Participation Factors

This material is related to VMAF, p. 281-284.

The mode shape, as indicated by the right eigenvector, gives the relative phase of each state in a particular mode.

However, it does ***not*** give the influence of each state on the mode.

We would like to be able to obtain the influence of states on modes because then we will know which states (machines) to control in order to increase damping of a certain problem mode.

Let's define a *new state variable* ("xi") as follows:

$$\xi_k = \underline{q}_k^T \underline{x} \Rightarrow \dot{\xi}_k = \underline{q}_k^T \underline{\dot{x}}$$

Notice that ξ_k is a scalar since the transpose of the left eigenvector is a $1\times n$ and the state vector is an $n\times 1$.

Thus, ξ_k is a combination of all states, but the manner in which all states are combined is through the left eigenvector elements of the k^{th} mode. In the words of VMAF (p.284), "The left eigenvector describes what weighted combination of state variables is needed to construct the mode, referred to as the *mode composition*."

An important attribute of ξ_k, and the reason why it is of great interest here, is that it is a state that is associated with the k^{th} mode and no other mode, proven as follows. Start with system state equations:

$$\underline{\dot{x}} = \underline{A}\underline{x}$$

Pre-multiply both sides by $\underline{q}_k^T$.

$$\underline{q}_k^T \underline{\dot{x}} = \underline{q}_k^T \underline{A}\underline{x} \qquad \text{(P-1)}$$

Recall that the left eigenvector is defined as

$$\underline{q}_k^T \underline{A} = \lambda_k \underline{q}_k^T \qquad \text{(P-2)}$$

Notice that the left-hand-side of eq. (P-2), is on the right-hand-side of eq. (P-1). Substituting the right-hand-side of eq. (P-2) into the right-hand-side of eq. (P-1), we obtain:

$$\underline{q}_k^T \underline{\dot{x}} = \lambda_k \underline{q}_k^T \underline{x} \tag{P-3}$$

Returning to the definition of our new state variable, which is $\xi_k = \underline{q}_k^T \underline{x} \Rightarrow \dot{\xi}_k = \underline{q}_k^T \underline{\dot{x}}$, we note the left-hand-side of eq. (P-3) is $\dot{\xi}_k$ while the right hand side of eq. (P-3) is $\lambda_k \xi_k$. Making these substitutions, eq. (P-3) becomes:

$$\dot{\xi}_k = \lambda_k \xi_k \tag{P-4}$$

Now eq. (P-4) is a time-domain expression. So let's take the LaPlace transform to obtain ("Ξ" is upper case "ξ", letter "xi" pronounced "zigh").

$$\begin{aligned} &s\Xi_k(s) - \xi_k(0) = \lambda_k \Xi_k(s) \\ &\Rightarrow s\Xi_k(s) - \lambda_k \Xi_k(s) = \xi_k(0) \\ &\Rightarrow (s - \lambda_k)\Xi_k(s) = \xi_k(0) \\ &\Rightarrow \Xi_k(s) = \frac{\xi_k(0)}{(s - \lambda_k)} \end{aligned}$$

Taking the inverse LaPlace transform, we find that

$$\xi_k(t) = \xi_k(0) e^{\lambda_k t} \tag{P-5}$$

This confirms that ξ_k is associated with only the k^{th} mode and no other mode. That is, the only dynamics associated with ξ_k are $e^{(\lambda_k)t}$.

<u>What is the implication of this fact</u>? The state variables that influence ξ_k are the state variables that influence the k^{th} mode. So we can study ξ_k to learn about the k^{th} mode.

Let's see if we can determine which state variables influence ξ_k….

