<u>Simulation of Multi-Machine Systems</u>

The parts of the text which we have yet to cover include:

- Chapter 3: System response to small disturbances
- Chapter 7: Simulation of multimachine systems
- Chapter 8: Small-signal stability analysis (linear models)
- Chapters 9: Excitation systems
- Chapter 10: Effect of excitation on stability
- Chapter 11: Dynamic modeling for wind and solar
- Chapter 12: Voltage stability
- Chapter 13: FACTS devices
- Chapter 14: Protection and monitoring associated with stability
- Chapters 15-18: Mechanical Dynamic Performance (speed governing and prime movers for steam/hydro/CTs/CC units)

The highlighted text above is what we will hope to study between now and the end of the course. Although Chapter 8 is not included, we will study small-signal stability except that we will focus on classical models only which is treated in Chapter 3.
(Note that Chapter 8 is to Chapter 3 as Chapter 4 is to Chapter 2, i.e., Chapter 4 extends the coverage of transient instability analysis done in Chapter 2 from the classical machine model to more elaborate machine models. Chapter 8 does the same thing, except instead of transient instability, it extends the coverage of small-signal instability done in Chapter 3).

We will study the first part of chapter 9 (9.1-9.3) and one part of chapter 10 (8.3) on excitation. Then we will spend a little time looking at Chapters 11 and 12. We will not have time to study any of chapter 13, 14, or 15-18 (turbine-governors) at all. We will spend some time on Chapter 9 and then move back to Chapter 3.

So here we look at Chapter 7.

Chapter 7 consists of the following sections:

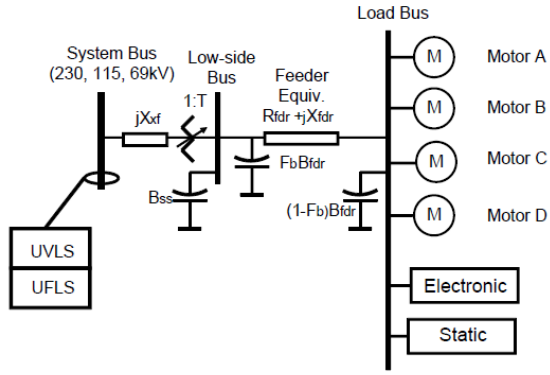
- 7.1: Introduction
- 7.2: Problem statement
- 7.3: Matrix representation of a passive network
    - Network in the transient state
    - Converting to a common reference frame
- 7.4: Converting machine coordinates to system reference
- 7.5: Relation between machine currents and voltages
- 7.6: System order
- 7.7: Machines represented by classical methods
- 7.8: Linearized model for the network
- 7.9: Hybrid formulation
- 7.10: Network equations with flux linkage model
- 7.11: Total system equations
- 7.12: Alternating solution method
    - 7.12.1 Nonlinear loads
    - 7.12.2 Network-machine interface
- 7.13 Simultaneous solution method
- 7.14 Design of numerical solvers

We will study sections 7.1-7.5 and may look briefly at section 7.14. Note that Padiyar's book also gives treatment of this in pp. 462-474.

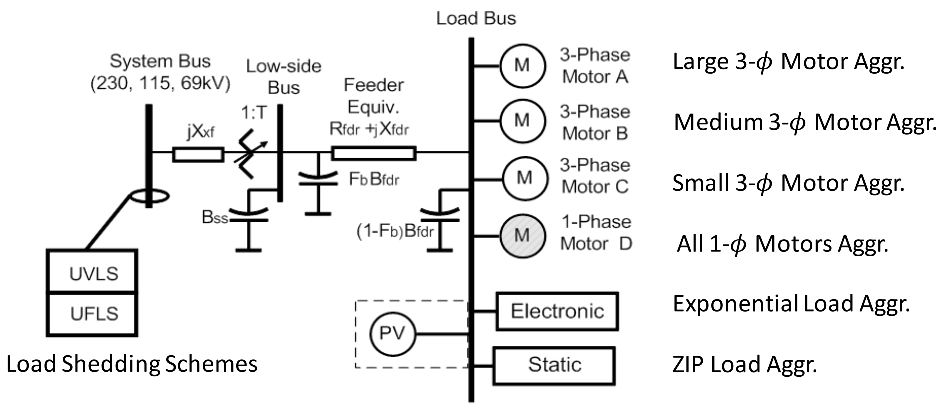
The first section of these notes, below (pp. 2-5) is a short summary on load modeling. We will skip this, since we just covered it.

Load modeling:

I will use this section to emphasize the importance of load modeling. Please read the 1993 Task Force paper on load modeling posted to the course website. Also, please review the WECC document on composite load model specifications, also posted to the website. This latter document shows the well-known illustration used for composite load modeling, shown below.



A more recent illustration illustrates that it accommodates distributed PV, as shown below.



There are two basic types of commonly used load models.

- Static:
    - Exponential
    - Polynomial
- Induction motor

The polynomial is probably the most common. One version of the polynomial is the so-called ZIP model:

$$P = P_0\left\{A + B\left(\frac{|V|}{|V_0|}\right) + C\left(\frac{|V|}{|V_0|}\right)^2\right\}(1 + L_P\Delta f)$$

$$P = P_0\left\{D + E\left(\frac{|V|}{|V_0|}\right) + F\left(\frac{|V|}{|V_0|}\right)^2\right\}(1 + L_Q\Delta f)$$

Typically, the frequency sensitivity coefficients obey $0<L_P<3$ and $-2<L_Q<0$ so that when frequency declines (meaning $\Delta f<0$), P decreases and Q increases, which tends to be the case for an induction motor.

The voltage sensitivity coefficients must obey A+B+C=1 and D+E+F=1. If we set A=B=D=E=0 and C=F=1, then we have a constant impedance model. This load model provides that power consumption of loads decreases as voltage drops. This characteristic typically decreases the severity of system response in terms of transient instability in that:

- We usually see voltage drop during and after a disturbance
- When voltage drops, constant Z loads consume less power according to the square of the voltage drop – which in turn improves the stability performance of the generators.

One advantage to using the constant Z-model is that it allows us to easily reduce the network to generator nodes as all loads are represented in the Y-bus. We obtain the impedance equivalents via $Z=|V_i|^2/S^*$.

One should note carefully here the difference between load modeling for transient analysis and load modeling for steady-state analysis.

Typically, for steady-state analysis (using power flow), we represent the load using constant power models. Some power flow programs do allow for using other load models, e.g., ZIP. However, if your power system contains under-load-tap-changing (ULTC) transformers connecting between the transmission system and the load (most commonly between the subtransmission and the distribution systems), and most do, then use of anything except a constant power model is usually inappropriate unless you are also representing the ULTC transformers.

The reason for this is as follows:

Steady-state analysis of disturbances using power flow is typically done to analyze the 3-10 minute time period following the disturbance. The value of 3 minutes is chosen because this is enough time for the ULTC to operate fully, restoring the voltage levels in the distribution system, so that the loads actually see a constant voltage and therefore behave as constant power loads.

<u>Section 7.2, Problem statement</u>:

Each machine is represented by the following relation:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{v}, T_m, t) \tag{7.1}$$

where $\underline{x}$ is the state vector (could be any number of states between 2-8 depending on the choice of machine model), $\underline{v}=[v_d, v_q, v_F]^T$, T_m is the mechanical torque, and t is time.

Recall that the input vector for each of our machine models included v_d and v_q (or V_d and V_q where $V_d = \frac{v_d}{\sqrt{3}}$ and $V_q = \frac{v_d}{\sqrt{3}}$), which are the d- and q- axis components of the machine terminal voltage. For example, the current-state-space model for model 1 is:

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_q \\ \dot{i}_G \\ \dot{i}_Q \\ \hline \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \left[\begin{array}{cccccc:cc} \multicolumn{6}{c:}{-\mathbf{L}^{-1}(\mathbf{R}+\omega\mathbf{N})} & \multicolumn{2}{c}{\mathbf{0}} \\ \hdashline \frac{-L_d i_q}{3\tau_j} & \frac{-kM_F i_q}{3\tau_j} & \frac{-kM_D i_q}{3\tau_j} & \frac{L_q i_d}{3\tau_j} & \frac{kM_G i_d}{3\tau_j} & \frac{kM_Q i_d}{3\tau_j} & \frac{-D}{\tau_j} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_G \\ i_Q \\ \hline \omega \\ \delta \end{bmatrix} + \begin{bmatrix} -\mathbf{L}^{-1}\mathbf{v} \\ \hdashline \frac{T_m}{\tau_j} \\ -1 \end{bmatrix}$$

where

$$\mathbf{R} = \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r_F & 0 & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_G & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix}; \qquad \mathbf{N} = \begin{bmatrix} 0 & 0 & 0 & L_q & kM_G & kM_Q \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -L_d & -kM_F & -kM_D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} L_d & kM_F & kM_D & 0 & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_G & kM_Q \\ 0 & 0 & 0 & kM_G & L_G & M_Y \\ 0 & 0 & 0 & kM_Q & M_Y & L_Q \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} v_d \\ -v_F \\ 0 \\ v_q \\ 0 \\ 0 \end{bmatrix}; \qquad \mathbf{i} = \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_G \\ i_Q \end{bmatrix}$$

These terms v_d and v_q (or V_d and V_q) are determined by the network, and we therefore need to interface the machine model with the network in order to account for them.

We assume here that v_F and T_m are fixed (they are actually governed by the excitation control and the turbine-governor control; we will study control of v_F in this course (Chapters 9-10), but we will not have time to study control of T_m (Chapters 15-18).

Let's assume that we are using the current state-space model of Model 1 (which is the "full" model including the G-circuit and two damper windings, so it is called model 2.2).

Note that VMAF make the following statements at the beginning of Section 7.2, pp. 239-240 (it references (7.1) given above and below):

> "Consider the set of equations (7.1). In Chapter 4, the current model that is developed represents a set of eight first-order differential equations *for each machine.*"
>
> "The number of the variables, however, is 10: 6 currents, ω and δ, and the voltages υ_d and υ_q."

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{v}, T_m, t) \qquad (7.1)$$

And so, with an 8-state model, we have the number of *state variables* is eight: six currents, ω and δ; and the number of *variables* is ten: eight state variables and the voltages v_d and v_q.

