HW #5 SOLUTIONS

1. (a) $F_1 = 180 + 6.72P_1 + .002P_1^2$

$F_2 = 743.5 + 6.426P_2 + .00826P_2^2$

$F_3 = 360 + 6.75P_3 + .00225P_3^2$

$$\frac{\partial F_1}{\partial P_{g_1}} = 6.72 + .004P_1 = \lambda$$

$$\frac{\partial F_2}{\partial P_{g_2}} = 6.426 + .01652P_2 = \lambda$$

$$\frac{\partial F_3}{\partial P_{g_3}} = 6.75 + .0045P_3 = \lambda$$

$$P_{g_1} + P_{g_2} + P_{g_3} = 450$$

$$\begin{bmatrix} .004 & 0 & 0 & -1 \\ 0 & .01652 & 0 & -1 \\ 0 & 0 & .0045 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{g_1} \\ P_{g_2} \\ P_{g_3} \\ \lambda \end{bmatrix} = \begin{bmatrix} -6.72 \\ -6.426 \\ -6.75 \\ 450 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P_{g_1} \\ P_{g_2} \\ P_{g_3} \\ \lambda \end{bmatrix} = \begin{bmatrix} 205.94 \\ 67.66 \\ 176.39 \\ 7.54 \end{bmatrix}$$

Note all values are within limits.

(b) $P_{g_1} = \dfrac{\lambda - 6.72}{.004}$

$P_{g_2} = \dfrac{\lambda - 6.426}{.01652}$ $\Rightarrow \dfrac{\partial P_T}{\partial \lambda} = \dfrac{1}{.004} + \dfrac{1}{.01652} + \dfrac{1}{.0045} = 532.7$

$P_{g_3} = \dfrac{\lambda - 6.75}{.0045}$ $\Rightarrow \Delta\lambda = \dfrac{\Delta P_T}{532.75}$

Try $\lambda = 10$

$$P_{g_1} = \frac{10 - 6.72}{.004} = 820$$

$$P_{g_2} = \frac{10 - 6.426}{.01652} = 216.34$$

$$P_{g_3} = \frac{10 - 6.75}{.0045} = 722.22$$

$1,758.6 \Rightarrow \Delta P_T = 900 - 1758.6 = -858$

$$\Rightarrow \Delta\lambda = \frac{-858.6}{532.75} = -1.61$$

$\lambda = 10 - 1.61 = 8.39$

$$P_{g1} = \frac{8.39 - 6.72}{.004} = 417.5$$

$$P_{g2} = \frac{8.39 - 6.426}{.01652} = 118.9$$

$$P_{g3} = \frac{8.39 - 6.75}{.0045} = \underline{364.4}$$

$900.8 \Rightarrow \Delta P_T = -.8$ OK!

But note! P_{g1} is above its limit by 117.5 MW. Therefore $P_{g1} = 300$ MW and remove it from iterative procedure.
This means we need to meet a loading condition of $900 - 300 = 600$ MW with the remaining two generators.

So we now have

$$P_{g2} = \frac{\lambda - 6.426}{.01652} \qquad \Rightarrow \frac{\partial P_T}{\partial \lambda} = \frac{1}{.01652} + \frac{1}{.0045} = 282.75$$

$$P_{g3} = \frac{\lambda - 6.75}{.0045}$$

Note from previous two iterations that

$\lambda = 10 \Rightarrow P_{g2} + P_{g3} = 938.56$

$\lambda = 8.39 \Rightarrow P_{g2} + P_{g3} = 483.3$

So lets try $\lambda = 9$

$$P_{g2} = \frac{9 - 6.426}{.01652} = 155.81$$

$$P_{g3} = \frac{9 - 6.75}{.0045} = \underline{500.0}$$

$655.81 \Rightarrow \Delta P_T = 600 - 655.81 = -55.8$

$$\Rightarrow \Delta \lambda = \frac{-55.81}{282.75} = -.197$$

$\Rightarrow \lambda = 9 - .197 = 8.80$

$$P_{g2} = \frac{8.8 - 6.426}{.01652} = 143.7$$

$$P_{g3} = \frac{8.8 - 6.75}{.0045} = \underline{455.6}$$

$599.26 \Rightarrow \Delta P_T = .744$ OK!

So final answer is

$$\boxed{\begin{aligned} P_{g1} &= 300 \text{ MW} \\ P_{g2} &= 143.7 \text{ MW} \\ P_{g3} &= 455.6 \text{ MW} \\ \lambda &= 8.8 \ \$/\text{MWhr} \end{aligned}}$$

(Note that the IC of gen 1 isn't meaningful since it is at its upper limit and cannot be increased further!)

