Production Costing (Chapter 8 of W&W)

1.0 Introduction

Production costs refer to the operational costs associated with producing electric energy. The most significant component of production costs are the fuel costs necessary to run the thermal plants.

A production cost program, also referred to as a production cost model, is widely used throughout the electric power industry for many purposes:

- Long-range system planning: Here, it is used to simulate a single future year following the planned expansion. For example, the Midwest ISO used a production cost program to understand the effect on energy prices of building HVDC from the Midwest US to the East coast.
- Fuel budgeting: Many companies run production cost programs to determine the amount of natural gas and coal they will need to purchase in the coming weeks or months.
- Maintenance: Production cost programs are run to determine maintenance schedules for generation.
- Energy interchange: Production cost programs are run to facilitate negotiations for energy interchange between companies.

There are two essential inputs for any production cost program:

1. Data characterizing future load
2. Data characterizing generation costs, in terms of:
    a. Heat rate curves and
    b. Fuel costs

All production cost programs require at least the above data. Specific programs will require additional data depending on their particular design.

The information provided by production costing includes the annual costs of operating the generation facilities, a cost that is dominated by the fuel costs but also affected by the maintenance costs. Production costing may also provide more time-granular estimates of fuel and maintenance costs, such as monthly, weekly, or hourly, from which it is then possible to obtain annual production costs.

A simplified way to consider a production cost program is as an hour-by-hour simulation of the power system over a duration of T hours, where at each hour,

- The load is specified;
- A unit commitment decision is made;
- A dispatch decision is made to obtain the production costs for that hour

The total production costs is then the sum of hourly production costs over all hours 1,…,T.

Some production programs do in fact simulate hour-by-hour operation in this manner. An important characterizing feature is how the program makes the unit commitment (UC) and dispatch decisions.

The simplest approach makes the UC decision based on priority ordering such that units with lowest average cost are committed first. Startup costs are added when a unit is started, but those costs do not figure into the optimization.

The simplest approach for making the dispatch decision is referred to as the block loading principle, where each unit committed is fully loaded before the next unit is committed. The last unit is dispatched at that level necessary to satisfy the load.

Greater levels of sophistication may be embedded in production cost programs, as described below:

- Unit commitment and dispatch: A full unit commitment program may be run for certain blocks of intervals at a time, e.g., a week.
- Hydro: Hydro-thermal coordination may be implemented.

- Network representation: The network may be represented using DC flow and branch limits.
- Locational marginal prices: LMPs may be computed.
- Maintenance schedules: Maintenance schedules may be taken into account.
- Uncertainty: Load uncertainty and generator unavailability may be represented using probabilistic methods. This allows for computation of reliability indices such as loss of load probability (LOLP) and expected unserved energy (EUE).
- Security constraints may be imposed using LODFs.

Below are some slides that Midwest ISO uses to introduce production cost models.

MidwestISO
Energizing the Heartland

What is a Production Cost Model?

- Captures all the costs of operating a fleet of generators
  - Originally developed to manage fuel inventories and budget in the mid 1970's
- Developed into an hourly chronological security constrained unit commitment and economic dispatch simulation
  - Minimize costs while simultaneously adhering to a wide variety of operating constraints.
  - Calculate hourly production costs and location-specific market clearing prices.

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MidwestISO Energizing the Heartland

What Are the Advantages of Production Cost Models?

- Allows simulation of all the hours in a year, not just peak hour as in power flow models.
- Allows us to look at the net energy price effects through
  - LMP’s and its components.
  - Production cost.
- Enables the simulation of the market on a forecast basis
- Allows us to look at all control areas simultaneously and evaluate the economic impacts of decisions.

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Disadvantages of Production Cost Models

- Require significant amounts of data
- Long processing times
- New concept for many Stakeholders
- Require significant benchmarking
- Time consuming model building process
  - Linked to power flow models
- Do not model reliability to the same extent as power flow

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Production Cost Model vs. Power Flow

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Production Cost Model	Power Flow
SCUC&ED: very detailed	Hand dispatch (merit Order)
All hours	One hour at a time
DC Transmission	AC and DC
Selected security constraints	Large numbers of security constraints
Market analysis/ Transmission analysis/planning	Basis for transmission reliability & operational planning

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2.0 Commercial grade production costing tools

We will describe in more detail the construction of production costing programs later. Here we simply mention some of the commercially available production costing tools.

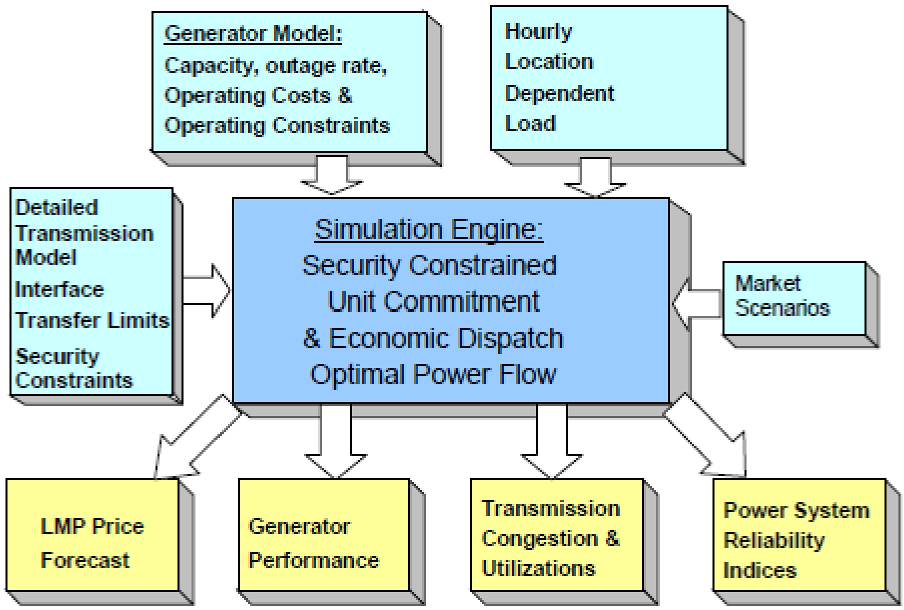
The Ventyx product Promod incorporates details in generating unit operating characteristics, transmission grid topology and constraints, unit commitment/ operating conditions, and market system operations. Promod can operate on nodal or zonal modes depending on the scope, timeframe, and simulation resolution of the problem. Promod is not a forecasting model and does not consider the price and availability of other fuels.

The ADICA product GTMAX, developed by Argonne National Labs, can be employed to perform regional power grade or national power development analysis. GTMax will evaluate system operation, determine optimal location of power sources, and assess the benefits of new transmission lines. GTMax can simulate complex electric market and operating issues, for both regulated and deregulated market.

The PowerCost, Inc. product GenTrader employs economic unit dispatch logic to analyze economics, uncertainty, and risk associated with individual generation resources and portfolios. GenTrader does not represent the network.

PROSYM is a multi-area electric energy production simulation model developed by Henwood energy Inc. It is an hourly simulation engine for least-cost optimal production dispatch based on the resources' marginal costs, with full representation of generating unit characteristics, network area topology and electrical loads. PROSYM also considers and respects operational and chronological constraints; such as minimum up and down times, random forced outages and transmission capacity. It is designed to determine the station generation, emissions and economic transactions between interconnected areas for each hour in the simulation period.

ABB produced the software called “GridView,” illustrated below [1].



PLEXOS, from Plexos Solutions, is a versatile software system that performs production cost simulation and other functions.

It is interesting to note that Global Energy Solutions (GES) in 2002 purchased Henwood Associates (owner of Prosym), then Ventyx (owners of Promod) purchased GES in 2008, then ABB (owners of Gridview) purchased Ventyx. At some point, Mark Henwood went to work for Plexos Solutions (see [2]). Energy Exemplar now owns Plexos.

3.0 Probability models

Key to use of production cost models is the ability to represent uncertainty in load and in generation availability.

3.1 Load duration curves

A critical issue for planning is to identify the total load level for which to plan. One extremely useful tool for doing this is the so-called load duration curve, which is formed as follows. Consider that we have obtained, either through historical data or through forecasting, a plot of the load vs. time for a period T, as shown in Fig. 3 below.

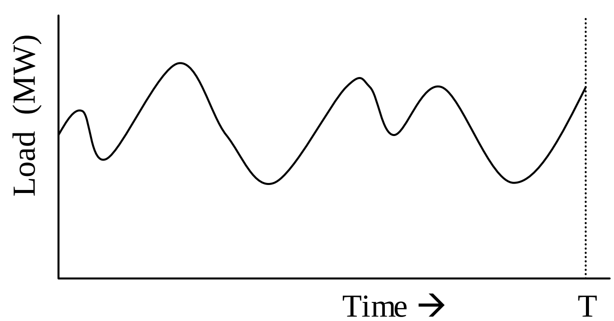


Fig. 3: Load curve (load vs. time)

Of course, the data characterizing Fig. 3 will be discrete, as illustrated in Fig. 4.

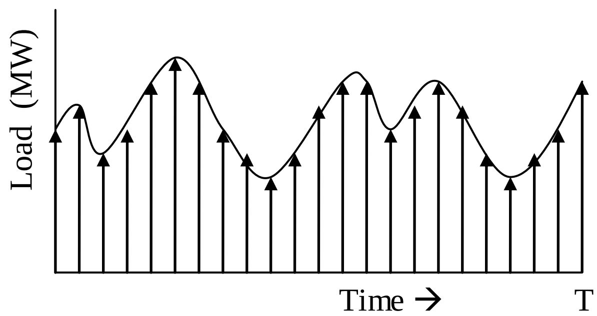


Fig. 4: Discretized Load Curve

We now divide the load range into intervals, as shown in Fig. 5.

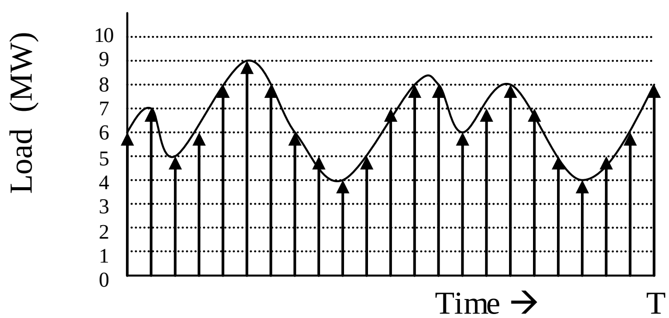


Fig. 5: Load range divided into intervals

This provides the ability to form a histogram by counting the number of time intervals contained in each load range. In this example, we assume that loads in Fig. 5 at the lower end of the range are "in" the range. The histogram for Fig. 5 is shown in Fig. 6.

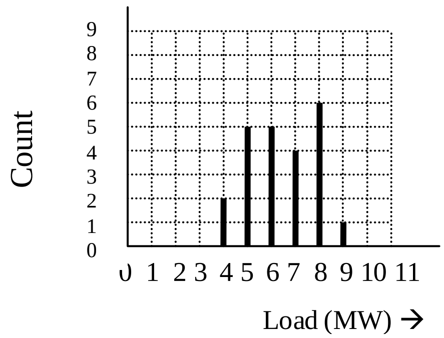


Fig. 6: Histogram

Figure 6 may be converted to a probability mass function, pmf, (which is the discrete version of the probability density function, pdf) by dividing each count by the total number of time intervals, which is 23. The resulting plot is shown in Fig. 7.

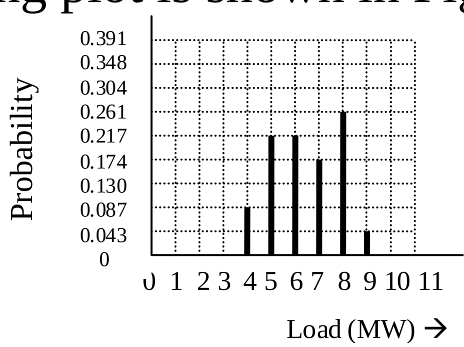


Fig. 7: Probability mass function

Like any pmf, the summation of all probability values should be 1, which we see by the following sum:

$$0.087+0.217+0.217+0.174+0.261+0.043=0.999$$

(It is not exactly 1.0 because there is some rounding error). The probability mass function provides us with the ability to compute the probability of the load being within a range according to:

$$\Pr(Load\ within\ Range) = \sum_{L\ in\ Range} \Pr(Load = L) \qquad (2)$$

We may use the probability mass function to obtain the cumulative distribution function (CDF) as:

$$\Pr(Load \geq Value) = \sum_{L \geq Value} \Pr(Load = L) \qquad (3)$$

From Fig. 7, we obtain:

$$\Pr(Load \geq 1) = \sum_{L \geq 1} \Pr(Load = L) = 1.0$$

$$\Pr(Load \geq 2) = \sum_{L \geq 2} \Pr(Load = L) = 1.0$$

$$\Pr(Load \geq 3) = \sum_{L \geq 3} \Pr(Load = L) = 1.0$$

$$\Pr(\text{Load} \geq 4) = \sum_{L \geq 4} \Pr(Load = L) = 1.0$$

$$\Pr(Load \geq 5) = \sum_{L \geq 5} \Pr(Load = L)$$

$$= 0.217 + 0.217 + 0.174 + 0.261 + 0.043 = 0.912$$

$$\Pr(Load \geq 6) = \sum_{L \geq 6} \Pr(Load = L)$$

$$= 0.217 + 0.174 + 0.261 + 0.043 = 0.695$$

$$\Pr(Load \geq 7) = \sum_{L \geq 6} \Pr(Load = L) = 0.174 + 0.261 + 0.043 = 0.478$$

$$\Pr(Load \geq 8) = \sum_{L \geq 8} \Pr(Load = L) = 0.261 + 0.043 = 0.304$$

$$\Pr(Load \geq 9) = \sum_{L \geq 9} \Pr(Load = L) = 0.043$$

$$\Pr(Load \geq 10) = \sum_{L \geq 10} \Pr(Load = L) = 0$$

Plotting these values vs. the load results in the CDF of Fig. 8.

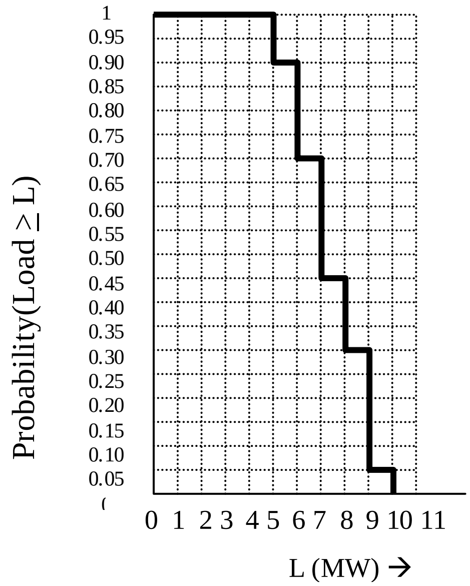


Fig. 8: Cumulative distribution function

The plot of Fig. 8 is often shown with the load on the vertical axis, as given in Fig. 9.

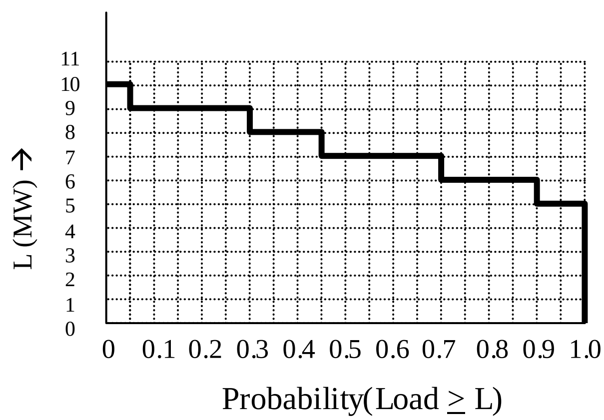


Fig. 9: CDF with axes switched

If the horizontal axis of Fig. 9 is scaled by the time duration of the interval over which the original load data was taken, T, we obtain the *load duration curve.* This curve provides the number of time intervals that the load equals, or exceeds, a given load level. For example, if the original load data had been taken over a year, then the load duration curve would show the number of hours out of that year for which the load could be expected to equal or exceed a given load level, as shown in Fig. 10a.

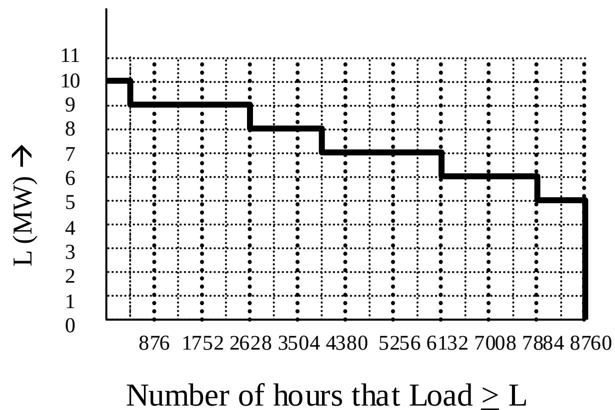


Fig. 10a: Load duration curve

Load duration curves are useful in a number of ways.

- They provide guidance for judging different alternative plans. One plan may be satisfactory for loading levels of 90% of peak and less. One sees from Fig. 10a that such a plan would be unsatisfactory for 438 hours per year (5% of the time).
- They identify the base load. This is the value that the load *always* exceeds. In Fig. 10a, this value is 5 MW. In Fig. 10b, which shows the LDC for the 2003 MISO region, the value is 40GW.

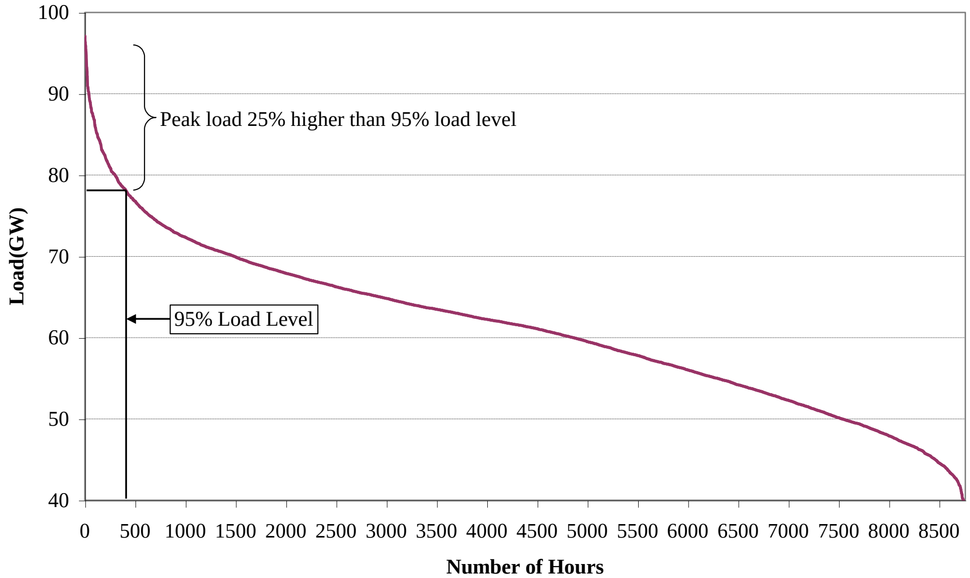


Fig. 10b: MISO LDC for 2003

- They provide convenient calculation of energy, since energy is just the area under the load duration curve. For example, Fig. 11 shows the area corresponding to the base load energy consumption, which is 5MWx8760hr=43800 MW-hrs.

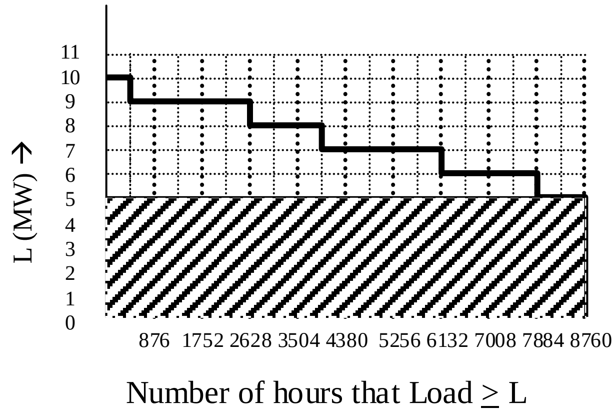


Fig. 11: Area corresponding to base load energy consumption

- They allow illustration of generation commitment policies and corresponding yearly unit energy production, as shown in Fig 12, where we see that the nuclear plant and coal plant #1 are base loaded plants, supplying 26280 MWhrs and 17520 MWhrs, respectively. Coal plant #2 and NGCC plant #1 are the mid-range plants, and CT #1 is a peaker.

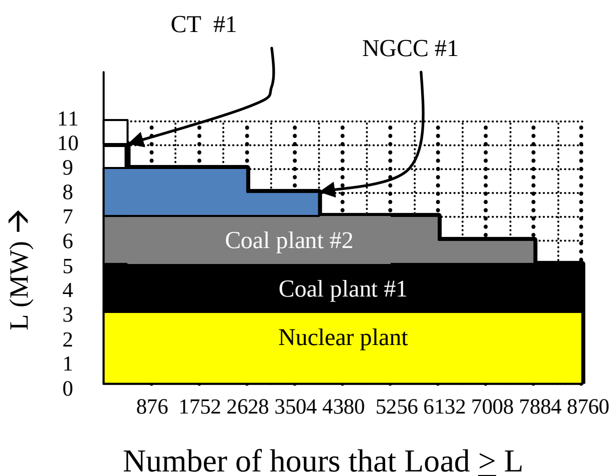


Fig. 12: Illustration of Unit commitment policy

Load duration curves are also used in reliability and production costing programs in computing different reliability indices, as we will see in Sections 4 and 5.

3.2 Generation probability models

We consider that generators obey a two-state model, i.e., they are either up or down, and we assume that the process by which each generator moves between states is Markov, i.e., the probability distribution of future states depends only on the current state and not on past states, i.e., the process is memoryless.