Recall from the notes called "Linear system theory" eq. (L-5), which was:

$$\underline{x}(t) = \sum_{k=1}^{n} \left[\underline{q}_k^T \underline{x}(0) e^{\lambda_k t} \right] \underline{p}_k \tag{L-5}$$

I will change the summation index so as to not confuse with the index (k) used previously in these notes:

$$\underline{x}(t) = \sum_{j=1}^{n} \left[\underline{q}_j^T \underline{x}(0) e^{\lambda_j t} \right] \underline{p}_j \tag{P-6}$$

Now substitute eq. (P-6) into the definition of ξ_k:

$$\xi_k = \underline{q}_k^T \underline{x} = \underline{q}_k^T \sum_{j=1}^{n} \left[\underline{q}_j^T \underline{x}(0) e^{\lambda_j t} \right] \underline{p}_j \tag{P-7}$$

Note that the part of the summation in brackets is a scalar. Therefore we can move the $\underline{q}_k^T$ inside the summation, beyond the brackets, so as to pre-multiply $\underline{p}_j$. This results in:

$$\xi_k = \underline{q}_k^T \underline{x} = \sum_{j=1}^{n} \left[\underline{q}_j^T \underline{x}(0) e^{\lambda_j t} \right] \underline{q}_k^T \underline{p}_j \tag{P-8}$$

Recalling that the matrices $\underline{\mathrm{P}}$ and $\underline{\mathrm{Q}}$ are orthogonal, we know that

$$\underline{q}_k^T \underline{p}_j = 0 \qquad \text{for } k \neq j \tag{P-9}$$

Therefore, there is only one non-zero term in the summation of eq. (P-8), and that is the term for which k=j. As a result, eq. (P-8) is:

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0) e^{\lambda_k t} \right] \, \underline{q}_k^T \underline{p}_k \tag{P-10}$$

Rearranging slightly, we have,

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0) \right] \, \left[\underline{q}_k^T \underline{p}_k \right] \, e^{\lambda_k t} \tag{P-11}$$

But note:

$$\underline{q}_k^T \underline{p}_k = \begin{bmatrix} q_{1k} & q_{2k} & \cdots & q_{nk} \end{bmatrix} \begin{bmatrix} p_{1k} \\ p_{2k} \\ \vdots \\ p_{nk} \end{bmatrix} \tag{P-12}$$

We may express the above vector product as a summation:

$$\underline{q}_k^T \underline{p}_k = \sum_{j=1}^{n} q_{jk} p_{jk} \qquad \text{(P-13)}$$

Substitution of eq. (P-13) into eq. (P-11), we obtain:

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0)\right]\left[\sum_{j=1}^{n} q_{jk} p_{jk}\right] e^{\lambda_k t} \qquad \text{(P-14)}$$

Now we are in a position to make a definition:

<u>Participation factor</u>:

$$\rho_{jk} = q_{jk} p_{jk} \qquad \text{(P-15)}$$

Substitution of eq. (P-15) in eq. (P-14) results in:

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0)\right]\left[\sum_{j=1}^{n} \rho_{jk}\right] e^{\lambda_k t} \qquad \text{(P-16)}$$

Three observations:

1. If all $\rho_{jk} = 0$, then the k^{th} mode would not exist, an observation that leads to a conclusion that existence, or prevalence, of a mode depends on the magnitudes of the various ρ_{jk}.
2. The first bracketed term of (P-16) and the exponential to the right are independent of subscript "*j*", that is, they are independent of states. However, the second bracketed term, the summation, depends not only on mode "*k*" but also on state "*j*", that is, it depends on both the mode and the state.
3. Inspection of (P-15) shows that ρ_{jk} depends on "state-related" terms, the j^{th} elements in the k^{th} left and right eigenvectors.

It is the intention that these three observations provide that the definition of the participation factor be intuitive:

> *The participation factor ρ_{jk} indicates the participation (influence) of the j^{th} state in the k^{th} mode.*

The participation factor is extremely useful. Consider you learn via eigenvalue calculation and/or time-domain simulation that mode *k* is a "problem mode," i.e., it is marginally damped or negatively damped. Then one can identify what to do about this problem mode (which state to control) by inspecting participation factors for it.

There will be "*n*" participation factors for each mode *k*, ρ_{jk}, $j=1,\ldots,n$ (where "*n*" is the number of states). The states having the larger participation factors (magnitudes) are the states which should be most strongly considered to control to affect problem mode *k*.
Note: in contrast to $\underline{q}_k^T \underline{x}(0)$, ρ_{jk} is less[1] dependent of the initial conditions and therefore serves as more of a structural indicator of participation than does $\underline{q}_k^T \underline{x}(0)$.

