Homework #2, EE 553, Fall 2012, Dr. McCalley, Due Monday, September 17, 2012

1. Solve for x in the below by hand, using LU-decomposition.

$$\begin{bmatrix} 4 & 1 & 1 & 1 \\ 0 & 2 & -1 & -2 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 4 \\ 8 \end{bmatrix}$$

Augment the A-matrix by adding $\underline{b}$ as the 5th column

$$\begin{array}{c} \begin{array}{ccccc} l_{i1} & & & & \\ \downarrow & & & & \end{array} \\ \left[\begin{array}{cccc|c} 4 & 1 & 1 & 1 & 3 \\ 0 & 2 & -1 & -2 & -1 \\ 1 & 0 & 3 & 1 & 4 \\ 0 & 1 & 1 & 6 & 8 \end{array}\right] \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & .25 & .25 & .25 & .75 \\ 0 & 2 & -1 & -2 & -1 \\ 1 & 0 & 3 & 1 & 4 \\ 0 & 1 & 1 & 6 & 8 \end{array}\right]$$

$$\Rightarrow \begin{array}{c} \begin{bmatrix} 1 & .25 & .25 & .25 & .75 \\ 0 & 2 & -1 & -2 & -1 \\ 0 & -.25 & 2.75 & .75 & 3.25 \\ 0 & 1 & 1 & 6 & 8 \end{bmatrix} \\ \begin{array}{ccccc} & \uparrow & & & \\ & l_{i2} & & & \end{array} \end{array} \Rightarrow \begin{bmatrix} 1 & .25 & .25 & .25 & .75 \\ 0 & 1 & -.5 & -1 & -.5 \\ 0 & -.25 & 2.75 & .75 & 3.25 \\ 0 & 1 & 1 & 6 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{array}{c} \begin{bmatrix} 1 & .25 & .25 & .25 & .75 \\ 0 & 1 & -.5 & -1 & -.5 \\ 0 & 0 & 2.625 & .5 & 3.125 \\ 0 & 0 & 1.5 & 7 & 8.5 \end{bmatrix} \\ \begin{array}{ccccc} & & \uparrow & & \\ & & l_{i3} & & \end{array} \end{array} \Rightarrow \begin{bmatrix} 1 & .25 & .25 & .25 & .75 \\ 0 & 1 & -.5 & -1 & -.5 \\ 0 & 0 & 1 & .1905 & 1.1905 \\ 0 & 0 & 1.5 & 7 & 8.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & .25 & .25 & .25 & .75 \\ 0 & 1 & -.5 & -1 & -.5 \\ 0 & 0 & 1 & .1905 & 1.1905 \\ 0 & 0 & 0 & 6.714 & 6.714 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & .25 & .25 & .25 & .75 \\ 0 & 1 & -.5 & -1 & -.5 \\ 0 & 0 & 1 & .1905 & 1.1905 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

l_{44}

$$\underline{L} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & -.25 & 2.625 & 0 \\ 0 & 1 & 1.5 & 6.714 \end{bmatrix} \qquad \underline{U} = \begin{bmatrix} 1 & .25 & .25 & .25 \\ 0 & 1 & -.5 & -1 \\ 0 & 0 & 1 & .1905 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \underline{z} = \begin{bmatrix} .75 \\ -.5 \\ 1.1905 \\ 1 \end{bmatrix}$$

$$\underline{L}\,\underline{U}\,\underline{x} = \underline{b} \quad ; \quad \underline{U}\,\underline{x} = \underline{z}$$

$$\begin{bmatrix} 1 & .25 & .25 & .25 \\ 0 & 1 & -.5 & -1 \\ 0 & 0 & 1 & .1905 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} .75 \\ -.5 \\ 1.1905 \\ 1 \end{bmatrix}$$

USE BACK SUBSTITUTION:

$x_4 = 1$

$x_3 + .1905\, x_4 = 1.1905 \Rightarrow x_3 = 1$

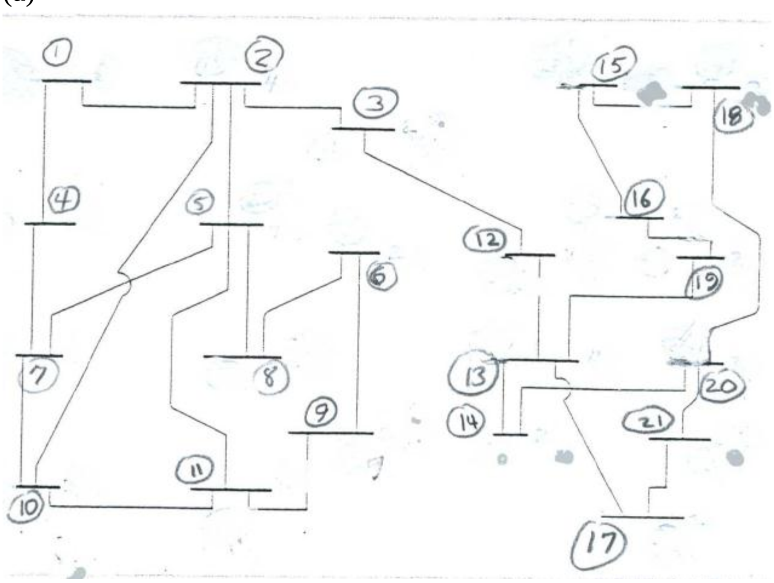
$x_2 - .5 x_3 - x_4 = -.5 \Rightarrow x_2 = -.5 + .5 + 1 = 1$

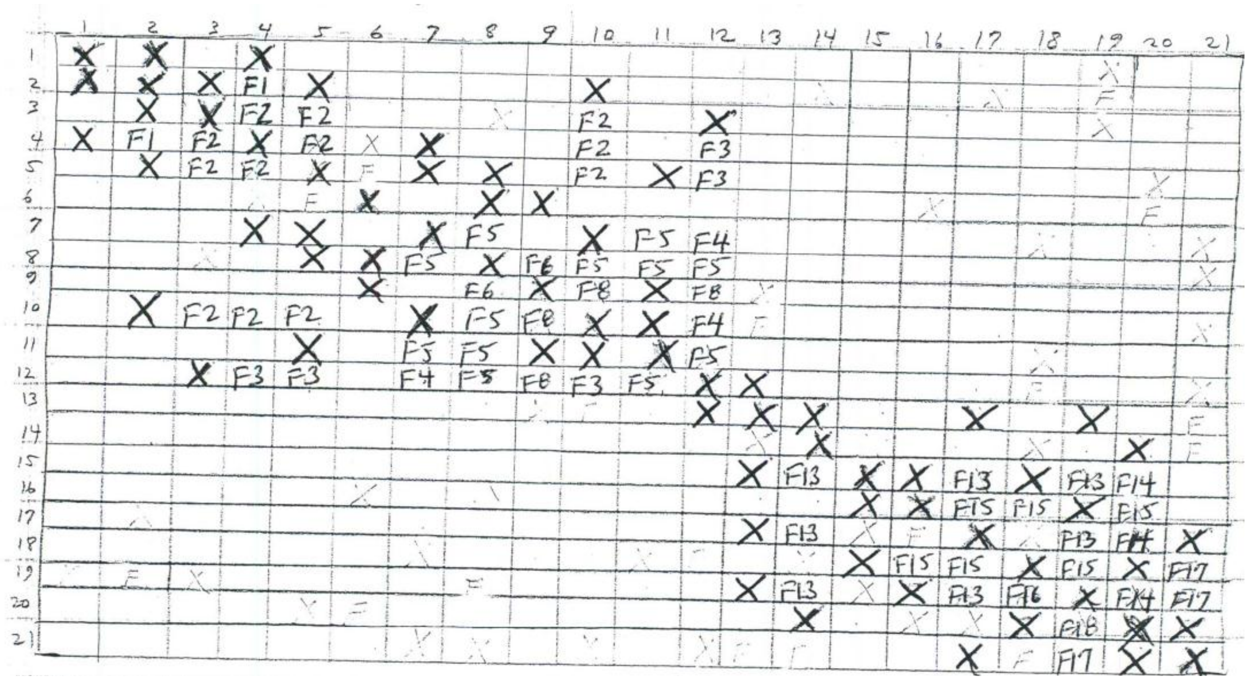
$x_1 + .25 x_2 + .25 x_3 + .25 x_4 = .75 \Rightarrow x_1 = 0$

$$\underline{x} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

2. Consider the two different numbering systems for the network given below. For each numbering system, determine the number of fill-ups and the number of row operations assuming no re-ordering is performed after the first row operation. Indicate which scheme is better and why. Describe a better scheme.