In this case, it is possible to show that unavailability (or *forced outage rate*, FOR) is the "steady-state" (or long-run) probability of a component not being available and is given by

$$U = q = \frac{\lambda}{\lambda + \mu} \tag{4}$$

and the availability is the long-run probability of a component being available and is given by

$$A = p = \frac{\mu}{\lambda + \mu} \tag{5}$$

where λ is the "failure rate" and μ is the "repair rate." See www.ee.iastate.edu/~jdm/ee653/U16-inclass.doc for complete derivation of these expressions.

Substituting $\lambda=1/MTTF$ and $\mu=1/MTTR$, where $MTTF$ is the mean time to failure, and $MTTR$ is the mean time to repair, we get that

$$U = q = \frac{MTTR}{MTTF + MTTR} \tag{6}$$

$$A = p = \frac{MTTF}{MTTF + MTTR} \tag{7}$$

The probability mass function representing the outaged capacity (8a) or available capacity (8b) corresponding to unit j is then given as $f_{Dj}(d_j)$, expressed as

$$f_{Dj}(d_j) = p_j\delta(d_j) + q_j\delta(d_j - C_j) \tag{8a}$$

$$f_{Dj}(d_j) = q_j\delta(d_j) + p_j\delta(d_j - C_j) \tag{8b}$$

and illustrated by Fig. 13 (we will use them both).

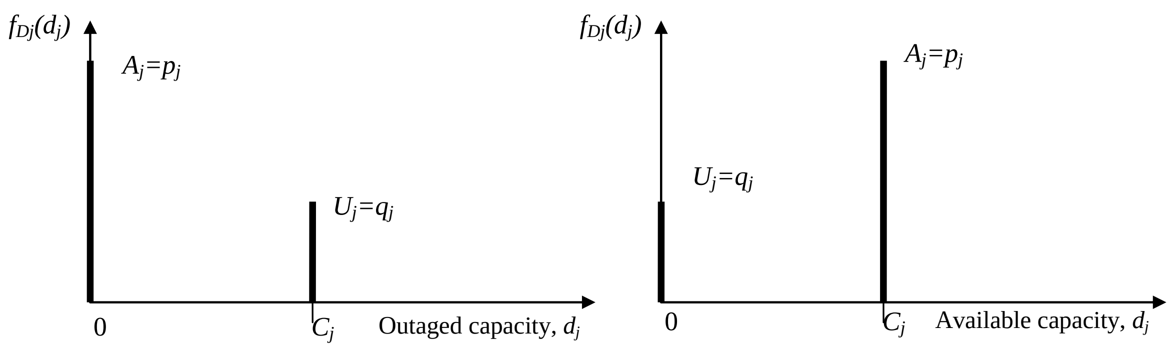


Fig. 13: Two state generator outage model

Unavailability U expresses the fraction of time (not including maintenance time) the generator has been forced out of service. Availability A is the fraction of time (not including maintenance time) the generator is available for service. U+A=1.

4.0 Preliminary definitions

Let's characterize the load shape curve with $t=g(d)$, as illustrated in Fig. 14. It is important to note that the load shape curve characterizes the (forecasted) future time period and is therefore a probabilistic characterization of the demand.

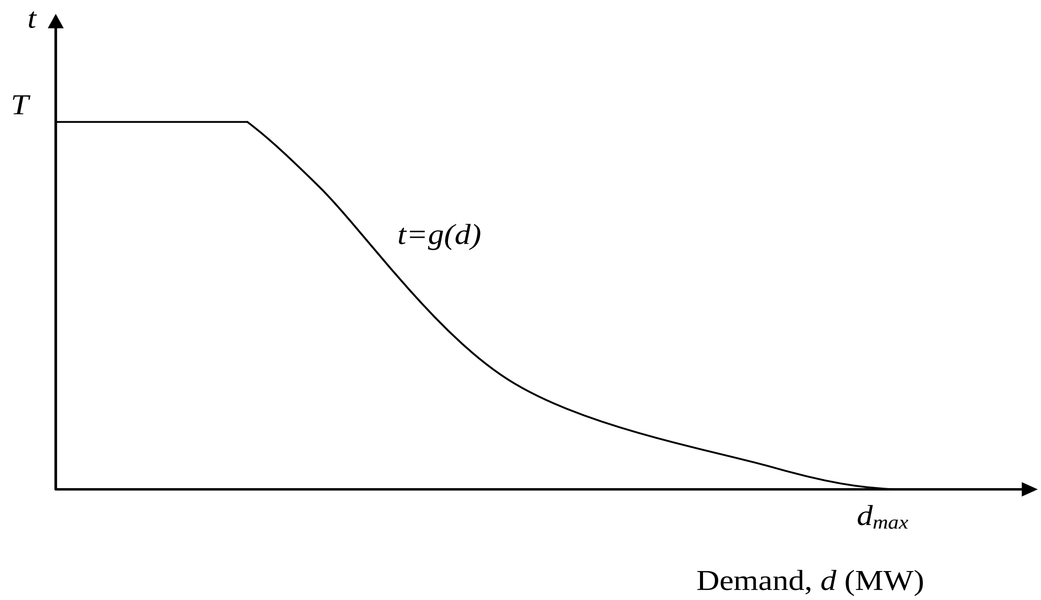


Fig. 14: Load shape $t=g(d)$

Here:

- d is the system load
- t is the number of time units in the interval T for which the load equals or exceeds d and is most typically given in hours or days
- $t=g(d)$ expresses functional dependence of t on d

- T represents, most typically, a year but can be any interval of time (week, month, season, years).

The cumulative distribution function (cdf) is given by

$$F_D(d) = P(D \geq d) = \frac{t}{T} = \frac{g(d)}{T} \quad (9)$$

One may also compute the *total energy* E_T consumed in the period T as the area under the curve, i.e.,

$$E_T = \int_0^{d_{\max}} g(\lambda) d\lambda \quad (10)$$

The *average demand* in the period T is obtained from

$$d_{avg} = \frac{1}{T} E_T = \frac{1}{T} \int_0^{d_{\max}} g(\lambda) d\lambda = \int_0^{d_{\max}} F_D(\lambda) d\lambda \quad (11)$$

Now assume the planned system generation capacity, i.e, the installed capacity, is C_T, and $C_T < d_{max}$. This is an undesirable situation, since we will not be able to serve some demands, even when there is no capacity outage! Nonetheless, it serves well to understand the relation of the load duration curve to several useful indices. The situation is illustrated in Fig. 15.

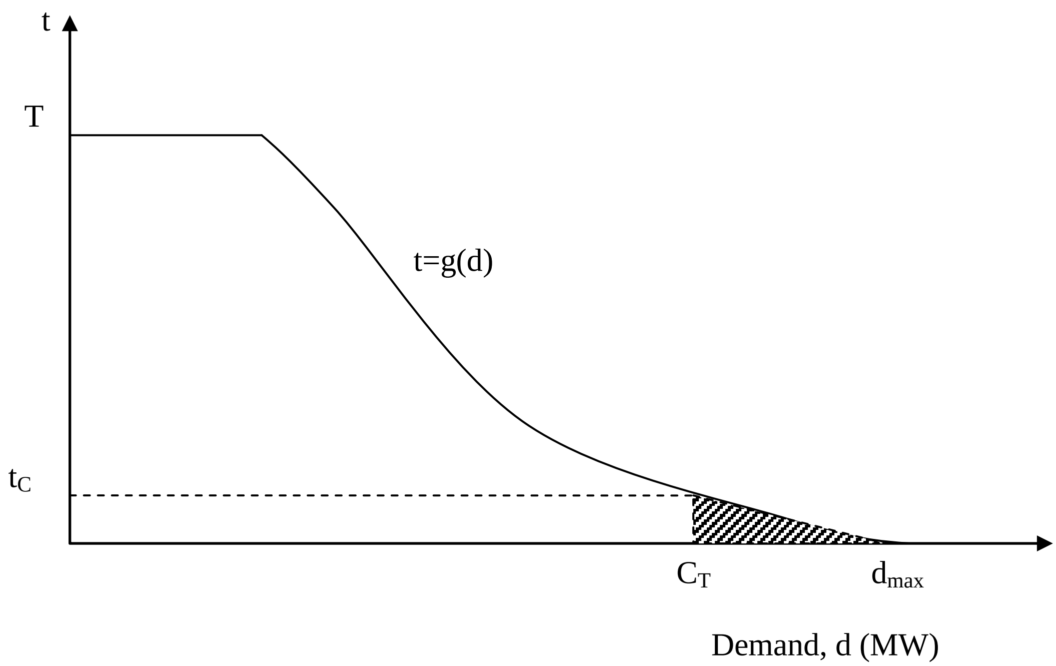


Fig. 15: Illustration of Unserved Demand

Then, under the assumption that the given capacity C_T is perfectly reliable, we may express three useful reliability indices:

- Loss of load expectation, *LOLE*: the expected number of time units that load will exceed capacity

$$LOLE = t_{C_T} = g(C_T) \qquad (12)$$

- Loss of load probability, *LOLP*: the probability that the demand will equal or exceed capacity during *T*:

$$LOLP = P(D \geq C_T) = F_D(C_T) \qquad (13)$$

We note that the condition $D=C_T$ is assumed here to represent a loss of load situation, which would be a conservative assumption.

One may think that, if $d_{max}>C_T$, then $LOLP=1$. However, if $F_D(d)$ is a true probability distribution, then it describes the event $D \geq C_T$ with uncertainty associated with what the load is going to be, i.e., only with a probability. One can take an alternative view, that the load duration curve is certain, which would be the case if we were considering a previous year. In this case, LOLP should be thought of not as a probability but rather as the percentage of time during the interval T for which the load equals or exceeds capacity.

It is of interest to reconsider (9), repeated here for convenience:

$$F_D(d) = P(D \geq d) = \frac{t}{T} = \frac{g(d)}{T} \tag{9}$$

Substituting $d=C_T$, we get:

$$F_D(C_T) = P(D \geq C_T) = \frac{t}{T} = \frac{g(C_T)}{T} \tag{*}$$

By (12), $g(C_T)=LOLE$; by (13), $P(D \geq C_T)=LOLP$, and so (*) becomes:

$$LOLP = \frac{LOLE}{T} \Rightarrow LOLE = LOLP \times T$$

which expresses that $LOLE$ is the expectation of the number of time units within T that demand will exceed capacity.

- Expected demand not served, *EDNS*: If the average (or expected) demand is given by (11), then it follows that expected demand not served is:

$$EDNS = \int_{C_T}^{d_{\max}} F_D(\lambda) d\lambda \qquad (14)$$

  which would be the same area as in Fig. 15 when the ordinate is normalized to provide $F_D(d)$ instead of t. Reference [3] provides a rigorous derivation for (14).

- Expected energy not served, EENS: This is the total amount of time multiplied by the expected demand not served, i.e.,

$$EENS = T \int_{C_T}^{d_{\max}} F_D(\lambda) d\lambda = \int_{C_T}^{d_{\max}} g(\lambda) d\lambda \qquad (15)$$

  which is the area shown in Fig. 15.

4.1 Effective load approach

The notion of effective load is used to account for the unreliability of the generation, and it is essential for understanding the view taken in [3].

The basic idea is that the total system capacity is always C_T, and the effect of capacity outages are accounted for by changing the load model in an appropriate fashion, and then the different indices are computed as given in (12), (13), (14), and (15).

A capacity outage of C_i is therefore modeled as an increase in the demand, not as a decrease in capacity!

We have already defined D as the random variable characterizing the demand. Now we define two more random variables:

- D_j is the random increase in load for outage of unit i.
- D_e is the random load accounting for outage of all units and represents the *effective* load.

Thus, the random variables D, D_e, and D_j are related:

$$D_e = D + \sum_{j=1}^{N} D_j \qquad (16)$$

It is important to realize that, whereas C_j represents the capacity of unit j and is a deterministic value, D_j represents the increase in load corresponding to outage of unit j and is a random variable. The probability mass function (pmf) for D_j is assumed to be as given in Fig. 16 below, i.e., a two-state model. We denote the pmf for D_j as $f_{Dj}(d_j)$. It expresses the probability that the unit experiences an outage of 0 MW as A_j, and the probability the unit experiences an outage of C_j MW as U_j.

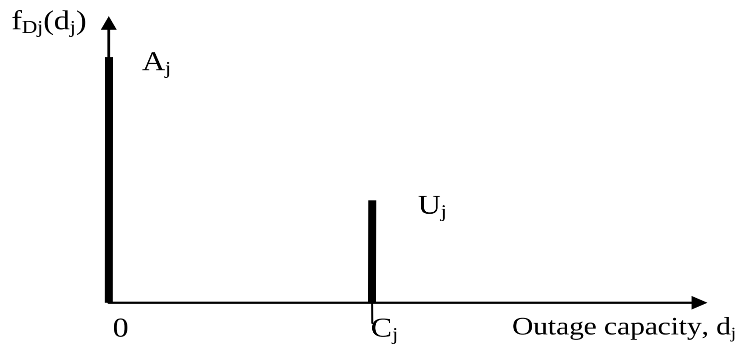


Fig. 16: Two state generator outage model

Recall from probability theory that the pdf of the sum of two independent random variables is the convolution of their individual pdfs, that is, for random variables *X* and *Y*, with *Z*=*X*+*Y*, then

$$f_Z(z) = \int_{\lambda=-\infty}^{\infty} f_X(z-\lambda) f_Y(\lambda) d\lambda \qquad (17)$$

Similarly, we obtain the cdf of two random variables by convolving the cdf of one of them with the pdf (or pmf) of the other, that is, for random variables *X* and *Y*, with *Z*=*X*+*Y*, then

$$F_Z(z) = \int_{\lambda=-\infty}^{\infty} F_X(z-\lambda) f_Y(\lambda) d\lambda \qquad (18)$$

Let's consider the case for only 1 unit, i.e., from (16),

$$D_e = D + D_j \qquad (19)$$

Then, by (18), we have that:

$$F_{D_e}^{(1)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(0)}(d_e - \lambda) f_{D_j}(\lambda) d\lambda \qquad (20)$$

where the notation $F_D^{(j)}(\cdot)$ indicates the cdf after the j^{th} unit is convolved in. Under this notation, then, (19) becomes

$$D_e^{(j)} = D_e^{(j-1)} + D_j \qquad (21)$$

and the general case for (20) is:

$$F_{D_e}^{(j)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(j-1)}(d_e - \lambda) f_{D_j}(\lambda) d\lambda \qquad (22)$$

which expresses the equivalent load after the j^{th} unit is convolved in.

Since $f_{Dj}(d_j)$ is discrete (a pmf), we rewrite (22) as

$$F_{D_e}^{(j)}(d_e) = \sum_{d_j=-\infty}^{\infty} F_{D_e}^{(j-1)}(d_e - d_j) f_{D_j}(d_j) \qquad (23)$$

From an intuitive perspective, (23) is providing the convolution of the cdf $F_D^{(j-1)}(\cdot)$ with the set of impulse functions comprising $f_{Dj}(d_j)$. When using a 2-state model for each generator, $f_{Dj}(d_j)$ is comprised of only 2 impulse functions, one at 0 and one at C_j. Recalling that the convolution of a function with an impulse function simply shifts and scales that function, (23) can be expressed for the 2-state generator model as:

$$F_{D_e}^{(j)}(d_e) = A_j F_{D_e}^{(j-1)}(d_e) + U_j F_{D_e}^{(j-1)}(d_e - C_j) \qquad (24)$$

So the cdf for effective load, following convolution with capacity outage pmf of the j^{th} unit, is the sum of

- the original cdf, scaled by A_j and
- the original cdf, scaled by U_j, right-shifted by C_j.

Example 1: Fig. 17 illustrates the convolution process for a single unit C_1=4 MW supplying a system having peak demand d_{max}=4 MW, with demand cdf given as in plot (a) based on a total time interval of T=1 year.

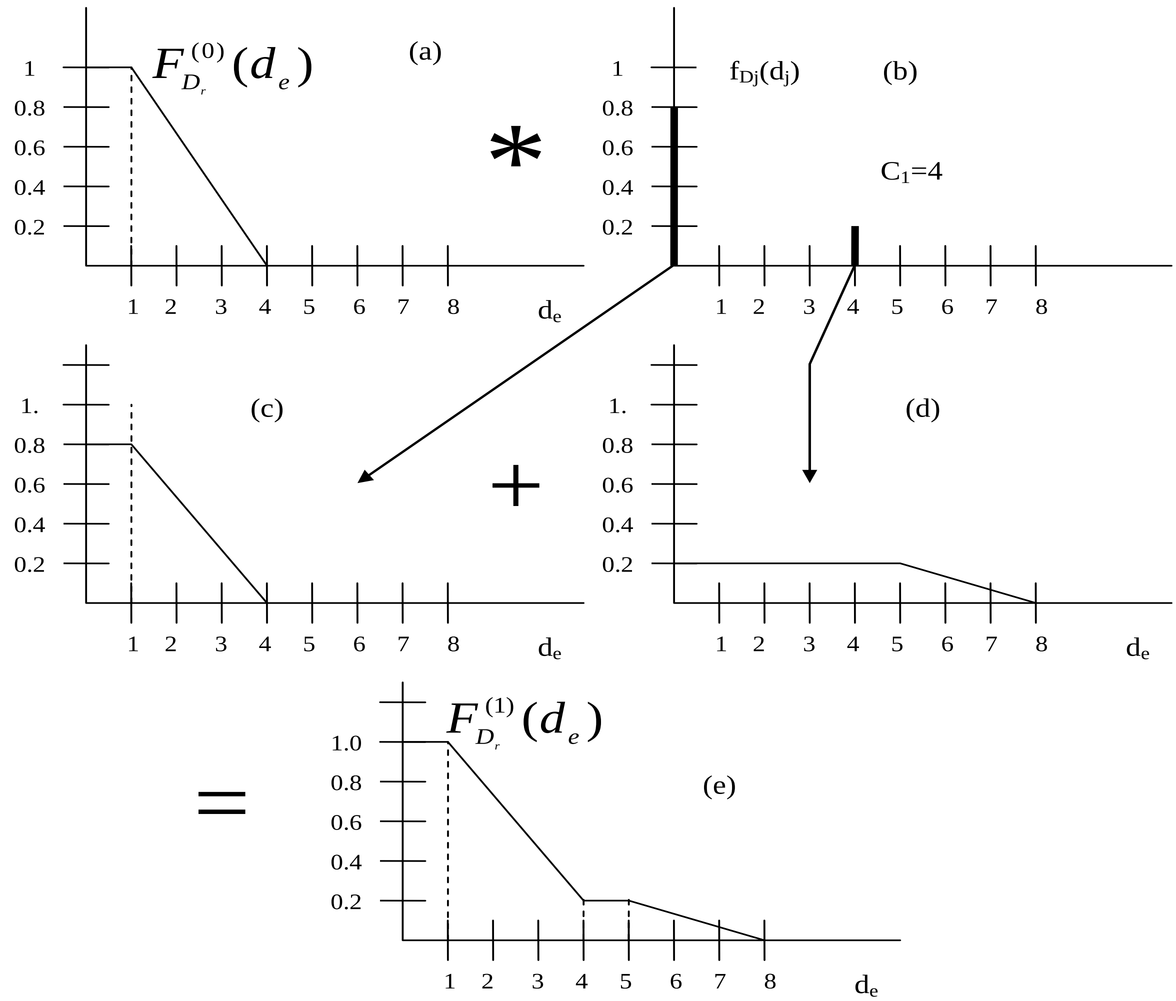


Fig. 17: Convolving in the first unit

Plots (c) and (d) represent the intermediate steps of the convolution where the original cdf $F_{D_e}^{(0)}(d_e)$ was scaled by A_1=0.8 and U_1=0.2, respectively, and right-shifted by 0 and C_1=4, respectively. Note the effect of convolution is to spread the original cdf.

Plot (d) may raise some question since it appears that the constant part of the original cdf has been extended too far to the left. The reason for this apparent discrepancy is that all of the original cdf, in plot (a), was not shown. The complete cdf is illustrated in Fig. 18 below, which shows clearly that $F_{D_e}^{(0)}(d_e)=1$ for $d_e<0$, reflecting the fact that $P(D_e>d_e)=1$ for $d_e<0$.

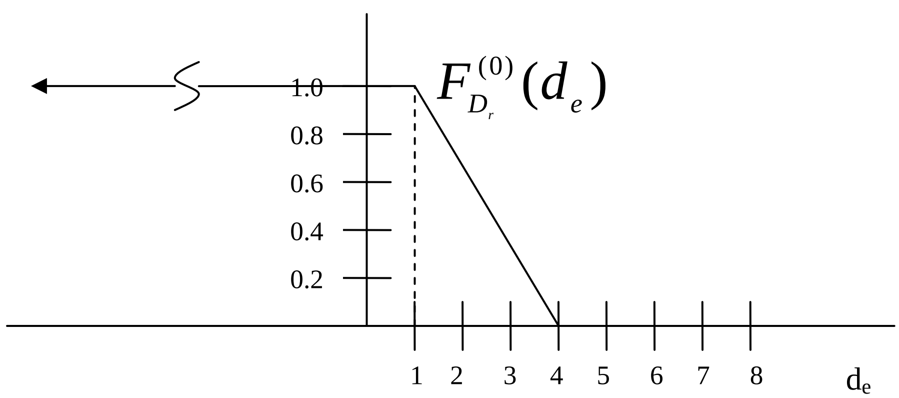


Fig. 18: Complete cdf including values for $d_e<0$

Let's consider that the "first" unit we just convolved in is actually the only unit. If that unit were perfectly reliable, then, because $C_1=4$ and $d_{max}=4$, our system would never have loss of load. This would be the situation if we applied the ideas of Fig. 15 to Fig. 17, plot (a).

However, Fig. 17, plot (e) tells a different story. Fig. 19 applies the ideas of Fig. 15 to Fig. 17, plot (e) to show how the cdf on the *equivalent load* indicates that, for a total capacity of $C_T=4$, we do in fact have some chance of losing load.