> "Assuming that there are *n* synchronous machines in the system,"

And assuming that all machines are modeled with the 8-state model…

> "we have a set of 8*n* differential equations with 10*n* unknowns."

> "Therefore, 2*n* additional equations are needed to complete the description of the system."

That is, the variables v_d and v_q result in the additional two unknowns per machine, and so we need an additional two equations per machine.

> "These equations are obtained from the load constraints."

Our objective is to derive expressions for v_d and v_q in terms of the state variables (and so avoid adding additional variables), which in the case of the current state-space model of Model 1 (with G-circuit), would be the six currents, ω and δ. We will do this from the "load constraints."

We begin by recalling the stator-side equivalents to v_d, v_q, i_d, and i_q, given by:

$$V_{di} = \frac{v_{di}}{\sqrt{3}} \qquad V_{qi} = \frac{v_{qi}}{\sqrt{3}}$$

$$I_{di} = \frac{i_{di}}{\sqrt{3}} \qquad I_{qi} = \frac{i_{di}}{\sqrt{3}}$$

where subscript "*i*" indicates that the relations apply to machine *i*.

We also have that

$$\bar{V}_i = V_{qi} + jV_{di} \qquad \bar{I}_i = I_{qi} + jI_{di} \tag{7.2}$$

for every machine *i*=1, …, *n*.

Thus we have a vector of nodal voltages and currents for every generator bus given by:

$$\underline{V} = \begin{bmatrix} V_{q1} + jV_{d1} \\ \vdots \\ V_{qn} + jV_{dn} \end{bmatrix} = \begin{bmatrix} \bar{V}_1 \\ \vdots \\ \bar{V}_n \end{bmatrix} \qquad \underline{I} = \begin{bmatrix} I_{q1} + jI_{d1} \\ \vdots \\ I_{qn} + jI_{dn} \end{bmatrix} = \begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_n \end{bmatrix} \tag{7.4}$$

where, says VMAF, "*the axis q_i is taken as the phasor reference in each case*" (p. 240).

(Note that we will use underlines to denote vectors and matrices, and we will use overbars to denote phasors, which differs from VMAF which uses bold to denote vectors and matrices).

Our problem is to express $\underline{V}$ in terms of $\underline{I}$. One might think that this is an easy problem, based on recollection of the Y-bus relation which has that $\underline{I}=\underline{YV}$.

However, there is a major issue in doing this…

The elements of these two vectors, e.g., $V_{q1}+jV_{d1}$ and $I_{q1}+jI_{d1}$, are, **by definition**, expressed on the d-q reference frame of the corresponding machines. We have done nothing at this point to relate the d-q frame of one machine to that of another. VMAF say it this way (p. 240, italics added):

> "*Note carefully that the voltage $\bar{V}_i$ and the current $\bar{I}_i$ are referred to the q and d axes of machine i. In other words the different voltages and currents are expressed in terms of different reference frames*. The desired relation is that which relates the vectors $\underline{V}$ and $\underline{I}$. When obtained, this will represent a set of n complex algebraic equations, or $2n$ real equations. These are the additional equations needed to complete the mathematical description of the system."

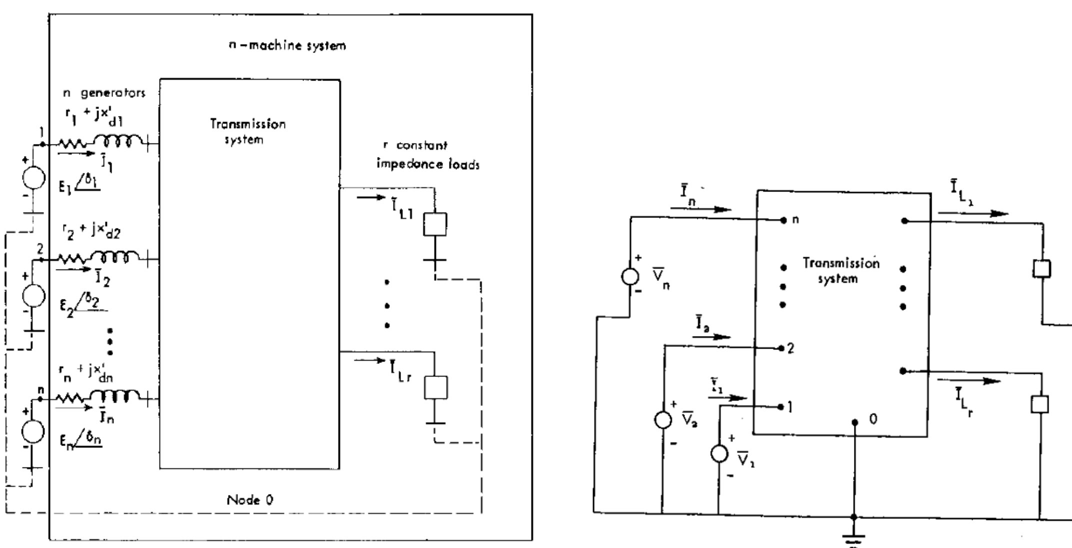
So the elements of $\underline{V}$ (and the elements of $\underline{I}$) are expressed on different reference frames. Any analysis using these numbers "as is" would have relative angles between nodes in the network that mean absolutely nothing. Since relative angles have a very large effect in determining power flow, this is highly unacceptable.

<u>Section 7.3, Matrix representation of a passive network</u>:

In consideration of a multimachine system in Chapter 2, using the classical machine representation, because the machine internal EMF is constant, we could reduce the network to its *internal machine nodes*, thus eliminating the nodes corresponding to each machine's terminal voltage V_a.

Now, however, we need to retain the node corresponding to each machine's terminal voltage V_a because all of our higher-order models require it through the presence in the models of v_d and v_q. The difference between these two approaches is illustrated by the Fig. 1 below from your text (left, internal nodes, Fig. 2.17, and right, terminal nodes, Fig. 7.2).

The figure on the left indicates that the only nodes in the system are the ones outside the box identified as black dots 1, 2, , …, n.



The figure on the right has internal nodes (as does the figure on the left), but also terminal nodes and possibly other nodes having loads not represented as only constant impedance (represented inside the box labeled "transmission system").

Fig. 1

We assume, for now, that we represent all loads using constant impedance shunts.

Then we *can* (but do not have to) use network reduction (Gaussian elimination) to eliminate all network nodes except machine terminal nodes.

We have already recognized that we cannot express $\underline{I}=\underline{YV}$ using eq. (7.4) because the various vector elements are all on different reference frames.

So let's consider a new set of nodal voltages and currents that are expressed to a common reference frame where one of the quantities, often one of the voltages, has an angle designated as 0°.

We will refer to this set of nodal voltages and currents as $\underline{\hat{V}}$ and $\underline{\hat{I}}$, articulated as V-hat and I-hat. So the underline indicates "vector," and the hat indicates that all elements are referred to the *network reference frame*.

So on the network reference frame, it is acceptable to write that

$$\underline{\hat{I}} = \underline{Y}\underline{\hat{V}} \tag{7.5}$$

where $\underline{Y}$ is the network admittance matrix. Of course, at this point, we are simply conjecturing that we can express all voltages and currents to a common reference frame, but we have not yet done it.

But Dr. Anderson is careful….. he recognizes that eq. (7.5) is a steady-state relation, and he takes a little aside to check: under what conditions can we use eq. (7.5) for transient analysis?

In the following, I simply cut out the part of VMAF which addresses this question, Section 7.3.1, and then, just after that, I give a summary.

7.3.1 Network in the Transient State

Consider a branch in the reduced network of Figure 7.2. Let this branch, located between any two nodes in the network, be identified by the subscript k. Let the branch resistance be r_k, its inductance be ℓ_k, and its impedance be $\bar{z}_k$. The branch voltage drop and current are v_k and i_k.

In the transient state the relation between these quantities is given by

$$v_k = \ell_k \dot{i}_k + r_k i_k \quad k = 1, 2, \ldots, b \tag{7.7}$$

where b is the number of branches.

Using subscripts abc to denote the phases abc, (7.7) can be written as

$$\mathbf{v}_{abck} = \ell_k \dot{\mathbf{i}}_{abck} + r_k \mathbf{i}_{abck} \quad k = 1, 2, \ldots, b \tag{7.8}$$

This branch equation could be written with respect to any of the n q axis references by using the appropriate transformation $\mathbf{P}$. Premultiplying (7.8) by the transformation $\mathbf{P}$ as defined by (4.5),

$$\mathbf{P}\mathbf{v}_{abck} = \ell_k \mathbf{P}\dot{\mathbf{i}}_{abck} + r_k \mathbf{P}\mathbf{i}_{abck} \tag{7.9}$$

Then from (4.31) and (4.32)

$$\mathbf{P}\dot{\mathbf{i}}_{abc} = \dot{\mathbf{i}}_{0dq} - \omega \begin{bmatrix} 0 \\ -i_q \\ i_d \end{bmatrix} \tag{7.10}$$

Substituting (7.10) in (7.9) and using (4.7),

$$\mathbf{v}_{0dqk} = \ell_k \left(\dot{\mathbf{i}}_{0dqk} - \omega \begin{bmatrix} 0 \\ -i_{qk} \\ i_{dk} \end{bmatrix} \right) + r_k \mathbf{i}_{0dqk} \tag{7.11}$$

which in the case of balanced conditions becomes

This 2nd assumption results in neglecting network transients, an assumption that may not be so good if there are many IBRs in the network.

$$\mathbf{v}_{dqk} = \ell_k \left(\dot{\mathbf{i}}_{dqk} + \omega \begin{bmatrix} i_{qk} \\ -i_{dk} \end{bmatrix} \right) + r_k \mathbf{i}_{dqk} \tag{7.12}$$

It is customary to make the following assumptions: (1) the system angular speed does not depart appreciably from the rated speed, or $\omega \cong \omega_R$, and (2) the terms $\ell \dot{i}$ are negligible compared with the terms $\omega \ell i$. The first assumption makes the term $\omega \ell_k i_k$ approximately equal to $x_k i_k$, and the second assumption suggests that the terms in $\dot{i}$ are to be neglected.

