Hydro-Thermal Scheduling (HTS)

1.0 Introduction

From an overall systems view, the single most important attribute of hydroelectric plants is that there is no fuel cost, therefore production costs, relative to that of thermal plants, are very small.

There are three basic types of hydroelectric plants: run-of-river, pumped storage, and reservoir systems. We will just introduce the first two in this section, and then the remainder of these notes will be dedicated to understanding reservoir systems.

<u>Run-of-river</u>

Here a dam is placed across a river to create a height differential between the upstream inlet and the downstream outlet, but without creating an expansive reservoir on the upstream side [1]. The turbine is rotated simply by the normal flow of the river. These plants run at a capacity associated with the natural river current. Figure 1 [2] illustrates a number of different run-of-the-river projects.

Fig. 1 [2]

Pump-storage

This kind of hydro plant is a specialized reservoir-type plant which has capability to act as both a source and a sink of electric energy. In the source or generation mode, it supplies power to the grid using the kinetic energy of the water as it falls from higher-lake to lower-lake as would a typical reservoir plant. In the sink or pumping mode, it consumes power from the grid in order to pump water from the lower lake to the higher lake. Thus, electric energy from the grid is converted into potential energy of the water at the higher elevation.

The original motivation for pumped storage plants was to valley-fill and peak-shave.

- Valleys: During low-load periods, the plant is used in pumping mode, thus increasing overall system load. This is beneficial because a decreased number of thermal plants will need to be shut-down (avoiding shut-down and start-up costs), and for those remaining on-line, they can be used at higher, more efficient generation levels.
- Peaks: During high-load periods, the plant is used in generating mode, thus decreasing the overall system load that must be met by thermal generation. This is beneficial because it avoids the need to start some of the expensive peaking plants.

Figure 2 [3] illustrates a typical 24 cycle for a northwestern region of the US.

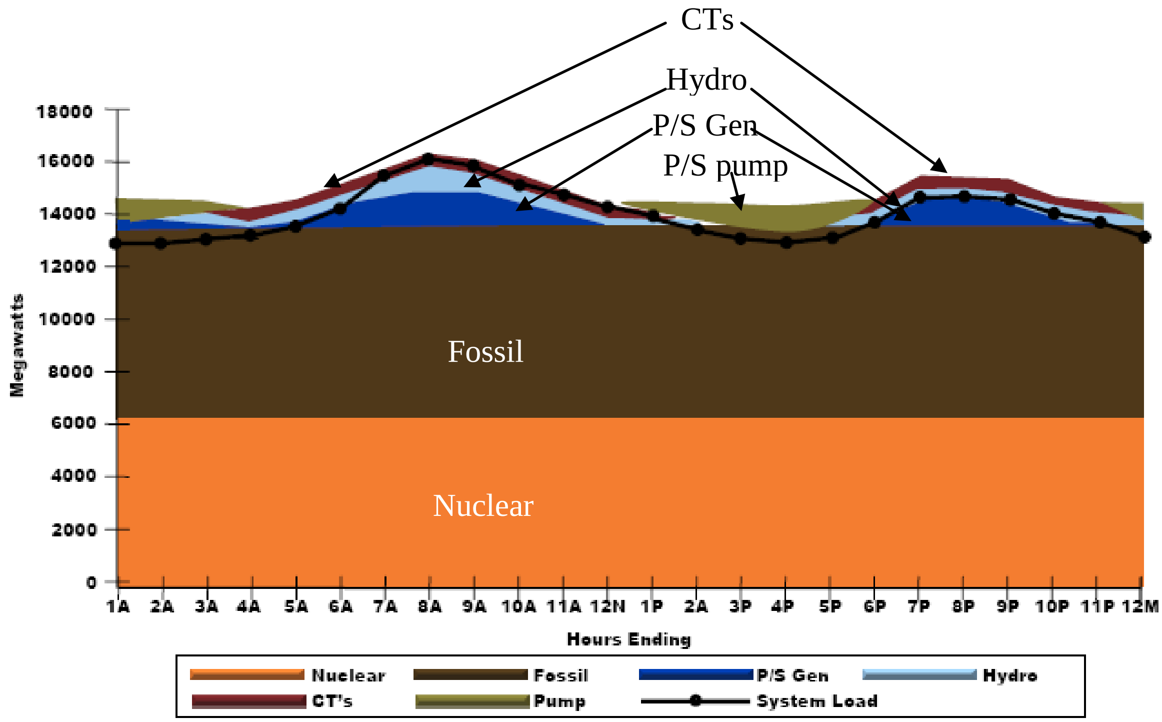


Fig. 2 [3]

Of course, the cycle of pumping and generating incurs a net loss. It is typical for the efficiency of a round-trip pump storage cycle to be about 70%; for every 100 MW used to pump water, only about 70 MW will be recovered by the grid. The cost of this loss is lessened by the fact that the energy is supplied by thermal plants operating at higher (and thus more efficient) loading levels because of the presence of the pumping. This cost is compensated by the savings incurred by avoiding shut-down and start up costs of the thermal plants during the valleys and by avoiding the start-up costs of the peaking plants during the peaks.

Pump storage has become of even greater interest today because it offers a way to store energy that is available from renewable resources (wind and solar) during off-peak times so that they can then be used during on-peak times. Figure 3 [3] illustrates a situation in the BPA region (which is seeing significant wind growth) where the wind plants are frequently generating when load is low and not generating when load is high.

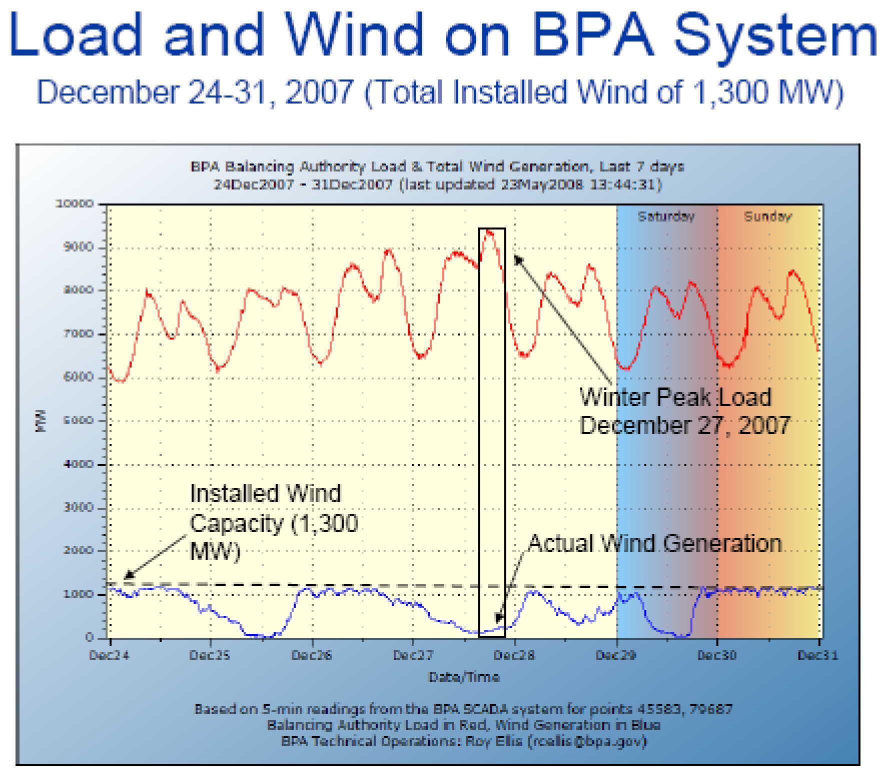


Fig. 3 [3]

Figure 4 indicates the manner in which pumped storage could be used with wind over a 24 hour period.

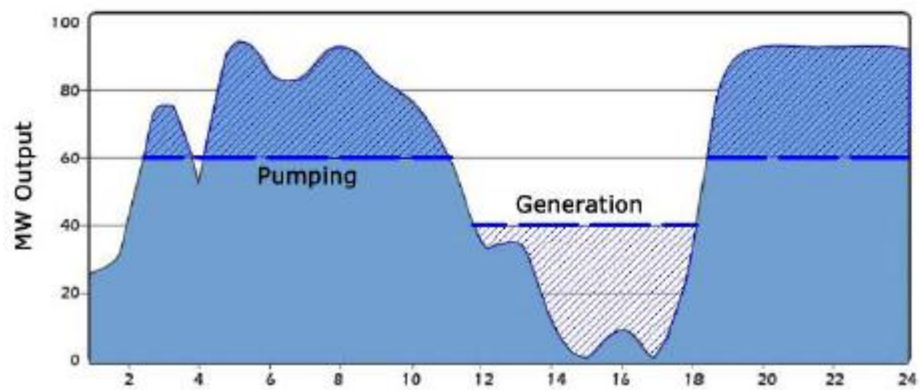


Fig. 4 [3]

Pump storage also supplies regulation and load following to which renewables generally do not contribute.

Figure 5 [4] illustrates a typical pump-storage set-up.

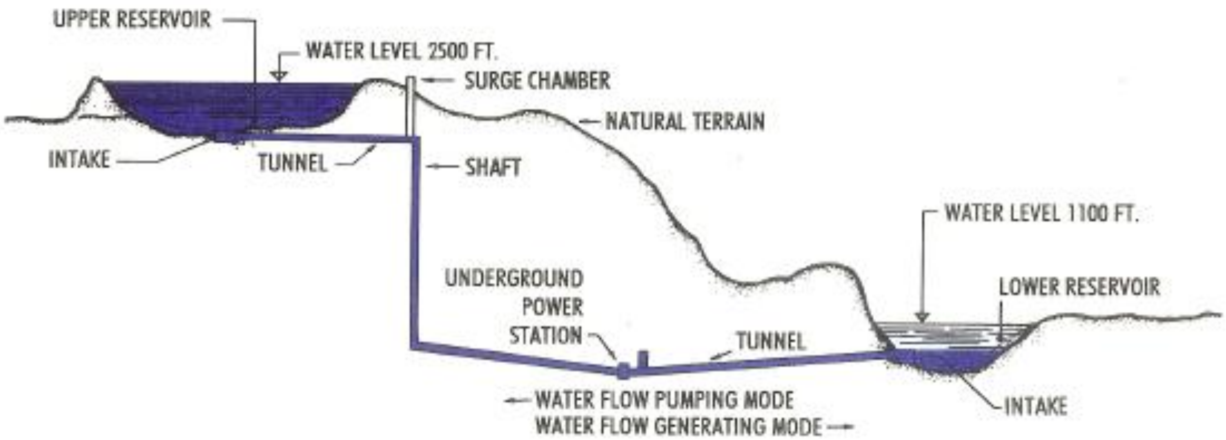
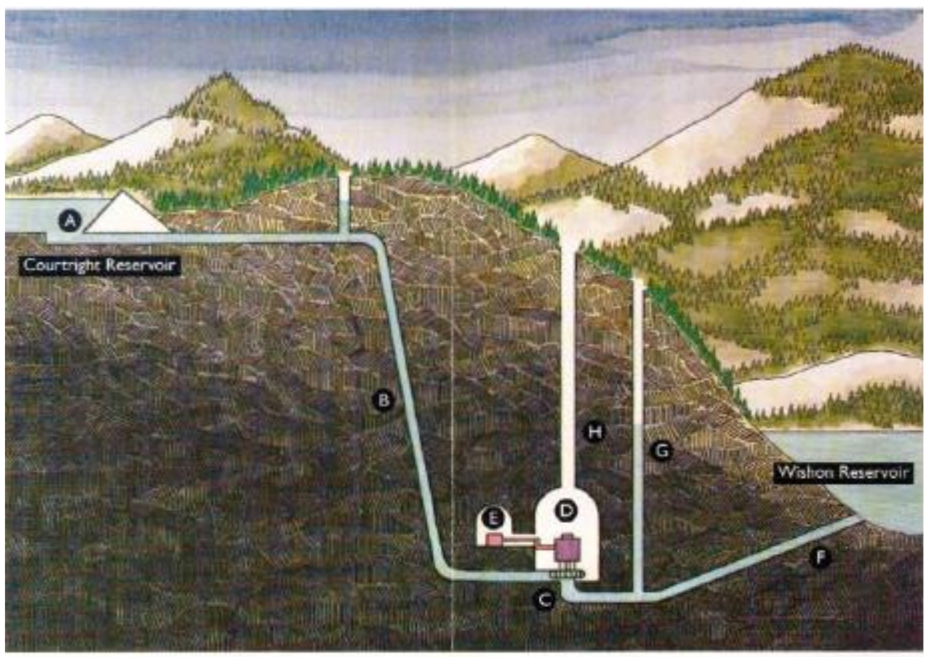


Fig. 5 [4]

One pump-storage plant of which I am familiar is called Helms pumped storage plant, commissioned in 1984. It consists of three units rated at 404 MW (1212 MW total) in the generating mode and 310 MW (930 MW total) in the pumping mode. Figure 6 [5] illustrates the overall setup of Helms which operates between Courtright and Wishon Lakes about 50 miles east of the city of Fresno California.