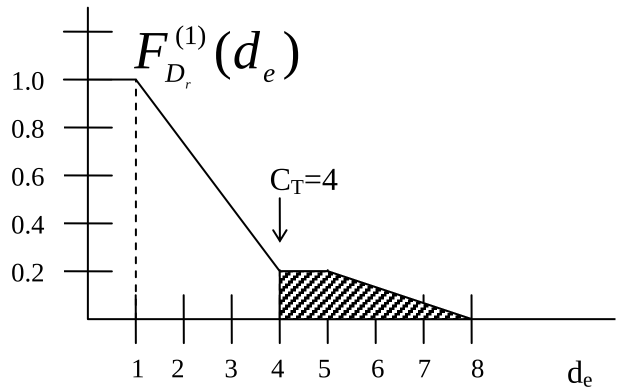


Fig. 19: Illustration of loss of load region

The desired indices are obtained from (12),(13), (14):

$$LOLE = t_{C_T} = g_e(C_T) = T \times F_{D_r}(C_T = 4) = 1 \times 0.2 = 0.2 years$$

A LOLE of 0.2 years is 73 days, a very poor reliability level that reflects the fact we have only a single unit with a high FOR=0.2.

The LOLP is given by:

$$LOLP = P(D_e > C_T) = F_{De}(C_T) = 0.2$$

and the EDNS is given by:

$$EDNS = \int_{C_T}^{d_{e,\max}} F_{De}(\lambda) d\lambda$$

which is just the shaded area in Fig. 19, most easily computed using the basic geometry of the figure, according to:

$$0.2(1)+\frac{1}{2}(3)(0.2)=0.5MW$$

The EENS is given by

$$EENS=T\int_{C_T}^{d_{e,\max}}F_{De}(\lambda)d\lambda=\int_{C_T}^{d_{e,\max}}g_e(\lambda)d\lambda$$

or $T\times EDNS$=1(0.5)=0.5MW-years,

or 8760(0.5)=4380MWhrs.

Example 2: This example is from [4].
A set of generation data is provided in Table 5.

Table 5

Unit No.	Unit name	Rated capacity (MW(e))	Forced outage (%)	Type of fuel	Variable cost ($/MW·h)
1	NUC1	200	20	Nuclear	6.5
2	NUC2	200	20	Nuclear	6.5
3	COAL1	200	10	Coal	27.0
4	COAL2	200	10	Coal	27.0
5	OIL1	100	10	Oil	58.1
6	OIL2	100	10	Oil	58.1
7	OIL3	100	10	Oil	58.1
8	OIL4	100	10	Oil	58.1
9	CT1	100	5	Distillate oil	113.2
System capacity		1300			

The 4th column provides the forced outage rate, which we have denoted by *U*. The two-state generator outage model for each unit is obtained from this value, together with the rated capacity, as illustrated in Fig. 20, for unit 1. Notice that the units are ordered from least cost to highest cost.

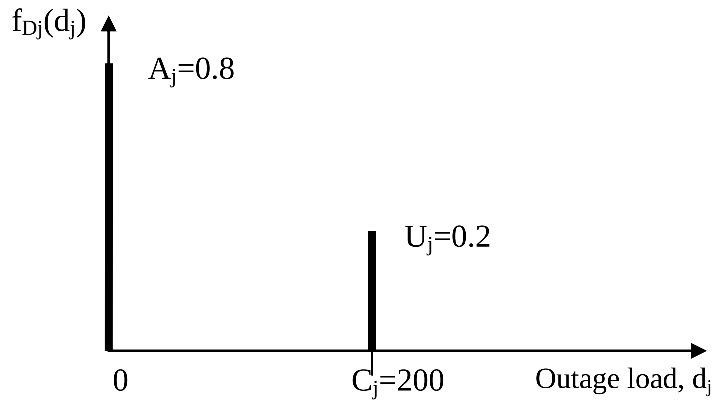


Fig. 20: Two-state outage model for Unit 1

Load duration data is provided in Table 6 and plotted in Fig. 21.

Table 6

Load MW(e)	Fraction of time load exceeds given load
0	1.00
100	1.00
200	1.00
300	1.00
400 (minimum load)	1.00
500	0.80
600	0.40
700	0.20
800	0.10
900	0.05
1000 (peak load)	0.00

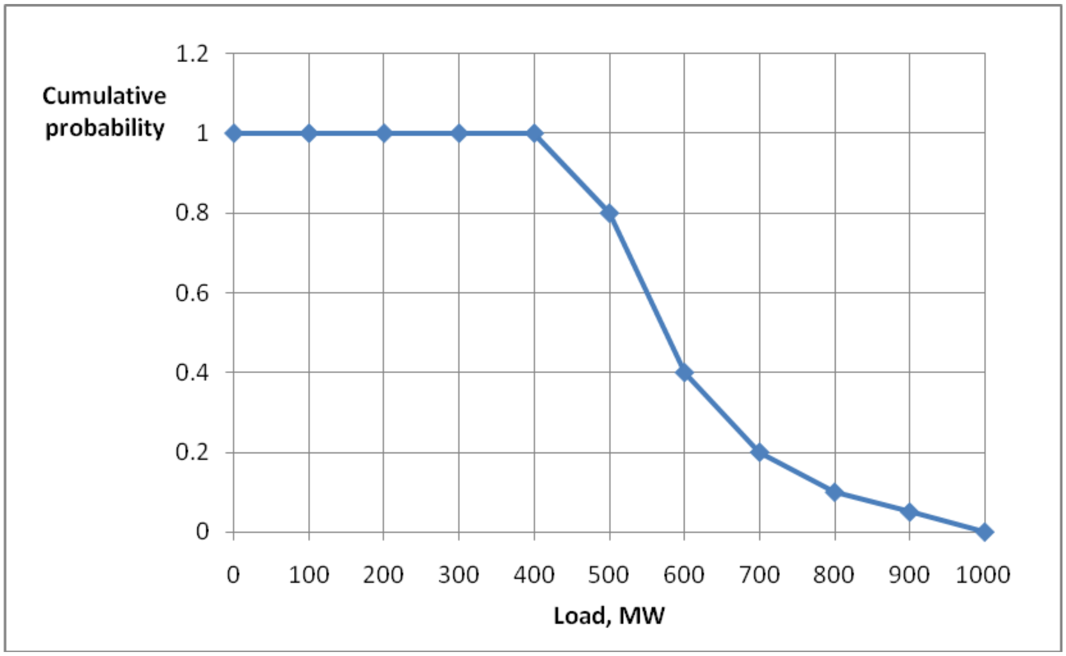


Fig. 21

We now deploy (24), repeated here for convenience,

$$F_{D_e}^{(j)}(d_e) = A_j F_{D_e}^{(j-1)}(d_e) + U_j F_{D_e}^{(j-1)}(d_e - C_j) \qquad (24)$$

to convolve in the unit outage models with the load duration curve of Fig. 21. The procedure is carried out in an Excel spread sheet, and the result is provided in Fig. 22. In Fig. 22, we have shown

- Original load duration curve, F0;
- Load duration curve with unit 1 convolved in, F1.
- Load duration curve with all units convolved in, F9

We could, of course, show the load duration curves for any number of units convolved in, but this would be a cluttered plot.

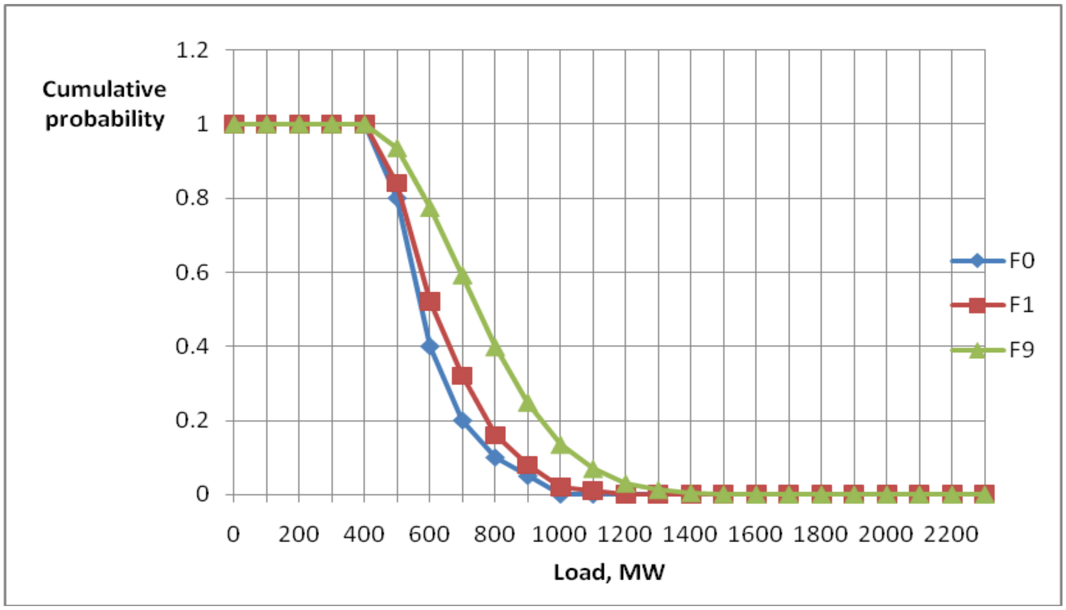


Fig. 22

We also show, in Table 7, the results of the calculations performed to obtain the series of load duration curves (LDC) F0-F9. Notice the following:

- Each LDC is a column FO-F9
- The first column, in MW, is the load.
  - It begins at -200 to facilitate the convolution for the largest unit, which is a 200 MW unit.
  - Although it extends to 2300 MW, the largest actual load is 1000 MW; the extension is to obtain the equivalent load corresponding to a 1000 MW load with 1300 MW of failable generation.
- The entries in the table show the % time the load exceeds the given value.
- LOLP is, for a particular column, the % time load exceeds the total capacity corresponding to that column, and is underlined.

For example, one observes that *LOLP=1* if we only have units 1 (F1, $C_T=200$) or only units 1 and 2 (F2, $C_T=400$). This is because the capacity would never be enough to satisfy the load, at any time. And *LOLP=0.6544* if we have only units 1, 2, and 3 (F3, $C_T=600$). This is because we would be able to supply the load for some of the time with this capacity. And *LOLP=0.012299* if we have all units (F9, $C_T=1300$), which is non-0 (in spite of the fact that $C_T>1000$) because units can fail.

Table 7

	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9
		0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.05
		0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.95
		200	200	200	200	100	100	100	100	100
Load (MW)	Fraction of time load exceeds given load									
-200	1	1	1	1	1	1	1	1	1	1
-100	1	1	1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1	1	1
100	1	1	1	1	1	1	1	1	1	1
200	1	1	1	1	1	1	1	1	1	1
300	1	1	1	1	1	1	1	1	1	1
400	1	1	1	1	1	1	1	1	1	1
500	0.8	0.84	0.872	0.8848	0.89632	0.906688	0.916019	0.924417	0.931976	0.935377
600	0.4	0.52	0.616	0.6544	0.68896	0.709696	0.729395	0.748058	0.765694	0.774008
700	0.2	0.32	0.424	0.4688	0.5104	0.528256	0.5464	0.5647	0.583035	0.592168
800	0.1	0.16	0.232	0.2704	0.3088	0.32896	0.34889	0.368641	0.388247	0.397986
900	0.05	0.08	0.128	0.1576	0.18872	0.200728	0.213551	0.227085	0.241241	0.248591
1000	0	0.02	0.048	0.0664	0.0868	0.096992	0.107366	0.117984	0.128894	0.134512
1100	0	0.01	0.024	0.0344	0.04672	0.050728	0.055354	0.060556	0.066298	0.069428
1200	0	0	0.004	0.0084	0.0142	0.017452	0.02078	0.024237	0.027869	0.02979
1300	0	0	0.002	0.0042	0.00722	0.007918	0.008871	0.010062	0.01148	0.012299
1400	0	0	0	0.0004	0.0012	0.001802	0.002414	0.003059	0.00376	0.004146
1500	0	0	0	0.0002	0.0006	0.00066	0.000774	0.000938	0.00115	0.001281
1600	0	0	0	0	0.00004	0.000096	0.000152	0.000215	0.000287	0.00033
1700	0	0	0	0	0.00002	0.000022	2.94E-05	4.17E-05	5.9E-05	7.04E-05
1800	0	0	0	0	0	0.000002	0.000004	6.54E-06	1.01E-05	1.25E-05
1900	0	0	0	0	0	0	2E-07	5.8E-07	1.18E-06	1.62E-06
2000	0	0	0	0	0	0	0	2E-08	7.6E-08	1.31E-07
2100	0	0	0	0	0	0	0	0	2E-09	5.7E-09
2200	0	0	0	0	0	0	0	0	0	1E-10
2300	0	0	0	0	0	0	0	0	0	0

5.0 Production cost modeling using effective load

The most basic production cost model obtains production costs of thermal units over a period of time, say 1 year, by building upon the equivalent load duration curve described in Section 5.

To perform this, we will assume that generator variable cost, in $/MWhr, for unit j operating at P_j over a time interval t, is expressed by

$$C_j(E_j)=b_jE_j$$

where $E_j=P_jt$ is the energy produced by the unit during the hour and b_j is the unit's average variable costs of producing the energy (we omit fixed costs because we are only trying to quantify production costs here).

The production cost model begins by assuming the existence of a loading (or merit) order, which is how the units are expected to be called upon to meet the demand facing the system. We assume for simplicity that each unit consists of a single "block" of capacity equal to the maximum capacity. It is possible, and more accurate, to divide each unit into multiple capacity blocks, but there is no conceptual difference to the approach when doing so.

Table 5, listed previously in Example 2, provides the variable cost for each unit in the appropriate loading order. This table is repeated here for convenience.

Table 5

Unit No.	Unit name	Rated capacity (MW(e))	Forced outage (%)	Type of fuel	Variable cost ($/MW·h)
1	NUC1	200	20	Nuclear	6.5
2	NUC2	200	20	Nuclear	6.5
3	COAL1	200	10	Coal	27.0
4	COAL2	200	10	Coal	27.0
5	OIL1	100	10	Oil	58.1
6	OIL2	100	10	Oil	58.1
7	OIL3	100	10	Oil	58.1
8	OIL4	100	10	Oil	58.1
9	CT1	100	5	Distillate oil	113.2
System capacity		1300			

The criterion for determining loading order is clearly economic. Sometimes it is necessary to alter the economic loading order to account for must-run units or spinning reserve requirements. We will not consider these issues in the discussion that follows.

To motivate the approach, we introduce the concept of a unit's *observed load* as the load "seen" by a unit just before it is committed in the loading order. Thus, it will be the case that all higher-priority units will have been committed.

If all higher-priority units would have been perfectly reliable ($A_j=1$), then the observed load seen by the next unit would have been just the total load less the sum of the capacities of the committed units.

However, all higher-priority units are not perfectly reliable, i.e., they may fail according to the forced outage rate U_j. This means we must account for their stochastic behavior over time. This can be done in a straight-forward fashion by using the equivalent load duration curve developed for the last unit committed.

In the notation of (24) unit *j* sees a load characterized by $F_{D_e}^{(j-1)}(d_e)$. Thus, the energy provided by unit *j* is proportional to the area under $F_{D_e}^{(j-1)}(d_e)$ from x_{j-1} to x_j, where

- x_{j-1} is the summed capacity over all previously committed units and
- x_j is the summed capacity over all previously committed units and unit *j*.

But unit *j* is only going to be available A_j% of the time. Also, since $F_{D_e}^{(j-1)}(d_e)$ is a probability function, we must multiply it by *T*, resulting in the following expression for energy provided by unit *j* [5]:

$$E_j = TA_j \int_{x_{j-1}}^{x_j} F_{D_e}^{(j-1)}(\lambda)d\lambda \tag{25}$$

where

$$x_j = \sum_{i=1}^{j} C_i \,, \quad x_{j-1} = \sum_{i=1}^{j-1} C_i \qquad (26)$$

Referring back to Example 2, we describe the computations for the first three entries. This description is adapted from [4].

For unit 1, the original load duration curve F0 is used, as forced outages of any units in the system do not affect unit l's observed load. The energy requested by the system from unit 1, excluding unit l's forced outage time, is the area under $F_{D_e}^{(0)}(d_e)$ over the range of 0 to 200 MW (unit 1 's position in the loading order) times the number of hours in the period (8760) times A_1. The area under $F_{D_e}^{(0)}(d_e)$ from 0 to 200, illustrated in Fig. 23 below, is 200.

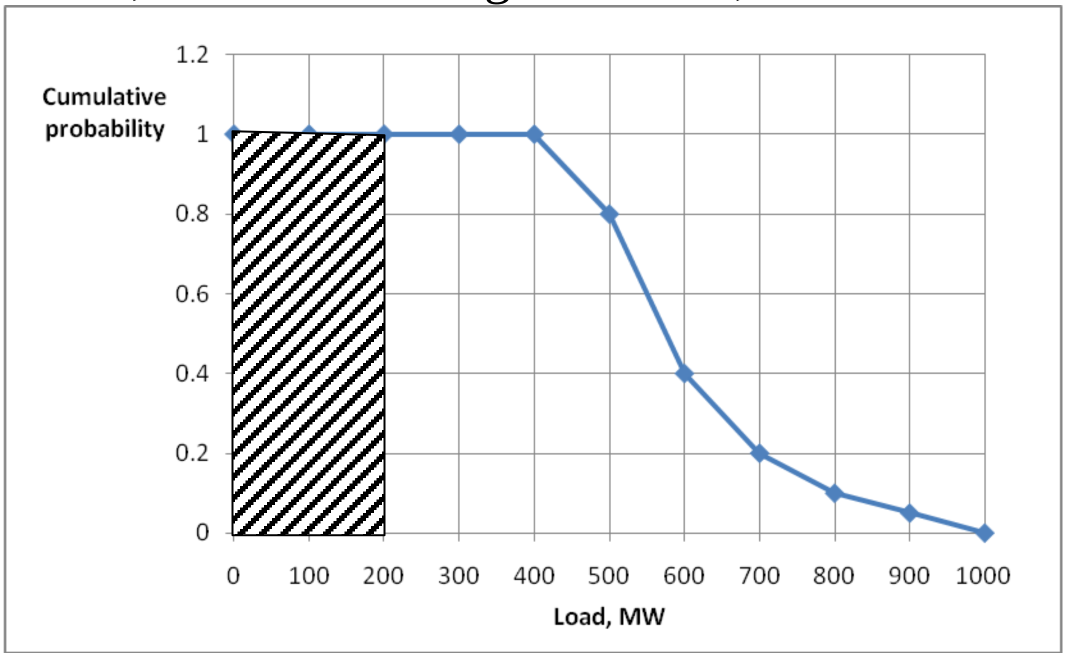


Fig. 23

Therefore,

$$E_1 = 8760 \times 0.8 \times 200 = 1{,}401{,}600 \text{ MWhrs}$$

For unit 2, the load duration curve F1 is used, as forced outage of unit 1 will affect unit 2's observed load. The energy requested by the system from unit 2, excluding unit 2's forced outage time, is the area under $F_{D_e}^{(1)}(d_e)$ over the range of 200 to 400 MW (unit 2 's position in the loading order) times the number of hours in the period (8760) times A_2. The area under $F_{D_e}^{(1)}(d_e)$ from 200 to 400, illustrated in Fig. 24 below, is 200.

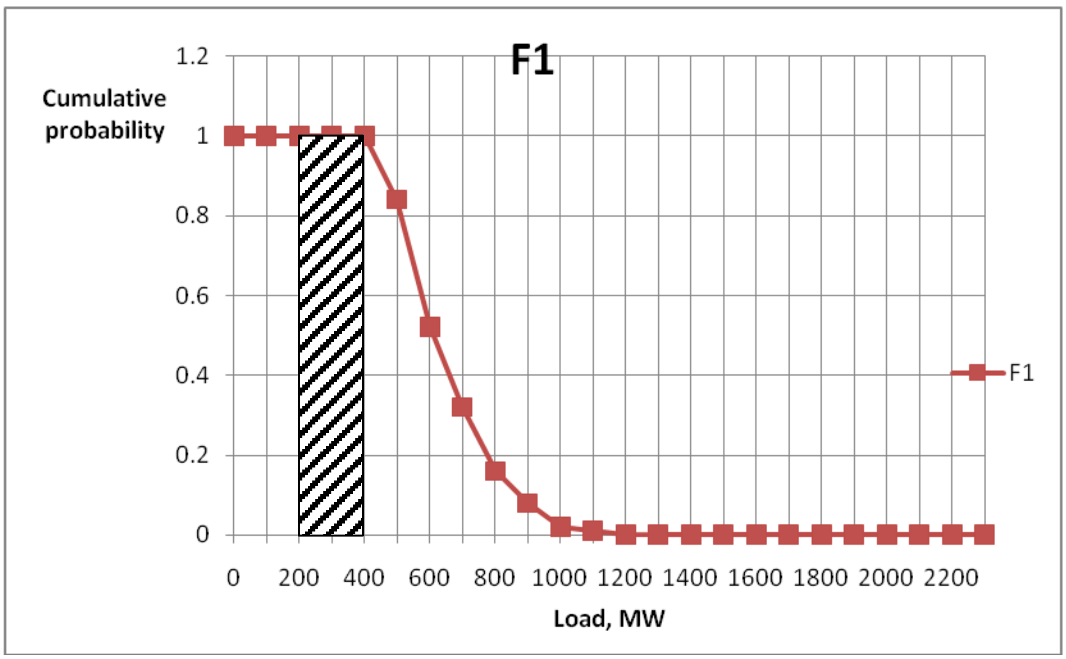


Fig. 24

Therefore,

$$E_2 = 8760 \times 0.8 \times 200 = 1{,}401{,}600 \text{ MWhrs}$$

For unit 3, the load duration curve F2 is used, as forced outage of units 1 and 2 will affect unit 3's observed load. The energy requested by the system from unit 3, excluding unit 3's forced outage time, is the area under $F_{D_e}^{(2)}(d_e)$ over the range of 400 to 600 MW (unit 3 's position in the loading order) times the number of hours in the period (8760) times A_3. The area under $F_{D_e}^{(2)}(d_e)$ from 400 to 600, illustrated in Fig. 25, is calculated below Fig 25. The coordinates on Fig. 25 are obtained from Table 7, repeated on the next page for convenience.