Under the above assumptions (7.12) becomes

$$\mathbf{v}_{dqk} = r_k \mathbf{i}_{dqk} + x_k \begin{bmatrix} i_{qk} \\ -i_{dk} \end{bmatrix} \quad k = 1, 2, \ldots, b \tag{7.13}$$

Equation (7.13) gives a relation between the voltage drop and the current in one branch of the network in the transient state. These quantities are expressed in the q-d frame of reference of any machine. Let the machine associated with this transformation be i. The rotor angle θ_i of this machine is given by

$$\theta_i = \omega_R t + \pi/2 + \delta_i \tag{7.14}$$

where δ_i is the angle between this rotor and a *synchronously rotating* reference frame.

From (7.13) multiply both sides by $1/\sqrt{3}$, and using (7.3),

$$V_{qk(i)} = r_k I_{qk(i)} - x_k I_{dk(i)} \qquad V_{dk(i)} = r_k I_{dk(i)} + x_k I_{qk(i)} \tag{7.15}$$

where the subscript i is added to indicate that the rotor of machine i is used as reference.

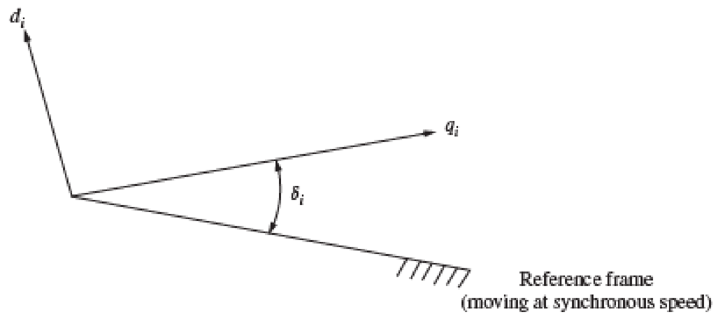


Figure 7.3 Position of axes of rotor k with respect to reference frame.

Expressing (7.15) in phasor notation,

$$\bar{V}_{k(i)} = V_{qk(i)} + \mathrm{j}V_{dk(i)} = \left(r_k I_{qk(i)} - x_k I_{dk(i)}\right) + \mathrm{j}\left(r_k I_{dk(i)} - x_k I_{qk(i)}\right) = (r_k + \mathrm{j}x_k)\left(I_{qk} + \mathrm{j}I_{dk}\right)$$

or

Conclusion➔

$$\boxed{\bar{V}_{k(i)} = z_k \bar{I}_{k(i)} \qquad k = 1, 2, \ldots, b, \qquad (7.16)}$$

Equation (7.16) expresses, in complex phasor notation, the relation between the voltage drop in branch k and the current in that branch. The reference is the q axis of some (hypothetical) rotor i located at angle δ_i, with respect to a synchronously rotating system reference, as shown in Figure 7.3.

<u>Summary of Section 7.3.1</u>: We write the time-domain voltage drop equation for a network branch, and then transform this equation using Park's transformation. This transformation is based on an assumed synchronously rotating reference frame which, at t=0, is aligned with the a-phase of a chosen machine. This action, then, locates the machine's rotor, and thus the machine's d-axis, at

$$\theta_i = \omega_{\mathrm{Re}} t + \pi/2 + \delta_i$$

Fig. 2 illustrates.

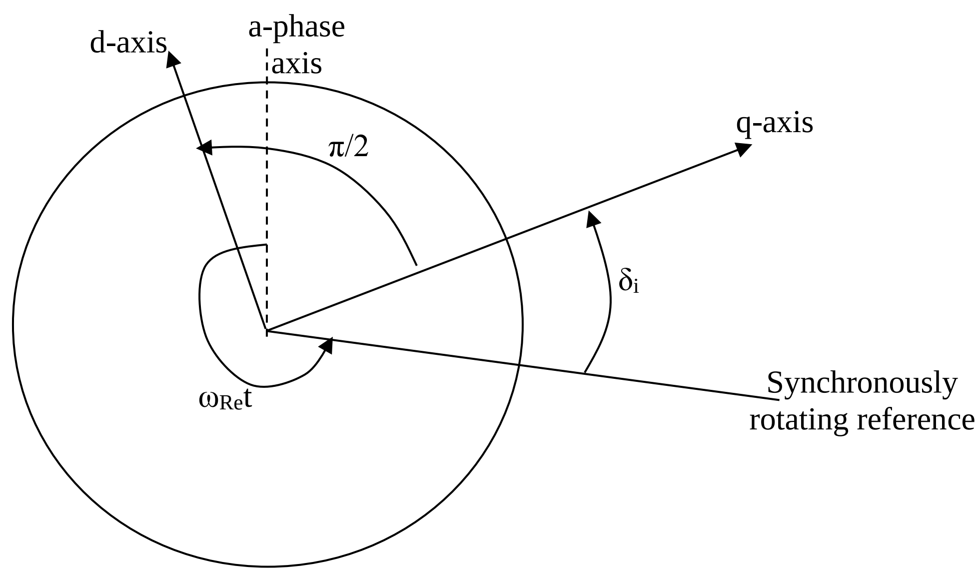


Fig. 2

I will not go through this analysis but rather will simply state the conclusions. Dr. Anderson's conclusion is that:

$$\bar{V}_{k(i)} = z_k \bar{I}_{k(i)}\text{, k=1, …, b} \tag{7.16}$$

where

- $\bar{V}_{k(i)}$ and $\bar{I}_{k(i)}$ are the *branch* voltage drops and *branch* currents, respectively,
- expressed on the d-q axis reference frame of machine i, that is, the reference is the q-axis of the i^{th} machine located at angle δ_i with respect to a synchronously rotating system reference,
- z_k is the impedance of branch k, and
- b is the total number of branches in the network.

Equation (7.16), which is our standard Ohm's Law relation, is applicable for transient analysis if the following two conditions are satisfied (these are the two "assumptions" on above p. 11):

1. The frequency, and therefore the reactances of the branches, are constant.
2. Current derivatives are much less than speed-current products.

$$\left|\dot{i}_d\right| << \left|\omega i_q\right|$$

$$\left|\dot{i}_q\right| << \left|\omega i_d\right|$$

This is analogous to where we assumed that transformer voltages are much less than speed voltage drops (svd), i.e., the d-q voltage components due to transformer action (i.e., variation in d-q currents or in d-q flux linkages) is much less than the d-q voltage components due to the speed. We used this in deriving the E'' model in our notes on "simplified models," expressed as:

$$\left|\dot{\lambda}_d\right| << \left|\omega \lambda_q\right|$$

$$\left|\dot{\lambda}_q\right| << \left|\omega \lambda_d\right|$$

We spent some time discussing this assumption in our notes on "simplified models," (under **Comment on $d\lambda_d/dt=d\lambda_q/dt=0$** which extended from pp. 11-13, which included the following statement:

> Section 7.3.1 of VMAF addresses this last point, which is further characterized by the following statement from [2]:

"In stability studies it has been found adequate to represent the network as a collection of lumped resistances, inductances, and capacitances, and to neglect the short-lived electrical transients in the transmission system.[8],[5],[9],[10] As a consequence of this fact, the terminal constraints imposed by the network appear as a set of algebraic equations which may be conveniently solved by matrix methods."

[2] K. Prabhashankar and W. Janischewsyj, "Digital simulation of multimachine power systems for stability studies," IEEE Trans. Power Apparatus and Systems, Vol. PAS-87, No. 1, January, 1968.
NOTE: This is ref [5] in ch 7 of your book.

We also said in "simplified models" (p. 13)

> Setting $d\lambda_d/dt=d\lambda_q/dt=0$ is referred to in the literature as "neglecting stator transients" or "neglecting network transients."

In addition to identifying the conditions under which we can use our familiar steady-state form of Ohm's Law (and thus the Y-bus relation), eq. (7.16) also provides that we may express the network to a particular machine's d-q reference frame.

But this does not do us too much good since we have all the machine models expressed to their own frame.

So a better approach is to express all of the machine d-q reference frames to a ***network*** reference frame. Let's try that (Section 7.3.2).

We have already defined the d-q reference frame of the machine.

Now we define the network reference frame, and we will denote the network reference frame as D-Q (*do **NOT** confuse this notation with the upper-case D,Q notation used for the damper windings*!!!!).

So our question is: how to convert a voltage (or current) on the d-q reference frame to a voltage (or current) on the D-Q (network) reference frame?

Fig. 3 (Fig 7.4 in text) illustrates.

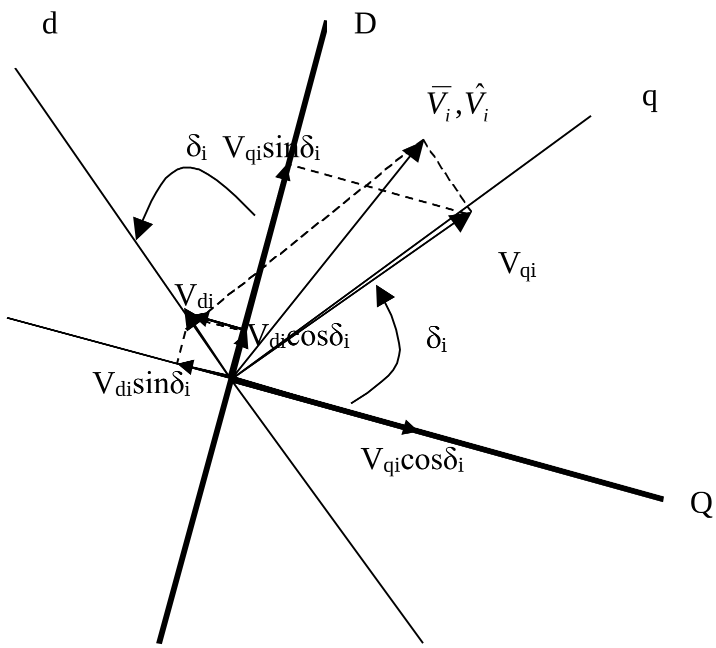


Fig. 3

Note two things with respect to Fig. 3:

- $\bar{V}_i$, $\hat{V}_i$ are drawn leading the q-axis, whereas we know that for generator action, the terminal voltage will lag the q-axis. This is because Fig. 3 is drawn to facilitate understanding of how to project any general quantity given on the d-q frame to a quantity given on the D-Q frame. It is *not* drawn to depict the operation of a generator.
- The angle δ_i has a new definition.
  - Whereas previously we have defined δ_i as the angle by which the machine internal voltage (and thus the q-axis) leads the (synchronously rotating) *machine terminal voltage*;
  - now, in Fig. 3, we define δ_i as the angle by which the machine internal voltage (and thus the q-axis) leads the (synchronously rotating) *Q-axis network reference frame*.