A-Courtright, B-Supply Tunnel, C-Turbine, D-Generator, E-Transformer, F-Wishon, G-Surge Chamber, H-Elevator

Fig. 6 [5]

Figure 7 [5] shows the powerhouse for Helms, where one can observe that it is underground (at a depth of 1000 ft!).

Fig. 7 [5]

Figure 8 [5] below shows the typical week-long cycles of Helms. Note that unit 2 is typically not used as a result of the fact that the region around Fresno has recently become transmission constrained. PG&E had to build new transmission to alleviate this problem.

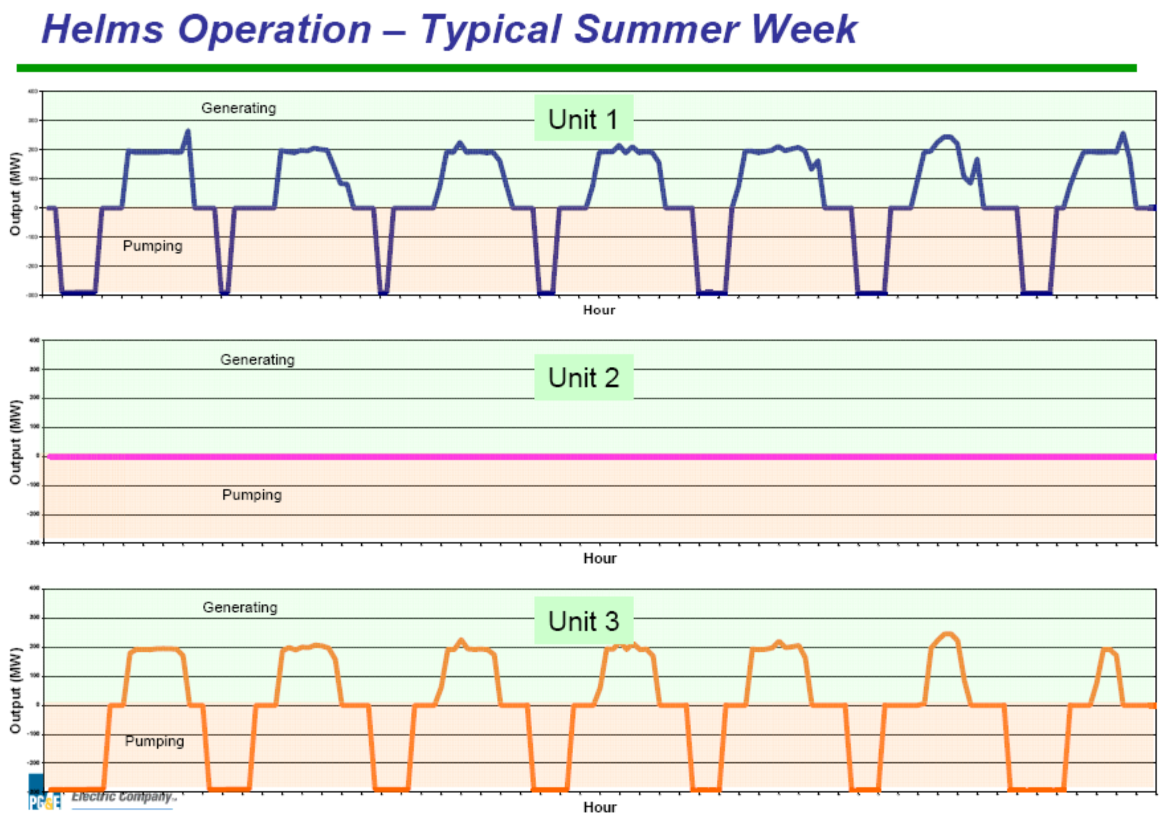


Fig. 8 [5]

In addition to the ability to peak shave and valley fill, Helms is a highly flexible plant with operating flexibility characterized by the following attributes:

- Dead stop to full generation in 8 minutes.
- Dead stop to full pump in 20 minutes.
- Ramp rate of 80 MW/min per unit (about 20% per minute!)

This level of operational flexibility is highly desirable for systems that have high wind penetration levels.

2.0 US reservoir systems

Reservoir systems are typically created where large water systems occur in highly mountainous terrain so that one or a series of cascading lakes, either natural or enlarged with dams, form reservoirs. Each lake has an associated penstock that runs down the mountainside and leads to one or more turbines. Fig. 9 [6] shows the 10 largest hydroelectric facilities in the US.

Facility Name	State	Waterway	Generation (MW)	Owner
Grand Coulee	*WA*	*Columbia*	*6,800 MW*	*USBR*
Chief Joseph	*WA*	*Columbia*	*2,457*	*USACE*
John Day	*WA-OR*	*Columbia*	*2,160*	*USACE*
Hoover	*AZ-NV*	*Colorado*	*2,100*	*USBR*
Bath County PS*	VA	Little Back	2,100	Dominion Power**
Robert Moses	NY	Niagara	1,950	NY Power Authority
The Dalles	*WA-OR*	*Columbia*	*1,807*	*USACE*
Ludington PS*	MI	Lake Michigan	1,872	Consumers/Detroit**
*Raccoon Mtn. PS**	*TN*	*Tennessee*	*1,618*	*TVA*
Glen Canyon	*AZ*	*Colorado*	*1,288*	*USBR*

* PS means "pumped storage." In a pumped storage plant, water is pumped uphill during times of low demand to a reservoir from where the water flows by gravity through turbine-generators during times of peak demand.

** Indicates private power utility. Dominion Power is based in Virginia. Consumers Energy and Detroit Edison in Michigan jointly own the Ludington plant.

Fig. 9: Ten largest hydroelectric facilities in the US

The major US reservoir systems are in the states of Washington, Nevada, California, and Tennessee. The Colombia River system which flows from British Columbia to Washington State to Oregon has 14 reservoir dams ranging from 185 MW to 6809 MW (Grand Coulee Dam) for a total capacity of 24,149 MW (the overall watershed which also includes the Snake River includes more than this). Figure 9a shows a map of the Colombia River System [7].

Fig. 10a [7]

The Colorado River which flows through seven states (Wyoming, Colorado, Utah, New Mexico, Arizona, Nevada, and California) begins in the Rocky Mountains at an altitude of 9019 feet. Figure 10 below shows its course. It is one of the most diverted water systems in the US, with the major use of the river being to irrigate 4 million acres of agricultural land in the US and 500,000 acres in Mexico. In addition, it is the water supply for Los Angeles, Las Vegas, Phoenix, San Diego, Denver, and Salt Lake City. Hardly any water actually reaches the Gulf of California.

The total capacity on the Colorado River is 4166MW. The largest of Colorado River dams is Hoover Dam, which when it was built in 1936 was the largest hydro plant in the world (now it is 36th). Its capacity is 2080MW. The second largest is Glen Canyon Dam at 1288MW. There are five other dams with total capacity of 798MW.

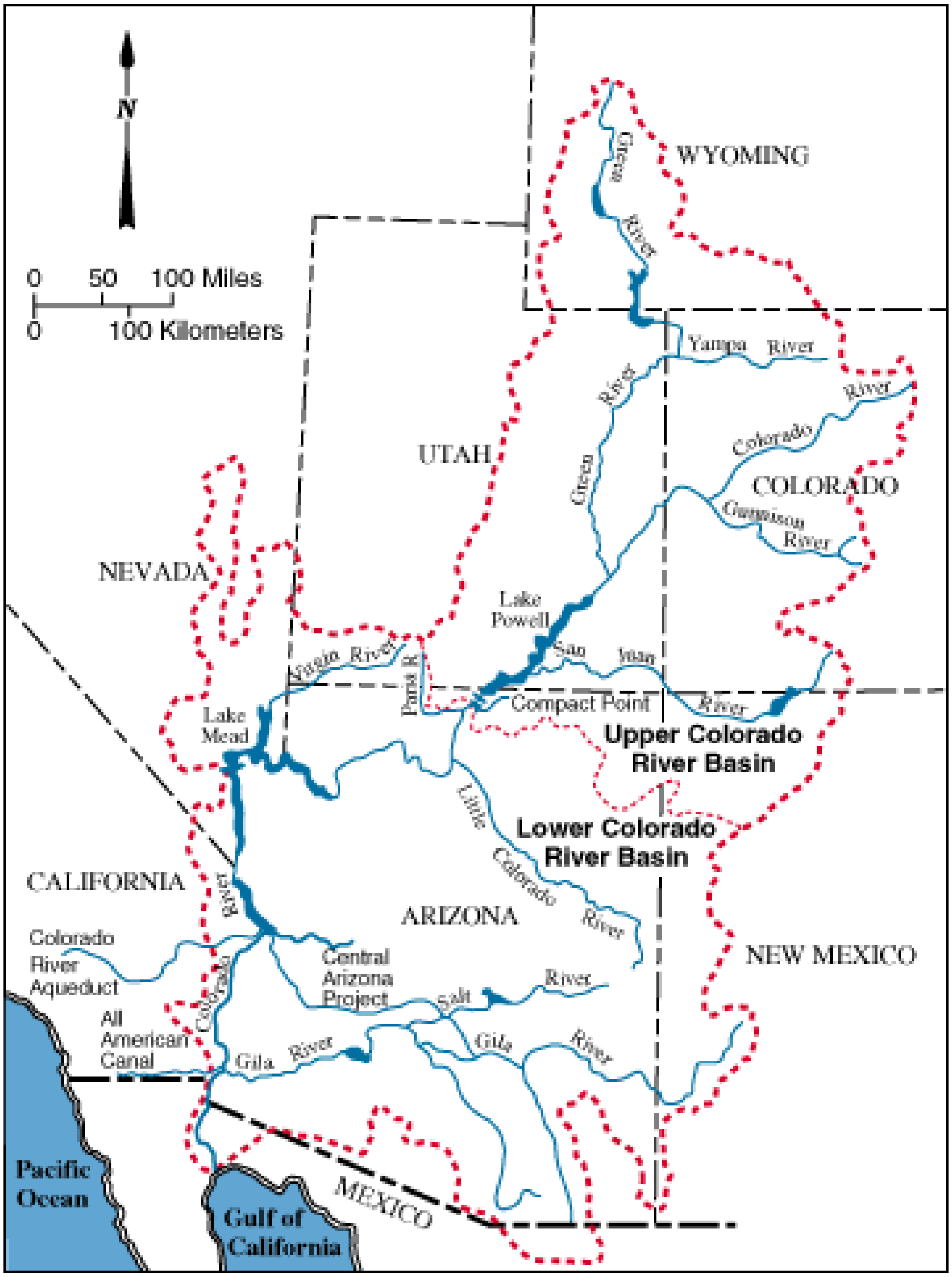


Fig. 10b

The Tennessee Valley Authority (TVA) operates 29 conventional hydroelectric dams throughout the Tennessee River system (4050MW), four dams on the Little Tennessee River (1452MW), and 8 US Army Corps of Engineers dams on the Cumberland River (707MW). These facilities are shown by the red dots in Fig. 11 below (the yellow dots are thermal plants).