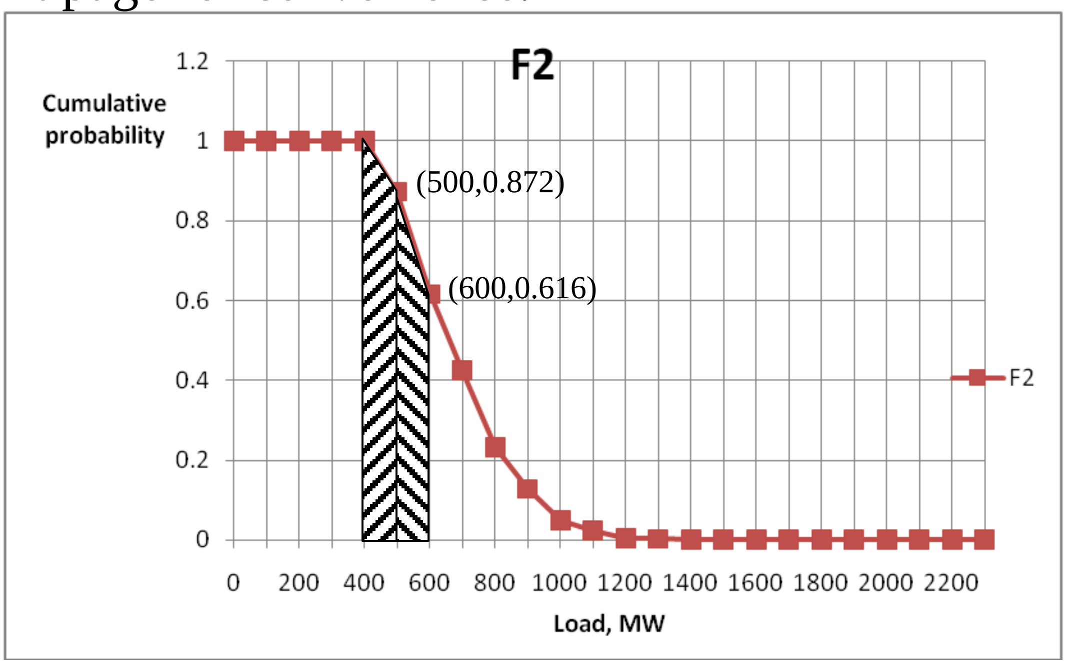


Fig. 25

The area, indicated in Fig. 25, is obtained as two applications of a trapezoidal area (1/2)(h)(a+b), as

$$\underbrace{\frac{1}{2}(100)(1+.872)}_{LeftPortion}+\underbrace{\frac{1}{2}(100)(.872+.616)}_{RightPortion}$$

$$=93.6+74.4=168$$

Therefore,

$$E_3=8760\times 0.9\times 168=1{,}324{,}512 \text{ MWhrs}$$

Table 7

	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9
		0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.05
		0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.95
		200	200	200	200	100	100	100	100	100
Load (MW)	Fraction of time load exceeds given load									
-200	1	1	1	1	1	1	1	1	1	1
-100	1	1	1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1	1	1
100	1	1	1	1	1	1	1	1	1	1
200	1	1	1	1	1	1	1	1	1	1
300	1	1	1	1	1	1	1	1	1	1
400	1	1	1	1	1	1	1	1	1	1
500	0.8	0.84	0.872	0.8848	0.89632	0.906688	0.916019	0.924417	0.931976	0.935377
600	0.4	0.52	0.616	0.6544	0.68896	0.709696	0.729395	0.748058	0.765694	0.774008
700	0.2	0.32	0.424	0.4688	0.5104	0.528256	0.5464	0.5647	0.583035	0.592168
800	0.1	0.16	0.232	0.2704	0.3088	0.32896	0.34889	0.368641	0.388247	0.397986
900	0.05	0.08	0.128	0.1576	0.18872	0.200728	0.213551	0.227085	0.241241	0.248591
1000	0	0.02	0.048	0.0664	0.0868	0.096992	0.107366	0.117984	0.128894	0.134512
1100	0	0.01	0.024	0.0344	0.04672	0.050728	0.055354	0.060556	0.066298	0.069428
1200	0	0	0.004	0.0084	0.0142	0.017452	0.02078	0.024237	0.027869	0.02979
1300	0	0	0.002	0.0042	0.00722	0.007918	0.008871	0.010062	0.01148	0.012299
1400	0	0	0	0.0004	0.0012	0.001802	0.002414	0.003059	0.00376	0.004146
1500	0	0	0	0.0002	0.0006	0.00066	0.000774	0.000938	0.00115	0.001281
1600	0	0	0	0	0.00004	0.000096	0.000152	0.000215	0.000287	0.00033
1700	0	0	0	0	0.00002	0.000022	2.94E-05	4.17E-05	5.9E-05	7.04E-05
1800	0	0	0	0	0	0.000002	0.000004	6.54E-06	1.01E-05	1.25E-05
1900	0	0	0	0	0	0	2E-07	5.8E-07	1.18E-06	1.62E-06
2000	0	0	0	0	0	0	0	2E-08	7.6E-08	1.31E-07
2100	0	0	0	0	0	0	0	0	2E-09	5.7E-09
2200	0	0	0	0	0	0	0	0	0	1E-10
2300	0	0	0	0	0	0	0	0	0	0

Continuing in this way, we obtain the energy produced by all units. This information, together with the average variable costs from Table 5, and the resulting energy cost, is provided in Table 8 below.

Table 8

Unit	MW-hrs	Avg. Variable Costs, $/MWhr	Energy Costs, $
1	1,401,600	6.5	9,110,400
2	1,401,600	6.5	9,110,400
3	1,324,500	27.0	35,761,500
4	734,200	27.0	19,823,400
5	196,100	58.1	11,393,410
6	117,400	58.1	6,820,940
7	64,100	58.1	3,724,210
8	33,400	58.1	1,940,540
9	16,400	113.2	1,856,480
Total	E_T= 5,289,300		99,541,280

It is interesting to note that the total energy supplied, E_T=5,289,300 MWhrs, is less than what one obtained when the original load duration curve is integrated. This integration can be done by applying our trapezoidal approach to curve F0 in Table 7. Doing so results in E_0=5,299,800 MWhrs. The difference is E_0-E_T=5,299,800-5,289,300=10,500 MWhrs.

What is this difference of 10,500 MWhrs?

➔To answer this question, consider:

- The total area under the original curve F0, integrated from 0 to 1000 (the peak load), is 5,299,800 MWhrs, as shown in Fig. 26. This is the amount of energy provided to the *actual* load if it were supplied by perfectly reliable generation having capacity of 1000 MW. As indicated above, we will denote this as E_0.

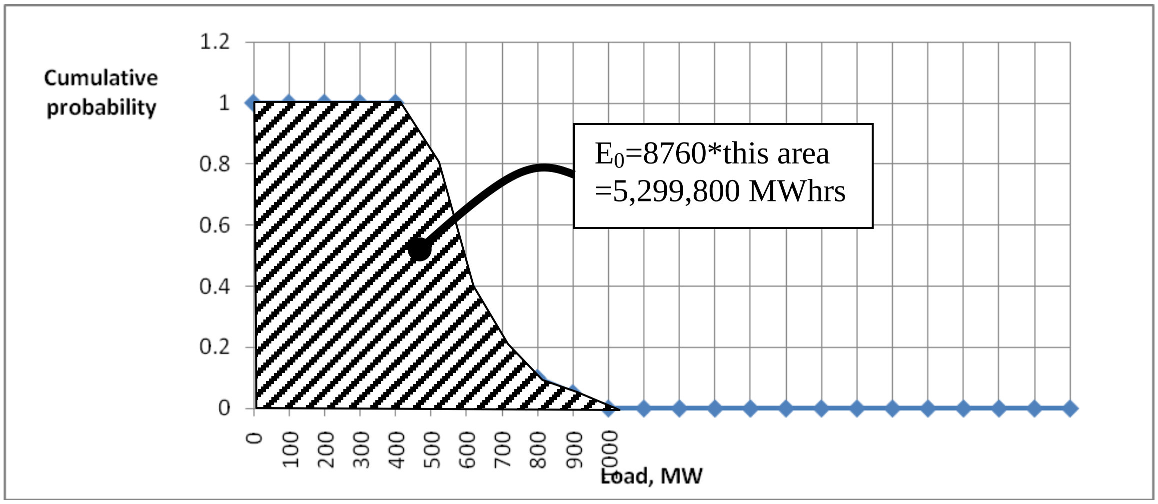


Fig. 26

- The total area under the final curve, F9, integrated from 0 to 1300 MW (the generation capacity) is E_{1300}=6,734,696 MWhrs, as shown in Fig. 27. This is the amount of energy provided to the *effective* load if it were supplied by perfectly reliable generation having capacity of 1300 MW.

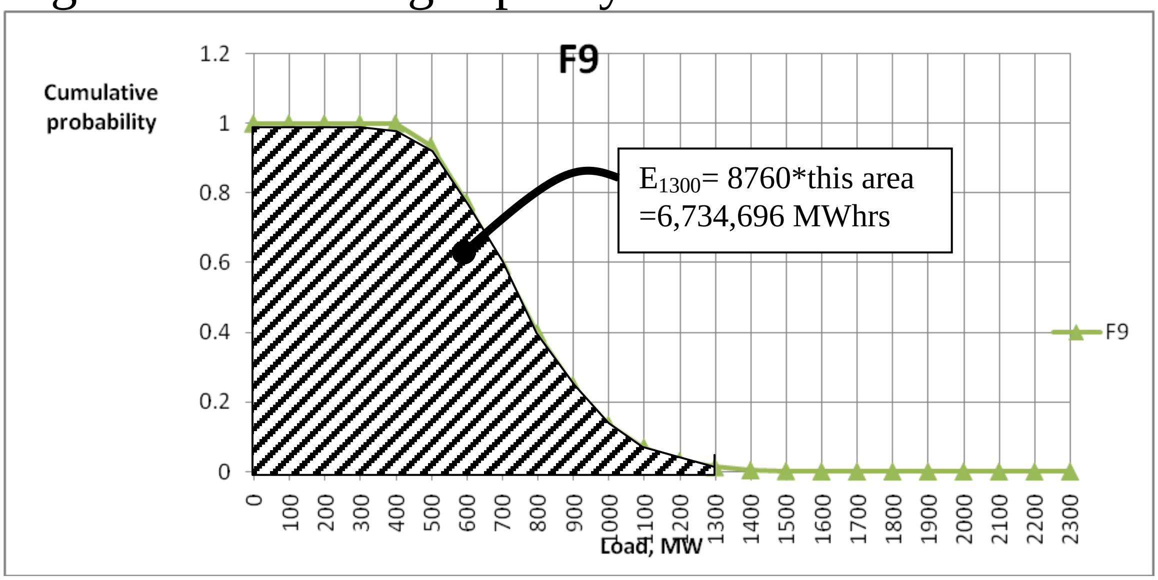


Fig. 27

The energy represented by the area of Fig. 27, which is the energy provided to the *effective* load if it were supplied by perfectly reliable generation having capacity of 1300 MW, is greater than the energy provided by the actual 1300 MW, that is

$$E_{1300} > E_T$$

because E_{1300} includes load required to be served when the generators are outaged, and this portion was explicitly removed from the calculation of Table 8 (E_T). One can observe this readily by considering a system with only a single unit. Recalling the general formula (25) for obtaining *actual energy supplied by a unit* per the method of Table 8:

$$E_j = TA_j \int_{x_{j-1}}^{x_j} F_{D_e}^{(j-1)}(\lambda)d\lambda \tag{25}$$

and applying this to the one-unit system, we get:

$$E_T = E_1 = TA_1 \int_{0}^{C_1} F_{D_e}^{(0)}(\lambda)d\lambda \tag{27}$$

In contrast, the energy E_e obtained when we integrate the effective load duration curve (accounting for only the one unit) is

$$E_e = T \int_0^{C_1} F_{D_e}^{(1)}(\lambda) d\lambda \qquad (28)$$

Recalling the convolution formula (24),

$$F_{D_e}^{(j)}(d_e) = A_j F_{D_e}^{(j-1)}(d_e) + U_j F_{D_e}^{(j-1)}(d_e - C_j) \qquad (24)$$

and for the one-unit case, we get

$$F_{D_e}^{(1)}(d_e) = A_1 F_{D_e}^{(0)}(d_e) + U_1 F_{D_e}^{(0)}(d_e - C_1) \qquad (29)$$

Substituting (29) into (28) results in

$$E_e = T \int_0^{C_1} A_1 F_{D_e}^{(0)}(\lambda) + U_1 F_{D_e}^{(0)}(\lambda - C_1) d\lambda \qquad (30)$$

Breaking up the integral gives

$$\begin{aligned} E_e &= T \int_0^{C_1} A_1 F_{D_e}^{(0)}(\lambda) d\lambda + T \int_0^{C_1} U_1 F_{D_e}^{(0)}(\lambda - C_1) d\lambda \\ &= TA_1 \int_0^{C_1} F_{D_e}^{(0)}(\lambda) d\lambda + TU_1 \int_0^{C_1} F_{D_e}^{(0)}(\lambda - C_1) d\lambda \end{aligned} \qquad (31)$$

Comparing (31) with (27), repeated here for convenience:

$$E_T = E_1 = TA_1 \int_0^{C_1} F_{D_e}^{(0)}(\lambda) d\lambda \qquad (27)$$

we observe the expressions are the same except for the presence of the second integration in (31). This proves that $E_e > E_T$, i.e.,

effective energy demanded > energy served by generation

- Now consider computing the energy consumed by the total effective load as represented by Fig. 28 (note that in this figure, the curve should go to zero at Load=2300 but does not due to limitations of the drawing facility used).

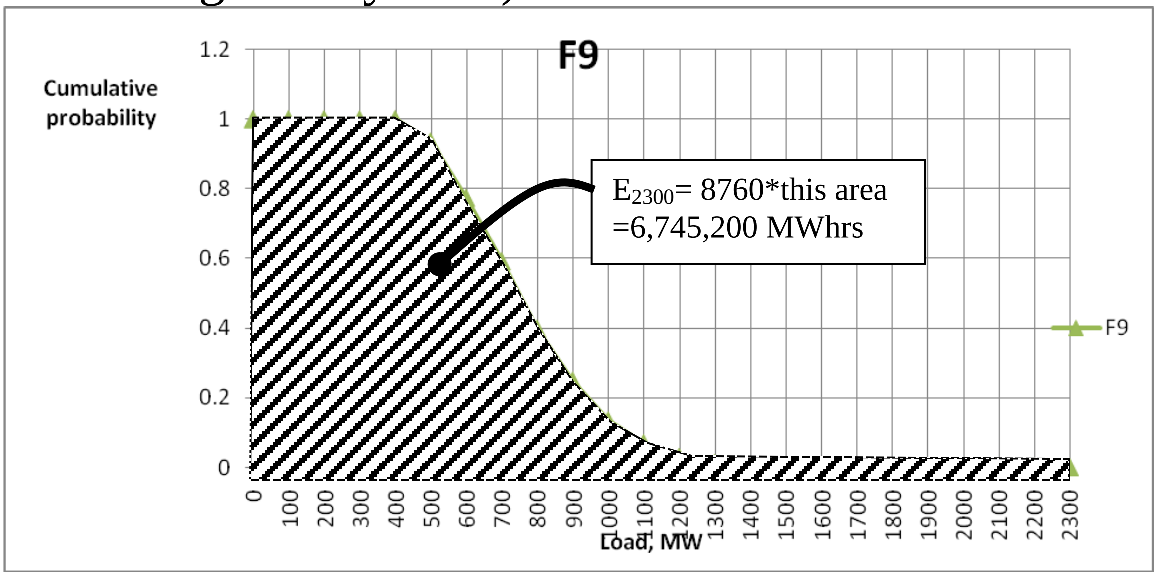


Fig. 28

Using the trapezoidal method to compute this area results in E_{2300}=6745200 MWhrs, which is the energy provided to the *effective* load if it were supplied by perfectly reliable generation having capacity of 2300 MW. This would leave zero energy unserved.

- The difference between
  - E_{2300}, the energy provided to the effective load if it were supplied by 2300 MW of perfectly reliable generation and
  - E_{1300}, the energy provided to the effective load if it were supplied by 1300 MW of perfectly reliable generation

is given by:

E_{2300}-E_{1300}=6,745,200-6,734,696=10,504 MWhrs

This is the expected energy not served (EENS), sometimes called the expected unserved energy (EUE).

We observe, then, that we can obtain EENS in two different approaches.

1. E_0-E_T=5,299,800-5,289,300=10,500 MWhrs
2. E_{2300}-E_{1300}=6,745,200-6,734,696=10,504 MWhrs

Approach 1 may be computationally more convenient for production costing because E_T is easily obtained as the summation of all the energy values.

Approach 2 may be more convenient conceptually as it is simply the area under the effective load curve from total capacity (I call it C_T) to infinity.

6.0 Comments on W&W approach

W&W, in section 8.3.2, refers to the “unserved load method.” It is somewhat different from the “effective load method” described above.

The main difference can be observed by comparing equation (8.2) in your text with equation (24) used above.

$$F_{D_e}^{(j)}(d_e) = A_j F_{D_e}^{(j-1)}(d_e) + U_j F_{D_e}^{(j-1)}(d_e - C_j) \qquad (24)$$

$$P_n'(x) = qP_n(x) + pP_n(x + C) \qquad (8.2)$$

Both left-hand expressions are the "new" cdf after "convolving in" a unit.

Specifically, the nomenclature relates as follows:

- d_e=x (value of equivalent load)
- C_j=C (capacity of unit j)
- A_j=p (availability of unit j)
- U_j=q (unavailability of unit j)

One observes that the two equations are almost the same, with two exceptions:

1. Shift: Whereas the "shift" on the second term of (24) is a "right-shift" by an amount C, the "shift" on the second term of (8.2) is a left-shift by an amount C.
2. A_j and U_j: Whereas the "unshifted" (first) term of (24) is multiplied by A_j=p, the "unshifted" (first) term of (8.2) is multiplied by U_j=q.

The difference should be understood.

Whereas

- the "effective load" method
  - *extends* or *increases* the load to probabilistically account for generator unavailability,
  - and uses *total capacity* under assumption of *perfect reliability* to assess metrics
- the "unserved load" method
  - reduces or decreases the load to probabilistically account for generator availability,
  - and uses zero capacity to assess metrics.

6.0 W&W (unserved load) method

We will maintain the notation used in describing the effective load method. The differences in notation relative to W&W are described in the previous section.

Define D as the random variable characterizing the demand. Now we define two more random variables:

- D_j is the random decrease in load for (probabilistic) availability of unit j.
- D_e is the random load accounting for the (probabilistic) availability of all units and represents the *unserved* load.

Thus, the random variables D, D_e, and D_j are related:

$$D_e = D - \sum_{j=1}^{N} D_j \tag{25}$$

Whereas C_j represents the capacity of unit j and is a deterministic value, D_j represents an effective decrease in load corresponding to (probabilistic) availability of unit j and is a random variable.

The probability mass function (pmf) for D_j is assumed to be as given in Fig. 29, i.e., a two-state model. We denote the pmf for D_j as $f_{Dj}(d_j)$. It expresses the probability that the unit experiences an outage of 0 MW as A_j, and the probability the unit experiences an outage of C_j MW as U_j.

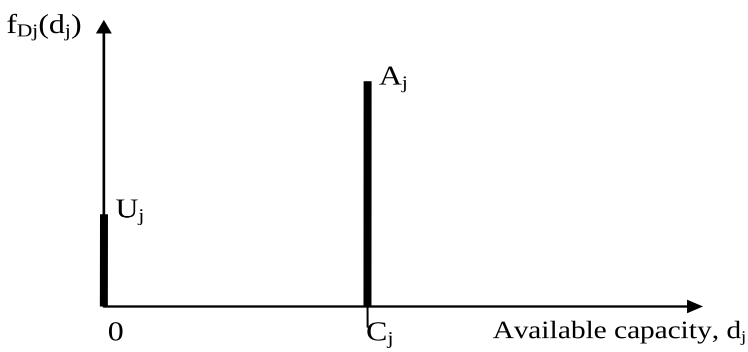


Fig. 29: Two state generator availability model

We saw in the above notes that the pdf of the sum of 2 independent random variables is the convolution of their individual pdfs, that is, for random variables X and Y, with $Z=X+Y$, then

$$f_Z(z) = \int_{\lambda=-\infty}^{\infty} f_X(z-\lambda) f_Y(\lambda) d\lambda \qquad (17)$$

which can also be written as:

$$f_Z(z) = \int_{\lambda=-\infty}^{\infty} f_X(\lambda) f_Y(z-\lambda) d\lambda$$

Likewise, the pdf of the difference of 2 independent random variables is also a convolution, that is, for random variables X and Y, with Z=X-Y, then

$$f_Z(z) = \int_{\lambda=-\infty}^{\infty} f_X(z) f_Y(z+\lambda) d\lambda \qquad (26)$$

In addition, it is true that the cdf of the difference between 2 random variables can be found by convolving the cdf of one of them with the pdf (or pmf) of the other, that is, for random variables X and Y, with $Z=X-Y$, then

$$F_Z(z) = \int_{\lambda=-\infty}^{\infty} F_X(z) f_Y(z+\lambda) d\lambda \qquad (27)$$

Let's consider the case for only 1 unit, i.e., from (25),

$$D_e = D - D_j \qquad (28)$$

Then, by (27), we have that:

$$F_{D_e}^{(1)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(0)}(\lambda) f_{D_j}(d_e + \lambda) d\lambda \qquad (20)$$

where the notation $F_D^{(j)}(\cdot)$ indicates the cdf after the j^{th} unit is convolved in. With this notation, (28) is

$$D_e^{(j)} = D_e^{(j-1)} - D_j \qquad (29)$$

and the general case for (29) is:

$$F_{D_e}^{(j)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(j-1)}(\lambda) f_{D_j}(d_e + \lambda) d\lambda \qquad (30)$$

which expresses the equivalent load after the j^{th} unit is convolved in, considering the (probabilistic) availability of unit j and all lower numbered units.