From this picture, it is easy to see how to compute V_{Qi} and V_{Di} from V_{qi} and V_{di}.

It is important to recognize that we are NOT getting V_{Qi} and V_{Di} from $\overline{V}_i$ (or $\hat{V}_i$) directly but rather getting it from V_{di} and V_{qi}, which are the d-q axis components of $\overline{V}_i$ (or $\hat{V}_i$).

For example, consider getting V_{Qi}. By inspection, we see that

$$V_{Qi} = V_{qi} \cos\delta_i - V_{di} \sin\delta_i$$

where, again, we emphasize that the angle δ_i is the angle by which machine *i* q-axis leads the synchronously rotating network reference frame.

Similarly, consider getting V_{Di}. Again, by inspection, we see that:

$$V_{Di} = V_{qi} \sin\delta_i + V_{di} \cos\delta_i$$

Therefore, the voltage $\overline{V}_i$ when expressed to the network reference frame, becomes $\hat{V}_i$, expressed as:

$$\hat{V}_i = V_{Qi} + jV_{Di} = (V_{qi} \cos\delta_i - V_{di} \sin\delta_i) + j(V_{qi} \sin\delta_i + V_{di} \cos\delta_i)$$

Collecting terms in V_{qi} and V_{di}, we have:

$$\hat{V}_i = V_{Qi} + jV_{Di} = V_{qi}\left(\cos\delta_i + j\sin\delta_i\right) + V_{di}\left(j\cos\delta_i - \sin\delta_i\right)$$

Factoring out a "*j*" from the last term:

$$\hat{V}_i = V_{Qi} + jV_{Di} = V_{qi}\left(\cos\delta_i + j\sin\delta_i\right) + jV_{di}\left(\cos\delta_i + j\sin\delta_i\right)$$

And finally, we observe the common sum which can be factored as:

$$\hat{V}_i = V_{Qi} + jV_{Di} = (V_{qi} + jV_{di})(\cos\delta_i + j\sin\delta_i) = \overline{V}_i e^{j\delta_i}$$

In summary, the transformation that we are making is from one set of coordinate axes

where the positive q-axis is assigned 0 degrees,

to another set of coordinate axes

$$\hat{V}_i = V_{Qi} + jV_{Di}$$

where the positive Q-axis is assigned 0 degrees.

Here, the +q-axis leads the +Q axis by δ_i degrees.

And we have found that

$$\hat{V}_i = \overline{V}_i e^{j\delta_i} \tag{7.17}$$

As an example, consider an arbitrary quantity $\overline{V}_i = 10\angle 30°$ (expressed on the d-q frame), and let q lead Q by δ_i=20°. Then

$$\hat{V}_i = \overline{V}_i e^{j20°} = 10\angle 30° e^{j20°} = 10\angle 50°$$

which is illustrated in Fig. 4 below.

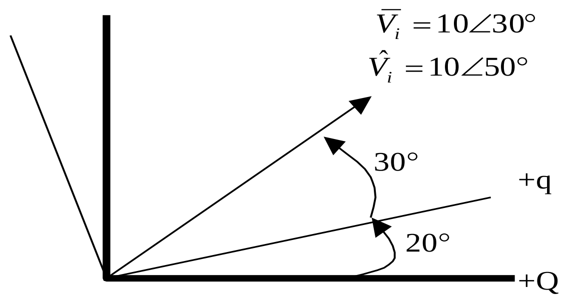


Fig. 3

Before we go further, let's clarify two things:

1. What is the angle δ_i?
2. How do we identify the system reference?

We will take these questions one at a time.

1. **What is the angle δ_i?**

   Several comments here:

   a. Value vs. variable: In notes on "Simulation of Synchronous Machines," we located the initial value of δ_i (for each machine i) by finding $\overline{E}_a$. But make sure you are clear in your mind that

   - this value (we could call it δ_{i0}) is an initial condition, and as such, we can refer to it as a specific value;
   - in general, δ_i is a variable (indeed a state variable); here, in Chapter 7, we no longer think only of δ_i as an initial condition but also (and primarily) as a variable that will vary through the course of our time-domain simulation.

b. <u>The meaning of the angle δ_i has been changed</u>. To understand this, we will review what δ_i was (item c below) and what δ_i is now (item d below).

c. <u>What δ_i was</u>: It is worth going back to the beginning of chapter 4 to make sure we understand what δ_i was. On p. 84, we were shown the below diagram.

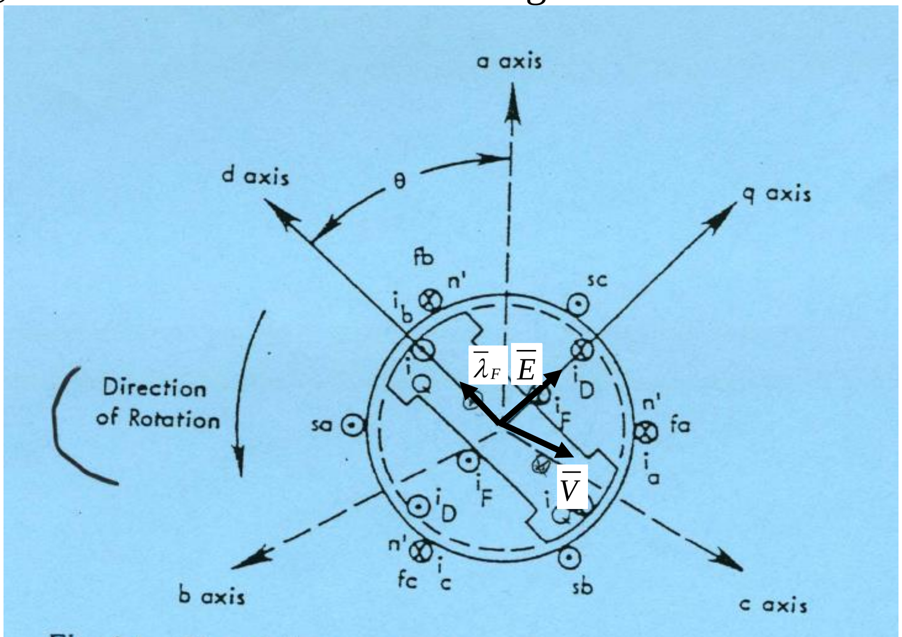


Fig. 4.1 Pictorial representation of a synchronous machine.

From A+F

It is useful to review what VMAF said about this figure (p. 93), which I have copied out below, in quotes, with (my) additional comments highlighted in yellow.

"The main field-winding flux is along the direction on the d-axis of the rotor."

➔This is $\bar{\lambda}_F$, which I added to Figure 4.1.

"It produces an EMF that lags this flux by 90°. Therefore the machine EMF E is primarily along the rotor q-axis."

➔I also added this to Figure 4.1.

"Consider a machine having a constant terminal voltage V. For the generator action the phasor $\bar{E}$ should be leading the phasor $\bar{V}$."

➔I also added this to Figure 4.1.

Key point: Previously, the angle δ has been the angle by which $\overline{E}$ leads $\overline{V}$.

"The angle between $\overline{E}$ and $\overline{V}$ is the machine torque angle δ if the phasor $\overline{V}$ is in the direction of the reference phase (phase a). At t=0 the phasor $\overline{V}$ is located at the axis of phase a, i.e., at the reference axis in Figure 4.1. The q-axis is located at an angle δ, and the d-axis is located at $\theta=\delta+\pi/2$."

➔I have redrawn Fig. 4.1, for t=0, as Fig. 4 below.

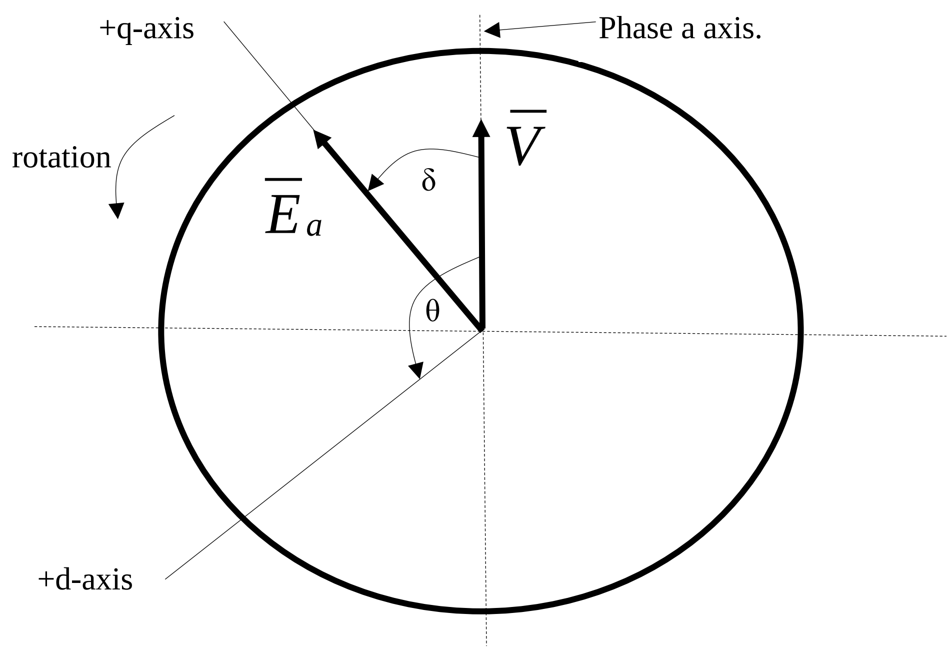


Fig. 4

"At t>0, the reference axis is located at an angle $\omega_R t$ with respect to the axis of phase a. The d-axis of the rotor is therefore located at

$$\theta= \omega_R t+\delta+\pi/2 \tag{4.6}$$

where ω_R is the rated (synchronous) angular frequency in rad/s and δ is the synchronous torque angle in electrical radians."