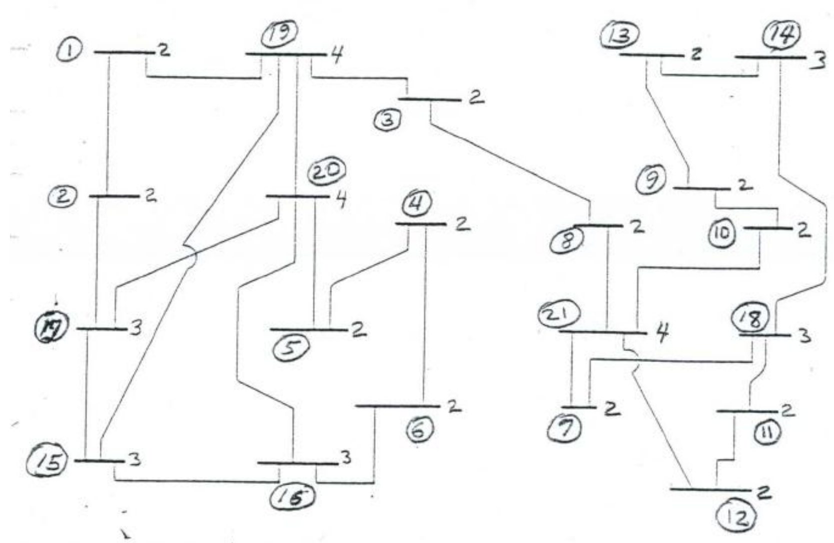
(a)