Since $f_{Dj}(d_j)$ is discrete (a pmf), we rewrite (30) as

$$F_{D_e}^{(j)}(d_e) = \sum_{d_j=-\infty}^{\infty} F_{D_e}^{(j-1)}(d_j) f_{D_j}(d_e + d_j) \qquad (31)$$

From an intuitive perspective, (31) is providing the convolution of the cdf $F_D^{(j-1)}(\cdot)$ with the set of impulse functions comprising $f_{Dj}(d_j)$. When using a 2-state availability model for each generator, $f_{Dj}(d_j)$ is comprised of only 2 impulse functions, one at 0 and one at C_j. Recalling that the convolution of a function with an impulse function simply shifts and scales that function, (31) can be expressed for the 2-state generator model shown in Fig. 23 as:

$$F_{D_e}^{(j)}(d_e) = U_j F_{D_e}^{(j-1)}(d_e) + A_j F_{D_e}^{(j-1)}(d_e + C_j) \qquad (32)$$

So the cdf for the effective load, following convolution with capacity outage pmf of the j^{th} unit, is the sum of

- the original cdf, scaled by U_j and
- the original cdf, scaled by A_j, left-shifted by C_j.

W&W say this (p. 287):

- The first term is the probability that new capacity C_j is unavailable times the probability of needing an amount of power d_e or more;
- The second term is the probability C_j is available times the probability d_e+C_j or more is needed.

Example 3: Fig. 30 illustrates the convolution process for a single unit C_1=4 MW supplying a system having peak demand d_{max}=4 MW, with demand cdf given as in plot (a) based on a total time interval of T=1 year.

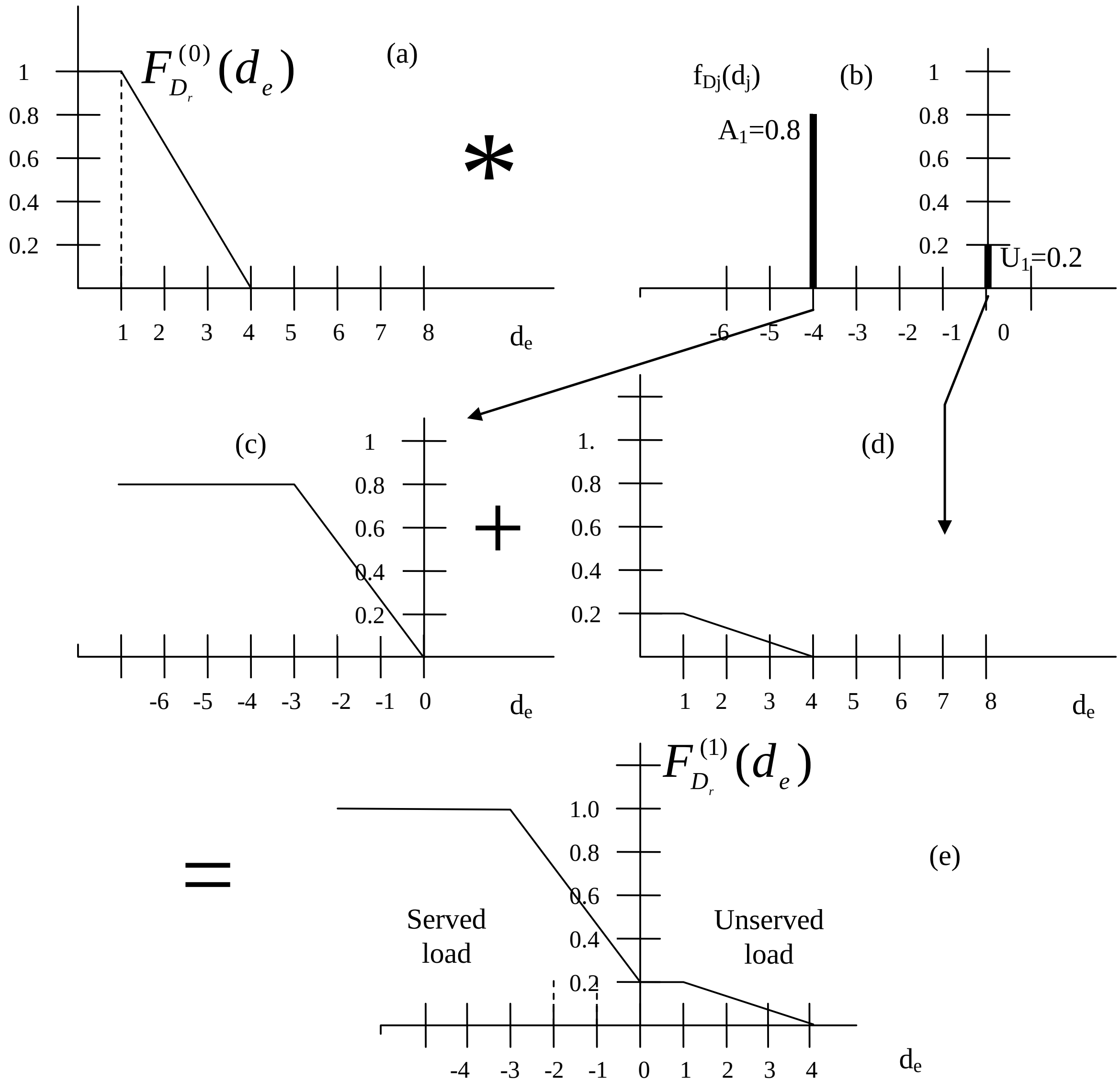


Fig. 30: Convolving in the first unit (not perfectly reliable)

Plots (d) and (c) represent the intermediate steps of the convolution where the original cdf $F_{D_e}^{(0)}(d_e)$ was scaled by U_1=0.2 and A_1=0.8, respectively, and left-shifted by 0 and C_1=4, respectively. Note the effect of convolution is to shift the original cdf to the left.

Plot (c) may raise some question since it appears that the constant part of the original cdf has been extended too far to the left. The reason for this is that all of the original cdf, in plot (a), was not shown. The complete cdf is illustrated in Fig. 31 below, which shows clearly that $F_{D_e}^{(0)}(d_e)=1$ for $d_e<0$, reflecting the fact that $P(D_e \geq d_e)=1$ for $d_e<0$.

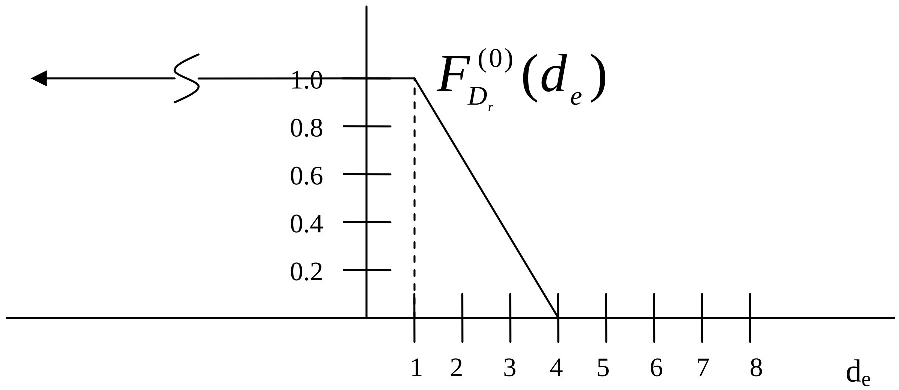


Fig. 31: Complete cdf including values for $d_e<0$

Let's consider that the "first" unit we just convolved in is actually the only unit. If that unit were perfectly reliable, then, because $C_1=4$ and $d_{max}=4$, our system would never have loss of load. In this case, with $A_1=1$ and $U_1=0$, the convolution process above would have resulted in Fig. 32 below. The fact that the final load duration curve $F_D^{(1)}(d_e)$ shows $Pr(d_e \geq 0)=0$ means that there is no chance we will encounter a load interruption for this system!

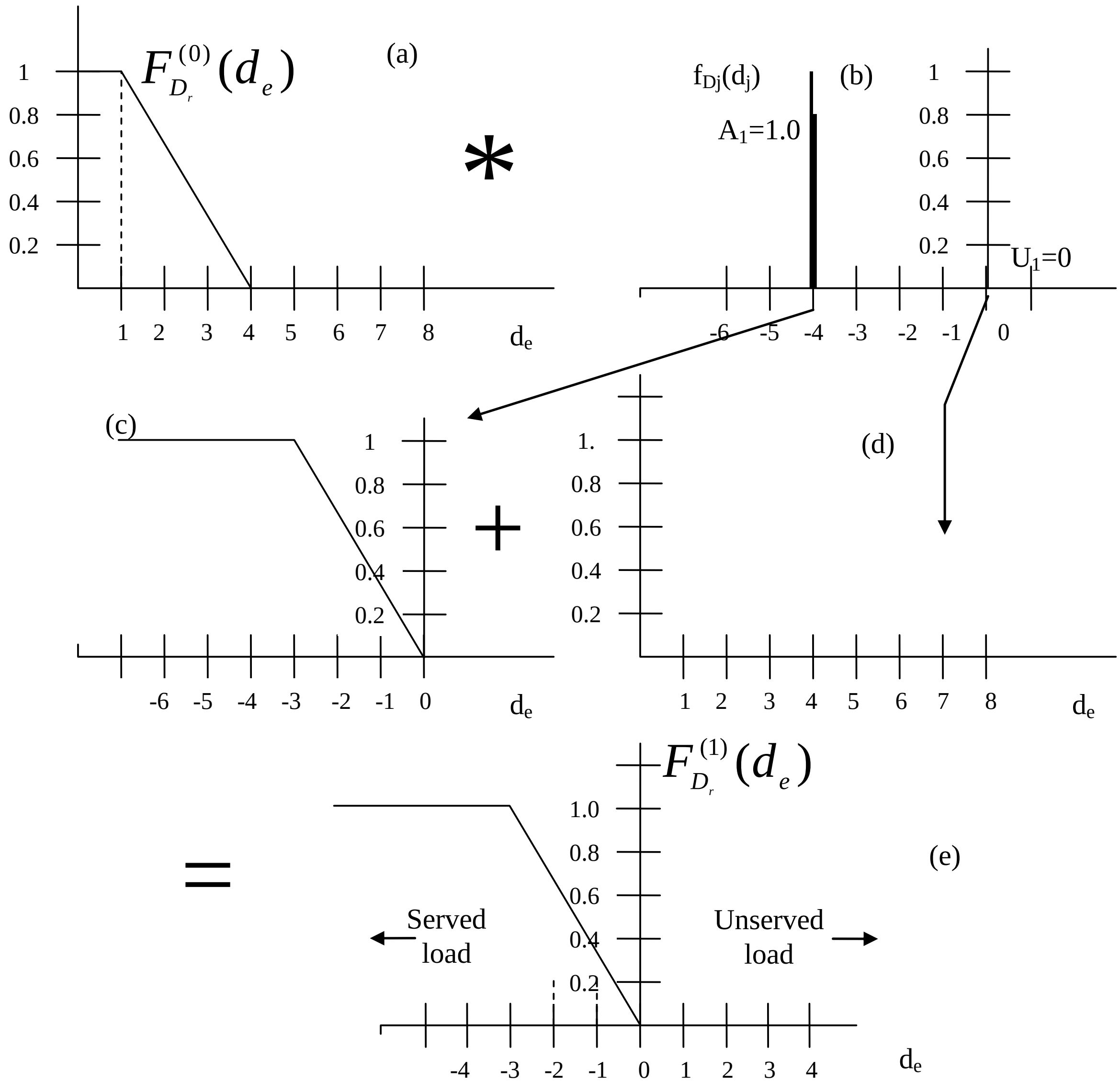


Fig. 32: Convolving in the first unit (perfectly reliable)

However, Fig. 30, plot (e) tells a different story. The fact that there is some part of the load duration curve to the right of d_e=0 is an indication that there is a possibility of load interruption.

Observe that positive d_e may be thought of as unserved load; negative d_e may be thought of as served load. In other words, Fig. 32 tells us

- Pr(unserved load $\geq$ 0 MW) = 0
- Pr(unserved load $\geq$ -4 MW)=1.0
- Pr(served load < 4 MW)=1.0

Fig. 30 applies the ideas of Fig. 15 to Fig. 30, plot (e) to show how the cdf on the *equivalent load* indicates that, for a total capacity of $C_T=0$, we do in fact have some chance of losing load.

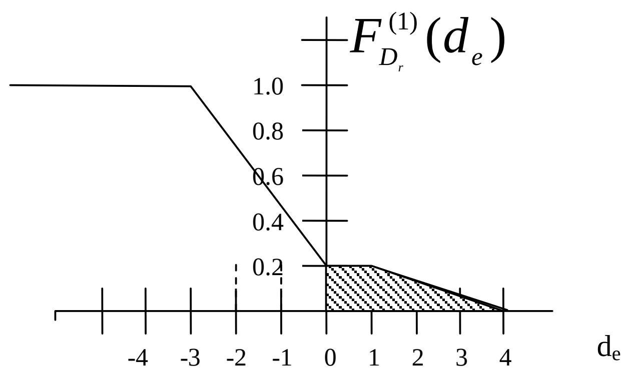


Fig. 33: Illustration of loss of load region

The desired indices are obtained from (12),(13), (14):

$$LOLE = t_{C_T} = g_e(C_T) = T \times F_{D_r}(C_T = 0) = 1 \times 0.2 = 0.2 years$$

A LOLE of 0.2 years is 73 days, a very poor reliability level that reflects the fact we have only a single unit with a high FOR=0.2.

The LOLP is given by:

$$LOLP = P(D_e \geq C_T) = F_{De}(C_T) = 0.2$$

and the EDNS is given by:

$$EDNS = \int_{C_T}^{d_{e,\max}} F_{De}(\lambda) d\lambda$$

which is just the shaded area in Fig. 33, most easily computed using the basic geometry of the figure, according to:

$$0.2(1) + \frac{1}{2}(3)(0.2) = 0.5MW$$

The EENS is given by

$$EENS = T \int_{C_T}^{d_{e,\max}} F_{De}(\lambda) d\lambda = \int_{C_T}^{d_{e,\max}} g_e(\lambda) d\lambda$$

or $T \times EDNS$=1(0.5)=0.5MW-years,

or 8760(0.5)=4380MWhrs.

Example 4: This example is from [6].
A set of generation data is provided in Table 8.

Table 8

Unit No.	Unit name	Rated capacity (MW(e))	Forced outage (%)	Type of fuel	Variable cost ($/MW·h)
1	NUC1	200	20	Nuclear	6.5
2	NUC2	200	20	Nuclear	6.5
3	COAL1	200	10	Coal	27.0
4	COAL2	200	10	Coal	27.0
5	OIL1	100	10	Oil	58.1
6	OIL2	100	10	Oil	58.1
7	OIL3	100	10	Oil	58.1
8	OIL4	100	10	Oil	58.1
9	CT1	100	5	Distillate oil	113.2
System capacity		1300			

Observe the units are ordered from least to highest cost. The 4th column provides the forced outage rate (FOR), which we have denoted by *U*. The two-state generator outage model for each unit (obtained from the FOR), together with the rated capacity, is illustrated in Fig. 34, for unit 1.

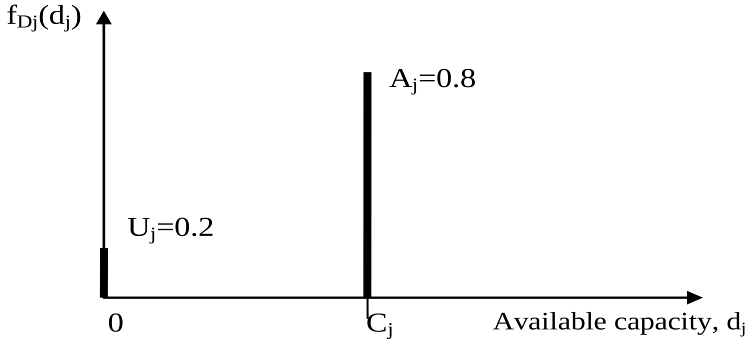


Fig. 34: Two-state outage model for Unit 1

Load duration data is provided in Table 9 and plotted in Fig. 35.

Table 9

Load MW(e)	Fraction of time load exceeds given load
0	1.00
100	1.00
200	1.00
300	1.00
400 (minimum load)	1.00
500	0.80
600	0.40
700	0.20
800	0.10
900	0.05
1000 (peak load)	0.00

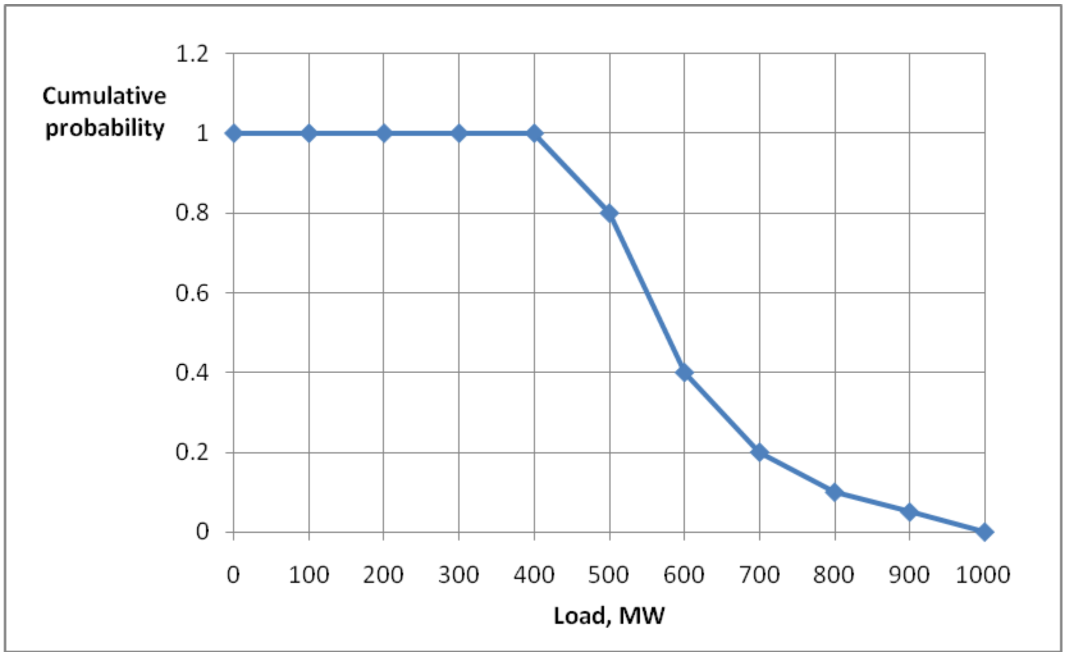


Fig. 35

We now deploy (32), repeated here for convenience,

$$F_{D_e}^{(j)}(d_e) = U_j F_{D_e}^{(j-1)}(d_e) + A_j F_{D_e}^{(j-1)}(d_e + C_j) \qquad (32)$$

to convolve in the unit outage models with the load duration curve of Fig. 35. The procedure is carried out in an Excel spread sheet, and the result is provided in Fig. 36. In Fig. 36, we have shown

- Original load duration curve, F0;
- Load duration curves with unit j convolved in, Fj, j=1,…,9.

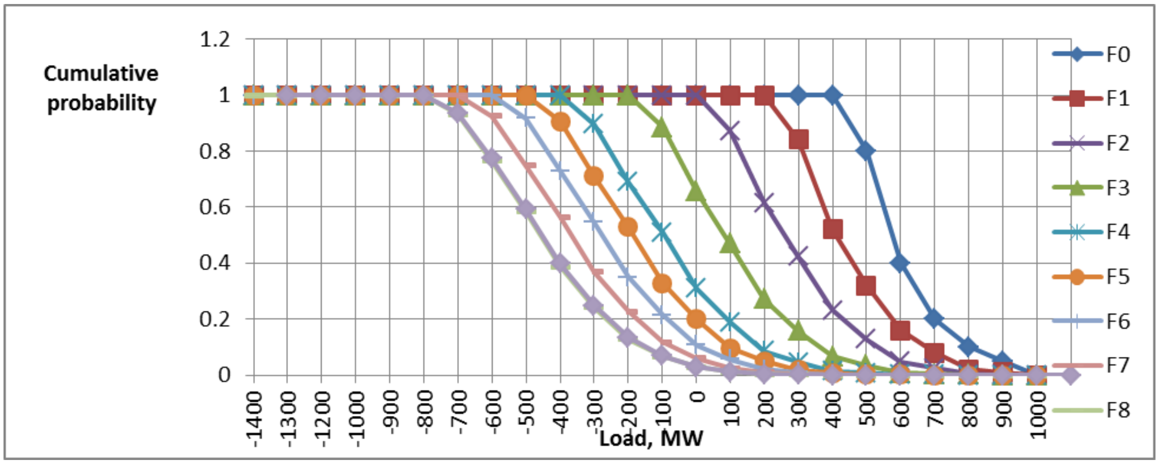


Fig. 36

We also show, in Table 10, the results of the calculations performed to obtain the series of load duration curves (LDC) F0-F9. Notice the following:

- Each LDC is a column FO-F9
- The first column, in MW, is the load.
    - It begins at -1400, an arbitrarily chosen large negative number to ensure each LDC begins from the left with an ordinate of 1.0 (we really only need to extend to -900).
    - The largest actual load is 1000 MW; the extension is to obtain the equivalent load corresponding to a 1000 MW load with 1300 MW of failable generation.
- The entries in the table show the % time the unserved load exceeds the given value.
- LOLP is, for a particular column, the % time load exceeds the total capacity corresponding to that column, and is underlined.