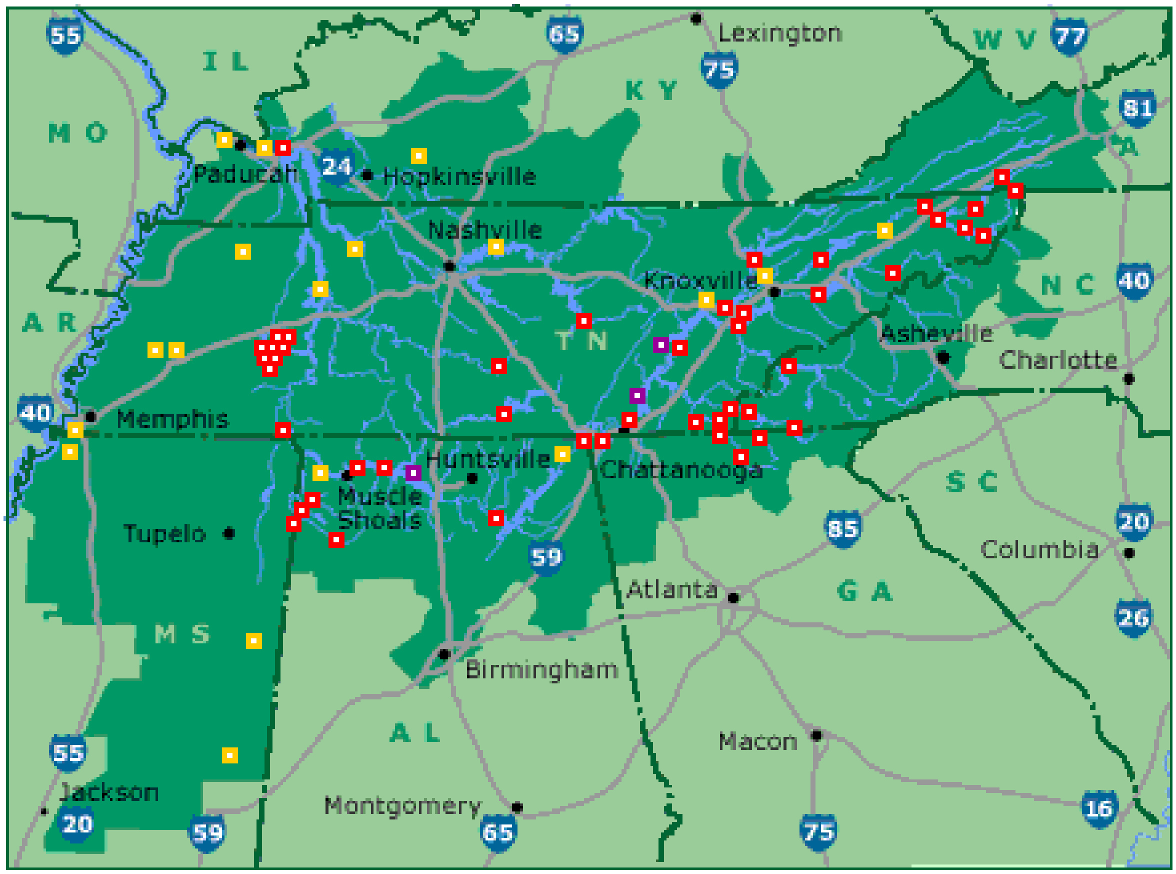
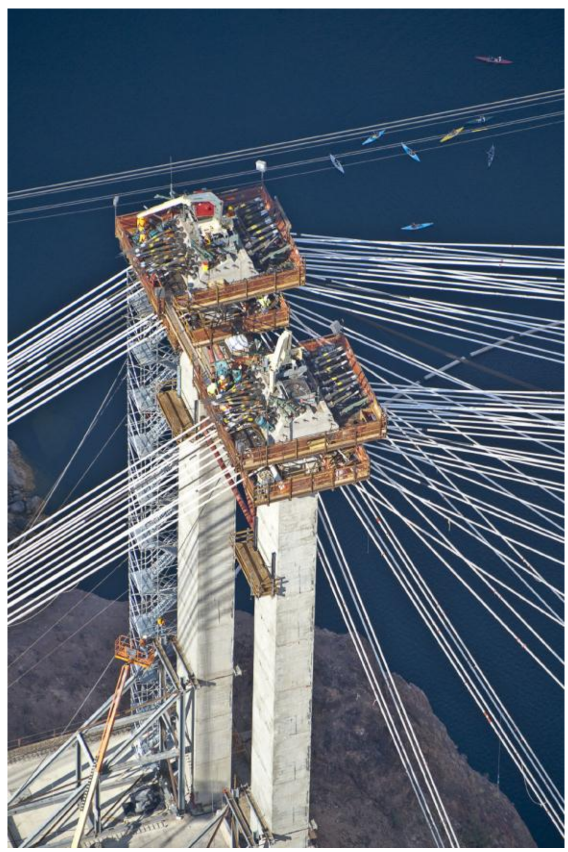
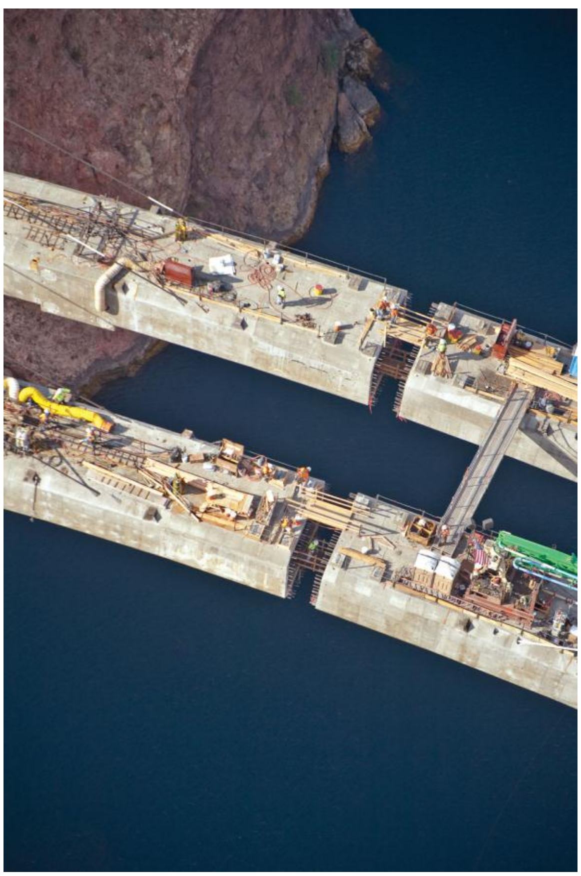
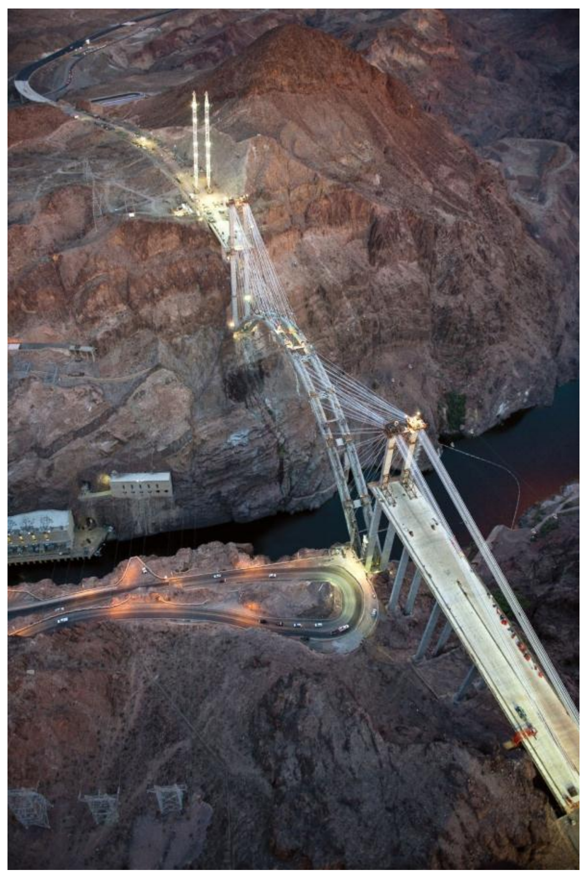


Fig. 11

Hoover Dam has a very interesting history that continues today as a new bridge across it is nearing completion. The following recent photos, although not necessarily pertinent to our class, are worth viewing. This new bridge construction was completed in 2010.







3.0 Basics of reservoir systems

Hydro turbines may be of two types: reaction and impulse. Reaction turbines are acted on by the water which changes pressure as it moves through the turbine and gives up energy. They must be encased or fully submerged in the water flow. Kaplan and Francis type turbines are both reaction types. Kaplan turbines have propellers and are used in very low head (2-40 m) turbines, typically where a flat stream or river is dammed, whereas Francis turbines may be used for applications where the head is up to 350 m. In Kaplan turbines, the water flow is axial, whereas Francis turbines, it is radial.

For impulse turbines, pressure change occurs only in the nozzles; it does not change while flowing across the blades. Impulse turbines change the velocity of a water jet and the resulting change in momentum causes a force on the turbine blades. In an impulse turbine, the water is fired through a narrow nozzle at the turbine blades; the blades are bucket-shaped so they catch the fluid and direct it off at an angle. Each "catch" of a blade is an impulse, and thus the name. Pelton turbines, the most common type of impulse turbine, are used for very high head (up to 1300 meter) facilities.

Different turbine types are illustrated in Fig. 12a, and their application ranges are illustrated in Fig. 12b. Some good explanations of the differences between turbine types may be found at [8].