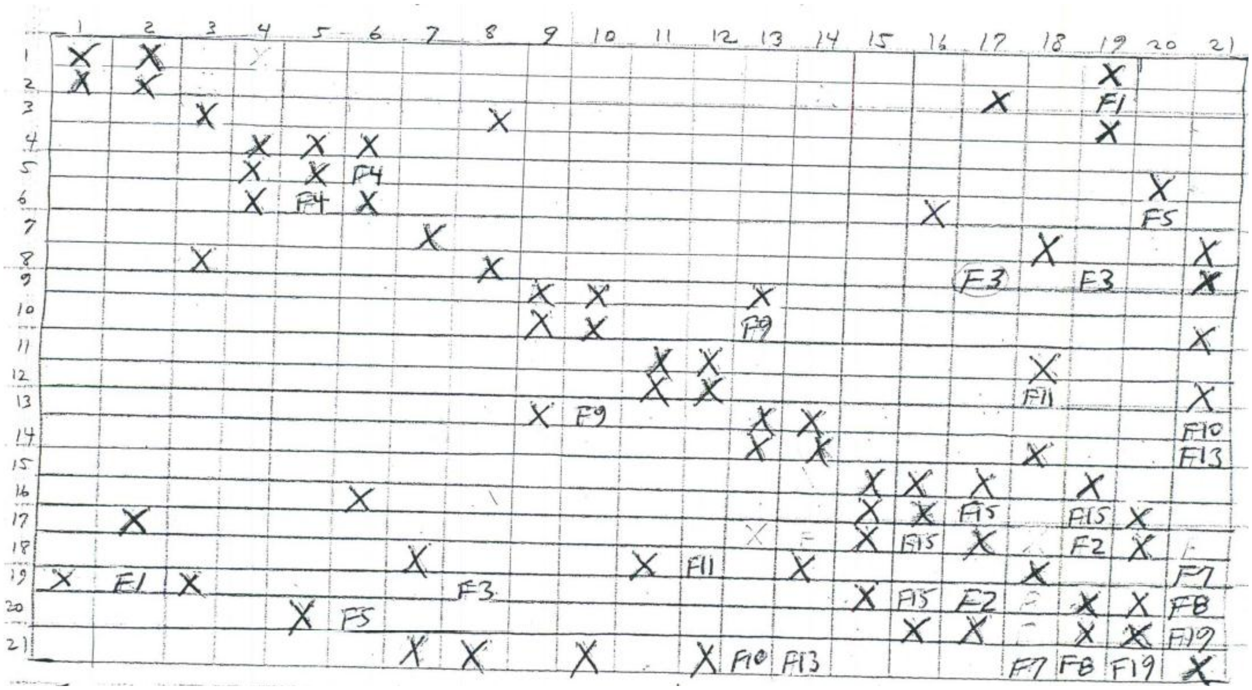




This scheme has 61 Fill-ups and the elimination procedure will require 55 row operations.

(b)

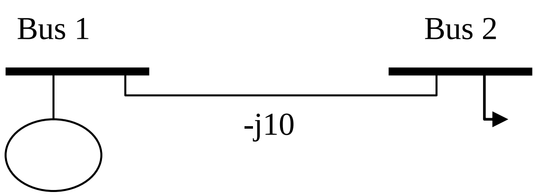




This scheme has 29 fill-ups, and the elimination procedure will require 40 row operations. Since the row number of row operations for this scheme is fewer, it is better than scheme (a).

A better scheme would be to start with scheme (b) and then re-order the buses after each pivot. If we would have applied this, for example, after pivoting on row 1 and getting the fill up in row 2, column 19, we would have then had 3 branches connected to bus 2. Therefore we would have reordered so that bus 2 followed bus 14.

3. For the lossless network shown below, the following data is given:



z1=V1=1.02, σ1=0.1
z2=V2=1.0, σ2=0.1
z3=P12=2.0, σ3=0.05
z4=Q12=0.2, σ4=0.05

Let

$$\underline{x}^{(0)} = \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ x_3^{(0)} \end{bmatrix} = \begin{bmatrix} \theta_2^{(0)} \\ V_1^{(0)} \\ V_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.02 \\ 1.0 \end{bmatrix}$$

and perform one iteration of the least squares state estimation solution procedure to find $x^{(1)}$.

The vector $h(\delta,V)$ is given by

$$h_1(\delta,V) = V_1$$

$$h_2(\delta,V) = V_2$$

$$h_3(\delta,V) = -10V_1V_2\sin\delta_2$$

$$h_4(\delta,V) = 10V_1^2 - 10V_1V_2\cos\delta_2$$

The Jacobian Matrix is given by

$$H(\delta,V) = \begin{pmatrix} \frac{\partial h_1}{\partial \delta_2} & \frac{\partial h_1}{\partial V_1} & \frac{\partial h_1}{\partial V_2} \\ \frac{\partial h_2}{\partial \delta_2} & \frac{\partial h_2}{\partial V_1} & \frac{\partial h_2}{\partial V_2} \\ \frac{\partial h_3}{\partial \delta_2} & \frac{\partial h_3}{\partial V_1} & \frac{\partial h_3}{\partial V_2} \\ \frac{\partial h_4}{\partial \delta_2} & \frac{\partial h_4}{\partial V_1} & \frac{\partial h_4}{\partial V_2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10V_1V_2\cos\delta_2 & -10V_2\sin\delta_2 & -10V_1\sin\delta_2 \\ 10V_1V_2\sin\delta_2 & 20V_1-10V_2\cos\delta_2 & -10V_1\cos\delta_2 \end{pmatrix}$$

Now we proceed with the iterative solution process:

<u>1st iteration</u>

$$\text{Let} \begin{pmatrix} \delta_2^o \\ \hat{V}_1^o \\ \hat{V}_2^o \end{pmatrix} = \begin{pmatrix} 0 \\ 1.02 \\ 1.0 \end{pmatrix}$$