For example, one observes that *LOLP=1* if we only have units 1 (F1, C_T*=200*) or only units 1 and 2 (F2, C_T*=400*). This is because the capacity would never be enough to satisfy the load, at any time. And *LOLP=0.6544* if we have only units 1, 2, and 3 (F3, C_T*=600*). This is because we would be able to supply the load for some of the time with this capacity. And *LOLP=0.012299* if we have all units (F9, C_T*=1300*), which is non-0 (in spite of the fact that C_T*>1000*) because units can fail.

Table 10

	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9
		0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.05
		0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.95
		200	200	200	200	100	100	100	100	100
Load (MW)	Fraction of time load exceeds given load									
-1400	1	1	1	1	1	1	1	1	1	1
-1300	1	1	1	1	1	1	1	1	1	1
-1200	1	1	1	1	1	1	1	1	1	1
-1000	1	1	1	1	1	1	1	1	1	1
-900	1	1	1	1	1	1	1	1	1	1
-800	1	1	1	1	1	1	1	1	1	0.935377
-700	1	1	1	1	1	1	1	1	0.931976	0.774008
-600	1	1	1	1	1	1	1	0.924417	0.765694	0.592168
-500	1	1	1	1	1	1	0.916019	0.748058	0.583035	0.397986
-400	1	1	1	1	1	0.906688	0.729395	0.5647	0.388247	0.248591
-300	1	1	1	1	0.89632	0.709696	0.5464	0.368641	0.241241	0.134512
-200	1	1	1	1	0.68896	0.528256	0.34889	0.227085	0.128894	0.069428
-100	1	1	1	0.8848	0.5104	0.32896	0.213551	0.117984	0.066298	0.02979
0	1	1	1	0.6544	0.3088	0.200728	0.107366	0.060556	0.027869	0.012299
100	1	1	0.872	0.4688	0.18872	0.096992	0.055354	0.024237	0.01148	0.004146
200	1	1	0.616	0.2704	0.0868	0.050728	0.02078	0.010062	0.00376	0.001281
300	1	0.84	0.424	0.1576	0.04672	0.017452	0.008871	0.003059	0.00115	0.00033
400	1	0.52	0.232	0.0664	0.0142	0.007918	0.002414	0.000938	0.000287	7.04E-05
500	0.8	0.32	0.128	0.0344	0.00722	0.001802	0.000774	0.000215	5.9E-05	1.25E-05
600	0.4	0.16	0.048	0.0084	0.0012	0.00066	0.000152	4.17E-05	1.01E-05	1.62E-06
700	0.2	0.08	0.024	0.0042	0.0006	0.000096	2.94E-05	6.54E-06	1.18E-06	1.31E-07
800	0.1	0.02	0.004	0.0004	0.00004	0.000022	0.000004	5.8E-07	7.6E-08	5.7E-09
900	0.05	0.01	0.002	0.0002	0.00002	0.000002	2E-07	2E-08	2E-09	1E-10
1000	0	0	0	0	0	0	0	0	0	0

7.0 Production cost modeling using unserved load

The most basic production cost model obtains production costs of thermal units over a period of time, say 1 year, by building upon the procedures described in Section 7.

The production cost model begins by assuming the existence of a loading (or merit) order, which is how the units are expected to be called upon to meet the demand facing the system. We assume for simplicity that each unit consists of a single "block" of capacity equal to the maximum capacity. It is possible, and more accurate, to divide each unit into multiple capacity blocks, but there is no conceptual difference to the approach when doing so.

Table 5, listed previously in Examples 2 and 4, provides the variable cost for each unit in the appropriate loading order. This table is repeated here for convenience.

Table 5

Unit No.	Unit name	Rated capacity (MW(e))	Forced outage (%)	Type of fuel	Variable cost ($/MW·h)
1	NUC1	200	20	Nuclear	6.5
2	NUC2	200	20	Nuclear	6.5
3	COAL1	200	10	Coal	27.0
4	COAL2	200	10	Coal	27.0
5	OIL1	100	10	Oil	58.1
6	OIL2	100	10	Oil	58.1
7	OIL3	100	10	Oil	58.1
8	OIL4	100	10	Oil	58.1
9	CT1	100	5	Distillate oil	113.2
System capacity		1300			

The criterion for determining loading order is clearly economic. Sometimes it is necessary to alter the economic loading order to account for must-run units or spinning reserve requirements. We will not consider these issues in the discussion that follows.

To motivate the approach, we introduce the concept of a unit's *observed load* as the load "seen" by a unit just before it is committed in the loading order. Thus, it will be the case that all higher-priority units will have been committed.

If all higher-priority units would have been perfectly reliable ($A_j=1$), then the observed load seen by the next unit would have been just the total load less the sum of the capacities of the committed units.

However, all higher-priority units are not perfectly reliable, i.e., they may fail according to the forced outage rate U_j. This means we must account for their stochastic behavior over time. This can be done in a straight-forward fashion by using the equivalent load duration curve developed for the last unit committed.

In the notation of (32) unit j sees the *unserved load* characterized by $F_{D_e}^{(j-1)}(d_e)$. Thus, the energy provided by unit j is proportional to the area under $F_{D_e}^{(j-1)}(d_e)$ from 0 to C_j, where C_j is the capacity of unit j.

In our example 4 above, Unit 1 sees the entire load, characterized by $F_{D_e}^{(0)}(d_e)$, illustrated as the white area in Fig. 37.

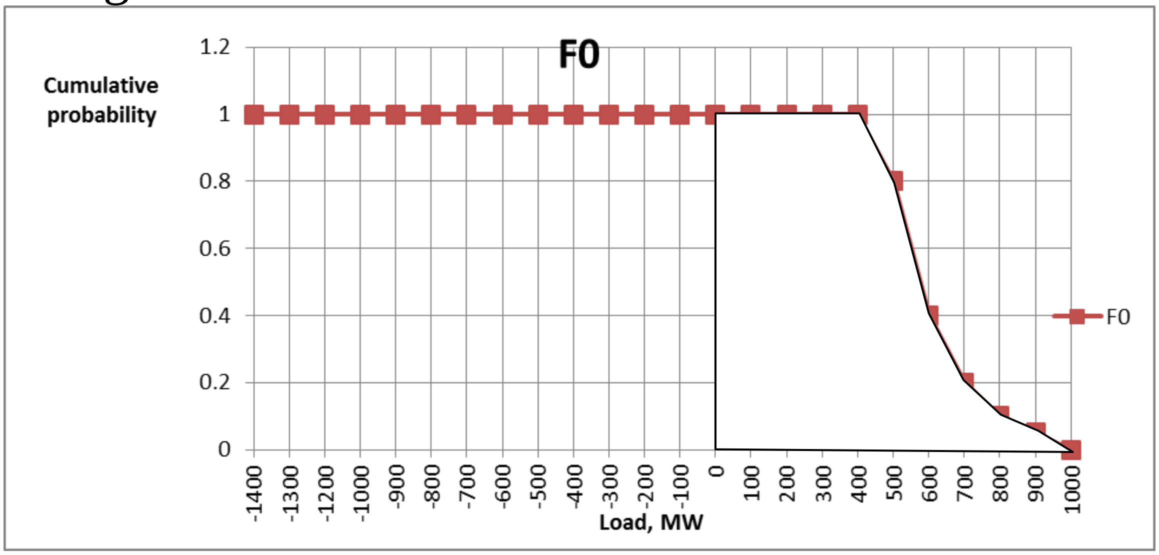


Fig. 37

Unit 2, however, will see the load after unit 1 has been convolved in (resulting in F1), which will have the effect of reducing the unserved load, illustrated in the white area in Fig. 38 (which is less area than the white area in Fig. 37).

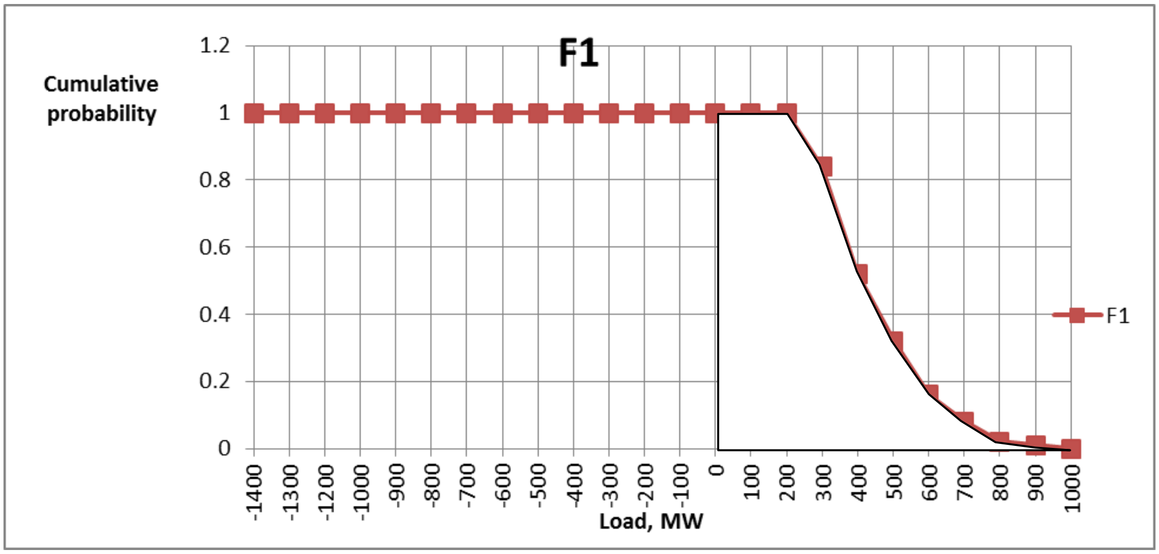


Fig. 38

We want to compute the cost of running each of the various units *j*.

We assume that generator cost rate, in \$/hr, for unit *j* operating at P_j, is linearized, expressed by

$$J_j = J_{j0} + J_{j1} P_j \quad (33)$$

where

- J_{j0} is the unit's no-load cost rate, \$/hr and
- J_{j1} is the unit's cost of energy production, \$/MWhr.

We also assume that unit *j* has capacity C_j.

We obtain the cost associated with scheduling unit *j* according to (33). Consideration of the no-load costs is easy, because we will incur them for every hour the unit is scheduled. Let's assume that the unit will be scheduled for the entire time period, *T* hours, where *T* is the number of hours characterized by the CDF $F_{D_e}^{(j-1)}(d_e)$. Therefore the no-load costs is (in \$):

$$NoloadCosts=J_{j0}T \tag{34}$$

The variable (fuel) costs could be computed (in \$) as:

$$VariableCosts=J_{j1}P_jT$$

However, this would require that the unit runs at P_j for all *T* hours. That may not happen. A better way to compute variable costs results from recognizing that J_{j1}, with units of \$/MWhr, is the cost per unit of energy. Therefore, if we can get the energy supplied by unit *j*, E_j, then this will allow us to compute the cost of supplying it from

$$VariableCosts=J_{j1}E_j \tag{35}$$

The energy supplied by unit *j* can be computed from the CDF $F_{D_e}^{(j-1)}(d_e)$ according to the following:

$$E_j = T\int_0^{C_j} F_{D_e}^{(j-1)}(\lambda)d\lambda \tag{36}$$

In our example, for unit 1 (a 200 MW unit), this would correspond to the area denoted by the hatched region in Fig. 39.

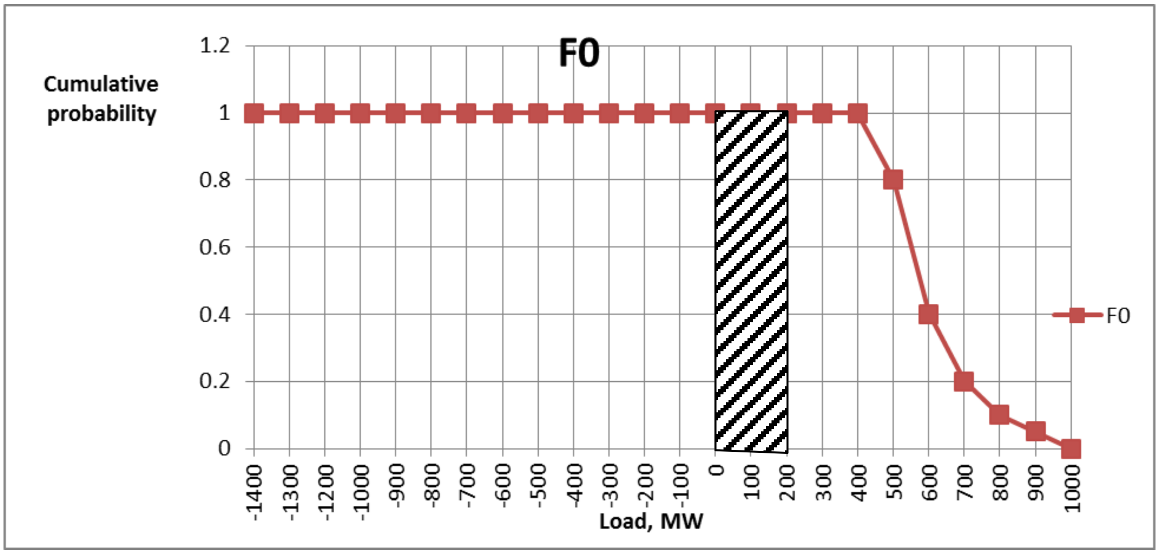


Fig. 39

In this particular case, the integration of (36) provides the same answer as P_jT, but this is because this unit is base-loaded and does in fact run all time at capacity.

And so the total cost of scheduling unit j can be evaluated as the sum of the no-load and variable costs, which is:

$$TotalCosts=NoloadCosts+VariableCosts=J_{j0}T+J_{j1}E_j \quad (37)$$

where E_j is given by (36).

There is just one problem with (37)…

Once we commit a unit, we do *intend* that it will be scheduled for all T hours, However, because the unit has an availability of A_j, we can only expect that the unit will be available for a number of hours equal to A_jT, i.e., unit j is only going to be available A_j% of the time. Therefore, we need to modify (37) to be

$$TotalCosts=A_jJ_{j0}T+ A_jJ_{j1}E_j \qquad (38)$$

where, as before, E_j is given by (36).

Referring back to Example 4, we describe the computations for the first three entries. This description is adapted from [4].

For unit 1, the original load duration curve F0 is used, as forced outages of any units in the system do not affect unit l's observed load. The energy requested by the system from unit 1 is the area under $F_{D_e}^{(0)}(d_e)$ over the range of 0 to 200 MW (unit 1's capacity) times the number of hours in the period (8760) times A_1=0.8. The area under $F_{D_e}^{(0)}(d_e)$ from 0 to 200, has already been illustrated in Fig. 37 above, and is 200.

Therefore,

$$E_1 = 8760 \times 0.8 \times 200 = 1{,}401{,}600 \text{ MWhrs}$$

and the total cost of unit 1 is

For unit 2, the load duration curve F1 is used, as forced outage of unit 1 will affect unit 2's observed load. The energy requested by the system from unit 2 is the area under $F_{D_e}^{(1)}(d_e)$ over the range of 0 to 200 MW (unit 2's capacity) times the number of hours in the period (8760) times $A_2=0.8$. The area under $F_{D_e}^{(1)}(d_e)$ from 0 to 200, illustrated in Fig. 40 below, is 200.

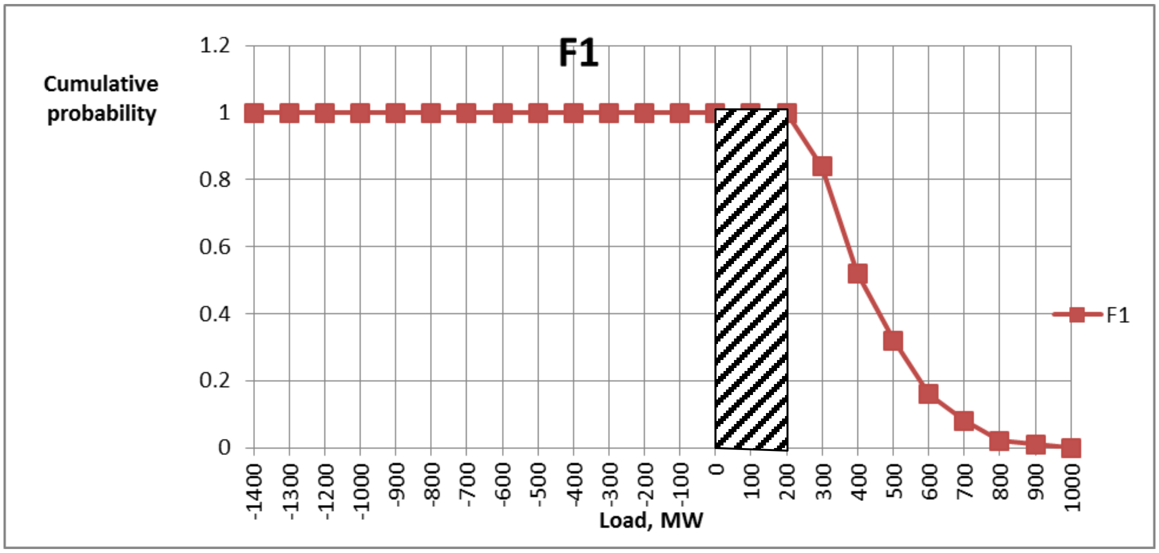


Fig. 40

Therefore,

$$E_2 = 8760 \times 0.8 \times 200 = 1{,}401{,}600 \text{ MWhrs}$$

For unit 3, the load duration curve F2 is used, as forced outage of units 1 and 2 will affect unit 3's observed load. The energy requested by the system from unit 3 is the area under $F_{D_e}^{(2)}(d_e)$ over the range of 0 to 200 MW (unit 3's capacity) times the number of hours in the period (8760) times $A_3=0.9$. The area under $F_{D_e}^{(2)}(d_e)$ from 0 to 200, illustrated in Fig. 41, is calculated below. The coordinates on Fig. 41 are obtained from Table 10, repeated on the next page for convenience.

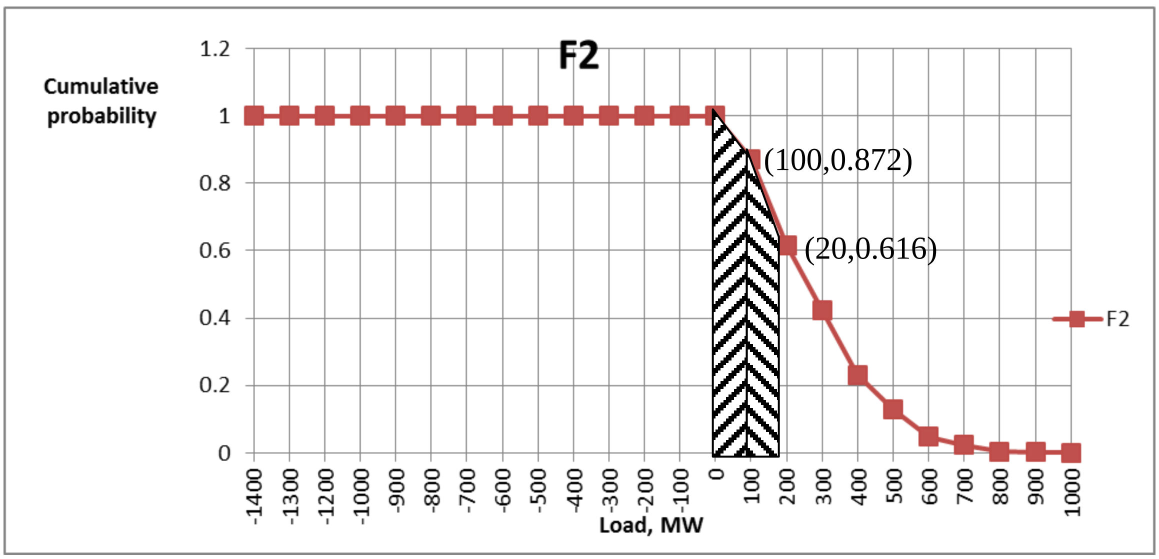


Fig. 41

The area, indicated in Fig. 41, is obtained as two applications of a trapezoidal area (1/2)(h)(a+b), as

$$\underbrace{\frac{1}{2}(100)(1+.872)}_{LeftPortion}+\underbrace{\frac{1}{2}(100)(.872+.616)}_{RightPortion}$$

$$=93.6+74.4=168$$

Therefore,

$$E_3 = 8760 \times 0.9 \times 168 = 1{,}324{,}512 \text{ MWhrs}$$

Table 10

	F0	F1	F2	F3	F4	F5	F6	F7	F8	F9
		0.2	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.05
		0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.9	0.95
		200	200	200	200	100	100	100	100	100
Load (MW)	Fraction of time load exceeds given load									
-1400	1	1	1	1	1	1	1	1	1	1
-1300	1	1	1	1	1	1	1	1	1	1
-1200	1	1	1	1	1	1	1	1	1	1
-1000	1	1	1	1	1	1	1	1	1	1
-900	1	1	1	1	1	1	1	1	1	1
-800	1	1	1	1	1	1	1	1	1	0.935377
-700	1	1	1	1	1	1	1	1	0.931976	0.774008
-600	1	1	1	1	1	1	1	0.924417	0.765694	0.592168
-500	1	1	1	1	1	1	0.916019	0.748058	0.583035	0.397986
-400	1	1	1	1	1	0.906688	0.729395	0.5647	0.388247	0.248591
-300	1	1	1	1	0.89632	0.709696	0.5464	0.368641	0.241241	0.134512
-200	1	1	1	1	0.68896	0.528256	0.34889	0.227085	0.128894	0.069428
-100	1	1	1	0.8848	0.5104	0.32896	0.213551	0.117984	0.066298	0.02979
0	1	1	1	0.6544	0.3088	0.200728	0.107366	0.060556	0.027869	0.012299
100	1	1	0.872	0.4688	0.18872	0.096992	0.055354	0.024237	0.01148	0.004146
200	1	1	0.616	0.2704	0.0868	0.050728	0.02078	0.010062	0.00376	0.001281
300	1	0.84	0.424	0.1576	0.04672	0.017452	0.008871	0.003059	0.00115	0.00033
400	1	0.52	0.232	0.0664	0.0142	0.007918	0.002414	0.000938	0.000287	7.04E-05
500	0.8	0.32	0.128	0.0344	0.00722	0.001802	0.000774	0.000215	5.9E-05	1.25E-05
600	0.4	0.16	0.048	0.0084	0.0012	0.00066	0.000152	4.17E-05	1.01E-05	1.62E-06
700	0.2	0.08	0.024	0.0042	0.0006	0.000096	2.94E-05	6.54E-06	1.18E-06	1.31E-07
800	0.1	0.02	0.004	0.0004	0.00004	0.000022	0.000004	5.8E-07	7.6E-08	5.7E-09
900	0.05	0.01	0.002	0.0002	0.00002	0.000002	2E-07	2E-08	2E-09	1E-10
1000	0	0	0	0	0	0	0	0	0	0

Continuing in this way, we obtain the energy produced by all units. This information, together with the average variable cost for each unit from Table 5, and the resulting variable cost for each unit, is provided in Table 11 below. Observe that in Table 11, the no-load costs are all zero, and so the total costs are the same as the variable costs.