➔I have redrawn Fig. 4.1, for t>0, as Fig. 5 below.

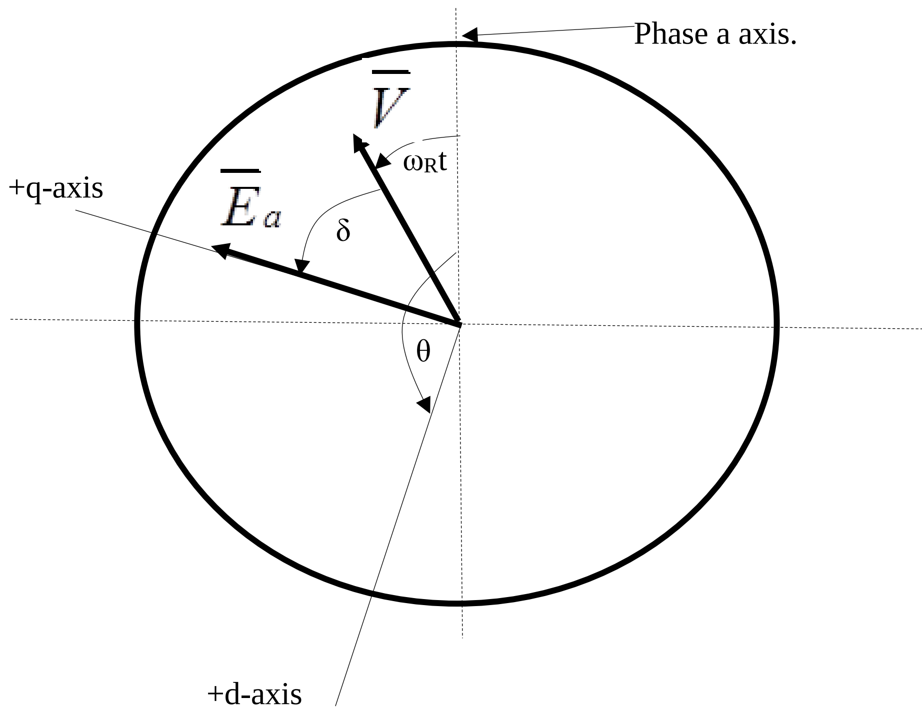


Fig. 5

d. <u>What δ_i is now</u>: In developing a system synchronously rotating reference frame, δ_i (for machine *i*) changes from
   - the angle by which the machine *i* q-axis leads the <u>*terminal voltage*</u> to
   - the angle by which the machine *i* q-axis leads the <u>*synchronously rotating system reference*</u>.

2. How do we identify the system reference?

The system reference is identified as a synchronously rotating vector having angle of 0 degrees at *t*=0. This is normally the reference bus in the power flow model.

Some additional clarifying comments: Consider the beginning of Section 7.4 (p. 244), where it reads (bold underline added):

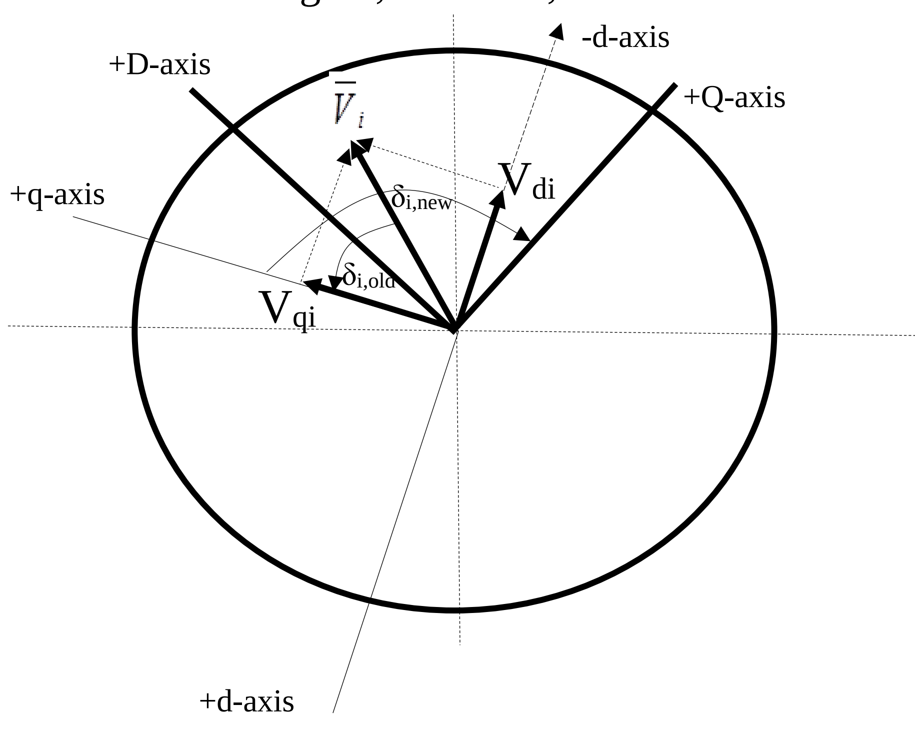
> "Consider a voltage $\underline{v}_{abci}$ at node *i*. We can apply Park's transformation to this voltage to obtain $\underline{v}_{dqi}$. From (7.2)
>
> $$\overline{V}_i = V_{qi} + jV_{di} \qquad \qquad \bar{I}_i = I_{qi} + jI_{di} \qquad (7.2)$$
>
> this voltage can be expressed in phasor notation as $\overline{V}_i$, <u>**using the rotor of machine *i* as reference**</u>."

➔ This statement is a little misleading. Since the d-axis is aligned with the rotor, "using the rotor as reference" implies using the d-axis as reference. However, the phasors of (7.2) are expressed with the q-components along the real axis (0°) and the d-components along the imaginary axis (90°), implying the q-axis is the reference. It may be that when A&F wrote "using the rotor as reference," they meant "using the rotor frame as reference," which could be interpreted as "using the q-axis as the reference." We will assume there that they meant to indicate they will use the q-axis as reference.

Pg. 244 continues by saying, "It can also be expressed to the system reference as V_i using the transformation (7.17).

$$\hat{V}_i = \overline{V}_i e^{j\delta_i} \qquad (7.17)$$

➔ I have redrawn the figure, as below, to illustrate:



➔Expression (7.17) can be understood as follows…Observe that the angle of phasor $\overline{V}_i$, identified as $\delta_{i,old}$, and given on the d-q frame, must be negative (the q-axis leads $\overline{V}_i$, and so if we express $\overline{V}_i$ relative to the q-axis, with the q-axis having a 0° angle, the angle of $\overline{V}_i$ must be negative).

➔On the other hand, if we express $\overline{V}_i$ relative to the Q-axis (to obtain $\hat{V}_i$), we observe that the angle must be positive as $\overline{V}_i$ is leading the Q-axis. We obtain this via:

$$\angle V_i = \delta_{i,old} + \delta_{i,new} \qquad (*)$$

Now, renaming $\delta_{i,old}$ and $\delta_{i,new}$ to be consistent with VMAF, we can write (*) as:

$$\angle V_i = \angle \overline{V}_i + \delta_i \qquad (**)$$

which is obtained from

$$\hat{V}_i = \overline{V}_i e^{j\delta_i} \qquad (7.17)$$

Now recall the equation relating branch voltage drops to branch currents:

$$\overline{V}_{k(i)} = z_k \bar{I}_{k(i)}, \quad k=1, \ldots, b \qquad (7.16)$$

Remember what the "*i*" notation indicates – that the quantity is expressed to the d-q coordinate axes of machine *i*.

But we want all quantities on the D-Q (network) coordinate axes, and now we know how to achieve this….

$$\hat{V}_k = \overline{V}_{k(i)} e^{j\delta_i} \Rightarrow\Rightarrow \overline{V}_{k(i)} = \hat{V}_k e^{-j\delta_i}$$

$$\hat{I}_k = \bar{I}_{k(i)} e^{j\delta_i} \Rightarrow\Rightarrow \bar{I}_{k(i)} = \hat{I}_k e^{-j\delta_i}$$

Substitution into (7.16) yields:

$$\hat{V}_k e^{-j\delta_i} = z_k \hat{I}_k e^{-j\delta_i}$$

And we see that the exponentials cancel so that:

$$\hat{V}_k = z_k \hat{I}_k \qquad k=1, \ldots, b \qquad (7.18)$$

Combining (7.18) with (7.16) we see that

$$z_k = \frac{\hat{V}_k}{\hat{I}_k} = \frac{\overline{V}_k}{\overline{I}_k} \qquad k=1, \ldots, b$$

This is expected – it says that the ratio of a voltage drop across an element to the current through the element will remain the same if we rotate all voltages and all currents by a particular angle.

Writing the above equation for every branch in the network results in the following matrix relation:

$$\begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \vdots \\ \hat{V}_b \end{bmatrix} = \begin{bmatrix} z_{11} & 0 & 0 & 0 \\ 0 & z_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & z_{bb} \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \vdots \\ \hat{I}_b \end{bmatrix}$$

We may write the above relation in more compact form:

$$\underline{\hat{V}}_b = \underline{z}_b \underline{\hat{I}}_b \qquad (7.19)$$

Some comments about the above:

- Since all off-diagonal elements are zero, we have assumed that there is no mutual coupling in the network. (Mutual coupling can exist, however, between lines that are physically parallel and located in close proximity, a condition that is found when several circuits share a common right-of-way.)
- The matrix $\underline{z}_b$ is square with non-zero values along the diagonal and is therefore invertible. We denote its inverse as $\underline{y}_b$, such that:

$$\underline{\hat{I}}_b = \underline{y}_b \underline{\hat{V}}_b \qquad (7.20)$$

- The matrix of impedances $\underline{z}_b$ is called the *primitive impedance matrix*, the matrix of admittances $\underline{y}_b$ the *primitive admittance matrix*, and the equations using the z- and y- forms are called the *primitive network equation*, named by Gabriel Kron (see pic).

