Problem 9.1

Given a single area with three generating units

![Diagram of three generating units connected to a load with Load Base: 1000 MVA]

<table>
<thead>
<tr>
<th>Unit</th>
<th>Rating</th>
<th>Speed Droop R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100 MVA</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>500 MVA</td>
<td>0.015</td>
</tr>
<tr>
<td>3</td>
<td>500 MVA</td>
<td>0.015</td>
</tr>
</tbody>
</table>

The units are initially loaded:

\[ P_1 = 80 \text{ MW} \]
\[ P_2 = 300 \text{ MW} \]
\[ P_3 = 400 \text{ MW} \]

a) Assume \( D = 0 \). What is the new generation on each unit for a 50 MW load increase?

\[ \Delta \omega = \frac{-\Delta P}{\sum_{i=1}^{3} \left( \frac{1}{R_i} + D \right)} \]

Under the common base of 1000 MVA

\[ R_1 = 0.01 \left( \frac{1000}{100} \right) = 0.1 \text{ pu} \]
\[ R_2 = 0.015 \left( \frac{1000}{500} \right) = 0.03 \text{ pu} \]
\[ R_3 = 0.015 \left( \frac{1000}{500} \right) = 0.03 \text{ pu} \]
\[ D = 0 \]
\[ \Delta P = \frac{50}{1000} = 0.05 \text{ pu} \]
\[ \Delta \omega = \frac{0.05}{0.1 + 0.03 + 0.03} = -652.17 \times 10^{-6} \text{ pu} \]

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Problem 9.1, continued

\[ \omega = \omega_0 + \Delta \omega \]

\[ \omega = 60 - 652.17 \times 10^{-6} \text{ (60) } = 59.96 \text{ Hz} \]

**Change in unit generation:**

\[ \Delta P_1 = -\frac{\Delta \omega}{R_1} = 0.00652 \text{ pu } = 6.52 \text{ MW} \]

\[ \Delta P_2 = -\frac{\Delta \omega}{R_1} = 21.739 \times 10^{-3} \text{ pu } = 21.74 \text{ MW} \]

\[ \Delta P_3 = -\frac{\Delta \omega}{R_1} = 21.739 \times 10^{-3} \text{ pu } = 21.74 \text{ MW} \]

**New Generation:**

\[ P_1 = 80 + 6.52 = 86.52 \text{ MW} \]

\[ P_2 = 300 + 21.74 = 321.74 \text{ MW} \]

\[ P_3 = 400 + 21.74 = 421.74 \text{ MW} \]

b) Repeat for \( D = 1 \text{ pu} \) (on load base)

\[ \Delta \omega = -\frac{0.05}{0.1 + \frac{1}{0.03} + \frac{1}{0.03} + 1} = -643.78 \times 10^{-6} \]

\[ \omega = \omega_0 + \Delta \omega \]

\[ = 60 - 643.78 \times 10^{-6} \text{ (60) } = 59.9614 \text{ Hz} \]

**Changes in units generation:**

\[ \Delta P_1 = -\frac{\Delta \omega}{R_1} = 6.4378 \times 10^{-3} = 6.44 \text{ MW} \]

\[ \Delta P_2 = -\frac{\Delta \omega}{R_2} = 21.46 \times 10^{-3} = 21.46 \text{ MW} \]

\[ \Delta P_3 = -\frac{\Delta \omega}{R_3} = 21.46 \times 10^{-3} = 21.46 \text{ MW} \]

\[ D \Delta \omega = \frac{.64 \text{ MW}}{50 \text{ MW}} \]

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Problem 9.1, continued

New Generation

\[ P_1 = 80 + 6.44 = 86.44 \text{ MW} \]
\[ P_2 = 300 + 21.46 = 321.46 \text{ MW} \]
\[ P_3 = 400 + 21.46 = 421.46 \text{ MW} \]
Problem 9.2

Using the values of R and D in each area, for example 9B, resolve for the 100 MW load change in area 1 under the following conditions:

Area 1 Base MVA = 2000 MVA
Area 2 Base MVA = 500 MVA

Then solve for a load change of 100 MW occurring in area 2 with R's and D's as in Example 9B and Base MVA for each area as above.