Table 11

Unit i	No-load cost coefficient J_{0i} (\$/hr)	No-load costs $J_{0i}*T$ (\$)	Energy E_i (MW-hr)	Variable cost coefficient J_{1i} (\$/MWhr)	Variable Cost $J_{1i}E_i$ (\$)	Total costs, $J_{0i}*T+J_{1i}E_i$ (\$)
1	0	0	1,401,600	6.5	9,110,400	9,110,400
2	0	0	1,401,600	6.5	9,110,400	9,110,400
3	0	0	1,324,500	27.0	35,761,500	35,761,500
4	0	0	734,200	27.0	19,823,400	19,823,400
5	0	0	196,100	58.1	11,393,410	11,393,410
6	0	0	117,400	58.1	6,820,940	6,820,940
7	0	0	64,100	58.1	3,724,210	3,724,210
8	0	0	33,400	58.1	1,940,540	1,940,540
9	0	0	16,400	113.2	1,856,480	1,856,480
Total	0	0	E_T= 5,289,300		99,541,280	99,541,280

It is interesting to note that the total energy supplied, E_T=5,289,300 MWhrs, is less than what one obtains when the original load duration curve is integrated. This integration can be done by applying our trapezoidal approach to curve F0 in Fig. 37, repeated here for convenience, to obtain the white area shown in the figure.

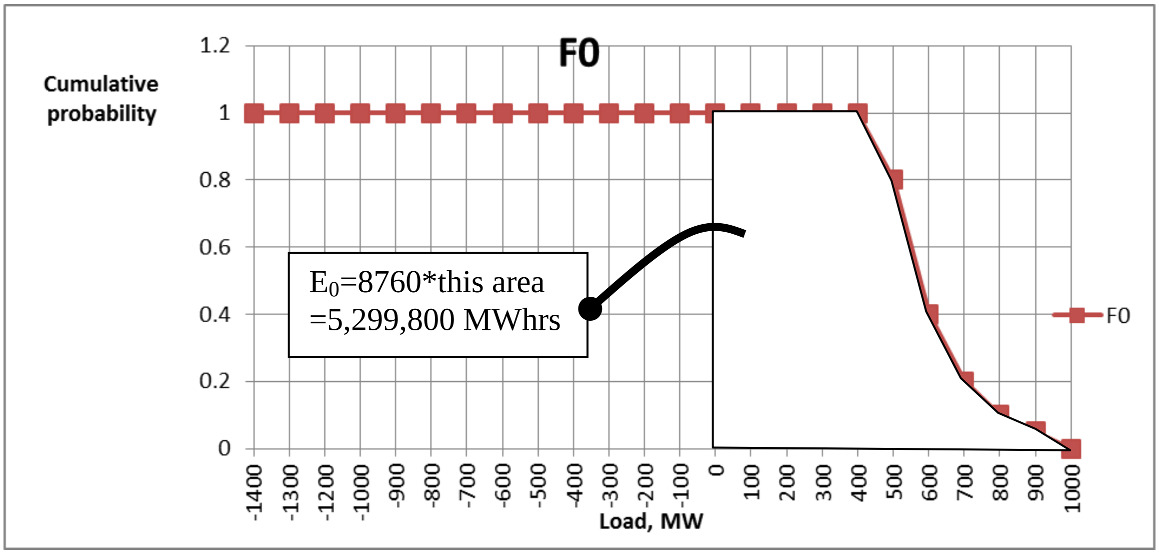


Fig. 37

Doing so results in E_0=5,299,800 MWhrs. The difference is

$$E_0 - E_T = 5{,}299{,}800 - 5{,}289{,}300 = 10{,}500 \text{ MWhrs.}$$

What is this difference of 10,500 MWhrs?

To answer this question, consider the load duration curve after the last unit has been convolved in, curve F9, as shown in Fig. 42.

- The total area under the original curve F0, integrated from 0 to 1000 (the peak load), is 5,299,800 MWhrs, as shown in Fig. 37. This is the amount of energy provided to the *actual* load if it were supplied by perfectly reliable generation having capacity of 1000 MW. As indicated above, we will denote this as E_0.

- The total area under the final curve, F9, integrated from -1300 (the total served load) to 0 (the generation capacity) is E_{es}=6,734,696 MWhrs, as shown in Fig. 42. This is the amount of energy provided to the *effective* load if it were supplied by perfectly reliable generation having capacity of 1300 MW. It is the served load.

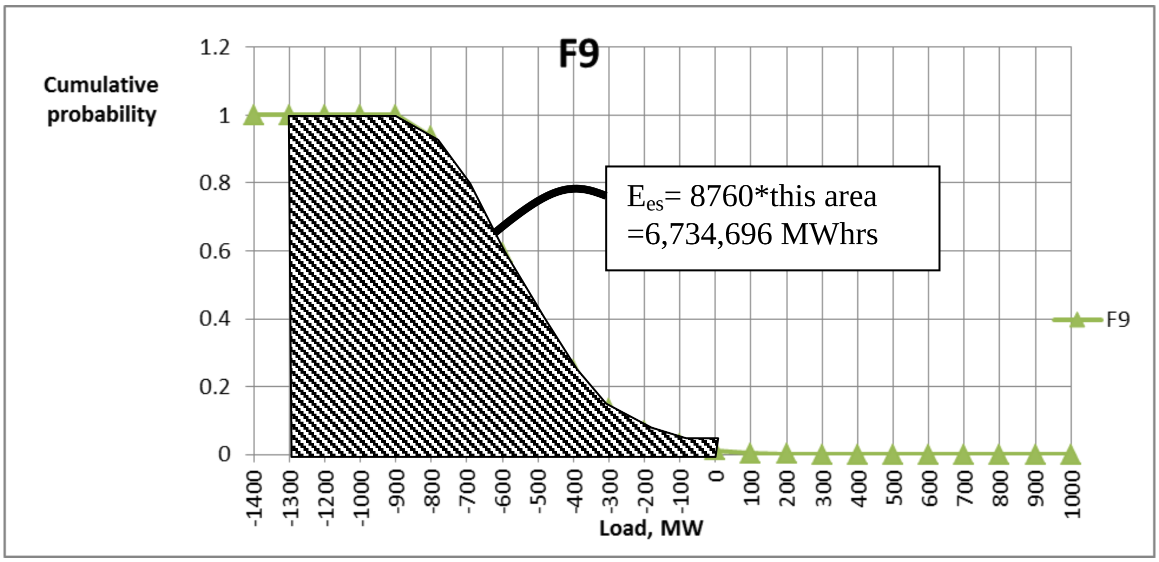


Fig. 42

The energy represented by the area of Fig. 42, which is the energy provided to the *effective* load if it were supplied by perfectly reliable generation having capacity of 1300 MW, is greater than the energy provided by the actual 1300 MW, that is

$$|E_{es}| > |E_T|$$

because E_{es} includes load required to be served when the generators are outaged, and this portion was explicitly removed from the calculation of Table 11 (E_T). One can observe this readily by considering a system with only a single unit. Combining the relations (36) and (38), we can obtain the actual energy supplied by a unit (same as method of Table 11):

$$E_j = TA_j \int_0^{C_j} F_{D_e}^{(j-1)}(\lambda)d\lambda \tag{39}$$

and applying this to the one-unit system, we get:

$$E_T = E_1 = TA_1 \int_0^{C_1} F_{D_e}^{(0)}(\lambda)d\lambda \tag{40}$$

In contrast, the energy served E_{es} obtained when we integrate the effective load duration curve (accounting for the one unit) is

$$E_{es} = T \int_{-C_1}^{0} F_{D_e}^{(1)}(\lambda)d\lambda \tag{41}$$

Recalling the convolution formula (32),

$$F_{D_e}^{(j)}(d_e) = U_j F_{D_e}^{(j-1)}(d_e) + A_j F_{D_e}^{(j-1)}(d_e + C_j) \tag{42}$$

and for the one-unit case, we get

$$F_{D_e}^{(1)}(d_e) = U_1 F_{D_e}^{(0)}(d_e) + A_1 F_{D_e}^{(0)}(d_e + C_1) \tag{43}$$

Substituting (43) into (41) results in

$$E_{es} = T \int_{-C_1}^{0} U_1 F_{D_e}^{(0)}(\lambda) + A_1 F_{D_e}^{(0)}(\lambda + C_1)d\lambda \tag{44}$$

Breaking up the integral gives

$$E_{es} = T\int_{-C_1}^{0} U_1 F_{D_e}^{(0)}(\lambda)d\lambda + T\int_{-C_1}^{0} A_1 F_{D_e}^{(0)}(\lambda + C_1)d\lambda$$

$$= TU_1\int_{-C_1}^{0} F_{D_e}^{(0)}(\lambda)d\lambda + TA_1\int_{-C_1}^{0} F_{D_e}^{(0)}(\lambda + C_1)d\lambda \qquad (45)$$

Reversing the order of integration and multiplying by -1 provides:

$$E_{es} = -\left(TU_1\int_{0}^{C_1} F_{D_e}^{(0)}(\lambda)d\lambda + TA_1\int_{0}^{C_1} F_{D_e}^{(0)}(\lambda + C_1)d\lambda \right) \qquad (46)$$

Comparing (46) with (40), repeated here for convenience:

$$E_T = E_1 = TA_1\int_{0}^{C_1} F_{D_e}^{(0)}(\lambda)d\lambda \qquad (40)$$

we observe the expressions are the same except for the presence of the second integration in (45). This proves that $|E_{es}| > |E_T|$

- Now consider computing the energy consumed by the total effective load, which includes the unserved load, as represented by Fig. 43.

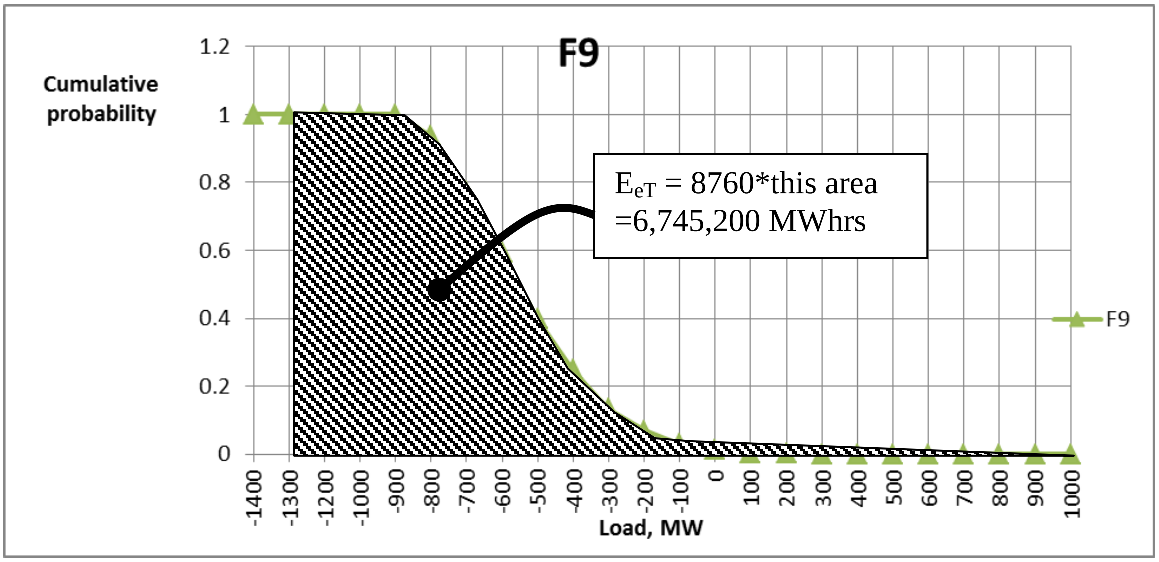


Fig. 43

Using the trapezoidal method to compute this area results in E_{eT}=6745200 MWhrs, which is the energy provided to the *effective* load if it were supplied by perfectly reliable generation having capacity of 2300 MW. This would leave zero energy unserved.

- The difference between
  - Total effective load, E_{eT}: the energy provided to the effective load if it were supplied by 2300 MW of perfectly reliable generation and
  - Effective load served, E_{es}: the energy provided to the effective load if it were supplied by 1300 MW of perfectly reliable generation

  is given by:

  E_{eT}-E_{es}=6,745,200-6,734,696=10,504 MWhrs

  This is the expected energy not served (EENS), sometimes called the expected unserved energy (EUE).

We observe, then, that we can obtain EENS in two different approaches.

1. E_0-E_T=5,299,800-5,289,300=10,500 MWhrs where
   - E_0 is the total energy demanded by the actual load as computed from the original load duration curve;
   - E_T is the energy served to the actual load by the 1300 MW of generation accounting for each unit's potential to fail.
2. E_{eT}-E_{es}=6,745,200-6,734,696=10,504 MWhrs where
   - E_{eT} is the total energy demanded by the effective load as computed from the complete effective load duration curve;
   - E_{es} is the energy served to the effective load by the 1300 MW of generation, assuming the 1300 MW is perfectly reliable.

Approach 1 may be computationally more convenient for production costing because E_T is easily obtained as the summation of all the energy values.

Approach 2 may be more convenient conceptually as it is simply the area under the effective load curve from 0 to total capacity (we can call it C_T).

8.0 Additional W&W comments of interest

A few other comments about the W&W text:

- Pg. 283: In reality, EENS is energy that would not be "not served" but rather provided via expensive interconnection or emergency backup (providing energy via interconnection was the original motivation behind interconnecting control areas).
- Pg. 286: An alternative method of handling EENS is to place "emergency sources" of very large capacity and high cost at the end of the priority list, so that they only get used if no other capacity is available.
- Pg. 284: Mentions NERC's database. It is called "GADS" (Generating Availability Data System). There is also a "TADS" (Transmission Availability Data System) and a "DADS" (Demand Response Availability Data System).
- Pg. 284: For very large systems, the convolution method described above can be computationally intensive. An alternative method is called the "method of cumulants."
- Pg. 318: All of what we have described also applies when generators are modeled as multi-state devices. This can account for the possibility of *de-rating* a unit which sometimes occurs when the unit requires a forced reduction in output due to some particular part of the plant becoming dysfunctional (e.g., one out of 6 boiler feedpumps goes down).

9.0 Industry-grade commercial production cost models

In the previous notes, we reviewed a relatively simple production cost model (PCM). This PCM required two basic kinds of input data:

- Annual load duration curve
- Unit data:
    - Capacity
    - Forced outage rate
    - Variable costs

It then computes load duration curves for *effective* load (which accounts for the unreliability of the generators supplying that load) through a convolution process and provides the following information:

- Reliability indices: LOLP, LOLE, EDNS, EENS (EUE)
- Annual energy produced by each unit
- Annual production costs for each unit
- Total system production costs

Another approach to PCMs is to simulate each hour of the year. This allows much more rigorous models and more refined results, which comes with a significant computational cost. Promod is one such model which you will hear about. I will describe the conceptual approach to such PCMs.

9.1 A refined production cost model

This PCM consists of the following loops:

1. Annual loop: Most PCMs have only one annual loop, i.e., the annual simulation is deterministic. But it is conceivable to make multiple runs through a particular year, each time selecting various variables based on probability distributions for those variables. Such an approach is referred to as a Monte Carlo approach, and it requires many loops in order to "converge" with respect to the average annual production costs.
2. UC loop: The program must have a way for deciding, in each hour, which units are committed. A UC program could be implemented within the PCM on a weekly basis, a 48 hour basis, or a day-ahead basis. The latter seems to be the preferred approach today because it is consistent with the fact that most electricity market structures today depend on the day-ahead using the security-constrained unit commitment.
3. Hourly loop: A security constrained optimal power flow (SCOPF) is implemented to dispatch available units. In addition, it is within the hourly loop that reliability indices are computed. There are two ways of doing this. Both ways depend on the fact that the load is deterministic during the hour and so is represented by a single number. The only randomness is in regards to the status of

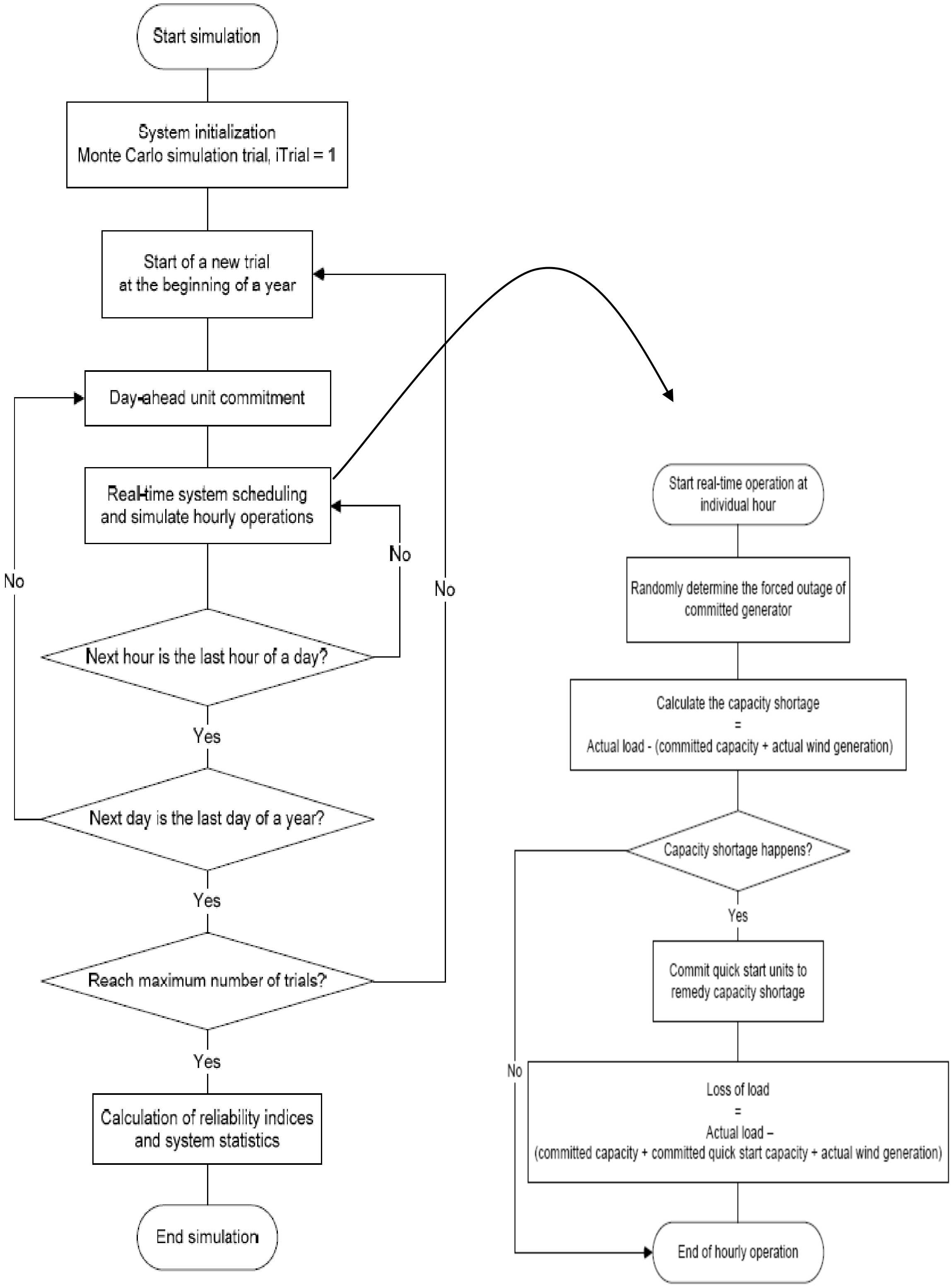
committed generators and whether they are in service or out of service due to a forced outage.

- Monte Carlo: Status of each committed generator is identified via random draw of a number between 0→1. If a number between 0 and the probability of the unit being down (e.g., 0→0.03) is chosen, the unit is outaged. If a number between probability of unit being down & 1 is chosen (e.g., 0.03→1), the unit is in up.
- Analytic: A convolution method similar to our effective load duration approach is employed to compute reliability indices for the hour. The method is simpler because the load is deterministic. The method is referred to as a capacity outage table approach; I can provide you with notes on this method if you want them.
- Network flows: This approach can also handle probabilistic treatment of transmission.

Comment: It is important to use outage replacement rate (ORR) as the probability of the unit being down, rather than the forced outage rate (FOR). The ORR is the probability that the unit will go down in the next hour given it is up at the beginning of the hour.

9.2 A reported model

A model is reported in [7] which captures some of the above attributes. I have lifted out two of the flow charts from this reference to illustrate.