Let's look at big picture - how do we proceed in a small-signal analysis study?

1. Compute eigenvalues & eigenvectors for an operating condition.
2. Choose an ε; If any $\lambda_k=\sigma_k \pm j\omega_k$ has $|\sigma_k|<\varepsilon$, or $\sigma_k>0$, then this is a problem mode at that operating condition.
3. Identify right $\underline{p}_k$ and left $\underline{q}_k$ eigenvectors for mode *k*.
   a. Identify "groups" of generators based on mode shape using $\underline{p}_k$ (use the angles of the elements of $\underline{p}_k$ corresponding to the speed deviation states).
   b. For each group, identify the speed deviation states (and thus the generators) most heavily participating (influencing) the mode, based on ρ_{jk}.
4. Install, or retune the power system stabilizer (PSS) on the generators identified in step 3-b using speed deviation as a control signal so that they increase damping of the k^{th} mode.

Last comment: This is for linearized (small-signal) analysis, not large-signal (fault) analysis.

[1] We say "less dependent" instead of "independent" because eigenvalues and both right/left eigenvectors are initially computed as a function of initial conditions, as our 3-generator example has shown.

Going back to our example (see notes on linear system theory), we recall that

$$\begin{bmatrix} \Delta\dot{\delta}_{13} \\ \Delta\dot{\delta}_{23} \\ \Delta\dot{\omega}_{13} \\ \Delta\dot{\omega}_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -104.096 & -59.524 & 0 & 0 \\ -33.841 & -153.460 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_{13} \\ \Delta\delta_{23} \\ \Delta\omega_{13} \\ \Delta\omega_{23} \end{bmatrix}$$

Observe the eigenvalues in Table 3.2.

Table 3.2. Frequencies of Oscillation of a Nine-Bus System

Quantity	Eigenvalue 1	Eigenvalue 2
λ	$\pm j8.807$	$\pm j13.416$
ω rad/s	8.807	13.416
f Hz	1.402	2.135
T s	0.713	0.468

Also observe the relative rotor angle plots of Fig. 3.3-b, where we see that one mode can be clearly observed having a period of about 0.7 sec (f=1.4 hz). The other mode is not readily observable, although its presence is probably responsible for the distortion seen in the δ_{31} plot.

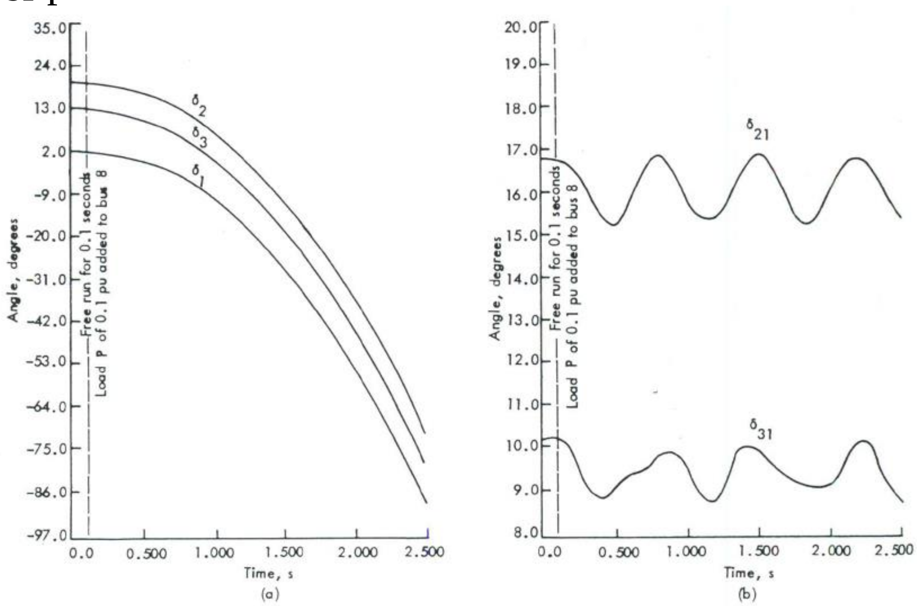


Fig. 3.3 Unregulated response of the nine-bus system to a sudden load application at bus 8: (a) absolute angles, (b) angles relative to δ_1.