For treatment of Kron's primitive matrices, see pp. 288-289 and pp. 366-368 of P. Anderson, "Analysis of Faulted Power Systems," The Iowa State University Press, 1973.

- The primitive network equation does not describe the network at all, i.e., it gives absolutely no information as to how the individual branches are interconnected in the network.

In order to provide network connection information, we need the node-incidence matrix $\underline{A}$, given by:

$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{b1} & a_{b2} & \cdots & a_{bn} \end{bmatrix}$$

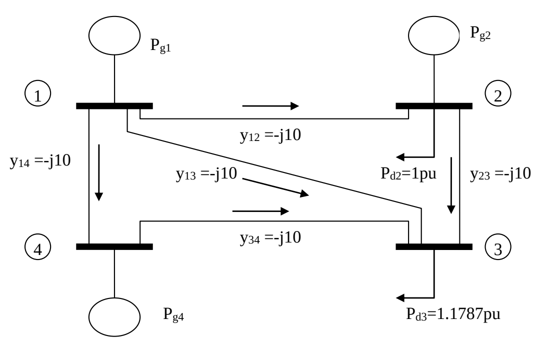
where

- b (number of rows) is the number of branches in the network.
- n (number of columns) is the number of nodes in the network.
- a_{ki} is given by:

$$a_{ki} = \begin{cases} +1 \text{ if current in branch k is leaving node i} \\ -1 \text{ if current in branch k is entering node i} \\ \ 0 \text{ if branch k is not connected to node i} \end{cases}$$

Note that $\underline{A}$ is a b × n matrix:

- Number of branches = number of rows
- Number of nodes = number of columns



$$\underline{A} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

Let's denote the nodal voltages and currents, expressed to the network frame, as $\underline{\hat{V}}$ and $\underline{\hat{I}}$.

The nodal currents may be related to the branch currents by summing over all currents leaving node *i*. Since a node corresponds to a *column* of the node-incidence matrix, we can relate the nodal currents to the branch currents through a multiplication of $\underline{A}^T$ with the branch current vector, i.e.,

$$\hat{I} = \underline{A}^T \hat{I}_b \qquad (*)$$

The matrix $\underline{A}^T$ has each row corresponding to a node, and therefore the elements of each row will pick out of $\hat{I}_b$ the appropriate branch flows emanating from that node to provide the total injected current into that node.

Note dimensions of terms in this relation, we obtain an $n \times 1$ matrix from the product of an $n \times b$ matrix with a $b \times 1$ matrix.

So the above relation illustrates that the node-incidence matrix can be used to sum quantities. In this particular case, we summed branch currents to get the nodal currents according to KCL.

What about relating nodal voltages to branch voltage drops? In this case, we consider KVL and recall that we need to "sum" the nodal voltages to obtain the voltage drops. So we need to express $\underline{\hat{V}}_b$ as a product of $\underline{\hat{V}}$ and $\underline{A}$ in some fashion.

If you toy with these matrices from purely a dimensional point of view, you will see that

$$\underline{A}\underline{\hat{V}} = \underline{\hat{V}}_b \qquad (**)$$

where the dimensions indicate that we obtain a $b \times 1$ from the product of a $b \times n$ with an $n \times 1$. We may also derive this from power relations (ref: P. Anderson, "Analysis of Faulted Power Systems," pp. 371-372).

But we observe in (**) that each row of $\underline{A}$ corresponds to a particular branch, and the non-zero elements of that row correspond to a bus that is connected to that branch. There will only be two such buses, and the product $\underline{AV}$ will pick off the two voltages at either end of the branch to find their difference, which is contained in $\underline{V}_b$.

Substitution of eq. (7.20), $\underline{\hat{I}}_b = \underline{y}_b \underline{\hat{V}}_b$,into eq. (*), $\hat{I} = \underline{A}^T \hat{I}_b$,yields:

$$\hat{I} = \underline{A}^T \hat{I}_b = \underline{A}^T \underline{y}_b \underline{\hat{V}}_b \qquad (***)$$

and substitution of eq. (**), $\underline{A}\underline{\hat{V}} = \underline{\hat{V}}_b$, into (***) yields:

$$\hat{I} = \underline{A}^T \underline{y}_b \underline{\hat{V}}_b = \underline{A}^T \underline{y}_b \underline{A}\underline{\hat{V}}$$

Here, we clearly see that the familiar Y-bus (admittance matrix) is obtained from the primitive admittance matrix from:

$$\underline{Y} = \underline{A}^T \underline{y}_b \underline{A}$$

so that we have, finally,

$$\underline{\hat{I}} = \underline{Y}\underline{\hat{V}} \qquad (7.21)$$

which relates nodal voltages and current injections given on the D-Q (network) coordinate axes.

Now define a square $n \times n$ transformation matrix $\underline{T}$ according to:

$$\underline{T} = \begin{bmatrix} e^{j\delta_1} & 0 & 0 & 0 \\ 0 & e^{j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{j\delta_n} \end{bmatrix} \Rightarrow \underline{T}^{-1} = \begin{bmatrix} e^{-j\delta_1} & 0 & 0 & 0 \\ 0 & e^{-j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{-j\delta_n} \end{bmatrix}$$

Then we can obtain the nodal currents and voltages expressed on D-Q (network) coordinate axes from the nodal currents and voltages expressed on d-q (individual machine i) coordinate axes from:

$$\underline{\hat{I}} = \underline{T}\,\underline{I} \text{ and } \underline{\hat{V}} = \underline{T}\,\underline{V}$$

Substitution into eq. (7.21), $\underline{\hat{I}} = \underline{Y}\,\underline{\hat{V}}$, yields:

$$\underline{T}\,\underline{I} = \underline{Y}\,\underline{T}\,\underline{V} \Rightarrow \underline{I} = \underline{T}^{-1}\underline{Y}\,\underline{T}\,\underline{V} = \underline{M}\,\underline{V} \Rightarrow \underline{I} = \underline{M}\,\underline{V}$$

where clearly,

$$\underline{M} = \underline{T}^{-1}\underline{Y}\,\underline{T}$$

What does the transformation do?

It allows us to relate currents in the d-q coordinate frame of one machine, $\bar{I}_1, \bar{I}_2, ..., \bar{I}_n$ to voltages in the d-q coordinate frame of all other machines.

You see, $\underline{I} = \underline{Y}\,\underline{V}$ (where the current and voltage vectors are given relative to the different q_i axes of the various machines) does not work!

$\underline{I} = \underline{M}\,\underline{V}$ is the replacement we need, where $\underline{M} = \underline{T}^{-1}\underline{Y}\,\underline{T}$.

Example 7.1: The matrix $\underline{M}$ can be evaluated by performing the appropriate matrix multiplications:

First, get $\underline{Y}$. Then…

$$M = \underline{T}^{-1}\underline{Y}\underline{T} =$$

$$\begin{bmatrix} e^{-j\delta_1} & 0 & 0 & 0 \\ 0 & e^{-j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{-j\delta_n} \end{bmatrix} \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j\theta_{12}} & \cdots & Y_{1n}e^{j\theta_{1n}} \\ Y_{21}e^{j\theta_{21}} & Y_{22}e^{j\theta_{22}} & \cdots & Y_{2n}e^{j\theta_{2n}} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n1}e^{j\theta_{n1}} & Y_{n1}e^{j\theta_{n2}} & \cdots & Y_{nn}e^{j\theta_{nn}} \end{bmatrix} \begin{bmatrix} e^{j\delta_1} & 0 & 0 & 0 \\ 0 & e^{j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{j\delta_n} \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j(\theta_{12}-\delta_{12})} & \cdots & Y_{1n}e^{j(\theta_{1n}-\delta_{1n})} \\ Y_{21}e^{j(\theta_{21}-\delta_{21})} & Y_{22}e^{j\theta_{22}} & \cdots & Y_{2n}e^{j(\theta_{2n}-\delta_{2n})} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n1}e^{j(\theta_{n1}-\delta_{n1})} & Y_{n1}e^{j(\theta_{n2}-\delta_{n2})} & \cdots & Y_{nn}e^{j\theta_{nn}} \end{bmatrix}$$

where $\delta_{ik}=\delta_i - \delta_k$.

With Y_{ik} as the Y-bus element magnitude, we have element in position i,k as $Y_{ik}e^{j\theta_{ik}} = G_{ik} + jB_{ik}$, and the general form of the term in row i, col k, in the matrix $\underline{M}$ is:

$$M_{ik} = Y_{ik}e^{j(\theta_{ik}-\delta_{ik})} = Y_{ik}e^{j\theta_{ik}}e^{-j\delta_{ik}} = \left(G_{ik} + jB_{ik}\right)\left(\cos\delta_{ik} - j\sin\delta_{ik}\right)$$

➔ Multiplying and then gathering real and imaginary parts:

$$M_{ik} = \underbrace{\left(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}\right)}_{F_{G+B}(\delta_{ik})} + j\underbrace{\left(B_{ik}\cos\delta_{ik} - G_{ik}\sin\delta_{ik}\right)}_{F_{B-G}(\delta_{ik})}$$

So the i-k^{th} term in matrix $\underline{M}$ is given by $F_{G+B}(\delta_{ik})+j\,F_{B-G}(\delta_{ik})$. This simplifies for the diagonal elements, since δ_{ii}=0, to $G_{ii}+jB_{ii}$. So

$$M_{ij}= F_{G+B}(\delta_{ik})+j\,F_{B-G}(\delta_{ik})$$
$$M_{ii}= G_{ii}+jB_{ii}$$

Separating real and imaginary parts, we obtain $\underline{M}=\underline{H}+j\underline{S}$ where

$H_{ik}= F_{G+B}(\delta_{ik})$
$H_{ii}=G_{ii}$

$S_{ik}=F_{B-G}(\delta_{ik})$
$S_{ii}=B_{ii}$

You should review examples 7.2 and 7.3 in the text.

Example 7.2: Derive the relations between the d and q machine voltages and currents for a two-machine system.

