Settlement

1.0 Settlement without congestion

Let's consider the base case solution obtained from the notes on demand-bidding. How would the suppliers and the loads be paid?

To answer this question, we repeat here the solution in terms of the one-line diagram and in terms of the table of Lagrange multipliers.

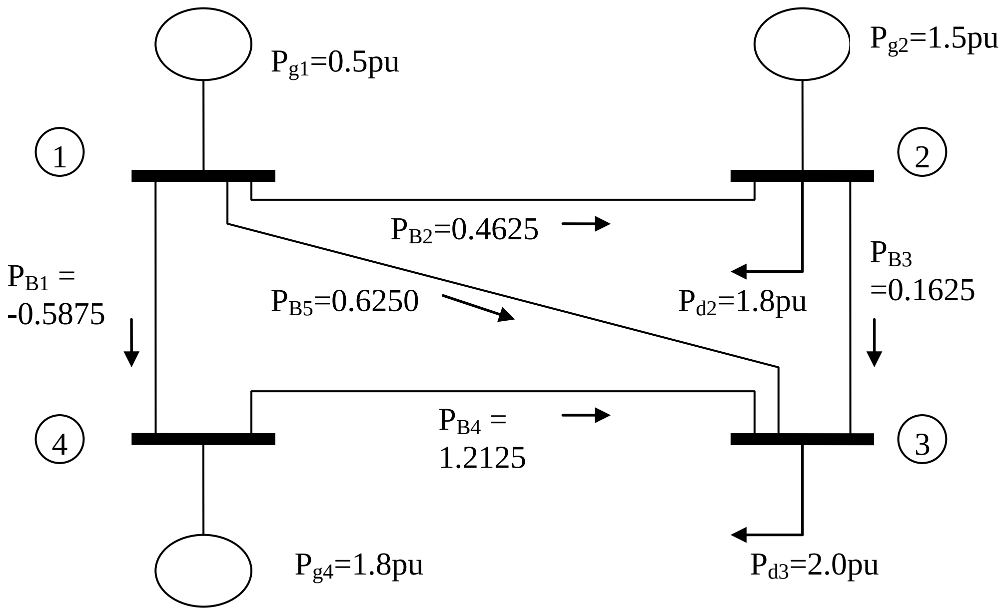

Fig. 1: Result in terms of generation levels, load levels and flows for base case

The objective function is Z=-12.8 \$/hr.

Table 2: Lagrange multipliers for ∞ transm. capacity

Equality constraints		Lower bounds		Upper bounds	
Equation	Value*10^3	Variable	value	variable	value
P_{B1}	-0.0000	P_{g1}	7.0000	P_{g1}	0.0000
P_{B2}	-0.0000	P_{g2}	0.0000	P_{g2}	89.0000
P_{B3}	0.0000	P_{g4}	0.0000	P_{g4}	46.0000
P_{B4}	0.0000	P_{d2}	0.0000	P_{d2}	0.0000
P_{B5}	-0.0000	P_{d3}	100.0000	P_{d2}	0.0000
P_1	1.3000	P_{B1}	0.0000	P_{B1}	0.0000
P_2	1.3000	P_{B2}	0.0000	P_{B2}	0.0000
P_3	1.3000	P_{B3}	0.0000	P_{B3}	0.0000
P_4	1.3000	P_{B4}	0.0000	P_{B4}	0.0000
		P_{B5}	0.0000	P_{B5}	0.0000
		θ_1	0.0000	θ_1	0.0000
		θ_2	0.0000	θ_2	0.0000
		θ_3	0.0000	θ_3	0.0000
		θ_4	0.0000	θ_4	0.0000

The settlement for this case would occur like this:
Amount paid to generators:

$$Payment_{g1} = P_{g1} \times LMP_1 = 50MW \times 13.00\$/MWhr = 650.00\$/hr$$

$$Payment_{g2} = P_{g2} \times LMP_2 = 150MW \times 13.00\$/MWhr = 1950.00\$/hr$$

$$Payment_{g4} = P_{g4} \times LMP_4 = 180MW \times 13.00\$/MWhr = 2340.00\$/hr$$

The total payments to the generators will be 650.00+1950.00+2340.00=4940.00$/hr.