Using matlab…
[P,D]=eig(A) where A is the matrix given above.

Then the matrix of eigenvalues D is given by

+13.4164i	0	0	0
0	-13.4164i	0	0
0	0	+8.8067i	0
0	0	0	- 8.8067i

And the matrix of right eigenvectors P is given by

-0.0459 - 0.0000i	-0.0459 + 0.0000i	-0.1030 - 0.0000i	-0.1030 + 0.0000i
-0.0585 - 0.0000i	-0.0585 + 0.0000i	0.0459 + 0.0000i	0.0459 - 0.0000i
0.0000 - 0.6154i	0.0000 + 0.6154i	0.0000 - 0.9075i	0.0000 + 0.9075i
0.0000 - 0.7847i	0.0000 + 0.7847i	-0.0000 + 0.4046i	-0.0000 - 0.4046i

And the matrix of left eigenvectors Q^T is given by P^{-1}, which is:

-2.8240 + 0.0000i	-6.3340 + 0.0000i	0.0000 + 0.2105i	0.0000 + 0.4721i
-2.8240 - 0.0000i	-6.3340 - 0.0000i	0.0000 - 0.2105i	0.0000 - 0.4721i
-3.5951 + 0.0000i	2.8194 - 0.0000i	0.0000 + 0.4082i	-0.0000 - 0.3201i
-3.5951 - 0.0000i	2.8194 + 0.0000i	0.0000 - 0.4082i	-0.0000 + 0.3201i

Note that here, the eigenvectors are along the rows. Taking transpose, we get Q, which is

-2.8240 + 0.0000i	-2.8240 - 0.0000i	-3.5951 + 0.0000i	-3.5951 - 0.0000i
-6.3340 + 0.0000i	-6.3340 - 0.0000i	2.8194 - 0.0000i	2.8194 + 0.0000i
0.0000 + 0.2105i	0.0000 - 0.2105i	0.0000 + 0.4082i	0.0000 - 0.4082i
0.0000 + 0.4721i	0.0000 - 0.4721i	-0.0000 - 0.3201i	-0.0000 + 0.3201i

Now I compute the *participation matrix* below.

```
a=[0 0 1 0; 0 0 0 1; -104.096 -59.524 0 0; -33.841 -153.46 0 0];
[P, D]=eig(a);
QT=inv(P);
Q=QT';
j=1;
% j is index on columns (modes)
% i is index on rows (states)
while j<5,
   i=1;
   while i<5,
      pf(i,j)=Q(i,j)*P(i,j);
      i=i+1;
   end
   j=j+1;
end
pf
```

This gives

DELTA13	0.1295 + 0.0000i	0.1295 - 0.0000i	0.3705 + 0.0000i	0.3705 - 0.0000i
DELTA23	0.3705 + 0.0000i	0.3705 - 0.0000i	0.1295 + 0.0000i	0.1295 - 0.0000i
OMEGA13	-0.1295 - 0.0000i	-0.1295 + 0.0000i	-0.3705 - 0.0000i	-0.3705 + 0.0000i
OMEGA23	-0.3705 - 0.0000i	-0.3705 + 0.0000i	-0.1295 - 0.0000i	-0.1295 + 0.0000i
	MODE 1	MODE 1	MODE 2	MODE2

From this, we see that ω_{23} participates most heavily in mode 1. ω_{13} participates most heavily in mode 2.
This is the information that we would use to decide where to place a PSS to enhance damping of a particular mode, although there is some ambiguity regarding whether ω_{1k} is a state associated with unit 1 or unit k.

Returning to Fig. 3.3 in the book (and given above), we observe that although mode 1 is clearly visible in both plots, mode 2 is only visible in the ω_{13} plot. This is consistent with the indication from the participation factors.
Recall that with uniform damping, we were able to eliminate one speed deviation state. In general, this is not possible, and so you end up with one speed generation state for each generator, a development which solves the ambiguity problem mentioned above.

As an example, the paper by Mansour provides participation factors for several cases, as indicated below. Note that for all participation "vectors" the participation factor is a normalized magnitude.

