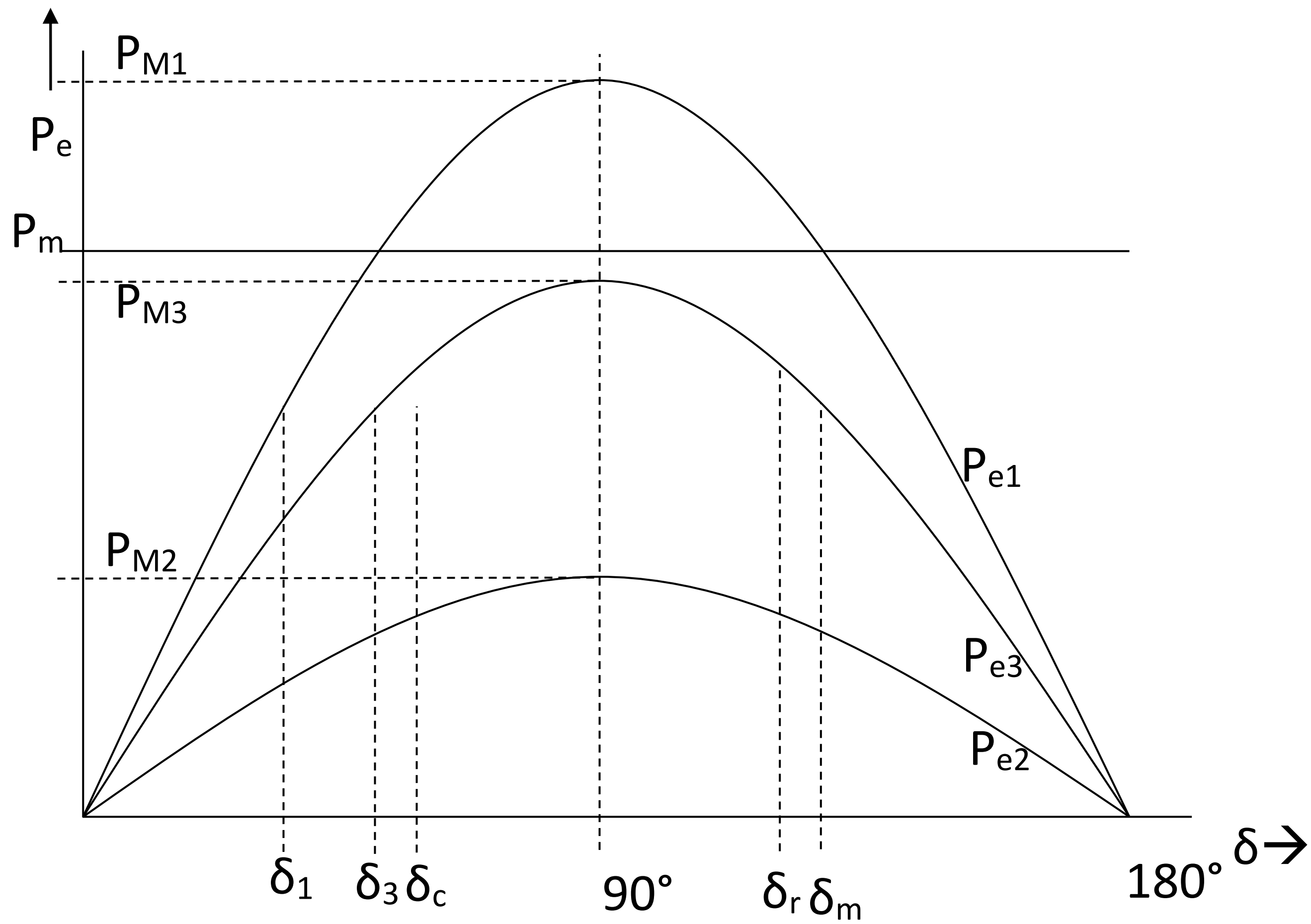


Fig. 16

- Is it possible that the machine reaches δ_m during a post-fault transient?

➔Yes! If the fault-on acceleration is enough, the angle could swing to δ_m, as shown in Fig. 17.

