HW 5, EE 553, Dr. McCalley, Fall 2012, Due Monday Oct. 22, 2012

For each of the problems 1-3,

a. Formulate the Lagrangian function.
b. State the necessary optimality conditions.
c. For problems 1 and 2, draw a contour in the x_1-x_2 plane using two different values for $f(\underline{x})$; also draw in the boundaries represented by the constraints.
d. Find the optimal solution using the equations from the optimality conditions. You should report all values of the vector $\underline{x}$, the vector λ (the Lagrange multipliers for the equality constraints, which is scalar for these problems), and all values of the vector μ (the Lagrange multipliers for the inequality constraints).

1. Equality constrained optimization:

$$\min \quad f(\underline{x}) = x_1^2 + 4x_2^2$$

subject to

$$x_1 + x_2 = 4$$

2. Inequality-constrained optimization

$$\min \quad f(\underline{x}) = x_1^2 + x_2^2$$

subject to

$$x_1 + x_2 \geq 4, \qquad x_1 \leq 3, \qquad x_2 \leq 5,$$

3. Equality and inequality-constrained optimization

$$\min \quad f(\underline{x}) = x_1^2 + 3x_2^2 + 4x_3^2$$

subject to

$$x_1 + x_2 + x_3 = 5, \qquad x_1 \leq 3, \qquad x_2 \leq 2, \qquad x_2 + x_3 \leq 5$$

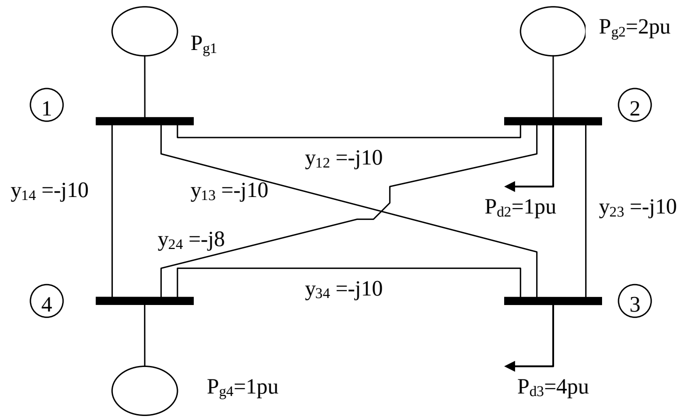
4. Assume that all three of the thermal units described below are connected and producing MW.

R, Fuel rate (MBTU/hr)	**P_{min}(MW)**	**P_{max} (MW)**	**K, Fuel cost ($/MBTU)**
R_1=225+8.4P_1+0.0025P_1^2	30	300	0.80
R_2=729+6.3P_2+0.0081P_2^2	45	350	1.02
R_1=400+7.5P_1+0.0025P_3^2	70	550	0.90

   a. Find a closed form (non-iterative) solution for the optimal dispatch given the load is 450MW.
   b. Use Lambda-iteration to find the optimal dispatch given that the load is 900 MW.

5. Work problem 4.3 in your W&W text. You can work part (a) using either closed form (non-iterative) or you may use Lambda-iteration. However, part (b) requires that you use the method depicted by the flow chart on page 118 of your text. Start the solution from an economic dispatch without losses.
6. For the system below:
   a. Obtain the Y-bus.
   b. Obtain the B' matrix.
   c. Set up and solve the DC power flow relation $\underline{P}=\underline{B}'\underline{\theta}$. Give all four angles.
   d. Obtain the $\underline{D}$ matrix and the node-arc incidence matrix $\underline{A}$.

e. Compute all line flows. Compare the line flows to those for the solution given in the class notes, and comment on the effect of the added line in terms of loading in other lines.



7. Using the same system you analyzed in problem 6, set up the optimal power flow as a linear program. Assume the objective function is exactly the same as used in the example in class, i.e., $Z = 1307P_{g1} + 1211P_{g2} + 1254P_{g4}$. Also, assume each unit has a lower limit of 100 MW and an upper limit of 300 MW, which will be (in per unit): $1 \le P_{g1} \le 3$, $1 \le P_{g2} \le 3$, $1 \le P_{g3} \le 3$.
   a. Write down the optimization problem you must solve. Assume infinite transmission capacity.
   b. Provide your CPLEX code used to solve the above optimization problem.
   c. Use CPLEX to solve the LP stated in part b. In answering the below questions, make sure you specify the units.
      i. Provide the value of the objective function at the optimal solution.
      ii. Provide the values of the decision variables at the optimal solution.
      iii. Provide the values of the auxiliary variables (angles and line flows) at the optimal solution. Make sure you specify line flows as $P_{bk}=P_{ij}$ where the flow direction is defined positive from bus i to bus j.
      iv. Identify the locational marginal prices (LMPs) at each bus.
      v. How much will the objective function increase if the load at bus 2 changes from 1.0 pu to 1.01 pu?
      vi. How much will the objective function increase if the load at bus 3 changes from 4.0 pu to 4.01 pu?
      vii. How much will the objective function increase if the lower generation limit for P_{g1} is increased to 101 MW?
   d. Constrain the flow limit on $P_{b3}=P_{23}$ to 1.4 pu and resolve using CPLEX.
      i. Provide the value of the objective function at the optimal solution.
      ii. Provide the values of the decision variables at the optimal solution. Typing "display solution variables - " provides
      iii. Provide the values of the auxiliary variables (angles and line flows) at the optimal solution. Make sure you specify line flows as $P_{bk}=P_{ij}$ where the flow direction is defined positive from bus i to bus j.
      iv. Identify the locational marginal prices (LMPs) at each bus.
      v. How much will the objective function increase if load at bus 2 changes from 1.0pu to 1.01 pu?
      vi. How much will the objective function increase if load at bus 3 changes from 4.0 pu to 4.01pu?
      vii. How much will the objective function increase if the transmission limit on $P_{b3}=P_{23}$ is increased by 1 MW from 1.4 to 1.41 pu?