Table 4

1988 HEAVY WINTER CASE
PALO VERDE - DEVERS LINE OUT OF SERVICE
0.72 Hz MODE (-0.057 + J 4.43)

EIGEN VECTOR				PARTICIPATION VECTOR		
GEN	AREA	MAG	PH	GEN	AREA	MAG
GEYSER34	PG&E	1.0	-176	HELMS G	PG&E	1.0
HELMS G	PG&E	0.9	169	BIGCREEK	S.CAL	0.6
GEYSER78	PG&E	0.9	-176	KEMANO	B.C.	0.5
BIGCREEK	S.CAL	0.8	-165	GEYSER D	PG&E	0.4
MELONES	PG&E	0.7	-156	FCNGN4CC	ARIZ	0.3
GEYSER B	PG&E	0.7	-175	RNCHSECO	PG&E	0.3
GEYSER D	PG&E	0.7	-175	GEYSER C	PG&E	0.3
GEYSER A	PG&E	0.6	-175	MOSS 7	PG&E	0.3
GEYSER C	PG&E	0.6	-175	GEYSER B	PG&E	0.2
FOLSOM1	PG&E	0.6	-178	MELONES	PG&E	0.2
FOLSOM23	PG&E	0.6	-178	GEYSER34	PG&E	0.2
KERCKHOF	PG&E	0.6	-179	DIABLO1	PG&E	0.2
	..			GEYSER78	PG&E	0.2
	..			DIABLO2	PG&E	0.2
---------				PTS7SWNG	PG&E	0.2
				COMANCHE	COLRD	0.2
INTERM2G	UTAH	0.3	19.5			
EHUNTR 1	UTAH	0.2	5.0			
INTERM1G	UTAH	0.3	19.5			
EHUNTR 2	UTAH	0.2	4.8			
NEWMANG6	N.MEX	0.3	-25.4			
MOHAV2CC	S.CAL	0.3	-22.3			
MOHAV1CC	S.CAL	0.3	-22.3			
APACHCT3	ARIZ	0.3	-21.5			
FCNGN4CC	ARIZ	0.3	-26.8			
HOVRA3A4	WAPA	0.4	-17.4			
NEWMAN45	N.MEX	0.5	-32.3			
BLNDL G1	UTAH	0.5	4.5			