This implies:

$$z - h(\hat{\delta}^o, \hat{V}^o) = \begin{bmatrix} 0 \\ 0 \\ 2.0 \\ -.004 \end{bmatrix}$$

$$H(\hat{\delta}^o, \hat{V}^o) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10.2 & 0 & 0 \\ 0 & 10.4 & -10 \end{bmatrix}$$

$$H^T R^{-1}(z - h(\hat{\delta}^o, \hat{V}^o))$$

$$= \begin{bmatrix} 0 & 0 & -10.2 & 0 \\ 1 & 0 & 0 & 10.4 \\ 0 & 1 & 0 & -10 \end{bmatrix} \begin{bmatrix} (.1)^2 & 0 & 0 & 0 \\ 0 & (.1)^2 & 0 & 0 \\ 0 & 0 & (.05)^2 & 0 \\ 0 & 0 & 0 & (.05)^2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 2.0 \\ -.004 \end{bmatrix}$$

$$= 100 \begin{bmatrix} 0 & 0 & -40.8 & 0 \\ (100)^2 & 0 & 0 & 41.6 \\ 0 & (100)^2 & 0 & -40 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2.0 \\ -.004 \end{bmatrix}$$

$$= 100 \begin{bmatrix} -81.6 \\ -.1664 \\ .16 \end{bmatrix}$$

$$H^T R^{-1} H = \begin{pmatrix} 0 & 0 & -40.8 & 0 \\ (100)^2 & 0 & 0 & 41.6 \\ 0 & (100)^2 & 0 & -40 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ -10.2 & 0 & 0 \\ 0 & 10.4 & -10 \end{pmatrix} \times 100$$

$$= \begin{pmatrix} 416.16 & 0 & 0 \\ 0 & 433.64 & -416 \\ 0 & -416 & 400 \end{pmatrix} \times 100$$

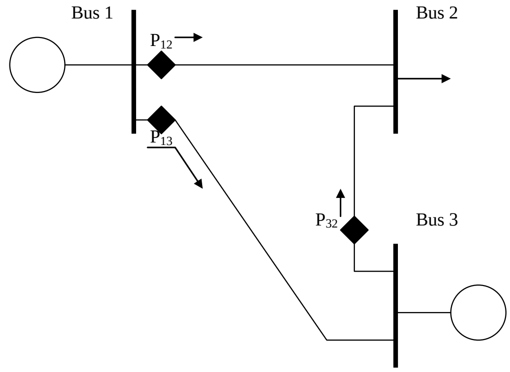
As a result we have

$$\begin{pmatrix} \hat{\delta}_2^1 \\ \hat{V}_1^1 \\ \hat{V}_2^1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1.02 \\ 1.0 \end{pmatrix} + \begin{pmatrix} 416.16 & 0 & 0 \\ 0 & 433.64 & 416 \\ 0 & 416 & 400 \end{pmatrix}^{-1} \begin{pmatrix} -81.6 \\ -.1664 \\ .16 \end{pmatrix}$$

$$= \begin{pmatrix} -.1960 \\ 1.0203 \\ 1.000728 \end{pmatrix}$$

From here we proceed to the second iteration, and so on.

4. Consider the system below. Real power measurements taken as follows: P_{12}=0.62 pu , P_{13}=0.06 pu, and P_{32}=0.37 pu . All voltages are 1.0 per unit, and all measurement devices have σ=0.01. Assume the bus 3 angle is reference. So the state vector is therefore $\underline{x}=[\theta_1 \;\; \theta_2]^T$. Your textbook solves this problem using DC power flow equations on pp. 467-471. Repeat, following the indicated steps below, but use AC power flow equations.



a) Determine the vector of measurement expressions $\underline{h}(\underline{x})$, the derivative expressions $\underline{H} = \dfrac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}$, and the weighting matrix $\underline{R}$.

b) Compute $\underline{H}(\underline{x}^{(0)})$, $\underline{h}(\underline{x}^{(0)})$ for an estimate of $\underline{x}^{(0)}$ =[0.024 -0.093]^T (units of radians).

c) Compute $\underline{A} = \underline{H}^T(\underline{x})\underline{R}^{-1}\underline{H}(\underline{x})\Big|_{\underline{x}^{(0)}}$, $\underline{b} = \underline{H}^T(\underline{x})\underline{R}^{-1}\left[\left(\underline{z}-\underline{h}(\underline{x})\right)\right]_{\underline{x}^{(0)}}$

d) Solve $\underline{A}\Delta\underline{x}=\underline{b}$ for $\Delta\underline{x}$.