Solution:
Get $\underline{Y}$.
First, compute **M** as in Example 7.1, and then express it as **M**=**H**+j**S.**
Then, express

$$\bar{\mathbf{I}} = \mathbf{MV} \quad = \quad (\mathbf{H} + \mathrm{j}\mathbf{S})\begin{bmatrix} V_{q1} & + & \mathrm{j}V_{d1} \\ & \vdots & \\ V_{qn} & + & \mathrm{j}V_{dn} \end{bmatrix} = (\mathbf{H} + \mathrm{j}\mathbf{S})(\mathbf{V}_q + \mathrm{j}\mathbf{V}_d) \tag{7.40}$$

$$\Rightarrow \mathbf{I}_q + j\mathbf{I}_d \quad = \quad (\mathbf{HV}_q - \mathbf{SV}_d) + \mathrm{j}(\mathbf{SV}_q + \mathbf{HV}_d)$$

For the real part,

$$\begin{bmatrix} I_{q1} \\ I_{q2} \end{bmatrix} = \begin{bmatrix} G_{11} & F_{G+B}(\delta_{12}) \\ F_{G+B}(\delta_{21}) & G_{22} \end{bmatrix}\begin{bmatrix} V_{q1} \\ V_{q2} \end{bmatrix} - \begin{bmatrix} B_{11} & F_{B-G}(\delta_{12}) \\ F_{B-G}(\delta_{21}) & B_{22} \end{bmatrix}\begin{bmatrix} V_{d1} \\ V_{d2} \end{bmatrix} \tag{7.40a}$$

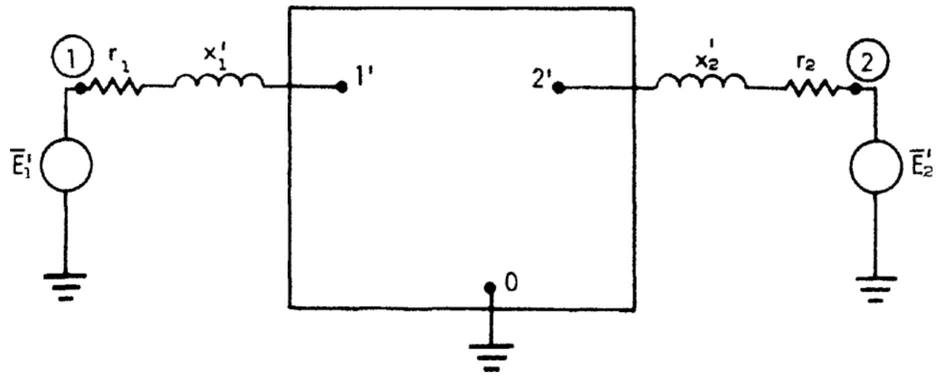
and the imaginary part

$$\begin{bmatrix} I_{d1} \\ I_{d2} \end{bmatrix} = \begin{bmatrix} B_{11} & F_{B-G}(\delta_{12}) \\ F_{B-G}(\delta_{21}) & B_{22} \end{bmatrix}\begin{bmatrix} V_{q1} \\ V_{q2} \end{bmatrix} + \begin{bmatrix} G_{11} & F_{G+B}(\delta_{12}) \\ F_{G+B}(\delta_{21}) & G_{22} \end{bmatrix}\begin{bmatrix} V_{d1} \\ V_{d2} \end{bmatrix} \tag{7.40b}$$

Example 7.3: Derive the complete system equations for a two-machine system. The machines are to be represented by a two-axis model of Section 4.15.3. Loads are to be represented by constant impedances.

Solution approach:

1. Get $\underline{Y}$, but because of the simplicity of the two-axis model (same as classical but internal voltage is not constant), we can include r_i+jx'_q in with it, as indicated below.



2. Reduce the network to its internal generator nodes (so that the internal impedance r_i+jx'_{qi} is included in the network **Y** matrix.
3. Compute **M=T^{-1}YT** and from this, **H** and **S**.
4. Express (7.40a) and (7.40b).
5. Write down the equations from the machine model of section 4.15.3, once for gen 1 and once for gen 2, using the given assumption that $x'_{qi}=x'_{di}$. This machine model, with this assumption, is

$$\begin{aligned}
\tau'_{q0i}\dot{E}'_{di} &= -E'_{di} - \left(x_{qi} - x'_i\right) I_{qi} \\
\tau'_{d0i}\dot{E}'_{qi} &= E_{FDi} - E'_{qi} + \left(x_{di} - x'_i\right) I_{di} \\
\tau_{ji}\dot{\omega}_i &= T_{mi} - \left(I_{di}E'_{di} + I_{qi}E'_{qi}\right) - D_i\omega_i \qquad (7.41) \\
\dot{\delta}_i &= \omega_i - 1 \quad i = 1,2
\end{aligned}$$

6. Substitute for I_{qi} and I_{di} from step 4.
7. This is an eighth-order system, but two equations for angle derivatives can be combined into one and so it is a seventh-order system.

<u>Additional comments</u>:

The overall problem is given by

$$\underline{\dot{x}} = \underline{f}(\underline{x}, \underline{v}, T_m, t)$$

$$\underline{I} = \underline{M}\,\underline{V}$$

where the current and voltage vectors are given relative to the different q_i axes of the various machines, and $\underline{M}$ is formulated as follows:

$$\underline{M} = \underline{T}^{-1}\underline{Y}\underline{T}$$

And because

$$\underline{Y} = \underline{A}^T \, \underline{y}_b \, \underline{A}$$

we have that

$$\underline{M} = \underline{T}^{-1}\underline{Y}\underline{T} = \underline{T}^{-1}\underline{A}^T \, \underline{y}_b \, \underline{A}\underline{T}$$

Now here is an issue. If we have entirely constant impedance loads, then all loads can be included into the matrix $\underline{\mathrm{Y}}$, and the above formulation is OK.

If we have constant current loads, then those loads may be included in the vector $\underline{\mathrm{I}}$. And clearly having both constant impedance and constant current loads can be handled according to these two approaches (use $\underline{\mathrm{Y}}$ for constant impedance loads and $\underline{\mathrm{I}}$ for constant current loads).

But if we have constant power loads, then those loads, when converted to a constant current representation through I=(S/V)*, are a function of voltage. In that case, the problem we are solving is

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{v}, T_m, t)$$

$$\underline{I}(\underline{V}) = \underline{M}\underline{V}$$

where the algebraic equations must be solved iteratively.

Either way, we have the interface problem (caused by the need to compute states at time *t* using algebraic values at time *t*-1), illustrated in a figure from Brian Stott’s paper below.

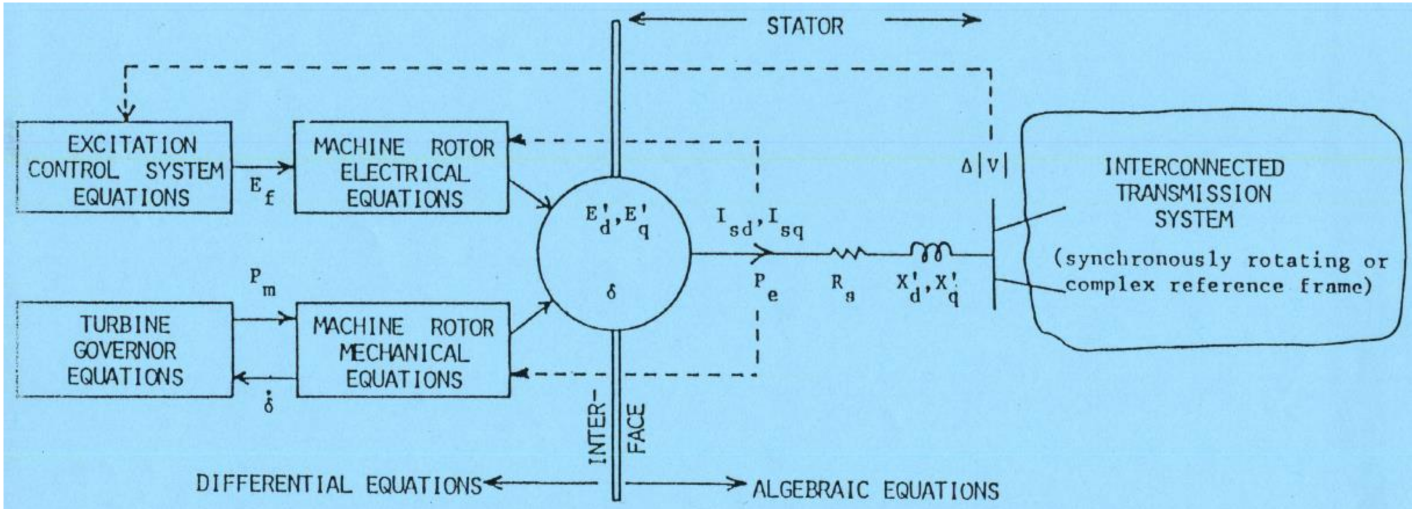


Fig. 1. Schematic of transient model of synchronous generator connected to transmission network.

Stott, Section IV of his paper, introduces a classification system for solving a differential-algebraic equation (DAE), which is what we have. He says that solution approaches are characterized by three attributes:

1. The way in which machine and network equations are interfaced with each other:
   a. Partitioned: alternating
   b. Simultaneous (combined or algebraically)
2. The integration method used:
   a. Explicit
   b. Implicit
3. The technique for solving the algebraic equations (an issue if you have constant power loads and you solve using the alternating method.

I provide some cutouts from Stott's paper below.

There is also material from the Powertech TSAT User Manual of interest here, below:

Solution of the entire differential-algebraic equation sets

The overall system differential and algebraic equations are solved using a partitioned approach:

- Numerical integration is performed to solve the differential equations.
- Iterative solution is performed to solve the algebraic equations.
- The current injections and the bus voltages are the interface variables.

This is shown in Figure 3-2.