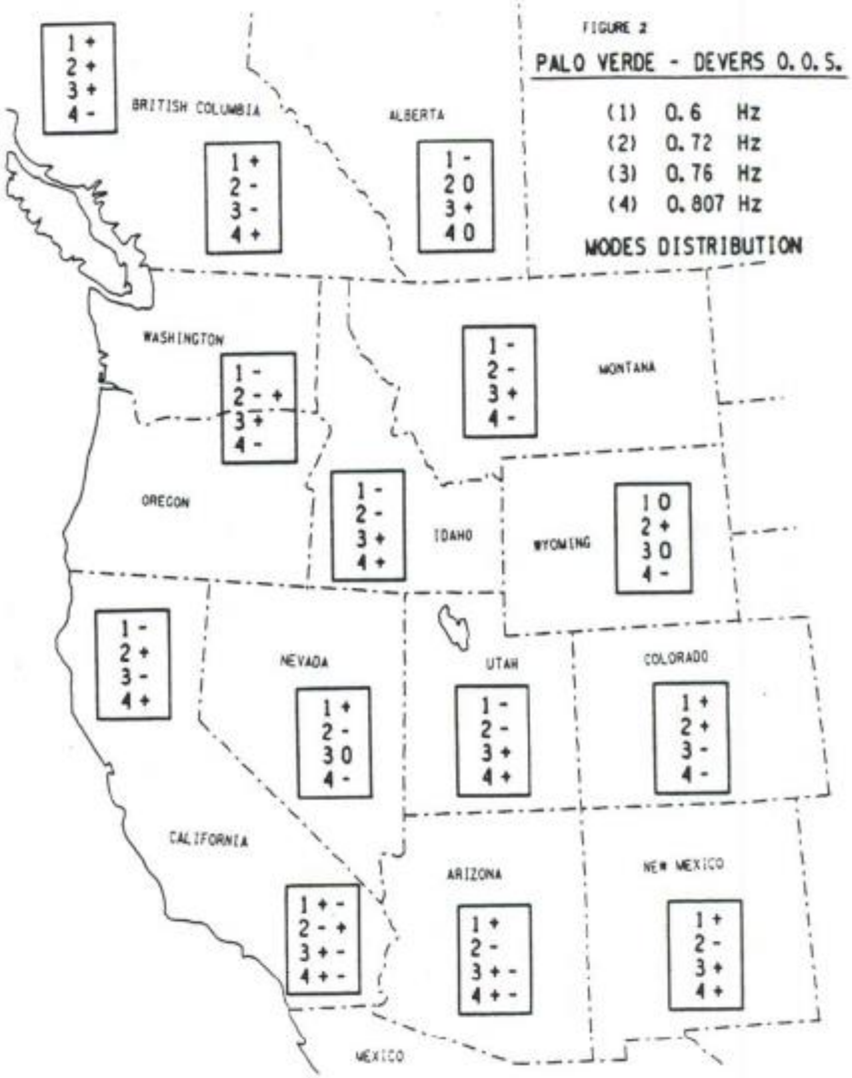
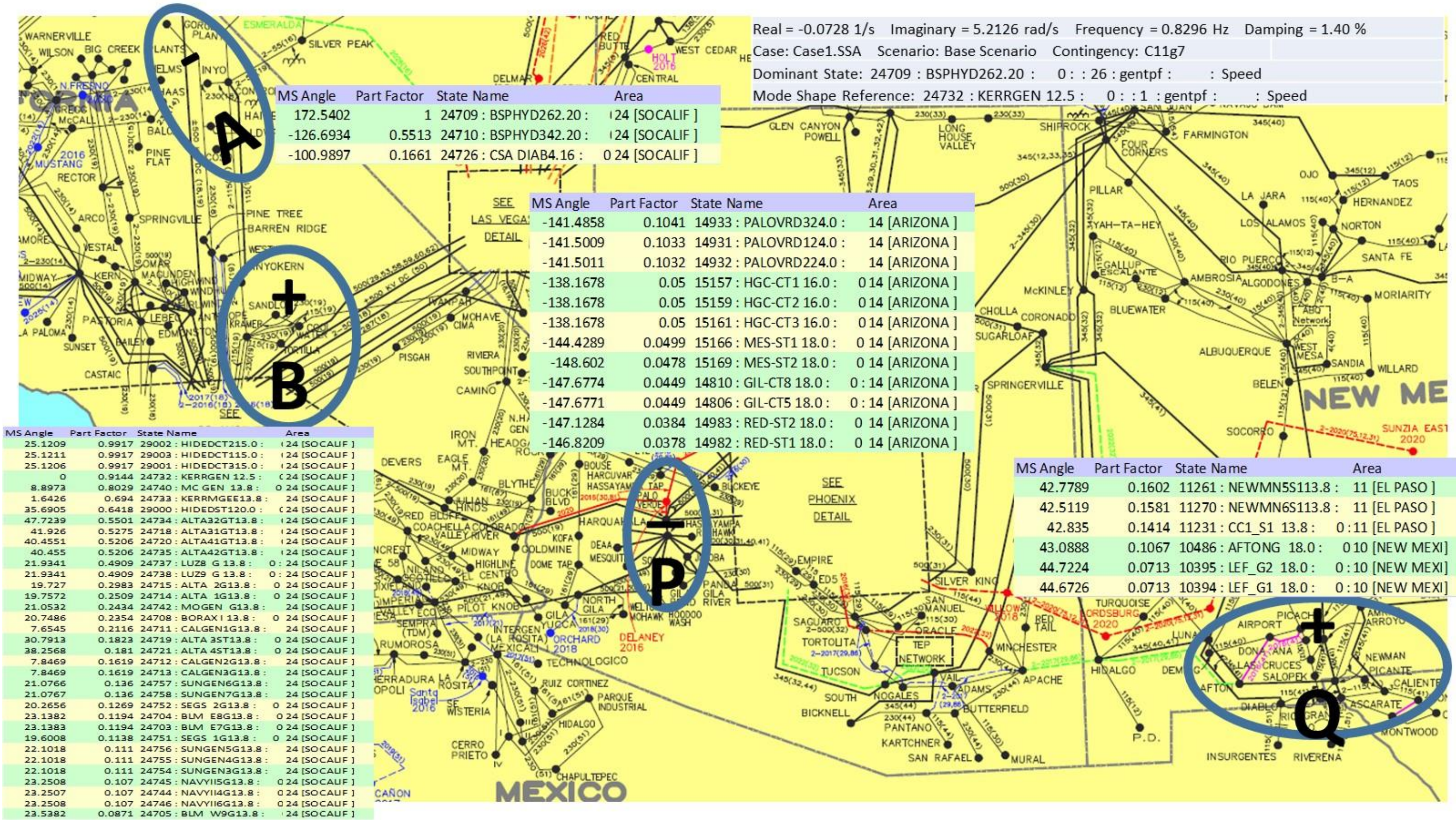