<u>Solution</u>:

a) Recall series admittance of $g_{pq}+jb_{pq}$; $g_{pq}>0$, $b_{pq}<0$ for inductive line

$$h_i(\underline{x}) = P_{p,inj} = \sum_{k=1}^{n} |V_p||V_k|\left(G_{pk}\cos(\theta_p-\theta_k)+B_{pk}\sin(\theta_p-\theta_k)\right)=0$$

$$h_1(\underline{x}) = P_{12} = |V_1|^2 g_{12} - |V_1||V_2|g_{12}\cos(\theta_1-\theta_2) - |V_1||V_2|b_{12}\sin(\theta_1-\theta_2) = 5\sin(\theta_1-\theta_2)$$

$$h_2(\underline{x}) = P_{13} = |V_1|^2 g_{12} - |V_1||V_3|g_{13}\cos(\theta_1-\theta_3) - |V_1||V_3|b_{13}\sin(\theta_1-\theta_3) = 2.5\sin(\theta_1-\theta_3)$$
$$= 2.5\sin(\theta_1)$$

$$h_3(\underline{x}) = P_{32} = |V_3|^2 g_{32} - |V_3||V_2|g_{32}\cos(\theta_3-\theta_2) - |V_3||V_2|b_{32}\sin(\theta_3-\theta_2) = 4\sin(\theta_3-\theta_2)$$
$$= 4\sin(-\theta_2) = -4\sin(\theta_2)$$

$$\underline{H}=\frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}=\begin{bmatrix}\frac{\partial h_1(\underline{x})}{\partial x_1} & \frac{\partial h_1(\underline{x})}{\partial x_2}\\ \frac{\partial h_2(\underline{x})}{\partial x_1} & \frac{\partial h_2(\underline{x})}{\partial x_2}\\ \frac{\partial h_3(\underline{x})}{\partial x_1} & \frac{\partial h_3(\underline{x})}{\partial x_2}\end{bmatrix}=\begin{bmatrix}\frac{\partial P_{12}(\underline{x})}{\partial \theta_1} & \frac{\partial P_{12}(\underline{x})}{\partial \theta_2}\\ \frac{\partial P_{13}(\underline{x})}{\partial \theta_1} & \frac{\partial P_{13}(\underline{x})}{\partial \theta_2}\\ \frac{\partial P_{32}(\underline{x})}{\partial \theta_1} & \frac{\partial P_{32}(\underline{x})}{\partial \theta_2}\end{bmatrix}=\begin{bmatrix}5\cos(\theta_1-\theta_2) & -5\cos(\theta_1-\theta_2)\\ 2.5\cos(\theta_1) & 0\\ 0 & -4\cos(\theta_2)\end{bmatrix}$$

$$\underline{R}=\begin{bmatrix}\sigma_1^2 & 0 & \cdots & 0\\ 0 & \sigma_2^2 & 0 & 0\\ \vdots & 0 & \ddots & 0\\ 0 & 0 & 0 & \sigma_m^2\end{bmatrix}=\begin{bmatrix}0.0001 & 0 & 0\\ 0 & 0.0001 & 0\\ 0 & 0 & 0.0001\end{bmatrix}$$

b) With $\underline{x}^{(0)}=[0.024\ \ -0.093]^T$, $\underline{H}(\underline{x}^{(0)})$, $\underline{h}(\underline{x}^{(0)})$ become:

$$h_1(\underline{x})=P_{12}=5\sin(\theta_1-\theta_2)=5\sin(0.024+0.093)=0.5837$$

$$h_2(\underline{x})=P_{13}=2.5\sin(\theta_1)=2.5\sin(0.024)=0.06$$

$$h_3(\underline{x})=P_{32}=-4\sin(\theta_2)=-4\sin(-.093)=0.3715$$

$$\underline{H}=\begin{bmatrix}5\cos(\theta_1-\theta_2) & -5\cos(\theta_1-\theta_2)\\ 2.5\cos(\theta_1) & 0\\ 0 & -4\cos(\theta_2)\end{bmatrix}=\begin{bmatrix}5\cos(0.024+0.093) & -5\cos(0.024+0.093)\\ 2.5\cos(0.024) & 0\\ 0 & -4\cos(-0.093)\end{bmatrix}=\begin{bmatrix}4.9658 & -4.9658\\ 2.4993 & 0\\ 0 & -3.9827\end{bmatrix}$$

c)

$$\underline{A}=\underline{H}^T(\underline{x})\underline{R}^{-1}\underline{H}(\underline{x})\Big|_{\underline{x}^{(0)}}=\begin{bmatrix}4.9658 & 2.4993 & 0\\ -4.9658 & 0 & -3.9827\end{bmatrix}\begin{bmatrix}0.0001 & 0 & 0\\ 0 & 0.0001 & 0\\ 0 & 0 & 0.0001\end{bmatrix}^{-1}\begin{bmatrix}4.9658 & -4.9658\\ 2.4993 & 0\\ 0 & -3.9827\end{bmatrix}$$

$$=\begin{bmatrix}4.9658 & 2.4993 & 0\\ -4.9658 & 0 & -3.9827\end{bmatrix}\begin{bmatrix}10000 & 0 & 0\\ 0 & 10000 & 0\\ 0 & 0 & 10000\end{bmatrix}\begin{bmatrix}4.9658 & -4.9658\\ 2.4993 & 0\\ 0 & -3.9827\end{bmatrix}=\begin{bmatrix}309060 & -246590\\ -246590 & 405210\end{bmatrix}$$

$$\underline{b}=\underline{H}^T(\underline{x})\underline{R}^{-1}\left[(\underline{z}-\underline{h}(\underline{x}))\right]\Big|_{\underline{x}^{(0)}}$$

$$=\begin{bmatrix}4.9658 & 2.4993 & 0\\ -4.9658 & 0 & -3.9827\end{bmatrix}\begin{bmatrix}10000 & 0 & 0\\ 0 & 10000 & 0\\ 0 & 0 & 10000\end{bmatrix}\begin{bmatrix}0.62-0.5837\\ 0.06-0.06\\ 0.37-0.3715\end{bmatrix}$$

$$=\begin{bmatrix}4.9658 & 2.4993 & 0\\ -4.9658 & 0 & -3.9827\end{bmatrix}\begin{bmatrix}10000 & 0 & 0\\ 0 & 10000 & 0\\ 0 & 0 & 10000\end{bmatrix}\begin{bmatrix}0.0363\\ 0\\ -0.0015\end{bmatrix}=\begin{bmatrix}1802.6\\ -1742.8\end{bmatrix}$$

d)

$$\left[\begin{bmatrix}309060 & -246590\\ -246590 & 405210\end{bmatrix}\right]\begin{bmatrix}\Delta\theta_1\\ \Delta\theta_2\end{bmatrix}=\begin{bmatrix}1802.6\\ -1742.8\end{bmatrix}\Rightarrow\begin{bmatrix}\Delta\theta_1\\ \Delta\theta_2\end{bmatrix}=\begin{bmatrix}0.0047\\ -.0015\end{bmatrix}$$

And the new value of $\underline{x}$ would then be

$[\theta_1\ \theta_2]^T=[0.024\ \ -0.093]^T+[.0047\ \ -.0015]^T=[0.0287\ \ -.0945]$

5. Work problem 12.3 in your W&W textbook.

Solution:

a) Let the state vector be $\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$

Then

$$H = \begin{bmatrix} 4 & -4 & 0 \\ 4 & 0 & -2 \\ -2 & 0 & 2 \end{bmatrix} \qquad R = \begin{bmatrix} (.02)^2 & & \\ & (.01)^2 & \\ & & (.01)^2 \end{bmatrix}$$