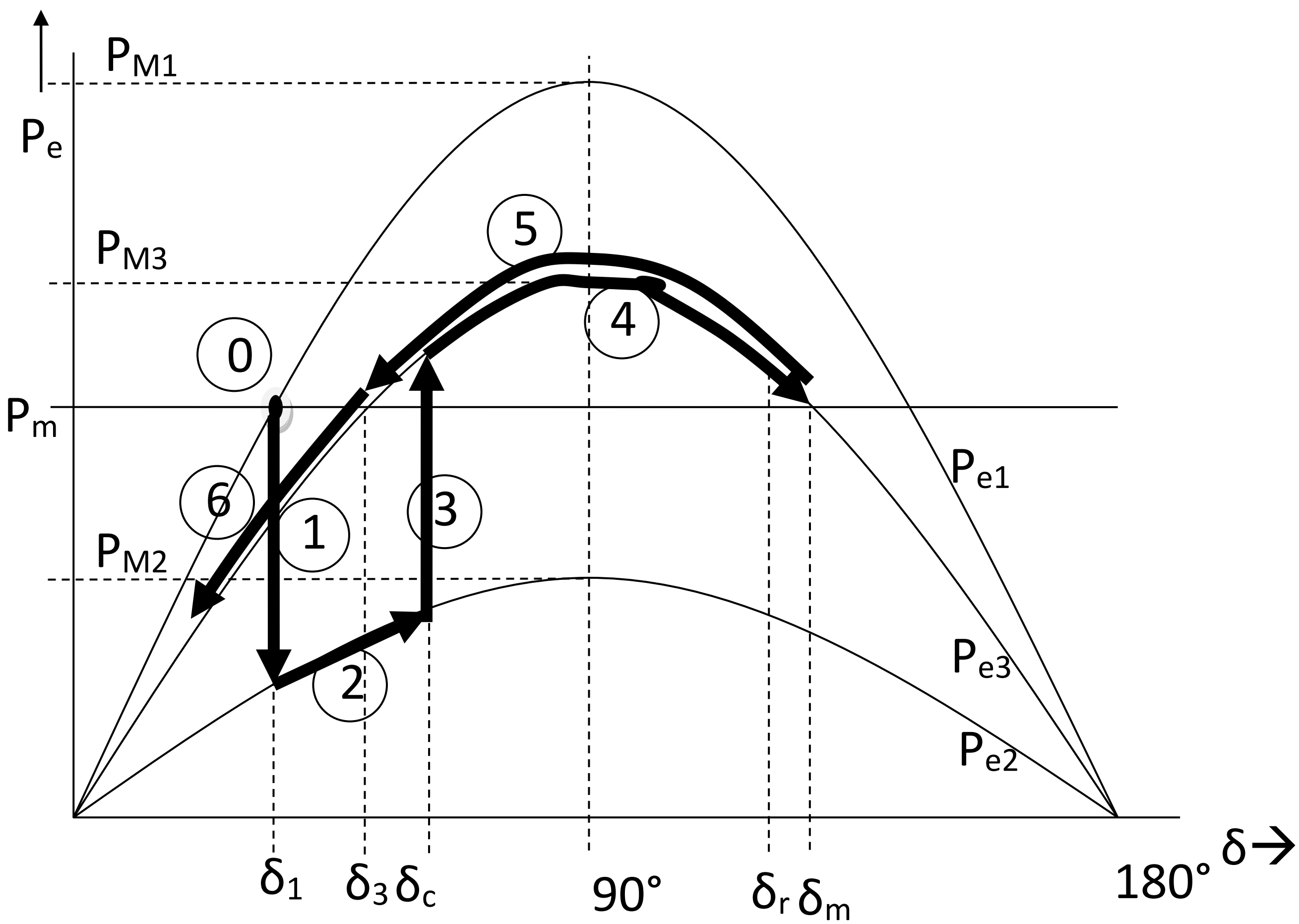


Fig. 17

- What is the significance of this point, δ_m, during a transient?

➔It is the maximum angular swing for stability.

If when $\delta=\delta_m$, $\omega>0$ (which it could be on a "first swing"), the angle increases beyond δ_m. At this point, the rotor begins to accelerate again, and there is no way to obtain deceleration because $P_m>P_{e3}$ permanently.

And so the condition for stability is that

- $\delta_{max}< \delta_m$ ➔STABLE!
- $\delta_{max}= \delta_m$ ➔MARGINALLY STABLE
- $\delta_{max}> \delta_m$ ➔UNSTABLE