FIGURE 2
PALO VERDE - DEVERS O.O.S.
(1) 0.6 Hz
(2) 0.72 Hz
(3) 0.76 Hz
(4) 0.807 Hz
MODES DISTRIBUTION
1 + 2 + 3 + 4 -
BRITISH COLUMBIA
1 + 2 - 3 - 4 +
ALBERTA
1 - 2 0 3 + 4 0
WASHINGTON
1 - 2 - + 3 + 4 -
MONTANA
1 - 2 - 3 + 4 -
OREGON
IDAHO
1 - 2 - 3 + 4 +
WYOMING
1 0 2 + 3 0 4 -
1 - 2 + 3 - 4 +
NEVADA
1 + 2 - 3 0 4 -
UTAH
1 - 2 - 3 + 4 +
COLORADO
1 + 2 + 3 - 4 -
CALIFORNIA
1 + - 2 - + 3 + - 4 + -
ARIZONA
1 + 2 - 3 + - 4 + -
NEW MEXICO
1 + 2 - 3 + 4 +
MEXICO

. . .



Real = -0.0728 1/s Imaginary = 5.2126 rad/s Frequency = 0.8296 Hz Damping = 1.40 %
Case: Case1.SSA Scenario: Base Scenario Contingency: C11g7
Dominant State: 24709 : BSPHYD262.20 : 0 : : 26 : gentpf : : Speed
Mode Shape Reference: 24732 : KERRGEN 12.5 : 0 : : 1 : gentpf : : Speed
- A
+ B
- P
+ Q
NEW ME
MEXICO

MS Angle	Part Factor	State Name	Area
172.5402	1	24709 : BSPHYD262.20 :	24 [SOCALIF]
-126.6934	0.5513	24710 : BSPHYD342.20 :	24 [SOCALIF]
-100.9897	0.1661	24726 : CSA DIAB4.16 :	0 24 [SOCALIF]

MS Angle	Part Factor	State Name	Area
-141.4858	0.1041	14933 : PALOVRD324.0 :	14 [ARIZONA]
-141.5009	0.1033	14931 : PALOVRD124.0 :	14 [ARIZONA]
-141.5011	0.1032	14932 : PALOVRD224.0 :	14 [ARIZONA]
-138.1678	0.05	15157 : HGC-CT1 16.0 :	0 14 [ARIZONA]
-138.1678	0.05	15159 : HGC-CT2 16.0 :	0 14 [ARIZONA]
-138.1678	0.05	15161 : HGC-CT3 16.0 :	0 14 [ARIZONA]
-144.4289	0.0499	15166 : MES-ST1 18.0 :	0 14 [ARIZONA]
-148.602	0.0478	15169 : MES-ST2 18.0 :	0 14 [ARIZONA]
-147.6774	0.0449	14810 : GIL-CT8 18.0 :	0 : 14 [ARIZONA]
-147.6771	0.0449	14806 : GIL-CT5 18.0 :	0 : 14 [ARIZONA]
-147.1284	0.0384	14983 : RED-ST2 18.0 :	0 14 [ARIZONA]
-146.8209	0.0378	14982 : RED-ST1 18.0 :	0 14 [ARIZONA]