Now what do the loads have to pay?

$$Payment_{d2} = P_{d2} \times LMP_2 = 180MW \times 13.00\$/MWhr = 2340.00\$/hr$$

$$Payment_{d3} = P_{d3} \times LMP_2 = 200MW \times 13.00\$/MWhr = 2600.00\$/hr$$

The total payments from the loads will be 2340.00+2600.00=4940.00$/hr, and so we see that the market settles with total payment to the generators equaling total payment from the loads, i.e., sum of total payments is 0.

Question: Why does this differ from the objective function of -12.80 $/hr?

Answer: We *optimize* on the offers. We *settle* at the LMPs.

➔The bus *k* LMP is the change in the objective function for increasing the load at bus *k* by a unit. It is determined by the regulating agent (generator or load), i.e., the marginal agent. So we are paying generators at the bid of the load at bus 2.

You can see this clearly by recomputing the total payment if we paid each generator and load according to the offers they make: In this case, it would be

$$\boldsymbol{Payment}_{g1} = \boldsymbol{P}_{g1} \times \boldsymbol{s}_{g1} = 50\boldsymbol{MW} \times 13.07\$/\boldsymbol{MWhr} = 653.50\$/\boldsymbol{hr}$$
$$\boldsymbol{Payment}_{g2} = \boldsymbol{P}_{g2} \times \boldsymbol{s}_{g2} = 150\boldsymbol{MW} \times 12.11\$/\boldsymbol{MWhr} = 1816.50\$/\boldsymbol{hr}$$
$$\boldsymbol{Payment}_{g4} = \boldsymbol{P}_{g4} \times \boldsymbol{s}_{g4} = 180\boldsymbol{MW} \times 12.54\$/\boldsymbol{MWhr} = 2257.20\$/\boldsymbol{hr}$$

$$\boldsymbol{Payment}_{d2} = \boldsymbol{P}_{d2} \times \boldsymbol{s}_{d2} = 180\boldsymbol{MW} \times 13.00\$/\boldsymbol{MWhr} = 2340.00\$/\boldsymbol{hr}$$
$$\boldsymbol{Payment}_{d3} = \boldsymbol{P}_{d3} \times \boldsymbol{s}_{d2} = 200\boldsymbol{MW} \times 12.00\$/\boldsymbol{MWhr} = 2400.00\$/\boldsymbol{hr}$$

In this case, if we paid according to the offers, the total payments to the generators will be 653.50+1816.50+2257.20=4727.20$/hr, and the total payments from the loads will be 2340.00+2400.00=4740.00, so the sum of total payments is 4727.20-4740.00=-12.80, which agrees with the value of the objective function.

So why do we settle at the LMPs rather than the offers? According to [1, pg. 26],

> "The primary reason for this conclusion is that under the pay-as-bid settlement scheme, market participants would bid substantially higher than their marginal costs (since there is no incentive for participants to bid their operating cost) to try to increase their revenue and, thus, offset and very likely exceed the expected consumer payment reduction. As a result, currently all ISOs in the United States adopt the pay-at-MCP principle."

In other words,

- A pay-as-bid settlement scheme incentivizes participants to bid high since the bid is what they will be paid if their bid is accepted. The disincentive to bidding high is that their bid might not be accepted.
- A pay-at MCP settlement scheme provides no incentive to bid high. The disincentive to bid high because their bid might not be accepted remains.

Side note: It is interesting to note that we may also obtain the objective function value of -12.80 by summing the product of each Lagrange multiplier and that value of its corresponding constrained variable. This is given by:
0.07*50-0.89*150-0.46*180+1.00*200=-12.80.
Why is this?