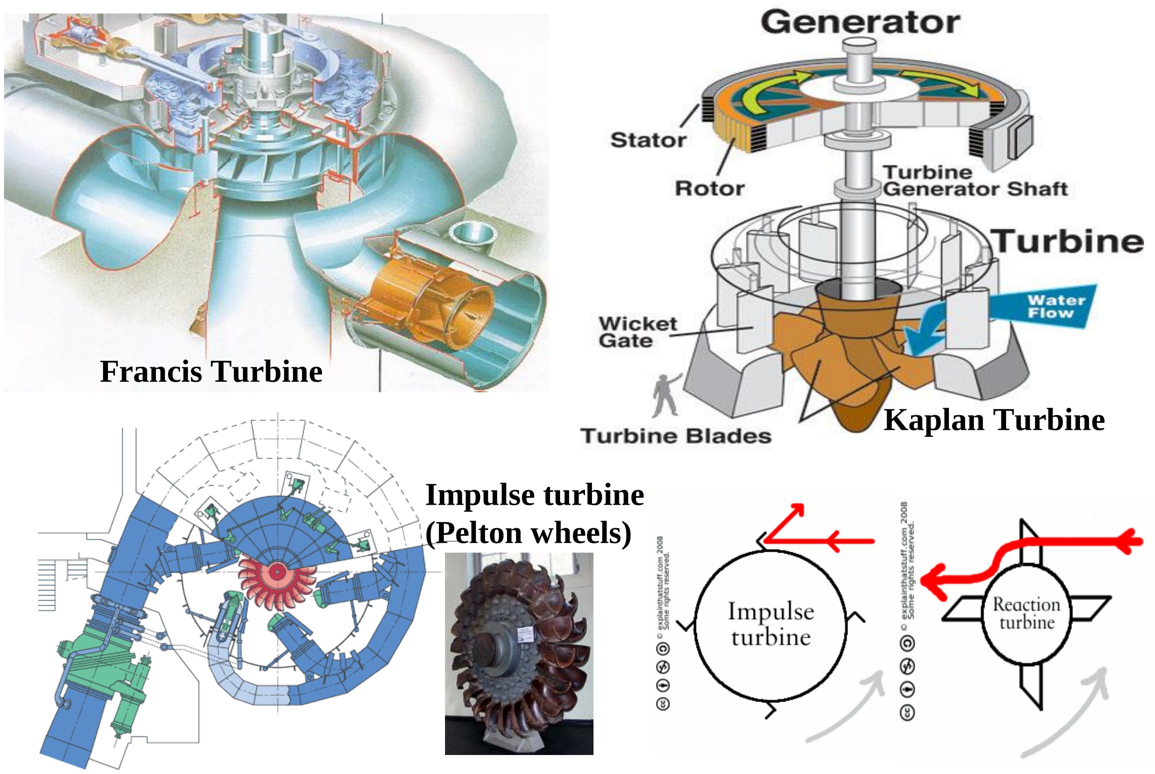


Fig. 12a: Reaction and impulse turbines

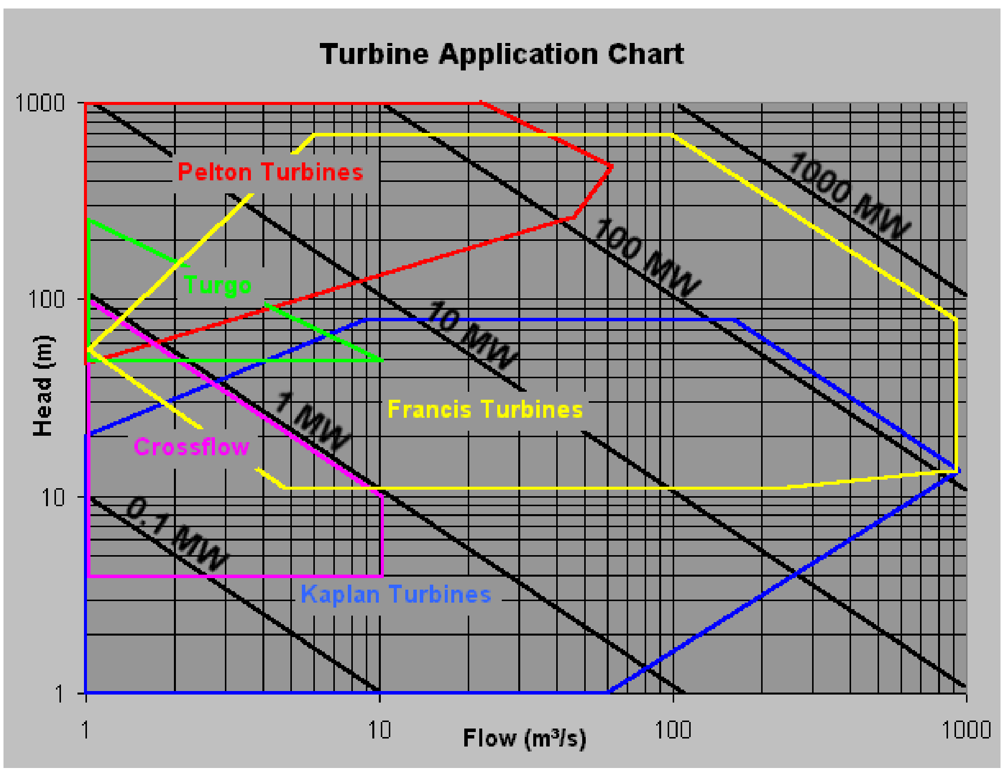
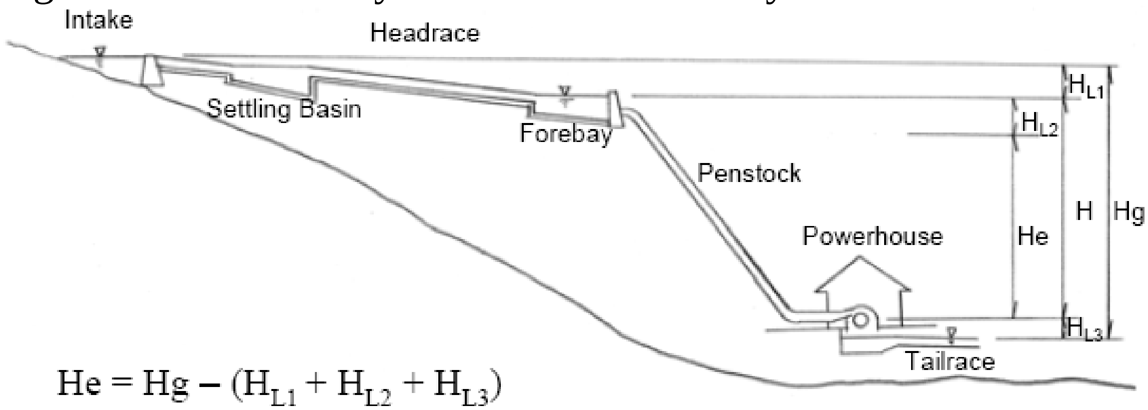


Fig. 12b: Typical application ranges of different hydro turbine types

Figure 13 shows a very basic reservoir facility.



$He = Hg - (H_{L1} + H_{L2} + H_{L3})$

where, He: Effective head

Hg: Gross head

H_{L1}: Loss from intake to forebay

H_{L2}: Loss at penstock

H_{L3}: Installation head and loss at tailrace

Fig. 13

Notice the attention given in Fig. 13 to the effective head. This is a very important quantity for any hydro facility, because it is a significant influence on the power production capacity of the plant.

A central concept in relation to reservoir plant operation is the available power. We can derive this by considering that the potential energy of a mass *m* of water at height *h* is given by

$$E = mgh \tag{1}$$

where *g* is the acceleration due to gravity. Differentiating to get power results in

$$P = \frac{dE}{dt} = \frac{d}{dt}(mgh) \tag{2}$$

Now the only thing in (2) that changes with time as water flows down the penstock is the mass of the water in the reservoir, and so (2) becomes:

$$P = \frac{dm}{dt} gh \tag{3}$$

where *dm/dt* is the mass flow rate, which is given as the product of density ρ (kg/m^3) and volume flow rate r (m^3/sec), that is,

$$P = \rho rgh \tag{4}$$

There are some friction losses in the penstock, and there are losses in the turbine, and so we are not able to convert all of the power of equation (4) to electric energy. This influence is considered via a multiplying factor η so that the electric power available from a turbine having head h is given by

$$P = \eta\rho rgh \tag{5}$$

Constraints cause the operation of reservoirs to be complex or very complex. The different kinds of constraints are listed below.

- Recreation
- Navigation
- Irrigation
- Environmental (fish kill)
- Sudden release
- Downstream water flow limitations

Another reason why hydro operation can be complex is that it must be operated within an overall system which may, and usually does, contain thermal generation. This requires coordination between the thermal plants and the hydro plants. This coordination is done in such a way that the use of hydroelectric power production is maximized within the constraints.

Of most importance to us in this context is the recognition that the reservoir is a limited energy system, not at all unlike our limited energy thermal systems which we have been studying.

A final note on hydro operation complexity. The complexity is minimized when the system is comprised of only a single power plant and a single reservoir. We may refer to this as the uncoupled case, i.e., all reservoir levels are independent of each other. Such a situation is illustrated in Fig. 14.

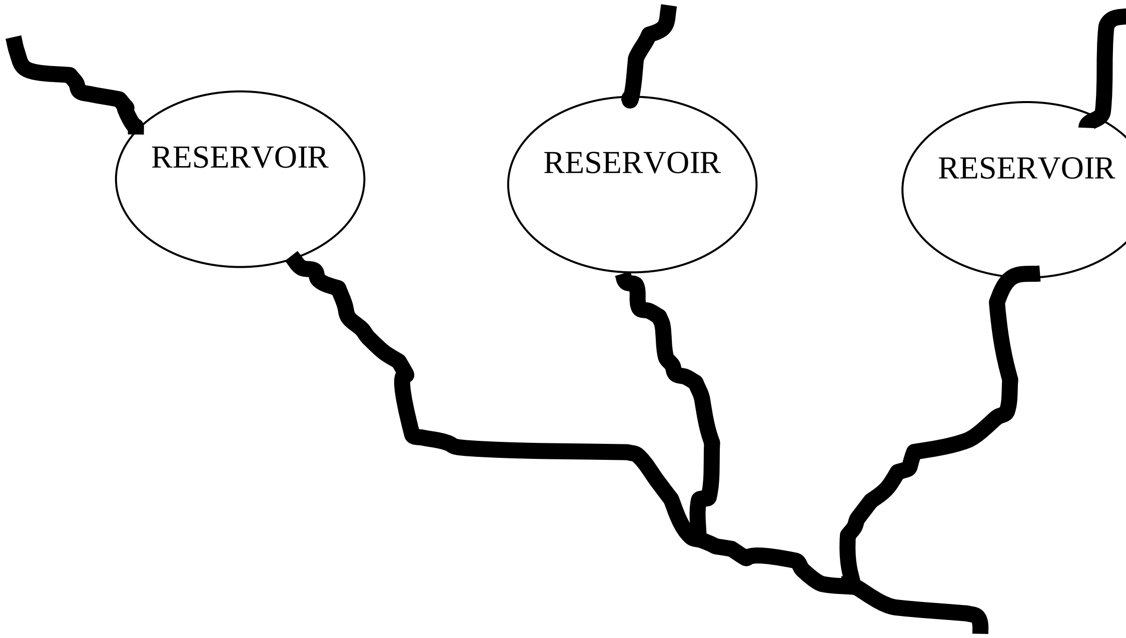


Fig. 14

In contrast, the coupled case can be much more complicated because the flow of one reservoir determines the level of another. Such a situation is illustrated in Fig. 15a.

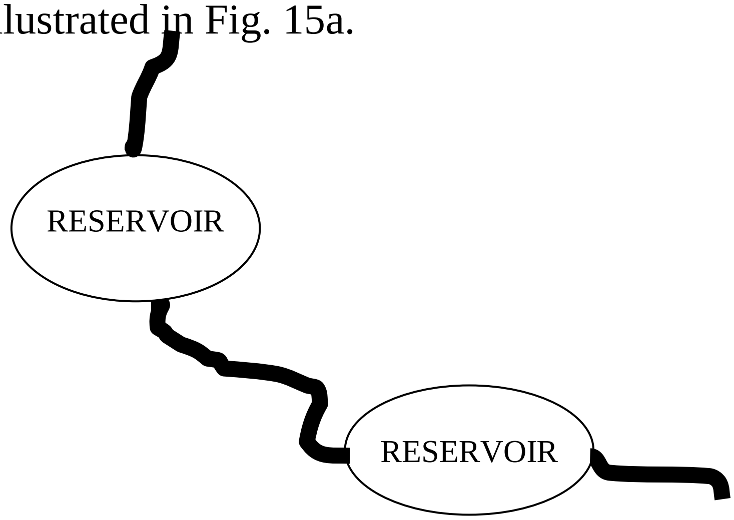


Fig. 15a

4.0 Preliminaries of hydro-thermal scheduling (HTS)

4.1 Taxonomy of problem types

The goal of reservoir plant scheduling depends on the time frame of interest.

1. Long-range (weeks to year): Here we need the load forecast and expected water flow from a rainfall forecast. Then we can predict the energy availability from the hydro facilities and compute the necessary thermal energy.
2. Short-range (day to 1 week): Using results from (1) and more precise load and water information, we can formulate a problem where the solution yields the minimum cost of running the thermal plants on an hour-to-hour basis.

In addition, there are 3 basic types of hydro scheduling problems.

1. All hydro system: Here we want to meet the demand with minimum overall decrease in reservoir water level (stored energy). For example, if you have two decoupled plants A and B, and you run A all the time and never B, then plant B may spill. The better approach is to run them both in such a way so that neither plant spills. There are not many all-hydro systems in the world, Nepal is one. But there are several that are close.
2. Mostly hydro system: New Zealand is such a system with 60% hydro, 32% thermal, and the other 8% from various sources (geothermal, wind, and biomass) [9]. Norway and Sweden comprise another mostly hydro system with 29GW out of 30.5 GW being hydro. One of the best examples of a mostly hydro system is Hydro Quebec. The Hydro Quebec region is shown in Fig. 15b below together with all hydro facilities in the US.

The below scale is has been zoomed out so as to read it. The largest bubble (2000MW & more) corresponds to the largest bubbles on the map.

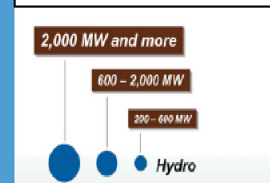


Fig 15b

This system uses hydro for producing over 97% of its energy, as indicated in Fig. 15c below.

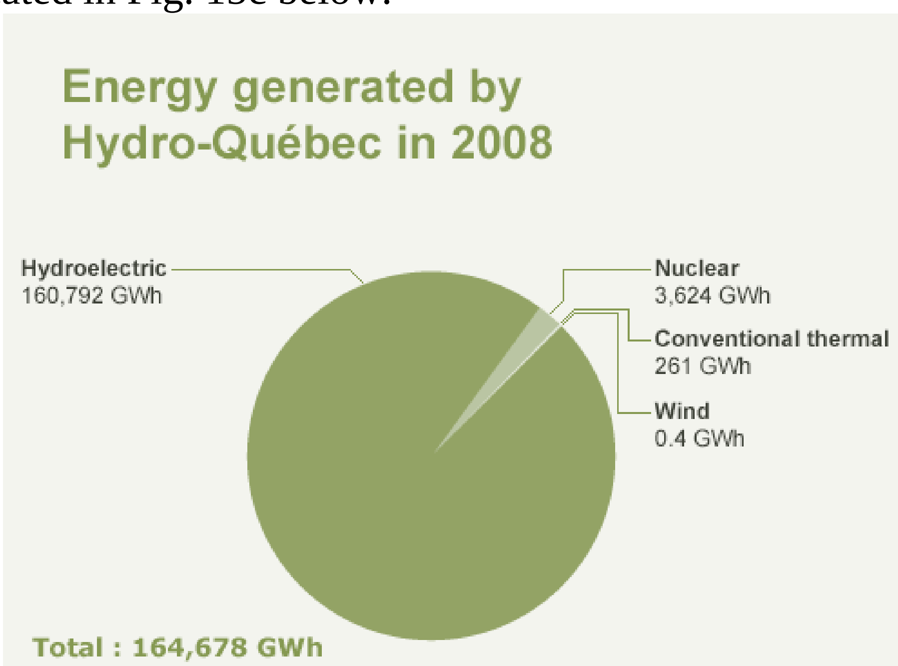


Fig. 15c

3. Mostly thermal system: Most systems with hydro qualify as this type. PG&E in California is an example, as indicated in Fig. 15d.