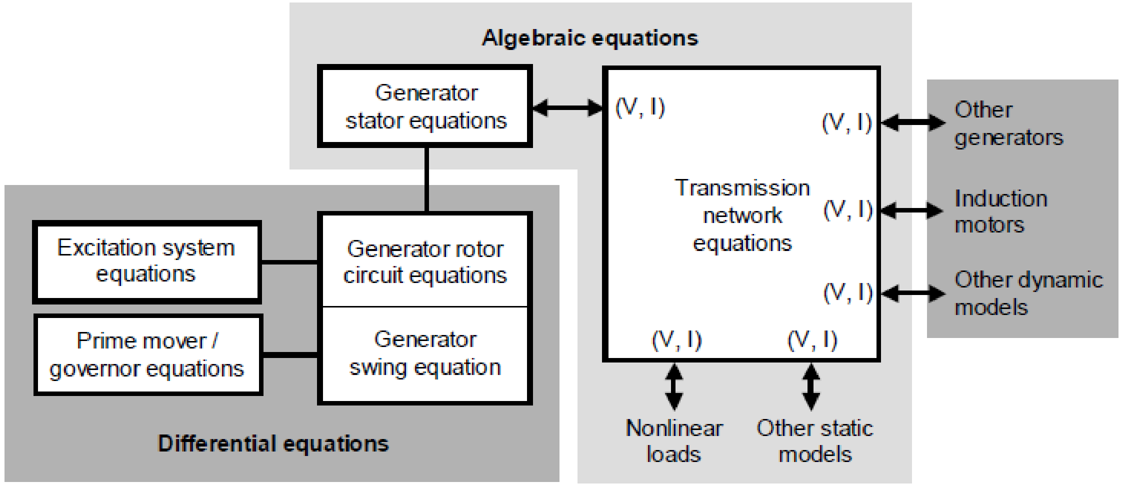


Figure 3-2: The Overall system model of time domain simulations

In addition, I encourage you to do three things: (1) Read section 7.11 to get a high-level view of the machine-network problem; (2) Read 7.12, where we take care of (a) the nonlinear load problem and (b) network-machine interface including machine saliency, when $x'_d \neq x'_q$, and things don't simplify; (3) Read sections 7.13 and 7.14 to remind you of what we learned in numerical solvers.

B. The Transmission Network

The network is described by a large sparse algebraic nodal admittance matrix equation. This matrix is usually complex and symmetrical, and constant in between infrequent branch-switching operations. Each bus load is modeled as an exponential or polynomial function of the bus voltage magnitude and occasionally of the frequency. Unless all loads have the simplest representation as fixed shunt admittances, the overall network/load equation set is nonlinear, with a similar structure to that of the standard load-flow problem.

C. General Overall Form of System Equations

The complete power-system model comprises a set of first-order differential equations

$$\dot{y} = f(y, x) \tag{1}$$

and an algebraic set

$$0 = g(y, x). \tag{2}$$

Set (1) comprises the differential equations of all machines. Since each machine is coupled to the other machines only through the network, set (1) is a collection of separate uncoupled subsets. In the model shown in Fig. 1, there are two such subsets per machine, but which become joined together whenever $\dot{\delta}$ is fed back to the excitation control.

Set (2) comprises the stator equations of each machine, transformed into the complex network reference frame, coupled to the equations of the network and loads, plus the equations defining the fed-back stator quantities $\boldsymbol{u}$.

D. Specific Form of System Equations

Equation (1) has a quasi-linear structure that can be shown as:

$$\dot{y} = f(y, u) = A \cdot y + B \cdot u. \tag{1a}$$

Matrix A is square, sparse, and block-diagonal. Matrix B is rectangular, sparse, and blocked. (Note that the matrix form is not necessarily retained in the programming.) When saturation is not represented, both A and B are constant in many of the most common specific models.

The algebraic set (2) can be subdivided into two parts:

$$I(E, V) = Y \cdot V \tag{2a}$$

and

$$u = u(E, V) \tag{2b}$$

where (2a) is the sparse bus admittance matrix equation of the loaded network. I is the vector of bus current injections. For a load, the injection is a function of the bus voltage, and for a generator, it is the stator current as a function of the stator internal and terminal voltages, transformed into the network frame. Equation (2b) simply serves to calculate $\boldsymbol{u}$.

B. The Partitioned-Solution Approach

This is the traditional approach, used in nearly all present-day industrial programs. The differential-equation set (1) is solved by integration separately for y, and the algebraic set (2) is solved separately for x. These solutions are alternated with each other in some manner. The respective solutions may or may not be iterative, and true elimination of interface error may or may not be achieved, depending on the specific methods and system models.

The two salient defining features of the Partitioned approach are: a) the integration method and the network-solution method may in principle be chosen independently of each other, and b) it is always possible, through the use of extrapolation/interpolation techniques (see Section V-D) to solve the network only every few integration steps. This latter is generally the case in mid- and long-term stability calculations, although it is much less common in the short-term mode.

C. Simultaneous-Solution Approach

Implicit integration methods convert (1) into a set of algebraic equations in the unknowns y_n and x_n, i.e., the values at the end of the step. In the Simultaneous-solution approach, these algebraized equations are lumped together with (2) to form a single larger algebraic set, all of whose variables are then solved simultaneously. Inherently in this approach, equation (1) is solved with the same frequency as (2), and there is no interface error. The Simultaneous-solution approach has been adopted in at least one routinely used industrial program, and a number of prototype test programs. It has been attracting interest as possibly a superior scheme.

VII. The Modeling and Solution of the Network

A. The Network Model

The network model comprises the loaded transmission system plus the machine stators. In order to construct and solve the network equation (2a), the d, q-axis stator equation (A.6) of each machine has to be expressed in the form (A.8), i.e., transformed into the network complex reference frame. From (A.8), the stator internal voltage is now $E'_{re} + jE'_{im}$ and the stator impedance is Z_s.

In (2a) as originally stated, the nodal injection at a machine terminal bus is the machine stator current, obtained by solving (A.8). There is some advantage in taking the Norton equivalent of each machine stator. Then a shunt impedance Z_s is inserted at the machine terminal bus, and the injected current becomes $(E'_{re} + jE'_{im})/Z_s$. The network equation (2a), restated here for convenience, now becomes:

$$I(E, V) = Y \cdot V \tag{8a}$$

where Y includes the machine-terminal Norton shunts. Vector I comprises the machine-terminal Norton injections that are functions of E and the load-bus currents that are functions of voltage magnitude and perhaps frequency.[1] This form of the network equation will be assumed henceforth, unless otherwise stated.

B. The Network Solution Problem

The problem is to solve either (8b) or (8c) for V. For a given value E, obtained from the solution of the differential equations (1), the machine-bus Norton injections and shunts are constant.[2] The nonlinearity of (8) is then due entirely or mainly to load currents that are functions of V. Unless all loads are represented as fixed shunt impedances (injected current always zero), an iterative solution of (8) not unlike a standard load-flow solution [31] needs to be performed.

For mid-term and long-term dynamic studies, excitation control is assumed to hold the machine terminal (or other bus) voltage magnitude constant [24], [25], which introduces a conventional constant-V load-flow constraint into (2a). When automatic transformer tap changing is represented in longer term studies, the relevant admittances in Y can change frequently. Very occasionally, network branch admittance variation with frequency is represented, in which case the elements of Y change continually. Such network changes can be dealt with by bus-injection techniques to avoid continual matrix alterations.

C. Network Solution Techniques

In this subsection, we consider four alternative methods for solving (8). Only the last two are now regarded as of interest for efficient modern large-scale industrial applications, but programs employing the first two are still in practical use.

1) Gauss–Seidel: This method has the merits of low storage, ease of programming, and of being able to accommodate any changes in the matrix elements with ease because the algorithm operates directly on the branch admittances.

The economical complex-symmetrical storage scheme can be used in the programming, even if nonbilateral elements are present. Since the Norton admittances are usually large, Y is better conditioned than in the standard load-flow case. Except at fault and switching times, each iterative solution has good starting values of V from the previous solution(s), preferably extrapolated.

Usually, the "load-flow" problem has no voltage-controlled (PV) buses, in which case the best convergent version seems to be the secondary correction method [31]. Nevertheless, convergence to acceptable accuracy can vary a great deal from problem to problem, from 2–3 iterations to hundreds (or no

4) Newton Method: The Newton method cannot be applied to most power network equations in complex form; therefore the expanded version (8c) is used. This equation can be written as

$$F^e = I^e - Y^e \cdot V^e \tag{12}$$

where F^e is zero at the solution. Each iteration of the Newton solution requires the construction of the Jacobian-matrix equation:

$$F^e = -J^e \cdot \Delta V^e \tag{13}$$

and its direct solution by sparse triangulation for the correction vector ΔV^e. This solution corresponds to the "rectangular current mismatch" Newton load-flow method, which is the natural version for the stability application although it is less so for conventional load flow [18], [31], [38]. When, as is usual, the series branches of the network have constant admittances, the Jacobian matrix J^e differs from Y^e only in the bus "self" terms—those in the 2×2 diagonal blocks.

3) Nonimpedance Loads: Nonimpedance loads are dealt with in a similar manner to machine saliency. In the factored-Y^c approach, a proportion of each load is represented as a fixed complex shunt in Y^c, and the residue of the load enters I^c as a nonlinear function of V^c to be iterated to convergence. Low-voltage cutoff must be provided so that for instance constant-power loads do not demand infinite current during a solid fault. In standard Z-matrix load flow, this "fringing current" technique was found to aid convergence considerably. Here, it is valuable though not equally successful because of the much greater bus voltage variation. Both [18] and the author have found that typical numbers of iterations are 2–6. In other words, nonimpedance loads cause more trouble than dynamic saliency.

In Newton's method, partial derivative terms are added to the 2×2 diagonal blocks in J^e to represent the loads incrementally, and this is better than the nonincremental fringing-current modeling. Newton's method is now noticeably superior, and the above-mentioned numbers of iterations reduce to 2–3.

How accurate the solution for nonimpedance loads must be is a matter for some conjecture, since the load characteristics are rarely well known. On the other hand, it is widely agreed that some improvement over the classical fixed-impedance model is necessary [12]. Reference [27] investigated the effect on accuracy of keeping the load current constant over the step (which was essential in that Runge–Kutta method with a noniterative network solution.) The results and the discussion of the paper suggest that the errors only become important for marginally stable longer duration studies. Using voltage extrapolations to estimate the required intermediate load currents would be more reliably accurate.