Start simulation
System initialization
Monte Carlo simulation trial, iTrial = 1
Start of a new trial
at the beginning of a year
Day-ahead unit commitment
Real-time system scheduling
and simulate hourly operations
No
No
No
Next hour is the last hour of a day?
Yes
Next day is the last day of a year?
Yes
Reach maximum number of trials?
Yes
Calculation of reliability indices
and system statistics
End simulation
Start real-time operation at
individual hour
Randomly determine the forced outage of
committed generator
Calculate the capacity shortage
=
Actual load - (committed capacity + actual wind generation)
Capacity shortage happens?
Yes
Commit quick start units to
remedy capacity shortage
No
Loss of load
=
Actual load –
(committed capacity + committed quick start capacity + actual wind generation)
End of hourly operation

10.0 MISO's use of Production Costing

Below are a few more slides that characterize how MISO utilizes production costing.

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Background

- PROMOD is a Production Cost Model developed by Ventyx (Formerly known as NewEnergy Associates, A Siemens Company).
- Detailed generator portfolio modeling, with both region zonal price and nodal LMP forecasting and transmission analysis including marginal losses

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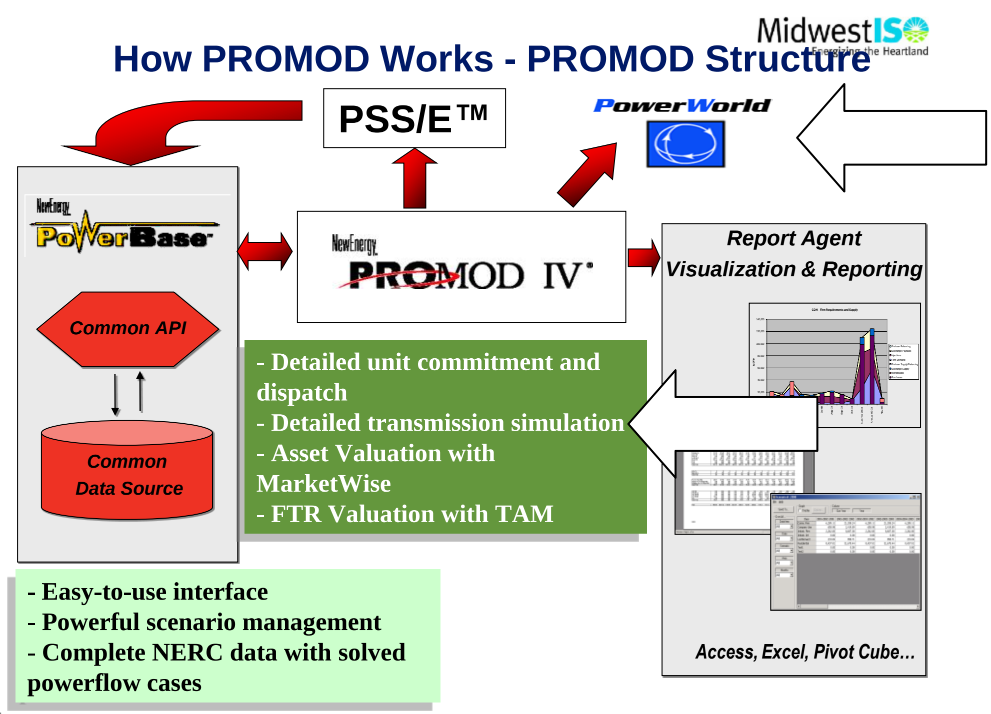

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How PROMOD Works – Input and Output of PROMOD

- Generation Data: heat rate, different costs, etc.
- Demand & Energy
- Fuel Forecasts: Gas, Coal, Oil
- Environmental Costs: Sox, Nox, Mercury
- Power Flow Case
- Monitored Flowgates
- Other Information: reserve requirement, market territory, etc.

→ **PROMOD** →

- Hourly LMP of buses and hubs, include energy, loss and congestion components.
- Hourly unit generation and production cost
- Hourly binding constraints and shadow prices
- Hourly line flows
- Hourly company purchase/sale
- Environmental emissions.
- Fuel consumptions.
- etc.

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Magnitude of the Challenge

Real System Dimensions – MTEP 08 PROMOD Cases

- Footprint: East interconnection excluding FRCC
- Generators: ~ 4,700
- Buses: ~ 47,500
- Branches: ~ 60,000
- Monitored Lines: ~ 1,500
- Contingencies: ~ 500
- Run Time: 60-90 Hrs (for one year 8760 hours)

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Data in PowerBase

- Generation
- Demand & Energy
- Transmission Network Data
- Fuel Forecasts
  - Coal, Uranium, Gas, Coal, Oil
- Environmental Effluent and Costs
  - CO2, Sox, Nox, Mercury

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PROMOD input files

- PFF file
  - Main input file, includes units, fuels, environmental and transmission data, pool configuration, reserve requirement, run option switches, etc.
- Load data file
  - Hourly load profiles for each company for a selected study period.
  - Based on the 8760 hour load shape and each year's peak load and annual energy for each company defined in PowerBase.
- Gen Outage Library and automatic maintenance schedule
  - Same outage library and maintenance schedule used by all cases

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PROMOD input files

- Event files
  - Define the monitored line/contingency pairs which are the transmission constraints
  - Combine MISO and NERC Book of Flowgates
  - Modify existing events or add new events according to member's comments.
  - Create new events which have the potential of overflow using PAT tool

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PROMOD Assumptions

- Study Footprint
  - East interconnection excluding Florida
  - Hourly fixed transactions modeled to include the influence of external areas to the study footprint
    - SETRANS sale to Florida

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PROMOD Assumptions (Cont')

- Pool Definition
  - a group of companies in which all its generators are dispatched together to meet its loads.
  - Hurdle rates are defined between pools to allow the energy exchange between pools.
  - Hurdle rates are based on the filed transmission through-and-out rates, plus a market inefficiency adder.
  - In current MISO cases, 11 pools are defined: MISO, PJM, TVA, MRO, East Canada, SPP, IMO, MHEB, ISONE,NYISO,SERC

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PROMOD Assumptions (Cont')

- Loss Calculation
  - Option1: Load is equal to actual load plus loss. Loss and LMP loss component are not calculated.
  - Option 2: Load is equal to actual load plus loss. Loss is not calculated while LMP loss component is calculated using an approximation method – Single Pass Loss Calculation.
  - Option 3: Load is equal to actual load. Loss and LMP loss component are calculated – Multi Pass Loss Calculation. Run time is 4 times of Option 2.

Option 2 is used in MISO PROMOD cases.

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PROMOD Assumptions (Cont')

- Wind Units – fixed load modifier transactions
  - Set at a same capacity factor for every hour (~ 33%);
  - Set different capacity factors for different months (15% for summer months, and 20% for winter months);
  - Set hourly profile for each unit to capture geographical diversity.
- Smelter Loads modeled as transactions

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PROMOD Output

- LMPs (include the energy, loss and congestion components):
  - Hourly LMP of selected buses, defined hubs.
  - Hourly Load Weighted and Gen Weighted LMP of defined zones.
- Constraints:
  - Hourly shadow price;
  - Number of hours at Pmax, total shadow price at Pmax;
  - Number of hours at Pmin, total shadow price at Pmin;

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PROMOD Output (Cont')

- Generators:
  - Hourly generation
  - Hourly production cost (sum of fuel, variable O&M, environmental cost)
  - Hourly fuel consumption, BTU consumption
  - Hours on line, hours of startup, hours at margin, Hours profitable.
  - Monthly variable O&M cost, fuel cost, emission, and emission cost.

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PROMOD Output (Cont')

- Fuel:
  - Hourly fuel consumption.
- Power Flow:
  - Hourly flow for selected lines, interfaces, and DC lines.
  - Monthly transmission losses (only for marginal loss calculation option)
- Company:
  - Hourly purchase/sale.
  - Hourly dump and emergency energy.

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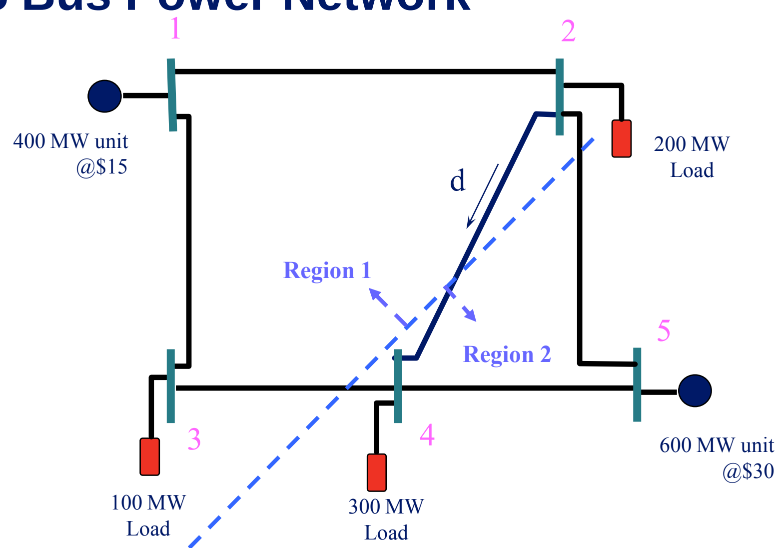
Economic Benefit

- To capture the economic benefit of transmission upgrade: run two PROMOD cases, one with transmission upgrade, one without. For each case, calculate (for each region):
  - Load Cost = Load LMP * Load
  - Adjusted Production Cost = Production Cost + Import * Load Weighted LMP (or) - Export *Gen Weighted LMP
- Economic Benefit:
  - Load Cost Saving: Load Cost difference between two cases;
  - Adjusted Production Cost Saving: Adjusted Production Cost difference between two cases
  - RECB II Benefit = sum over all regions (30%* Load Cost Saving + 70%*Adjusted Production Cost Saving)

RECB: Regional expansion criteria and benefits

Example:
5 Bus Power Network



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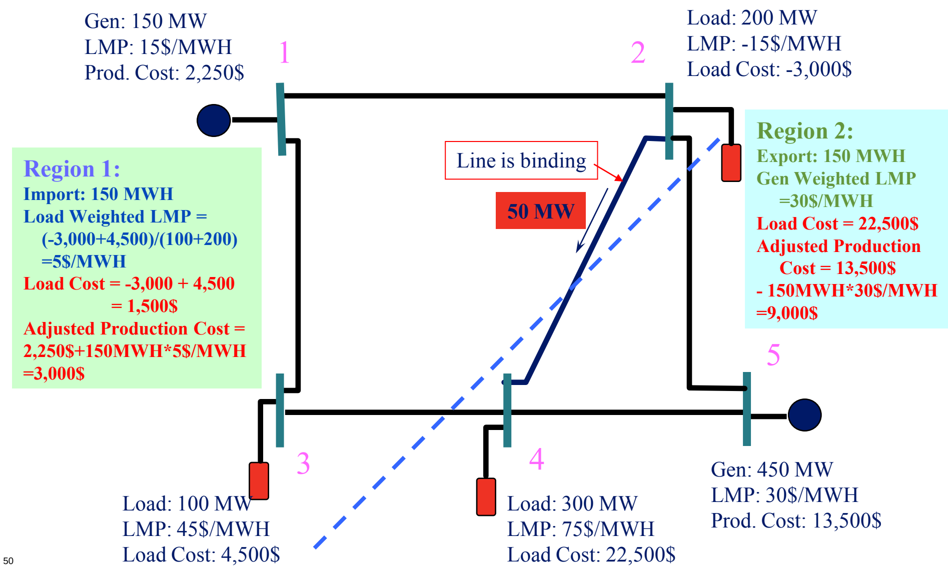
5 Bus Power Network (Original) – PROMOD result



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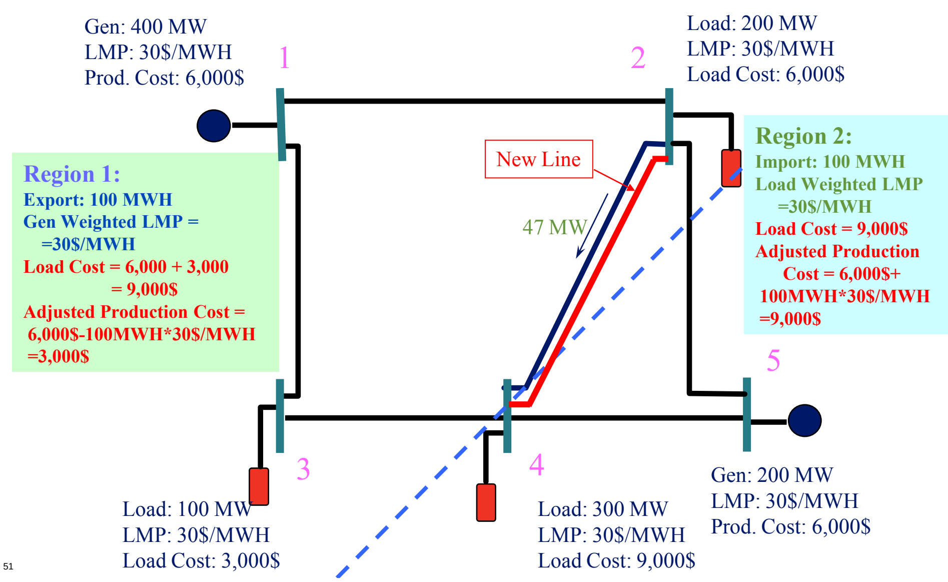
5 Bus Power Network (After upgrade) – PROMOD result



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5 Bus Power Network
– New Transmission RECB II Benefit

		Original Case	Case with New Line	Saving
Region 1	Load Cost	$1,500	$9,000	$-7,500
	Adjusted Production Cost	$3,000	$3,000	$0
Region 2	Load Cost	$22,500	$9,000	$13,500
	Adjusted Production Cost	$9,000	$9,000	$0

RECB II Benefit = 70% * 0 +30% * (-7,500+13,500) = $1,800

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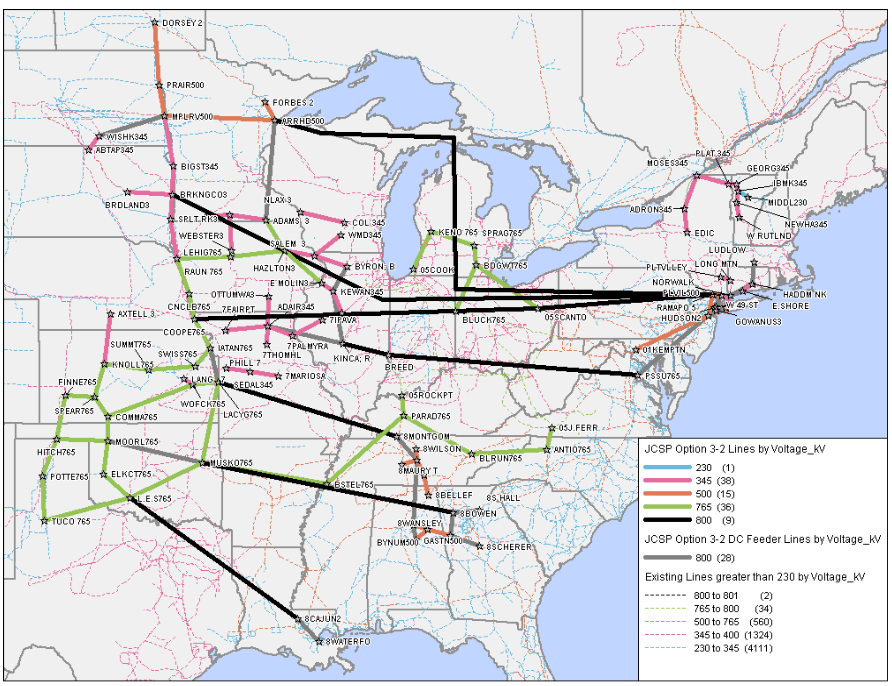
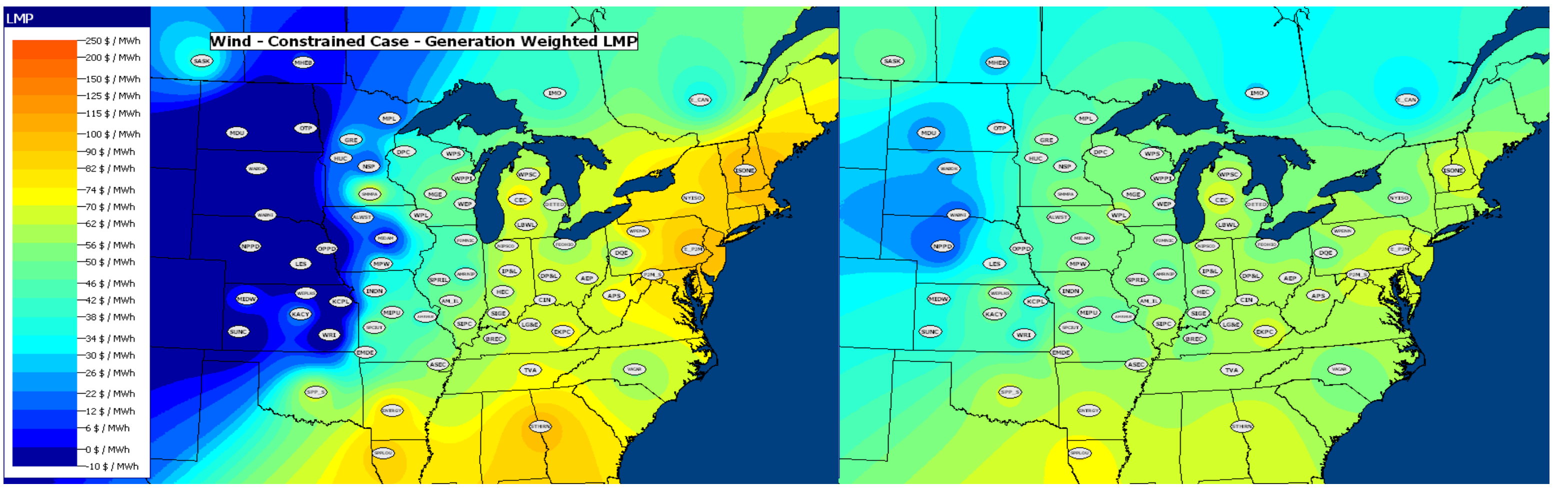
Joint Coordinated System Plan Overlay



20% Wind Energy Scenario

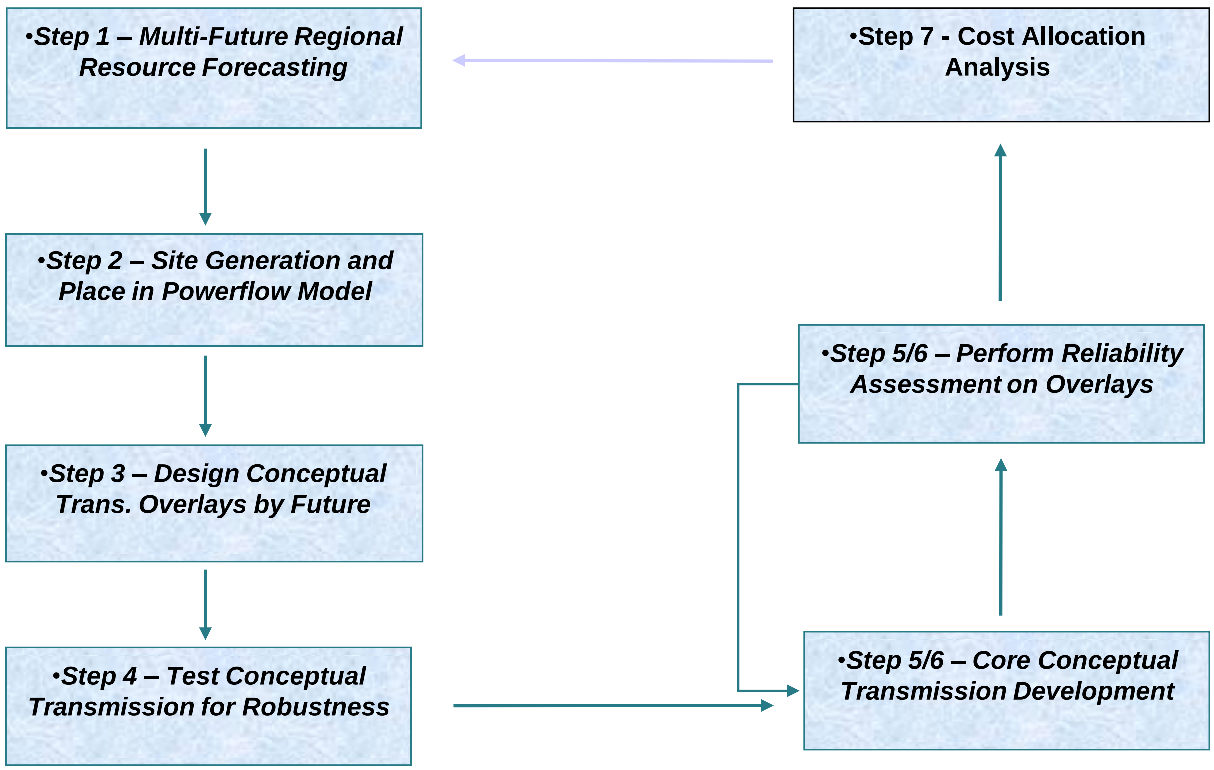
Without and With the JCSP

Overlay



Annual Generator Location Marginal Prices

Value-based Planning Process at Midwest ISO



MidwestISO
Energizing the Heartland

2

Appendix

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[1]http://www05.abb.com/global/scot/scot221.nsf/veritydisplay/581366a0c212c93ac1256fda00488562/$file/Gridview%20Brochure.pdf

[2] http://seekingalpha.com/author/mark-henwood

[3] R. Sullivan, "Power System Planning," McGraw Hill, 1977.

[4] International Atomic Energy Agency, "Expansion planning for electrical generation systems," Technical Reports Series No 241, 1984, available at http://www.energycommunity.org/documents/IAEATRS241.pdf.

[5] X. Wang and J. McDonald, "Modern Power System Planning," McGraw-Hill, 1994.

[6] International Atomic Energy Agency, "Expansion planning for electrical generation systems," Technical Reports Series No 241, 1984.

[7] S. Wang and M. Baran, "Reliability assessment of power systems with wind power generation," Proceedings of the IEEE Power and Energy Society General Meeting, 2010.