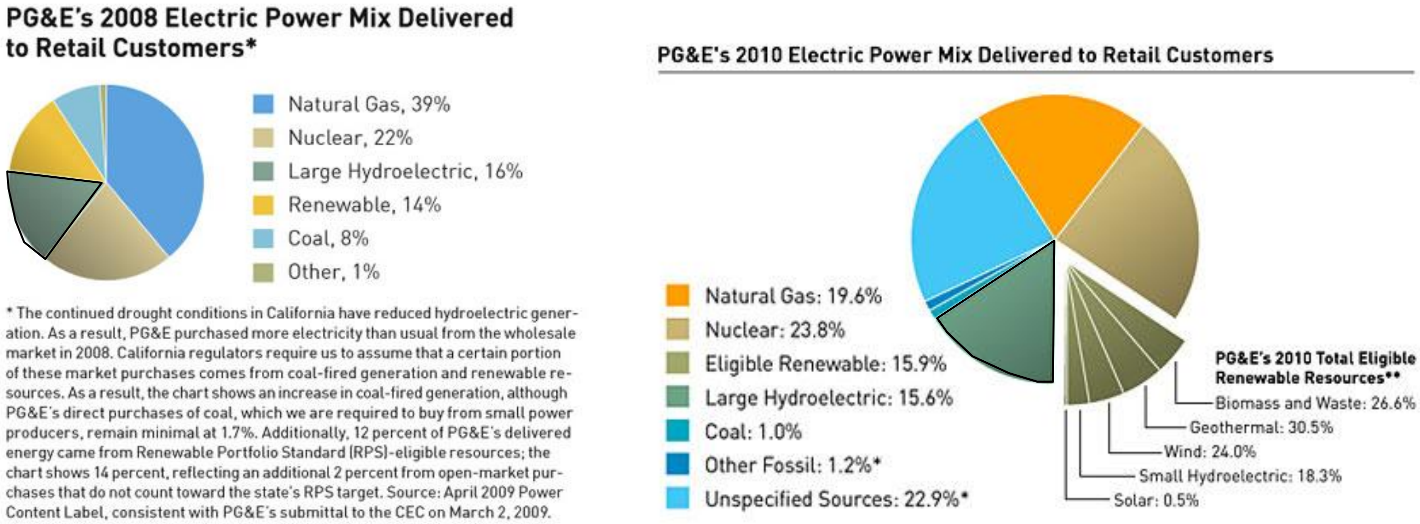


Fig. 15d

Both (2) and (3) represent hydro-thermal coordination problems.

The table below provides a breakdown of several characteristics that we have discussed – a taxonomy of problem types (note that no coupling is intended between the different columns). We will study a problem characterized by the bolded categories in the table below.

Plant characteristics	Time frame	Coupling	Problem types
Run of river	Long range	Coupled	All Hydro
Pumped storage	**Short-range**	**Decoupled**	**Mostly hydro**
Reservoir			**Mostly thermal**

4.2 Terminology

Some basic terminology is illustrated in Fig. 16a and defined below.

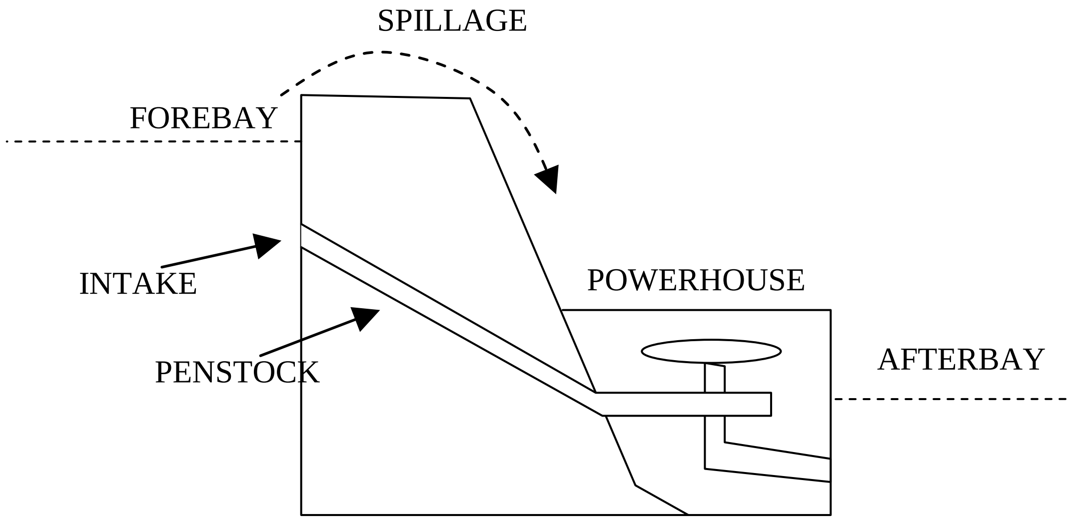


Fig. 16a

- Forebay: A lake or water impoundment (reservoir) before the entrance to the power plant.
- Afterbay: A lake or water impoundment downstream from the power plant that receives the water after it has passed through the turbines.
- Penstock: The pipe leading from the water intake to the turbine.
- Intake: The entrance from the forebay to the penstock.
- Spillage: releasing water over the dam rather than through the penstock. Some dams have spillways, as shown in Fig. 17, which allow smolts (adolescent salmon) to pass without transiting through the turbines.
- Fishladders: stepped runways that allow salmon to migrate upstream, as shown in Fig. 16b (left). Fig 16b (right) also shows the salmon's amazing jumping ability.

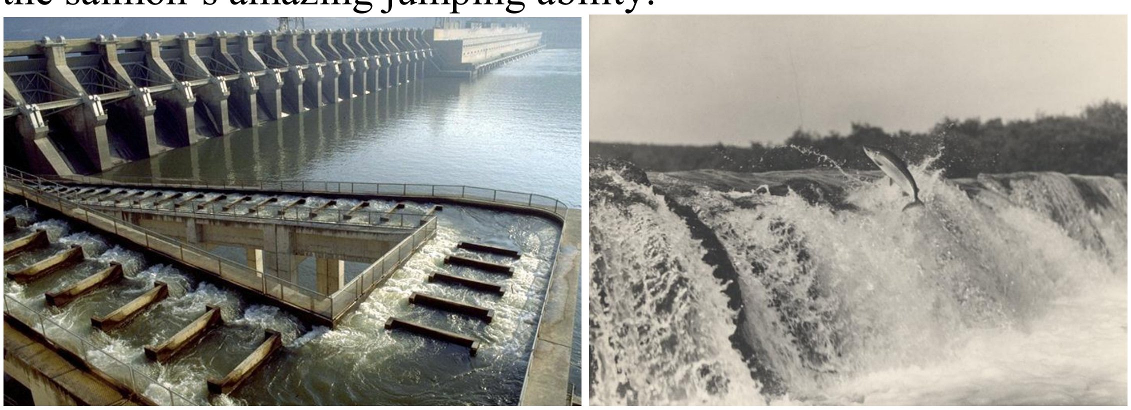


Fig. 16b

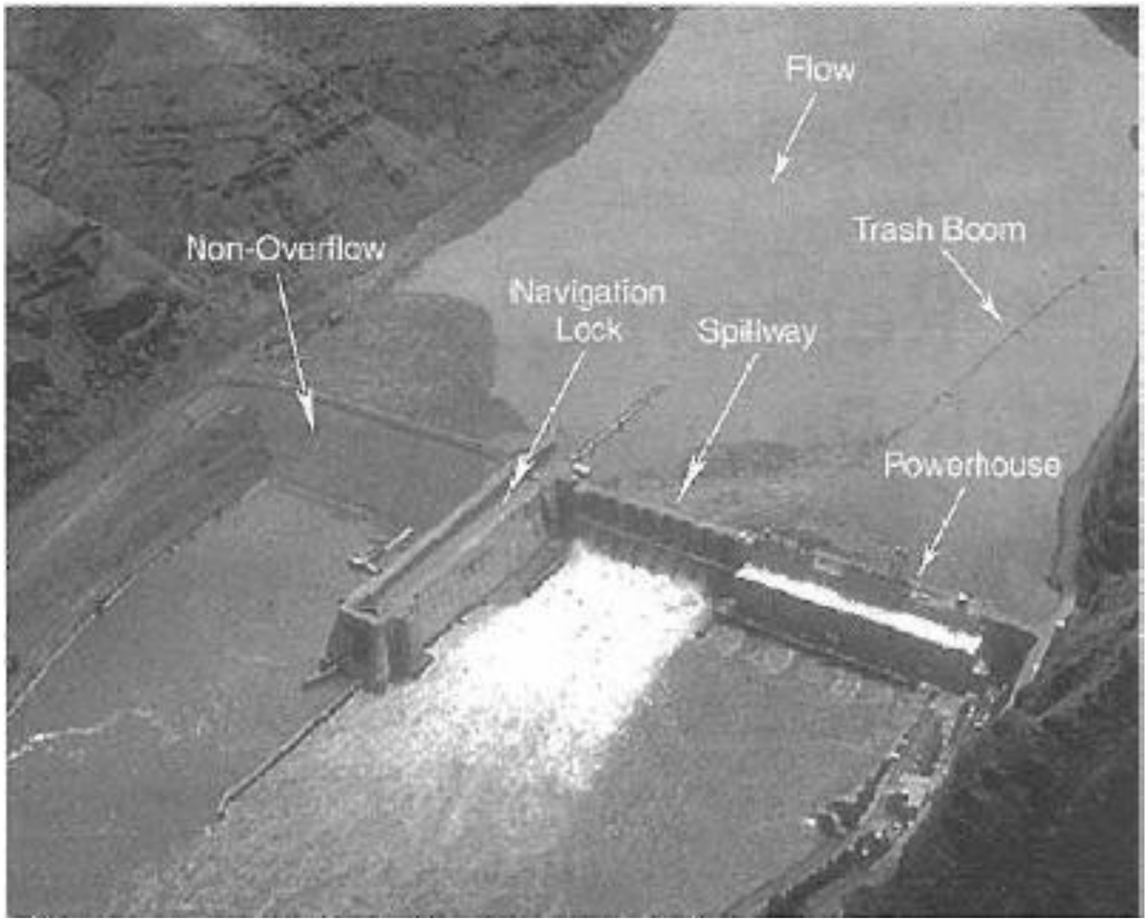


Fig. 17

4.0 The mostly-hydro case (Section 7.3.2 in book)

We assume in our development of this problem that there is one thermal plant and one hydro plant. The one thermal plant is not a restriction since we can always form a composite cost function $F_S(P_S)$ for our thermal plants. The one hydro plant represents a simplification that we will eliminate later.

In the mostly-hydro case, the total rated hydro *power* is assumed to be enough to meet the demand at any one period, but…the total hydro *energy* is not enough to meet the demand over all periods, and as a result, we must use the thermal generation.

We define the following nomenclature:

"*Available water volume for use*:" We can think of E_H as a computed volume of water that will not draw down the reservoir below a certain level, accounting for the upstream inflow.

- E_H: total hydro energy available. It is proportional to the head (gives distance water falls) and the volume of water above the intake (gives mass of water to fall) or designated "*available water volume for use*" in the time period of interest. Recall from (1):

$$E_H = mgh \tag{1}$$

And because $m=V\rho$, where V is water volume and ρ is the water density (about 1000kg/m^3 at normal temperatures), we have that

$$E_H = V\rho gh \tag{6}$$

We desire to use all of E_H in a certain time period T_{MAX}, where

$$T_{MAX} = \sum_{j=1}^{j_{MAX}} n_j \tag{7}$$

Therefore

$$E_H = \sum_{j=1}^{j_{MAX}} P_{Hj} n_j \tag{8}$$

where P_{Hj} is the hydro generation level at time period j.