Area 1 Area 2
Base 2000 MVA Base 500 MVA
R = 0.01 pu R = 0.02 pu
D = 0.8 pu D = 1.0 pu

Using 2000 MVA as common base:

\[ R_1 = 0.01 \quad R_2 = 0.02 \times \frac{2000}{500} = 0.08 \text{ pu} \]

\[ D = 0.8 \quad D_2 = 1.0 \times \frac{500}{2000} = 0.25 \text{ pu} \]

\[ \Delta P_L = \frac{100}{2000} = 0.05 \text{ pu} \]

\[ \Delta \omega = \frac{\frac{\Delta P_L}{2000} + \frac{\Delta P_{tie}}{0.08} + 0.8 + 0.25}{0.01} = -440.33 \times 10^{-6} \]

\[ f = 60 - 440.33 \times 10^{-6} \times 60 = 59.974 \text{ Hz} \]

\[ \Delta P_{tie} = \Delta \omega \left( \frac{1}{R_2} + D_2 \right) = 440.33 \times 10^{-6} \left( \frac{1}{0.08} + 0.25 \right) = -5.614 \times 10^{-3} \]

\[ \Delta P_{tie} = -11.2285 \text{ MW} \]

\[ \Delta P_{mech \ 1} = \frac{\Delta \omega}{R_1} = 44.033 \times 10^{-3} \text{ pu} = 88.067 \text{ MW} \]

\[ \Delta P_{mech \ 2} = \frac{\Delta \omega}{R_2} = 5.504 \times 10^{-3} \text{ pu} = 11.008 \text{ MW} \]

Load

\[ -D_1 \Delta \omega = .7045 \text{ MW} \]

\[ -D_2 \Delta \omega = .2202 \text{ MW} \]

\[ \Delta P_L = \Delta P_{mech \ 1} + \Delta P_{mech \ 2} + \left( D_1 + D_2 \right) \Delta \omega \]

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Problem 9.2, continued

b) Solve for a load change of 100 MW occurring in area 2 for the same MVA base as the first part:

\[ \Delta \omega = 440.335 \times 10^{-6} \text{ pu} \]
\[ f = 59.9734 \text{ Hz} \]

\[ \Delta P_{\text{tie}} = \Delta \omega \left( \frac{1}{R_1} + D_1 \right) = 440.335 \times 10^{-6} \left( \frac{1}{0.01} + 0.8 \right) = 0.04439 \text{ pu} \]

\[ \Delta P_{\text{tie}} = 88.78 \text{ MW} \]

All the other values of \( \Delta P_{\text{mech 1}}, \Delta P_{\text{mech 2}}, D_1 \Delta \omega \) and \( D_2 \Delta \omega \) are the same as the first part of this problem.
Problem 9.3

\[ \Delta \omega_1 = \left( \frac{1}{m_1 S + D_1} \right) (- \Delta P_L - \Delta P_{tie}) \]

\[ \Delta \omega_2 = \left( \frac{1}{m_2 S + D_2} \right) (\Delta P_{tie}) \]

\[ \Delta P_{tie} = (T/S) (\Delta \omega_1 - \Delta \omega_2) \]

after much algebra:

\[ \Delta \omega_1 = \frac{(m_2^2 S^2 + D_2 S + T) (- \Delta P_L)}{(m_1 m_2 S^3 + (m_1 D_2 + m_2 D_1) S + (m_1 T + D_1 D_2 + m_2 T)) S + (D_1 + D_2) T} \]

An interesting result may be seen if we divide by \( T \):

\[ \Delta \omega_1 = \frac{m_2^2 S^2 + D_2 S}{ \left( \frac{m_1 m_2 S^3}{T} + \frac{(m_1 D_2 + m_2 D_1) S^2}{T} + \frac{(m_1 + m_2 + \frac{D_1 D_2}{T}) S}{T} + (P_1 + P_2) \right)} (- \Delta P_L) \]