2.0 Settlement with congestion

Now let's consider the congested case solution obtained from the last set of notes. This is the case where line 3 was congested. How would the suppliers and the loads be paid?

To answer this question, we repeat here the solution in terms of the one-line diagram and in terms of the table of Lagrange multipliers.

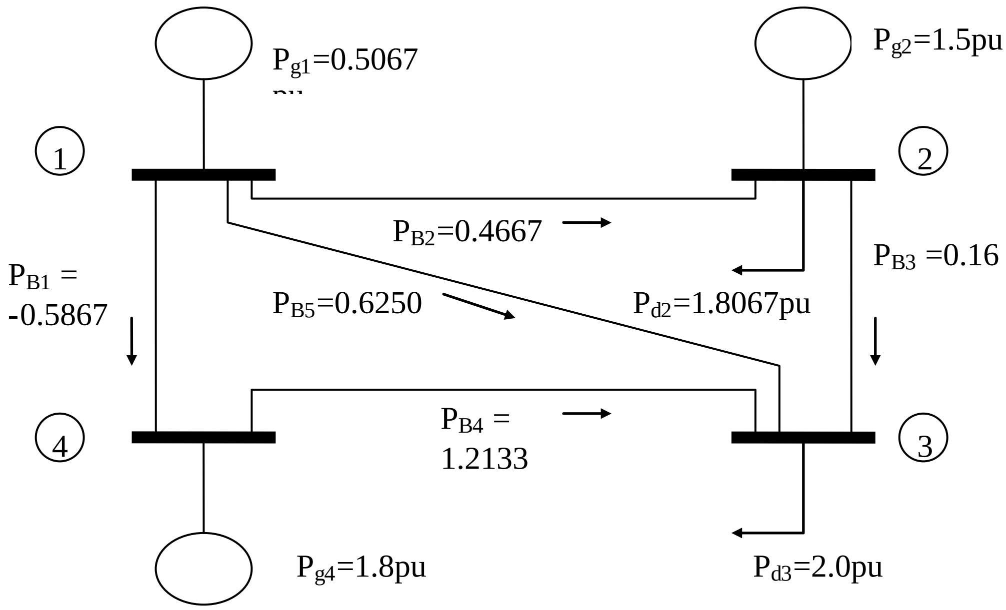

Fig. 6: Cases 2 flows

Table 3: Lagrange multipliers for constrained transm.

Equality constraints		Lower bounds		Upper bounds	
Equation	Value*10^3	Variable	value	variable	value
P_{B1}	-0.0000	P_{g1}	0.0000	P_{g1}	0.0000
P_{B2}	0.0000	P_{g2}	0.0000	P_{g2}	89.0000
P_{B3}	0.0187	P_{g4}	0.0000	P_{g4}	55.3333
P_{B4}	0.0000	P_{d2}	0.0000	P_{d2}	0.0000
P_{B5}	0.0000	P_{d3}	111.6667	P_{d3}	0.0000
P_1	1.3070	P_{B1}	0.0000	P_{B1}	0.0000
P_2	1.3000	P_{B2}	0.0000	P_{B2}	0.0000
P_3	1.3117	P_{B3}	0.0000	P_{B3}	18.6667
P_4	1.3093	P_{B4}	0.0000	P_{B4}	0.0000
		P_{B5}	0.0000	P_{B5}	0.0000
		θ_1	0.0000	θ_1	0.0000
		θ_2	0.0000	θ_2	0.0000
		θ_3	0.0000	θ_3	0.0000
		θ_4	0.0000	θ_4	0.0000

The settlement for this case would occur like this:
Amount paid to generators:

$$Payment_{g1} = P_{g1} \times LMP_1 = 50.67MW \times 13.07\$/MWhr = 662.26\$/hr$$

$$Payment_{g2} = P_{g2} \times LMP_2 = 150MW \times 13.00\$/MWhr = 1950.00\$/hr$$

$$Payment_{g4} = P_{g4} \times LMP_4 = 180MW \times 13.093\$/MWhr = 2356.70\$/hr$$

The total payments to the generators will be 662.26+1950.00+2356.70=4969.00$/hr.