- E_S: total thermal energy used. Then

$$E_S = \sum_{j=1}^{j_{MAX}} P_{Sj} n_j \tag{9}$$

  where P_{Sj} is the thermal generation level at time period j.

  We will also assume that, in order to avoid start-up and shut-down costs, we will utilize the thermal generation in only one continuous period, that is, we will run the thermal generation for the first N_S periods and then shut it down, so that total thermal run-time is:

$$T_S = \sum_{j=1}^{N_S} n_j \tag{10}$$

At this point, we do not know N_S; therefore we do not know T_S.

With the last assumption and the definition of N_S, we may express (9) as:

$$E_S = \sum_{j=1}^{N_S} P_{Sj} n_j \tag{11}$$

Equations (9) and (11) are equivalent, i.e.,

$$E_S = \sum_{j=1}^{j_{MAX}} P_{Sj} n_j = \sum_{j=1}^{N_S} P_{Sj} n_j$$

because, under the last assumption, the thermal generation levels P_{Sj}, $j=N_S+1,\ldots,j_{MAX}$, will be zero.

- E_L: Total energy used by the load.

$$E_L = \sum_{j=1}^{j_{MAX}} P_{Lj} n_j \tag{12}$$

where P_{Lj} is the load during time interval j.

Observe that E_S is a known quantity since

- we know P_{Lj} for all j which gives us E_L, by (12);
- if we know the head and the water volume, then we know E_H from (6); and

$$E_S = E_L - E_H \tag{13}$$

Our problem is then to minimize the cost of the thermal generation subject to the requirement that we must supply an amount of thermal energy equal to E_S, i.e.,

$$\min F_T = \sum_{j=1}^{N_s} F_S(P_{Sj}) n_j$$

$$\text{subject to:}$$

$$E_S - \sum_{j=1}^{N_s} P_{Sj} n_j = 0 \qquad (14)$$

The Lagrangian is then

$$\mathcal{L}(P_{S1}, \ldots P_{SN_s}, \alpha)$$

$$= \sum_{j=1}^{N_s} F_S(P_{Sj}) n_j + \alpha \left[E_S - \sum_{j=1}^{N_s} P_{Sj} n_j = 0 \right] \qquad (15)$$

where α is the Lagrange multiplier on the energy constraint.

Applying the first-order optimality conditions, we obtain:

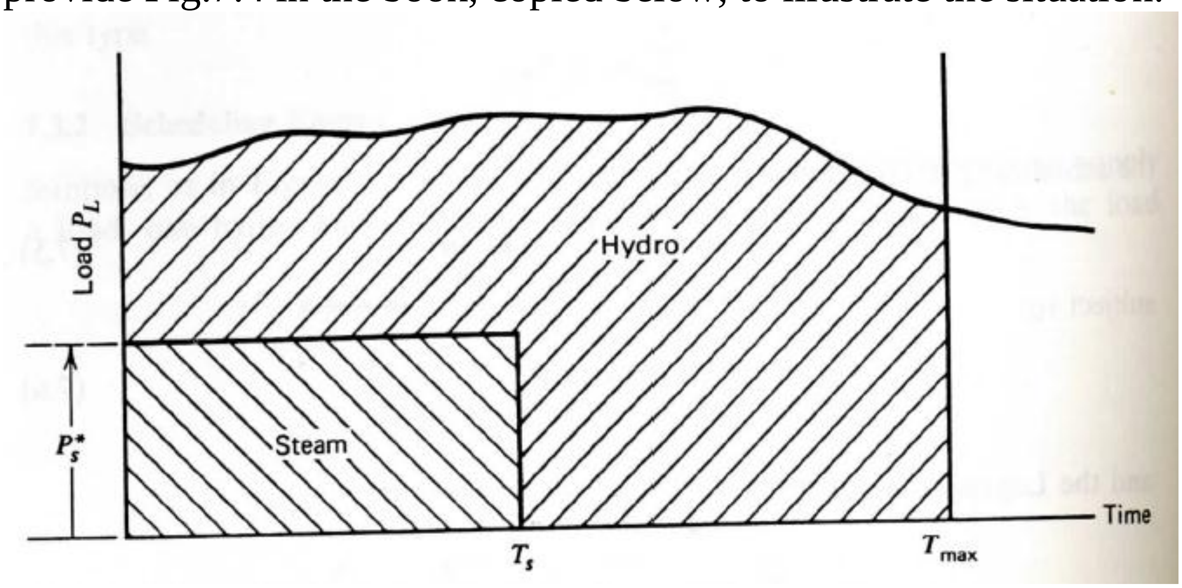
$$\frac{\partial \mathcal{L}}{\partial P_{Sj}} = n_j \frac{\partial F(P_{Sj})}{\partial P_{Sj}} - \alpha n_j = 0, \qquad j = 1, \ldots, N_S \qquad (16)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = E_S - \sum_{j=1}^{N_S} P_{Sj} n_j = 0 \tag{17}$$

From (16), we observe that

$$\alpha = \frac{\partial F(P_{Sj})}{\partial P_{Sj}}, \qquad j = 1,\ldots,N_S \tag{18}$$

Equation (18) implies that the thermal unit must be run at the same generation level for the entire duration it is on-line, T_S. Call this power generation level P_S*. We need to find this value. W&W provide Fig.7.4 in the book, copied below, to illustrate the situation.



To do so, first observe that, with P_{Sj}=P_S* for all j=1,…,N_S, where

$$T_S = \sum_{j=1}^{N_S} n_j \tag{10}$$

then (17) becomes

$$\frac{\partial \mathcal{L}}{\partial \alpha} = E_S - P_S^* \sum_{j=1}^{N_S} n_j = E_S - P_S^* T_S = 0 \tag{19}$$

so that

$$E_S = P_S^* T_S \Rightarrow T_S = \frac{E_S}{P_S^*} \tag{20}$$

Let’s now assume that the thermal plant has a cost-rate function given by

$$F(P_S) = A + BP_S + CP_S^2 \tag{21}$$

Since (18) requires the generation level to be the same throughout the duration T_S, the total cost of running the thermal plant is:

$$F_T = F(P_S^*)T_S = \left[A + BP_S^* + C(P_S^*)^2\right]T_S \tag{22}$$

Now substitute (20) into (22) to obtain

$$F_T = F(P_S^*)T_S = \left[A + BP_S^* + C(P_S^*)^2\right]\frac{E_S}{P_S^*}$$

$$= E_S\left[A\left(P_S^*\right)^{-1} + B + CP_S^*\right] \tag{23}$$

Equation (23) is of value because, unlike (22) which is a function in two unknowns (P_S* and T_S), (23) is a function in only a single unkown, P_S*, and it incorporates the equality constraint (19). So we have transformed an equality-constrained optimization (14) to an unconstrained optimization.

We can solve the unconstrained optimization using basic calculus, which requires that the minimum of F_T to occur when:

$$\frac{\partial F_T}{\partial P_S^*} = E_S\left[-A\left(P_S^*\right)^{-2} + C\right] = 0 \tag{24}$$

Manipulating (24) results in

$$\frac{A}{\left(P_S^*\right)^2} = C \tag{25}$$

Solving for P_S* results in

$$P_S^* = \pm\sqrt{\frac{A}{C}} \tag{26}$$

Clearly we must have a positive value of generation, therefore

$$P_S^* = \sqrt{\frac{A}{C}} \tag{27}$$

And we have solved our problem.
However, we should consider our solution – what does it mean?

To gain some insight into what (27) is telling us, recall that a thermal plant cost-rate function *F* is related to its fuel-rate function *R* (units of MBTU/hr) via:

$$F(P_S) = KR\left(P_S\right) \tag{28}$$

where *K* is the fuel cost in $/MBTU. The fuel-rate function is then

$$R\left(P_S\right) = \frac{F(P_S)}{K} \tag{29}$$

Then the (average, not incremental) heat rate of the plant is given by

$$H\left(P_S\right) = \frac{R(P_S)}{P_S} = \frac{F(P_S)}{KP_S} \tag{30}$$

ASIDE (Potential source of confusion): In W&W, both in Chapter 2 (see pg 8) and in Chapter 7 (see pg 217), they use *H* for *fuel rate* and therefore *H/P* is their heat rate, and they use *f* for fuel cost. In the above equations, and in what follows, I am sticking to the same notation that I used in my Cost-Curve notes earlier in the semester.

Assuming a quadratic form for $F(P_S)$, as we did in (21) above, (30) becomes

$$H(P_S) = \frac{A + BP_S + CP_S^2}{KP_S} \tag{31}$$

or

$$H(P_S) = \frac{1}{K}\left[A(P_S)^{-1} + B + C(P_S)\right] \tag{32}$$

Differentiating, we obtain

$$\frac{dH(P_S)}{dP_S} = \frac{1}{K}\left[-A(P_S)^{-2} + C\right] = 0 \tag{33}$$

Solving (33) for P_S results in

$$A = CP_S^2 \tag{34}$$

which results in

$$P_S = \pm\sqrt{\frac{A}{C}} \tag{35}$$

Again, we require positive value of generation, therefore

$$P_S = +\sqrt{\frac{A}{C}} \tag{36}$$

which is the same result obtained when minimizing cost rate.
Note the second derivative of (32) is

$$\frac{d^2 H(P_S)}{dP_S^2} = \frac{1}{K}\left[2A(P_S)^{-3}\right] \tag{37}$$

and this expression must be positive for positive values of P_S, therefore the extreme point found by (36) must be a minimum, that is, we found the value of generation that minimizes the heat rate function for the thermal plant.

Now you may recall from our Cost-Curve notes that Heat Rate, with units of MBTU/MWHr, is proportional to the inverse of efficiency, that is

$$H(P_S) \propto \frac{1}{\eta} \tag{38}$$

So if our solution (36) minimizes heat rate, it maximizes efficiency. This result provides us with the ability to understand the solution to the hydro-thermal coordination problem that we have posed in this section, for which we arrived at the same solution.

The implication is that by (13), repeated here for convenience,

$$E_S = E_L - E_H \tag{13}$$

we know the energy required by the thermal plant, our optimal solution is to run the thermal plant at its point of maximum efficiency for as long as it takes to produce this energy level E_S.

Now that we have found this solution, it is hardly a surprise that it should be this way, since operating at maximum efficiency is the least-fuel usage way of supplying a fixed amount of energy.

Once we know P_S*, then $T_S=E_S/P_S$*, and then

$$N_S = \left\{ \min k : \quad \sum_{j=1}^{k} n_j \geq T_S \right\}$$

which reads, "N_S equals the minimum value of k such that the sum of the durations from 1 to k is greater than or equal to T_S."

The hydro levels are then found from

PLEASE WORK THROUGH EXAMPLE 7A IN THE TEXT.

$$P_{Hj} = P_{Lj} - P_S^*, \qquad j = 1, \ldots, N_S$$
$$P_{Hj} = P_{Lj}, \qquad j = N_S + 1, \ldots, j_{MAX} \tag{39}$$

5.0 The mostly-thermal case (Section 7.4 in book)

We will again assume one hydro generator and one thermal plant.

Our basic assumption for this problem is that we have a specified amount of water energy we want to use over a given time, but we never have enough hydro power to supply the entire load. Therefore we must use some thermal generation at all times.

Let's first consider the hydro input/output curve.

We have previously discussed (see notes on cost-curves) the input/output (I/O) characteristic for a thermal plant, where we plotted fuel input in MBTU/hr as a function of the power output Pg in MW. The derivative of the I/O characteristic gives the incremental fuel rate, in MBTU/MW-hr[1].

[1] When the I/O curve for a thermal plant is multiplied by the fuel cost in $/MBTU, we obtain the cost rate curve in $/hr vs. MW. The derivative of the cost rate curve gives the incremental cost rate, in $/MW-hr vs. MW.

We can do a similar thing for a hydro plant. In regards to the I/O characteristic, the “fuel input” is given as q_T, in volume of water/hr. In the US, volume of water is often measured in Acre-ft (an Acre is a unit of area equal to 4,046.87m^2 or 43,560 ft^2). Figure 18a illustrates a representation of a hydroelectric plant I/O curve, which shows that the power output is directly proportional to the water flow rate up to the power rating of the plant, beyond which it increases steeply due to increased water friction losses at the higher flows.