MS Angle	Part Factor	State Name	Area
25.1209	0.9917	29002 : HIDEDCT215.0 :	24 [SOCALIF]
25.1211	0.9917	29003 : HIDEDCT115.0 :	24 [SOCALIF]
25.1206	0.9917	29001 : HIDEDCT315.0 :	24 [SOCALIF]
0	0.9144	24732 : KERRGEN 12.5 :	24 [SOCALIF]
8.8973	0.8029	24740 : MC GEN 13.8 :	0 24 [SOCALIF]
1.6426	0.694	24733 : KERRMGEE13.8 :	24 [SOCALIF]
35.6905	0.6418	29000 : HIDEDST120.0 :	24 [SOCALIF]
47.7239	0.5501	24734 : ALTA32GT13.8 :	24 [SOCALIF]
41.926	0.5275	24718 : ALTA31GT13.8 :	24 [SOCALIF]
40.4551	0.5206	24720 : ALTA41GT13.8 :	24 [SOCALIF]
40.455	0.5206	24735 : ALTA42GT13.8 :	24 [SOCALIF]
21.9341	0.4909	24737 : LUZ8 G 13.8 :	0 : 24 [SOCALIF]
21.9341	0.4909	24738 : LUZ9 G 13.8 :	0 : 24 [SOCALIF]
19.727	0.2983	24715 : ALTA 2G13.8 :	0 24 [SOCALIF]
19.7572	0.2509	24714 : ALTA 1G13.8 :	0 24 [SOCALIF]
21.0532	0.2434	24742 : MOGEN G13.8 :	24 [SOCALIF]
20.7486	0.2354	24708 : BORAX I 13.8 :	0 24 [SOCALIF]
7.6545	0.2116	24711 : CALGEN1G13.8 :	24 [SOCALIF]
30.7913	0.1823	24719 : ALTA 3ST13.8 :	0 24 [SOCALIF]
38.2568	0.181	24721 : ALTA 4ST13.8 :	0 24 [SOCALIF]
7.8469	0.1619	24712 : CALGEN2G13.8 :	24 [SOCALIF]
7.8469	0.1619	24713 : CALGEN3G13.8 :	24 [SOCALIF]
21.0766	0.136	24757 : SUNGEN6G13.8 :	24 [SOCALIF]
21.0767	0.136	24758 : SUNGEN7G13.8 :	24 [SOCALIF]
20.2656	0.1269	24752 : SEGS 2G13.8 :	0 24 [SOCALIF]
23.1382	0.1194	24704 : BLM E8G13.8 :	0 24 [SOCALIF]
23.1383	0.1194	24703 : BLM E7G13.8 :	0 24 [SOCALIF]
19.6008	0.1138	24751 : SEGS 1G13.8 :	0 24 [SOCALIF]
22.1018	0.111	24756 : SUNGEN5G13.8 :	24 [SOCALIF]
22.1018	0.111	24755 : SUNGEN4G13.8 :	24 [SOCALIF]
22.1018	0.111	24754 : SUNGEN3G13.8 :	24 [SOCALIF]
23.2508	0.107	24745 : NAVYII5G13.8 :	0 24 [SOCALIF]
23.2507	0.107	24744 : NAVYII4G13.8 :	0 24 [SOCALIF]
23.2508	0.107	24746 : NAVYII6G13.8 :	0 24 [SOCALIF]
23.5382	0.0871	24705 : BLM W9G13.8 :	24 [SOCALIF]

MS Angle	Part Factor	State Name	Area
42.7789	0.1602	11261 : NEWMN5S113.8 :	11 [EL PASO]
42.5119	0.1581	11270 : NEWMN6S113.8 :	11 [EL PASO]
42.835	0.1414	11231 : CC1_S1 13.8 :	0 : 11 [EL PASO]
43.0888	0.1067	10486 : AFTONG 18.0 :	0 10 [NEW MEXI]
44.7224	0.0713	10395 : LEF_G2 18.0 :	0 : 10 [NEW MEXI]
44.6726	0.0713	10394 : LEF_G1 18.0 :	0 : 10 [NEW MEXI]

..