Problem 4.3

Given the following loss formula:

$$B_{ij} = \begin{bmatrix} 1.36255\times10^{-4} & 1.753\times10^{-5} & 1.8394\times10^{-4} \\ 1.753\times10^{-5} & 1.5448\times10^{-4} & 2.82765\times10^{-4} \\ 1.8394\times10^{-4} & 2.82765\times10^{-4} & 1.6147\times10^{-3} \end{bmatrix}$$

B_{io} and B_{oo} are neglected

a) Scheduling without losses:

$$F_1 = H_1 \times \text{Fuel Cost} = 328.125 \times 8.663\ P_1 \times 0.0053\ P_1^2$$

$$\frac{dF_1}{dP_1} = 8.663 + 0.0105\ P_1$$

$$F_2 = H_2 \times \text{Fuel Cost} = 136.913 + 10.04\ P_2 + 0.0061\ P_2^2$$

$$\frac{dF_2}{dP_2} = 10.04 + 0.0122\ P_2$$

$$F_3 = H_3 \times \text{Fuel Cost} = 59.155 + 9.761\ P_3 + 0.0059\ P_3^2$$

$$\frac{dF_3}{dP_3} = 9.761 + 0.0118\ P_3$$

Coordination equation:

$$\lambda = \frac{dF_1}{dP_1} = 8.663 + 0.0105\ P_1$$

$$\lambda = \frac{dF_2}{dP_2} = 10.04 + 0.0122\ P_2$$

$$\lambda = \frac{dF_3}{dP_3} = 9.761 + 0.0118\ P_3$$

$$P_1 + P_2 + P_3 = 190\ \text{MW}$$

From the solution of those equations:

$P_1 = 143.99$ MW

$P_2 = 11.01$ MW

$P_3 = 35.03$ MW

Total Cost:

$F_t + F_1 + F_2 + F_3$

$F_1 = 328.125 + 8.663(143.99) + 0.0053(143.99)^2$

$F_1 = 1685.4$ R/h

$F_2 = 136.913 + 10.04(11.01) + 0.0061(11.01)^2$

$F_2 = 248.19$ R/h

$F_3 = 59.155 + 9.761(35.03) + 0.0059(35.03)^2$

$F_3 = 408.32$ R/h

$F_t = 2340.81$ R/h

Calculate system losses: (Using this dispatch and the loss formula).

$P_{loss} = P^{GT} B_{ii} P^G$

$$P^G = \begin{bmatrix} 143.99 \\ 11.01 \\ 35.03 \end{bmatrix} \qquad P^{GT} = [143.99 \quad 11.01 \quad 35.03]$$

$$P_{loss} = [143.99 \quad 11.01 \quad 35.03] \begin{bmatrix} 1.36255\times10^{-4} & 1.753\times10^{-5} & 1.8394\times10^{-4} \\ 1.753\times10^{-5} & 1.5448\times10^{-4} & 2.82765\times10^{-4} \\ 1.8394\times10^{-4} & 2.82765\times10^{-4} & 1.6147\times10^{-3} \end{bmatrix} \begin{bmatrix} 143.99 \\ 11.01 \\ 35.03 \end{bmatrix}$$

$P_{loss} = 6.9538$ MW

b) Note: Don't tell students to iterate this more than twice unless they are using a computer. If the solution is started from the solution to the first problem, one gets the following:

Start: ~~$P_1 = 143.99$ MW~~
~~$P_2 = 11.01$ MW~~
~~$P_3 = 35.03$ MW~~

This should start at ED solution w/o losses. You get:

$P_1 = 114.5$
$P_2 = -14.4 \Rightarrow P_2 = 5$
$P_3 = 8.8 \qquad P_3 = 15$
$\lambda = 9.59 \qquad \Rightarrow P_1 = 109 - 20 = 89$

Problem 4.3, continued

$$\frac{dP_{loss}}{dP_1} = .05251$$

$$\frac{dP_{loss}}{dP_2} = .02821$$

$$\frac{dP_{loss}}{dP_3} = .17232$$

Losses = 6.4538 MW

Total cost = 2340.81 R/h

After 1st Iteration: (see flowchart in Figure 4.17)

$P_1 = 89.$ MW

$P_2 = 5.$ MW

$P_3 = 15.$ MW

$$\frac{dP_{loss}}{dP_1} = .02995$$

$$\frac{dP_{loss}}{dP_2} = .01312$$

$$\frac{dP_{loss}}{dP_3} = .08401$$

Losses = 1.9954 MW

Total Cost = 1534.83 R/h

The second iteration produces no changes to this dispatch and therefore the logic in the flowchart in Figure 4.17 will terminate the calculation.