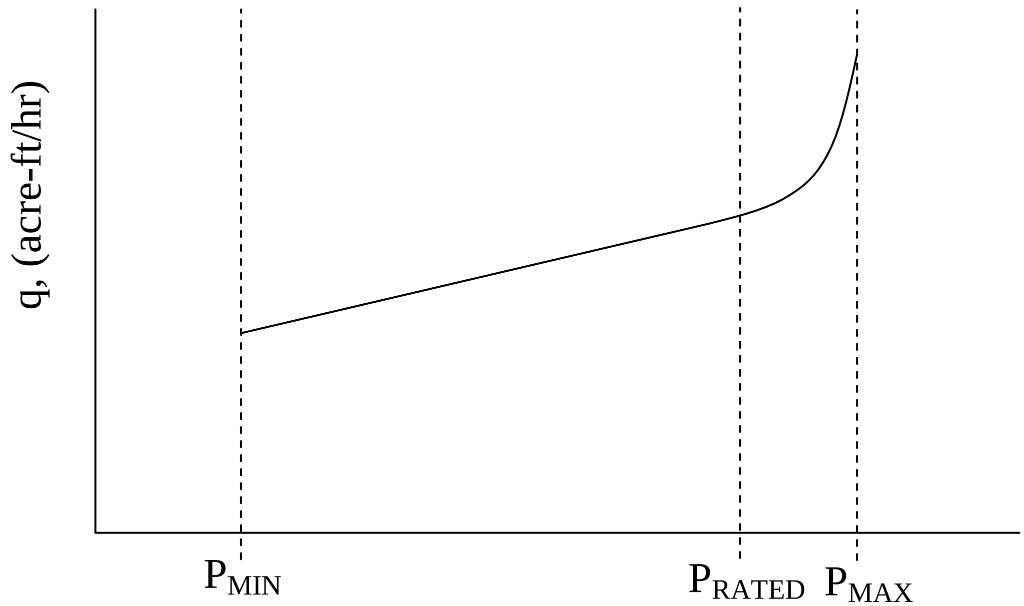


Fig. 18a

Figure 18b plots the incremental flow rate dq/dt as a function of P.

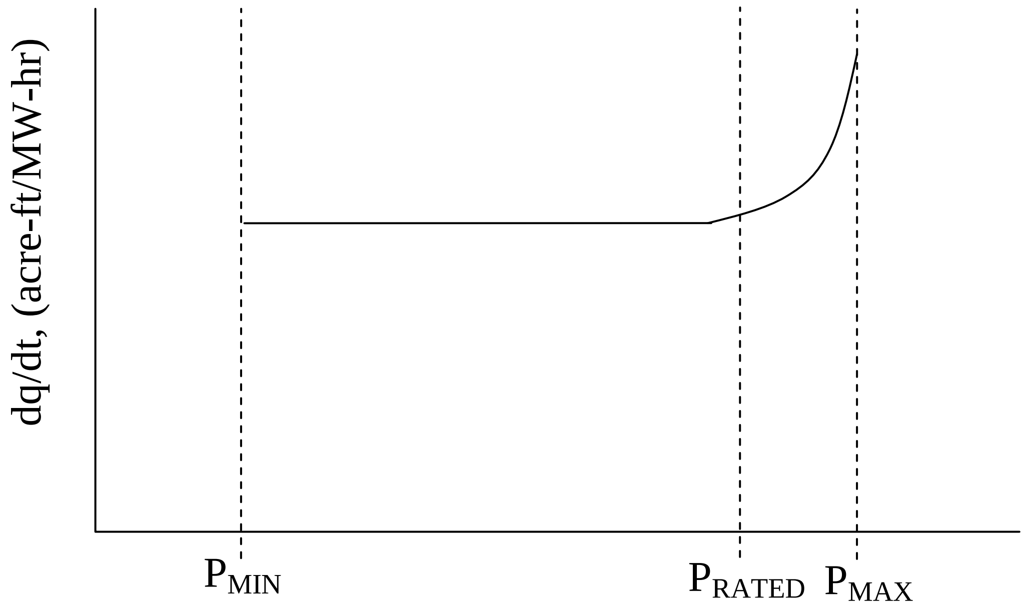


Fig. 18b

We let q_j be the water flow in acre-ft/hr for time interval n_j. Then

$$q_{TOT} = \sum_{j=1}^{j_{MAX}} n_j q_j \tag{40}$$

where q_{TOT} is the total water volume available for the hydro generation in units of acre-feet.

Let's also assume we have an I/O curve given by

$$q = q(P_H) \tag{41}$$

and that it has the following form:

$$q = q(P_H) = a + bP_H \tag{42}$$

We can now form our problem.

$$\min F_T = \sum_{j=1}^{N_S} F_S(P_{Sj}) n_j$$

subject to:

$$\sum_{j=1}^{j_{MAX}} n_j q_j(P_{Hj}) = q_{TOT} \tag{43}$$

$$P_{Lj} - P_{Hj} - P_{Sj} = 0, \qquad j = 1, \ldots, j_{MAX}$$

Recall the fuel scheduling problem (see (5) of FS notes), which was

$$\min \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij})$$

subject to

$$P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} = 0, \qquad j = 1, \ldots, j_{\max}$$

$$\sum_{j=1}^{j_{\max}} n_j q_{Tj} = q_{TOT}$$

This is the exact same problem except in hydro-thermal scheduling (HTS) we use composite thermal plant representation, whereas in the above stated fuel scheduling (FS) problem, we used individual thermal plant representation.

The Lagrangian of our HTS problem is

$$
\mathcal{L}(P_{S1},...,P_{Sj\max},\lambda_1,...,\lambda_{j\max},\gamma)
= \sum_{j=1}^{N_S}\left[F_S(P_{Sj})n_j + \lambda_j\left(P_{Lj} - P_{Hj} - P_{Sj}\right)\right]
+ \gamma\left(\sum_{j=1}^{j_{MAX}} n_j q_j(P_{Hj}) - q_{TOT}\right) \tag{44}
$$

And applying first-order conditions, we find for each interval that

$$
\lambda_k = \gamma\, n_k \frac{\partial q(P_{Hk})}{\partial P_{Hk}} = n_k \frac{\partial F_{Sk}(P_{Sk})}{\partial P_{Sk}} \tag{45}
$$

As in the FS problem, we have

$$
F_{Sk}(P_{Sk}) = A + BP_{Sk} + CP_{Sk}^2 \tag{46}
$$

and upon differentiating, we obtain

$$
\frac{\partial F_{Sk}(P_{Sk})}{\partial P_{Sk}} = B + 2CP_{Sk} \tag{47}
$$

Substitution into (45) results in

$$
\lambda_k = n_k \frac{\partial F_{Sk}(P_{Sk})}{\partial P_{Sk}} = n_k\left(B + 2CP_{Sk}\right) \tag{48}
$$

from which we obtain

$$
P_{Sk} = \frac{\lambda_k - n_k B}{2Cn_k} \tag{49}
$$

Equation (49) is the same as that which we obtained in the FS problem (see (18) of FS notes).

There is a small difference in the equation for q (input/output curve, MBTU/hr for FS or Acre-ft/hr for HTS), however, because in the fuel scheduling problem, the energy constrained unit was thermal, and therefore q was quadratic. Here, we use a linear expression for q, per (42), repeated here for convenience:

$$q = q(P_H) = a + bP_H \tag{42}$$

Differentiating, we obtain (compare to (22) of FS notes)

$$\frac{\partial q}{\partial P_H} = b \tag{50}$$

Substitution into (45) results in

$$\lambda_k = \gamma\, n_k \frac{\partial q(P_{Hk})}{\partial P_{Hk}} = \gamma\, n_k b \tag{51}$$

from which we obtain

$$\gamma = \frac{\lambda_k}{n_k b} \tag{52}$$

Substituting (48) into (52) we obtain

$$\gamma = \frac{n_k \left(B + 2CP_{Sk}\right)}{n_k b} = \frac{B + 2CP_{Sk}}{b} \tag{53}$$

Solving (53) for P_{SK} results in

$$P_{Sk} = \frac{\gamma\, b - B}{2C} \tag{54}$$

Equation (54) indicates that the thermal generation should be constant across all time periods k=1,…,j_{max}, The only thing we need to choose is γ. How do we choose γ?

W&W, pg. 220, states, “This is solved using the same techniques as in Chapter 6.” This is true, but in this case, those techniques can be considerably simplified because the “fuel” (water) is, in this case, a linear function of the corresponding generation P_H per (42). Because of this, the incremental flow rate of hydro generation is a constant, independent of the hydro generation level P_H, as indicated in (50). This means that the optimality condition of (45) does not directly impose any requirement on P_H (as it does P_S); i.e., P_H does not appear in (45) since the differentiation with respect to P_H is a constant. This differs from the fuel-constrained problem where we found that the optimality condition did impose a requirement on P_T. See equations (23) and (24) of the FS notes, which are repeated below:

$$\frac{\partial \mathcal{L}}{\partial P_{Tk}} = -\lambda_k + \gamma\, n_k \left(b_T + 2c_T P_{Tk}\right) = 0$$

$$P_{Tk} = \frac{\lambda_k - \gamma\, n_k b_T}{2c_T \gamma\, n_k}$$

We will describe the simplified procedure in what follows. However, you should be clear that the simplification arises not because of some inherent difference between the hydro scheduling problem and the FS problem but rather because we here represent the fuel I/O curve as linear instead of quadratic.

As in the fuel scheduling problem, γ may be thought of as a fuel (water in this case) price, if we raise it, the supply of the fuel will decrease; if we lower it, the supply of fuel will increase.

One can see this directly by substituting (54) into the power balance equation

$$P_{Lj} - P_{Hj} - P_{Sj} = 0, \qquad j = 1,\dots, j_{MAX}$$

which results in

$$P_{Lj} - P_{Hj} - \frac{\gamma\, b - B}{2C} = 0, \qquad j = 1, \ldots, j_{MAX}$$

Solving for P_{Hj} gives

$$P_{Hj} = P_{Lj} - \frac{\gamma\, b - B}{2C}, \qquad j = 1, \ldots, j_{MAX} \tag{55}$$

Substituting (55) into (42) results in

$$q_j = q(P_{Hj}) = a + b\left(P_{Lj} - \frac{\gamma\, b - B}{2C}\right) \tag{56}$$

Now it must be the case that the total water usage is q_{TOT} according to (40), repeated here for convenience:

$$q_{TOT} = \sum_{j=1}^{j_{MAX}} n_j q_j \tag{40}$$

Therefore (56) and (40) offer a simple way to solve the problem:

1. Guess γ
2. Use (56) to compute q_j for all $j=1,\ldots,j_{max}$
3. Compute

$$Q = \sum_{j=1}^{j_{MAX}} n_j q_j$$

4. If $|\Delta q_{TOT}| = |q_{TOT} - Q| < \varepsilon$, stop.
5. Else, if $\Delta q_{TOT} > 0$ ($q_{TOT} > Q$), decrease γ. If $\Delta q_{TOT} < 0$ ($q_{TOT} < Q$), increase γ.
6. Go to 2.

Step 5 requires an expression for how much to increase/decrease γ. This can be found in a similar way as we did for the FS problem, by substituting (56) into (40) and then differentiating $\partial q_{TOT}/\partial\gamma$, approximating as $\Delta q_{TOT}/\Delta\gamma$, and solving for $\Delta\gamma$. The result is

$$\Delta\gamma = \frac{-\Delta q_{TOT}}{\sum_{j=1}^{j_{\max}} \frac{n_j b^2}{2c}} = \frac{-\Delta q_{TOT}}{\frac{b^2 T_{\max}}{2c}}$$

Observe in the update equation that positive Δq_{TOT} (we have not used enough water) results in a price decrease; negative Δq_{TOT} (we have used too much water) results in a price increase.

where $T_{MAX} = \sum_{j=1}^{j_{MAX}} n_j$

In comparison to the FS solution approach (see Fig. 2 of FS notes), this problem is simpler because it eliminates the need to iterate to get P_S and P_H, which in the FS problem is required by the optimality conditions of (45).