If we now let \( T \) go to infinity we get:

\[ \Delta \omega_1 = \frac{- \Delta P_L}{(m_1 + m_2) S + (D_1 + D_2)} \]

Which says that when machines are "tightly coupled" (i.e. \( T \to \infty \)), the equivalent model has \( m = m_1 + m_2 \) and \( D = D_1 + D_2 \). (see Figure 9.6).
(b) with \( \Delta P_L(s) = \frac{\Delta P_L}{s} \), then

\[
\Delta \omega_i = \frac{(M_2 s^2 + D_2 s + T)}{(M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T)} \left( -\frac{\Delta P_L}{s} \right)
\]

Then the steady state frequency is

\[
\Delta \omega_{i,0} = \lim_{s \to 0} s \Delta \omega_i(s) = -\frac{\Delta P_L T}{(D_1 + D_2) T} = -\frac{\Delta P_L}{D_1 + D_2}
\]

Therefore, using the given data, we have that

\[
\Delta \omega_{i,0} = \frac{-0.2}{1 + 0.25} = -0.1143
\]

(C) From part (a), we know \( \Delta \omega_i = \frac{1}{M_1 s + D_1} \left( -\Delta P_L(s) - \Delta P_{\text{re}}(s) \right) \)

\[
\Rightarrow \frac{\Delta P_{\text{re}}(s)}{M_1 s + D_1} = \frac{\Delta P_L(s)}{M_1 s + D_1} - \Delta \omega_i
\]

Substituting the expression from (a) for \( \Delta \omega_i \) leads to

\[
\begin{align*}
\Delta P_{\text{re}}(s) &= \left( \frac{M_1 s + D_1}{M_1 s + D_1} \right) \left( \frac{M_2 s^2 + D_2 s + T}{M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T} \right) (-\Delta P_L(s)) \\
\Delta P_{\text{re}}(s) &= \left( M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T \right) (-\Delta P_L(s))
\end{align*}
\]

\[
\begin{align*}
\Delta \omega_i &= \frac{-1}{s} \left( M_1 s + D_1 \right) \left( \frac{M_1 s + D_1}{M_1 s + D_1} \right) \left( \frac{M_2 s^2 + D_2 s + T}{M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T} \right) \left( M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T \right)
\end{align*}
\]

\[
\begin{align*}
\Delta P_{\text{re}}(s) &= \left( M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T \right) \left( M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T \right)
\end{align*}
\]

\[
\Delta(s) = 2 M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T
\]

\[
\Delta(s) = 2 M_1 M_2 s^3 + 2 (M_1 D_2 + M_2 D_1) s^2 + (2 M_1 T + M_2 T + 2 D_1 D_2) s + (2 D_1 + D_2) T
\]

where \( \Delta(s) = M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1) s^2 + (M_1 T + D_1 D_2 + M_2 T) s + (D_1 + D_2) T \) is the characteristic equation for the transfer function.
The frequency of oscillation is given by the roots of the characteristic equation. Using the data from the problem statement, this is

\[ \Delta(s) = (3.5)(4)s^3 + (3.5+0.75+4.1)s^2 + (3.5+2.54+1.0.75+4.754)s+16 \]

\[ = 14s^3 + 6.25s^2 + 57.3s + 13.195 \]

Using the "roots" command in Matlab provides the roots of the above characteristic equation:

\[ \lambda_1, \lambda_2 = -0.1199 \pm j2.0056 \]

\[ \lambda_3 = -0.2335 \]

So the frequency of oscillation is 2.0056 rad/sec or 0.3192 Hz.

To see what happens as the stiffness increases (T \to \infty is the same as X \to 0 when X is the tie-line reactance) to the frequency of oscillation, we do the same thing as was done in part (a), divide \( \Delta(s) \) by \( T \) and then take the limit \( T \to \infty \). Since \( \Delta(s) \) here is the same as \( \Delta(s) \) in part (a), we obtain the same result, i.e., \( \Delta(s) = (M_1+M_2)s + (D_1+D_2) \). This just says that a two machine system connected by a tie-line looks like a one-machine system when the reactance of the tie line goes to zero.