Now what do the loads have to pay?

$$Payment_{d2} = P_{d2} \times LMP_2 = 180.67MW \times 13.00\$/MWhr = 2348.70\$/hr$$

$$Payment_{d3} = P_{d3} \times LMP_2 = 200MW \times 13.117\$/MWhr = 2623.40\$/hr$$

The total payments by the loads will be 2348.70.00+2623.40=4972.10$/hr.

Notice: Amount paid by loads exceeds that paid to generators by 4972.10-4969.00=3.10$/hr. Why gen/load payments not balance?
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Before we answer that, let's look at what happens when we pay at bid: In this case, it would be

$$Payment_{g1} = P_{g1} \times s_1 = 50.67MW \times 13.07\$/MWhr = 662.257\$/hr$$

$$Payment_{g2} = P_{g2} \times s_2 = 150MW \times 12.11\$/MWhr = 1816.50\$/hr$$

$$Payment_{g4} = P_{g4} \times s_4 = 180MW \times 12.54\$/MWhr = 2257.20\$/hr$$

$$Payment_{d2} = P_{d2} \times LMP_2 = 180.67MW \times 13.00\$/MWhr = 2348.70\$/hr$$

$$Payment_{d3} = P_{d3} \times LMP_2 = 200MW \times 12.00\$/MWhr = 2400.00\$/hr$$

In this case, if we paid according to the offers, the total payments to the generators will be 662.257+1816.50+2257.20=4736.00$/hr, and the total payments from the loads will be 2348.70+2400.00=4748.70, so the sum of total payments is 4736.00-4748.70=-12.70, which agrees with the value of the objective function.

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Back to question: Why do gen and load payments not balance?

This is due to the *congestion charges*, denoted as CC and given by:

$$CC = \sum_{j=1}^{M} \mu_j P_{bj}$$

In our example, since we have only one congested line, this is:

$$CC = \sum_{j=1}^{M} \mu_j P_{bj} = 18.6667 * 0.16 = 2.99$$

Note that the units of μ_j are $/per-unit hr and the units of P_{bj} are per unit, and so the units of CC are $/hr. Congestion charges are allocated by the market operator to holders of financial transmission rights (FTRs).

Comparison to the difference between payment to the generators and payment by the loads, 3.10$/hr, indicates that this accounts for it (within roundoff error).

Thus we are led to conclude that

$$CC = \sum_{j=1}^{M} \mu_j P_{bj} = \sum_{k \in load} LMP_j * P_{dk} - \sum_{k \in gen} LMP_k * P_{gk}$$

Let's try to prove this.

We show in previous notes that the LMPs at each bus are given by:

$$\begin{aligned} k \in load: \quad LMP_k &= \lambda && \text{Energy component} \\ &+ \lambda \frac{\partial P_{loss}}{\partial P_{dk}} && \text{Loss component} \\ &+ \sum_{j=1}^{M} \mu_j t_{jk} && \text{Congestion component} \end{aligned}$$

If we ignore the loss component, then LMP's are given by:

$$LMP_k = \lambda + \sum_{j=1}^{M} \mu_j t_{jk}$$

where t_{jk} are called *shift factors* and give the change in flow on circuit *j* to a change in real power injection at bus *k*, under a specified slack distribution, according to

$$t_{jk} = \frac{\Delta F_j}{\Delta P_k} \qquad (1)$$

If the network is linear over its entire operating range, then (1) applies even when

$$\Delta F_j = F_j - 0, \quad \Delta P_k = P_k - 0 \qquad (2)$$

so that

$$t_{jk} = \frac{F_j}{P_k} \qquad (3)$$

or

$$F_j = t_{jk} P_k \qquad (4)$$

In matrix form, (4) becomes:

$$\underline{F} = \underline{T}\,\underline{P} \qquad (5)$$

Then the congestion charge is:

$$CC = \sum_{k=1}^{N} LMP_k * P_{gk} - \sum_{k=1}^{N} LMP_k * P_{dk}$$

$$= \sum_{k=1}^{N} \left(\lambda + \sum_{j=1}^{M} \mu_j t_{jk} \right) * P_{gk} - \sum_{k=1}^{N} \left(\lambda + \sum_{j=1}^{M} \mu_j t_{jk} \right) * P_{dk}$$

$$= \sum_{k=1}^{N} \left[\left(\lambda + \sum_{j=1}^{M} \mu_j t_{jk} \right) * P_{gk} - \left(\lambda + \sum_{j=1}^{M} \mu_j t_{jk} \right) * P_{dk} \right]$$

$$= \sum_{k=1}^{N} \left[\left(\lambda + \sum_{j=1}^{M} \mu_j t_{jk} \right) \left(P_{gk} - P_{dk} \right) \right]$$

$$= \sum_{k=1}^{N} \left[\lambda \left(P_{gk} - P_{dk} \right) + \sum_{j=1}^{M} \mu_j t_{jk} \left(P_{gk} - P_{dk} \right) \right]$$

$$= \sum_{k=1}^{N} \lambda \left(P_{gk} - P_{dk} \right) + \sum_{k=1}^{N} \sum_{j=1}^{M} \mu_j t_{jk} \left(P_{gk} - P_{dk} \right)$$

$$= \lambda \sum_{k=1}^{N} \left(P_{gk} - P_{dk} \right) + \sum_{k=1}^{N} \sum_{j=1}^{M} \mu_j t_{jk} \left(P_{gk} - P_{dk} \right)$$

$$= 0 + \sum_{k=1}^{N} \sum_{j=1}^{M} \mu_j t_{jk} \left(P_{gk} - P_{dk} \right)$$

Now interchange the summation to obtain:

$$CC = \sum_{j=1}^{M} \sum_{k=1}^{N} \mu_j t_{jk} \left(P_{gk} - P_{dk} \right)$$

$$= \sum_{j=1}^{M} \mu_j \sum_{k=1}^{N} t_{jk} \left(P_{gk} - P_{dk} \right)$$

$$= \sum_{j=1}^{M} \mu_j \sum_{k=1}^{N} t_{jk} P_k = \sum_{j=1}^{M} \mu_j F_j$$

So the conclusion here is that we can get congestion charges by either the bus LMPs (the formula at the beginning of the derivation) or the branch flow Lagrange multipliers (the formula at the end of the derivation).

Let's summarize our above exercises in the following table.

Value	Without congestion	With congestion
Z	-12.80	-12.75
Pay at MC		
Pay to gens	4940.00	4969.00
Pay from loads	4940.00	4972.10
Balance	0	3.10
Pay at Bid		
Pay to gens	4727.20	4736.00
Pay from loads	4740.00	4748.70
Balance	-12.80	-12.70
Gen "constraint cost"	-12.80	-9.75
Branch "constraint cost" (congestion charge)	0	-3.10
Sum of gen & branch "constraint costs"	-12.80	-12.85
Total Load	3.8 pu	3.8067pu

Loads paid more than gens were paid.

The same as Z.

Gen constraint cost differs from Z due to branch constraint cost.

Gen constraint cost same as Z w/o congestion due to gen constraints are only binding constraints.

Gen + branch constraint cost same as Z with or w/o congestion .

Some comments:

1. Differences among the objective function in the two cases is due to round-off error and should not be interpreted to be of significance $\rightarrow$objective function is same$\rightarrow$Social surplus is the same.
2. Load is more in congested case.
3. Load pays more than gen receives in congested case.
4. Sum of constraint costs equals Z. This says that primal objective will be the same as dual objective.

[1] J. Yan, G. Stern, P. Luh, and F. Zhao, "Payment versus bid cost," IEEE Power and Energy Magazine, March/April 2008.