Mostly-thermal case with losses (pp 220-222)

W&W also consider representing losses in the problem of the mostly-thermal case. In contrast to (43), the lossless problem

$$
\begin{aligned}
&\min F_T = \sum_{j=1}^{N_S} F_S(P_{Sj}) n_j \\
&\text{subject to:} \\
&\qquad \sum_{j=1}^{j_{MAX}} n_j q_j(P_{Hj}) = q_{TOT} \\
&\qquad P_{Lj} - P_{Hj} - P_{Sj} = 0, \qquad j = 1, \ldots, j_{MAX}
\end{aligned}
\tag{43}
$$

our problem now becomes

$$
\begin{aligned}
&\min F_T = \sum_{j=1}^{N_S} F_S(P_{Sj}) n_j \\
&\text{subject to:} \\
&\qquad \sum_{j=1}^{j_{MAX}} n_j q_j(P_{Hj}) = q_{TOT} \\
&\qquad P_{Lj} + P_{loss,j} - P_{Hj} - P_{Sj} = 0, \qquad j = 1, \ldots, j_{MAX}
\end{aligned}
\tag{57}
$$

The Lagrangian becomes

$$\mathcal{L}(P_{S1},\ldots,P_{Sj\max},\lambda_1,\ldots,\lambda_{j\max},\gamma)$$
$$= \sum_{j=1}^{N_S}\left[F_S(P_{Sj})n_j + \lambda_j\left(P_{Lj} + P_{loss,j} - P_{Hj} - P_{Sj}\right)\right]$$
$$+\gamma\left(\sum_{j=1}^{j_{MAX}} n_j q_j(P_{Hj}) - q_{TOT}\right) \tag{58}$$

And applying first-order conditions, we find for each interval k that

$$\frac{\partial \mathcal{L}}{\partial P_{Sk}} = n_k \frac{\partial F_{Sk}(P_{Sk})}{\partial P_{Sk}} + \lambda_k\left(\frac{\partial P_{loss,k}}{\partial P_{Sk}} - 1\right) = 0 \tag{59a}$$

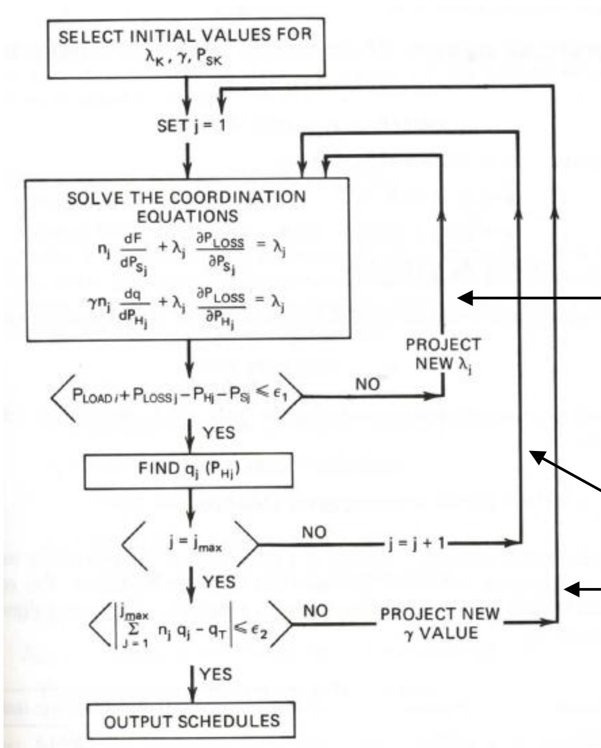
$$\frac{\partial \mathcal{L}}{\partial P_{Hk}} = \lambda_k\left(\frac{\partial P_{loss,k}}{\partial P_{Hk}} - 1\right) + \gamma\, n_k \frac{\partial q_k(P_{Hk})}{\partial P_{Hk}} = 0 \tag{59b}$$

Solving the above for λ_k and rearranging slightly results in

$$n_k \frac{\partial F_{Sk}(P_{Sk})}{\partial P_{Sk}} + \lambda_k \frac{\partial P_{loss,k}}{\partial P_{Sk}} = \lambda_k \tag{60a}$$

$$\gamma\, n_k \frac{\partial q_k(P_{Hk})}{\partial P_{Hk}} + \lambda_k \frac{\partial P_{loss,k}}{\partial P_{Hk}} = \lambda_k \tag{60b}$$

which are identical to (7.28) and (7.29) in the text, which W&W calls the coordination equations. In this case, the loss function $P_{loss,k}$ in (60b) will normally depend on P_{Hk}, and so (60b) will contain P_{Hk}. Therefore we will need to use lambda-iteration to solve (60a) and (60b). Figure 7.7 in W&W provides a general algorithm for solving the overall HTS problem with losses, copied below.



Note that there are three iterative loops:

- First (inner) loop: Lambda iteration solves the coordination equations for a given time interval. Actually, there is another loop here which would, after computing lambda (and corresponding generation levels) for a given value of losses, would recompute losses and iterate again.
- Second loop: Step through all of the time intervals $j=1,\ldots,j_{max}$.
- Third (outer) loop: Gamma adjustment to use the desired amount of water.

REVIEW EXAMPLE 7B IN TEXT.

We now consider a series of increasingly complex HTS problems for the mostly thermal case. We utilize a gradient approach for the solution procedures.

Problem 1: This problem is the same as what we have addressed above (mostly thermal case without losses), but we will utilize slightly different notation, and we will be more explicit about our assumptions, that is:

- Ignore inflow
- Starting volume=V_0
- Ending volume=V_{jmax}
- Time intervals duration n_j, $j=1,\ldots,j_{max}$.

The presence of the starting and ending volume specifies a total water volume usage which we will here denote as $Q_{TOT}=V_0-V_{jmax}$.

Solution of this problem using the gradient search approach is given below:

Step 1: Obtain starting solution as

$$q_k = q_{avg} = \frac{Q_{TOT}}{\sum_{j=1}^{j_{MAX}} n_j} = \frac{V_0 - V_{j\max}}{\sum_{j=1}^{j_{MAX}} n_j} \tag{61}$$

Step 2: Obtain the hydro generation level from q_{Hk} for each time interval k=1,…,j_{max}, using (this just comes from the linear water rate I/O equation (42)):

$$P_{Hk} = \frac{q_k - a}{b} \tag{62}$$

Also obtain the thermal generation levels from:

$$P_{Sk} = P_{Lk} - P_{Hk} \tag{63}$$

Step 3: Compute γ_k for all intervals k=1,…, j_{max} using:

$$\gamma_k = \frac{B + 2CP_{Sk}}{b} \tag{64}$$

which is the same as (53). If all γ_k are within ε of each other, then the algorithm stops.

Step 4: Identify maximum and minimum γ_k (denoted q_{j+} and q_{j-}) from the computations made in Step 3, and then compute the change in the total water released according to:

$$\Delta Q = \left[\gamma_{j+} - \gamma_{j-}\right]\Delta Q_0 \tag{65}$$

where ΔQ_0 is chosen to be relatively small. Note that this relation was used in the fuel scheduling notes as well. Justification was given there as below:

> The flow chart step "$\Delta q=(\gamma_{j+}-\gamma_{j-})\Delta q_0$ WHERE Δq_0 IS A CHOSEN SMALL STEP" is not dimensionally correct as it stands, because gamma has units of $/RE, and when multiplied by RE, gives $, consistent with the above discussion regarding (43). You can assume,

however, that the relation is really $\Delta q=[(\gamma_{j+}-\gamma_{j-})/1][\Delta q_0]$, where the "1" has the same units as γ. Then we observe that if Δq_0 has units of RE, then so will Δq. Basically, this relation is just telling us that if we want to correct two intervals j- and j+ for their fuel (or water) usage, we should choose an amount of water to shift that is proportional to the difference between the two interval's gamma values.

Step 5: Correct the water flow for intervals j+ and j- according to

$$q_{j+} = q_{j+} + \frac{\Delta Q}{n_{j+}} \tag{66}$$

$$q_{j-} = q_{j-} - \frac{\Delta Q}{n_{j-}} \tag{67}$$

Once this is done, we return to step 2.

Problem 2: This is the same as Problem 1, except we will assume a rate of inflow to the reservoir of r_k during interval k.

The only change in our previous solution strategy is in Step 1, where we now need to include the inflow into our starting solution according to

$$q_k = q_{avg} = \frac{V_0 - V_{j\max}}{\sum_{j=1}^{j_{MAX}} n_j} + r_k \tag{68}$$

This makes sense because an inflow is simply going to increase the available outflow by that same amount.

Once this change is made, the procedure may proceed as indicated in Problem 1.

<u>Problem 3</u>: Now assume that we have problem 2 except we will account for limitations on the reservoir level of

$$V_{\min} \le V_k \le V_{\max} \tag{69}$$

We must not violate this constraint in any time period.

To address this, we first must identify V_k, the water volume in period k. This will be given by

$$V_k = V_{k-1} + [\text{Water In}]_k - [\text{Water Out}]_k \tag{70}$$

But the "water in" is the rate of inflow r_k times the duration n_k, and the water out is the water flow rate q_k times the duration n_k, so that (70) becomes:

$$\begin{aligned} V_k &= V_{k-1} + r_k n_k - q_k n_k \\ &= V_{k-1} + (r_k - q_k) n_k \end{aligned} \tag{71}$$

Equation (71) is referred to as the *hydraulic continuity equation*.

Solving (71) for q_k results in

$$q_k = r_k + \frac{V_{k-1} - V_k}{n_k} \tag{72}$$

Now recall that we have constraints on V_k as specified in (69). Given a certain water level in the last period V_{k-1},

- Letting V_k go to $V_{\min}$ will result in the largest possible water flow rate q_k during interval k.
- Letting V_k go to $V_{\max}$ will result in the least possible water flow rate q_k during interval k.

Therefore we may write that

$$q_{k\min} \le q_k \le q_{k\max} \tag{72}$$

where

$$q_{k\max} = r_k + \frac{V_{k-1} - V_{\min}}{n_k}$$

$$q_{k\,\min} = r_k + \frac{V_{k-1} - V_{\max}}{n_k}$$

so that (72) becomes

$$r_k + \frac{V_{k-1} - V_{\max}}{n_k} \le q_k \le r_k + \frac{V_{k-1} - V_{\min}}{n_k} \tag{73}$$

Note that, in addition to (72) (or (73)), there are also constraints on water flow rate q_k imposed the physical capabilities of the facilities (water intake and penstock), i.e.,

$$q_{\min} \le q_k \le q_{\max} \tag{74}$$

Now what should be changed in our algorithm?

Recall Step 5 of our algorithm when we compute new values of q_{j+} and q_{j-}. When considering constraints on water volume, we need to check whether a new value of q_{j+} or q_{j-} is in violation. If so, then the water flow rate should be set to its limit. And so step 5 should read as follows:

==

Step 5: Correct the water flow for intervals j+ and j- according to

$$q_{j+} = q_{j+} + \frac{\Delta Q}{n_{j+}} \tag{66}$$

$$q_{j-} = q_{j-} - \frac{\Delta Q}{n_{j-}} \tag{67}$$

If $q_{j+} > \min(q_{max}, q_{kmax})$, then $q_{j+} = \min(q_{max}, q_{kmax})$.
If $q_{j-} < \max(q_{min}, q_{kmin})$, then $q_{j-} = \max(q_{min}, q_{kmin})$.

Once this is done, we return to step 2.

==

The other change that is necessary is due to the fact that hitting a constraint decouples the time intervals so that γ on one side of the binding constraint will differ from γ on the other side of the binding constraint.

You can think about what this means to our algorithm in two different ways.

1. It makes no sense to try and draw the gamma values closer together if they are chosen from separate sides of a binding constraint, since there is no reason why they should be the same. Therefore we need to choose the gamma values to draw closer from the same side of the constraint.
2. Augmenting water release from two sides of a binding constraint will necessarily result in violation of the constraint, since the fact that the constraint is binding means it wants more release in the direction of the violation. Augmenting water release from only one side of the binding constraint will necessarily result in no change to the flow rate at its limit, since the net change in water release is zero, and since both changes occur either before or after the binding constraint, it is not possible that the bound flow rate should change.

Therefore the other change that should be made is in Step 4, where we define the term "coherent" to include all time intervals for which the gamma values should be the same, i.e., there should be no time interval included within the chosen range for which a constraint is binding.

Step 4 should now read as follows.

==

Step 4: For a given coherent set of time intervals, identify maximum and minimum γ_k (denoted q_{j+} and q_{j-}) from the computations made in Step 3, and then compute the change in the total water released according to:

$$\Delta Q = \left[\gamma_{j+} - \gamma_{j-}\right]\Delta Q_0 \tag{65}$$

where ΔQ_0 is chosen to be relatively small.

==

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