Note: The order of the measurements in the H matrix is m_{12}, m_{13}, m_{31}

$$[H^T R^{-1} H] = \begin{bmatrix} 4 & 2 & -2 \\ -4 & 0 & 0 \\ 0 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2500 & & \\ & 10000. & \\ & & 10000. \end{bmatrix} \begin{bmatrix} 4 & -4 & 0 \\ 2 & 0 & -2 \\ -2 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10,000. & 20,000. & -20,000. \\ -10,000. & 0 & - \\ 0 & -20,000. & 20,000. \end{bmatrix} \begin{bmatrix} 4 & -4 & 0 \\ 2 & 0 & -2 \\ -2 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 120,000. & -40,000. & -80,000. \\ -40,000. & 40,000. & 0 \\ -80,000. & 0 & 80,000. \end{bmatrix}$$

This matrix is singular thus indicating that the network is unobservable.

b) With the new measurement:

$$P_3 = P_{31} + P_{34} = 4(\theta_3-\theta_1) + 10(\theta_3-\theta_4)$$

$$= 14\theta_3 - 4\theta_1$$

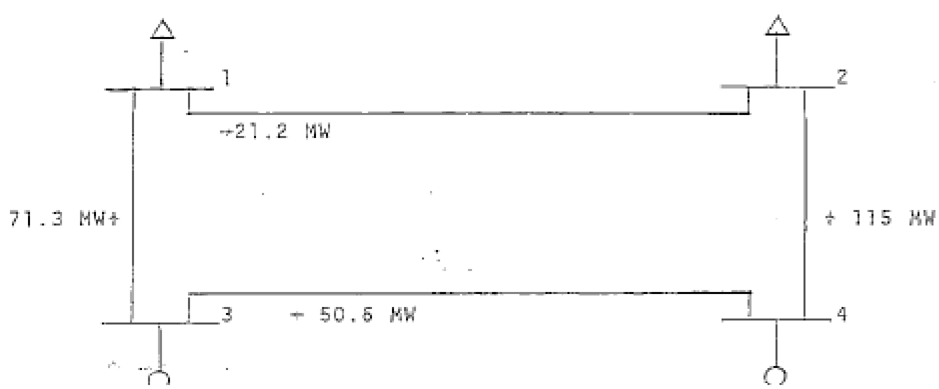
$$\text{Then } H = \begin{array}{c} \\ 12 \\ 13 \\ 31 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} & N & \end{array} \\ \begin{bmatrix} 4 & -4 & 0 \\ 2 & 0 & -2 \\ -2 & 0 & 2 \\ -4 & 0 & 14 \end{bmatrix} \end{array} \quad R = \begin{bmatrix} (.02)^2 & & & \\ & (.01)^2 & & \\ & & (.01)^2 & \\ & & & (.015)^2 \end{bmatrix}$$

$$[H^T R^{-1} H] = \begin{bmatrix} .19111\cdot10^6 & -.4\cdot10^5 & -.32889\cdot10^6 \\ -4\cdot10^5 & .4\cdot10^5 & 0 \\ -.32889\cdot10^6 & 0 & .95111\cdot10^6 \end{bmatrix}$$

$$[H^T R^{-1} H]^{-1} = \begin{bmatrix} .2675\cdot10^{-4} & .2675\cdot10^{-4} & .925\cdot10^{-5} \\ .2675\cdot10^{-4} & .5175\cdot10^{-4} & .925\cdot10^{-5} \\ .925\cdot10^{-5} & .925\cdot10^{-5} & .425\cdot10^{-5} \end{bmatrix}$$

$$\begin{bmatrix} \theta_1^{est} \\ \theta_2^{est} \\ \theta_3^{est} \\ \theta_4^{est} \end{bmatrix} = \begin{bmatrix} -.407 \\ -.460 \\ -.051 \\ 0 \end{bmatrix}$$

The estimated flows are shown below:



Note: The flow est. on line 1-2 is the same value as its measurement. This is because flow measurement m_{12} is a "critical" measurement. A "critical" measurement is one which if removed would immediately render the network unobservable. Critical measurements are not therefore redundant with any other measurements and will always return the measured value as the estimated value.