Homework #1, EE 553, Fall, 2012, Dr. McCalley

SOLUTIONS

1. Problem 4.1a.
2. Before working this problem, you may review page 102 of your W&W text (note that in class, we use V to denote voltages whereas your W&W text uses E to denote voltages).
   Observe that Jacobian derivative expressions given by equations T7.47 and T7.53 in your notes are almost the same. Also observe that Jacobian derivative equations T7.49 and T7.51 in your notes are almost the same. These two sets of equations are given below.

$$J_{jk}^{P\theta} = \frac{\partial P_j(\underline{x})}{\partial \theta_k} = |V_j||V_k|\left(G_{jk}\sin(\theta_j-\theta_k) - B_{jk}\cos(\theta_j-\theta_k)\right) \tag{T7.47}$$

$$J_{jk}^{QV} = \frac{\partial Q_j(\underline{x})}{\partial |V_k|} = |V_j|\left(G_{jk}\sin(\theta_j-\theta_k) - B_{jk}\cos(\theta_j-\theta_k)\right) \tag{T7.53}$$

$$J_{jk}^{Q\theta} = \frac{\partial Q_j(\underline{x})}{\partial \theta_k} = -|V_j||V_k|\left(G_{jk}\cos(\theta_j-\theta_k) + B_{jk}\sin(\theta_j-\theta_k)\right) \tag{T7.49}$$

$$J_{jk}^{PV} = \frac{\partial P_j(\underline{x})}{\partial |V_k|} = |V_j|\left(G_{jk}\cos(\theta_j-\theta_k) + B_{jk}\sin(\theta_j-\theta_k)\right) \tag{T7.51}$$

   a. What is the difference between the right-hand-side expression of T7.47 and that of T7.53?
      Solution: T7.47 has an extra $|V_j|$.
      What is the difference between the right-hand-side expression of T7.49 and that of T7.51?
   b. Modify (T7.53) to revise the Jacobian element to $J_{jk}^{QV} = |V_k|\dfrac{\partial Q_j(\underline{x})}{\partial |V_k|}$

      Solution:

$$J_{jk}^{QV,new} = |V_k|\frac{\partial Q_j(\underline{x})}{\partial |V_k|} = |V_k||V_j|\left(G_{jk}\sin(\theta_j-\theta_k) - B_{jk}\cos(\theta_j-\theta_k)\right)$$

   c. Modify (T7.51) to revise the Jacobian element to $J_{jk}^{PV} = -|V_k|\dfrac{\partial P_j(\underline{x})}{\partial |V_k|}$

      Solution:

$$J_{jk}^{PV,new} = |V_k|\frac{\partial P_j(\underline{x})}{\partial |V_k|} = |V_k||V_j|\left(G_{jk}\cos(\theta_j-\theta_k) + B_{jk}\sin(\theta_j-\theta_k)\right)$$

d. Express the relations in (T7.42e) below to account for the two modifications made above. This requires that you express the new Jacobian submatricies $J^{PV,new}$ and $J^{QV,new}$ in terms of its revised elements (from parts (a) and (b) above). It also requires that you modify the elements of the voltage-related part of solution vector $\Delta|\underline{V}|^{new}$.

$$\begin{aligned}\underline{J}^{P\theta}\Delta\underline{\theta}+\underline{J}^{PV}\Delta|\underline{V}| &= -\Delta\underline{P}\\ \underline{J}^{Q\theta}\Delta\underline{\theta}+\underline{J}^{QV}\Delta|\underline{V}| &= -\Delta\underline{Q}\end{aligned} \qquad \text{(T7.42e)}$$

Solution:

Define:

$$\underline{J}^{PV,new}=\begin{bmatrix} |V_{N_g+1}|\dfrac{\partial P_2(\underline{x})}{\partial|V_{N_g+1}|} & |V_{N_g+2}|\dfrac{\partial P_2(\underline{x})}{\partial|V_{N_g+2}|} & \cdots & |V_N|\dfrac{\partial P_2(\underline{x})}{\partial|V_N|}\\ |V_{N_g+1}|\dfrac{\partial P_3(\underline{x})}{\partial|V_{N_g+1}|} & |V_{N_g+2}|\dfrac{\partial P_3(\underline{x})}{\partial|V_{N_g+2}|} & \cdots & |V_N|\dfrac{\partial P_3(\underline{x})}{\partial|V_N|}\\ \vdots & \vdots & \cdots & \vdots\\ |V_{N_g+1}|\dfrac{\partial P_N(\underline{x})}{\partial|V_{N_g+1}|} & |V_{N_g+2}|\dfrac{\partial P_N(\underline{x})}{\partial|V_{N_g+2}|} & \cdots & |V_N|\dfrac{\partial P_{3N}(\underline{x})}{\partial|V_N|}\end{bmatrix}$$

$$\underline{J}^{QV,new}=\begin{bmatrix} |V_{N_g+1}|\dfrac{\partial Q_{N_g+1}(\underline{x})}{\partial|V_{N_g+1}|} & |V_{N_g+2}|\dfrac{\partial Q_{N_g+1}(\underline{x})}{\partial|V_{N_g+2}|} & \cdots & |V_N|\dfrac{\partial Q_{N_g+1}(\underline{x})}{\partial|V_N|}\\ |V_{N_g+1}|\dfrac{\partial Q_{N_g+2}(\underline{x})}{\partial|V_{N_g+1}|} & |V_{N_g+2}|\dfrac{\partial Q_{N_g+2}(\underline{x})}{\partial|V_{N_g+2}|} & \cdots & |V_N|\dfrac{\partial Q_{N_g+2}(\underline{x})}{\partial|V_N|}\\ \vdots & \vdots & \cdots & \vdots\\ |V_{N_g+1}|\dfrac{\partial Q_N(\underline{x})}{\partial|V_{N_g+1}|} & |V_{N_g+2}|\dfrac{\partial Q_N(\underline{x})}{\partial|V_{N_g+2}|} & \cdots & |V_N|\dfrac{\partial Q_{3N}(\underline{x})}{\partial|V_N|}\end{bmatrix}$$

$$\Delta|\underline{V}|^{new}=\begin{bmatrix}\dfrac{\Delta|V_{N_g+1}|}{|V_{N_g+1}|} & \dfrac{\Delta|V_{N_g+2}|}{|V_{N_g+2}|} & \cdots & \dfrac{\Delta|V_N|}{|V_N|}\end{bmatrix}^T$$

Then

$$\begin{aligned}\underline{J}^{P\theta}\Delta\underline{\theta}+\underline{J}^{PV,new}\Delta|\underline{V}|^{new} &= -\Delta\underline{P}\\ \underline{J}^{Q\theta}\Delta\underline{\theta}+\underline{J}^{QV,new}\Delta|\underline{V}|^{new} &= -\Delta\underline{Q}\end{aligned}$$

This can be expressed in matrix form as

$$\begin{bmatrix}\underline{J}^{P\theta} & \underline{J}^{PV,new}\\ \underline{J}^{Q\theta} & \underline{J}^{QV,new}\end{bmatrix}\begin{bmatrix}\Delta\underline{\theta}\\ \Delta|\underline{V}|^{new}\end{bmatrix}=-\begin{bmatrix}\Delta\underline{P}\\ \Delta\underline{Q}\end{bmatrix}$$

e. What is the advantage of this new formulation? Hint: Express and compare $J_{jk}^{P\theta}$ to $J_{jk}^{QV,new}$ and express and compare $J_{jk}^{Q\theta}$ to $J_{jk}^{PV,new}$.

Solution:
To see the advantage, one must observe:

$$J_{jk}^{P\theta} = \frac{\partial P_j(\underline{x})}{\partial \theta_k} = |V_j||V_k|\left(G_{jk}\sin(\theta_j - \theta_k) - B_{jk}\cos(\theta_j - \theta_k)\right) \quad \text{(T7.47)}$$

$$J_{jk}^{QV,new} = |V_k|\frac{\partial Q_j(\underline{x})}{\partial |V_k|} = |V_k||V_j|\left(G_{jk}\sin(\theta_j - \theta_k) - B_{jk}\cos(\theta_j - \theta_k)\right) \quad \text{(T7.53)}$$

$$J_{jk}^{Q\theta} = \frac{\partial Q_j(\underline{x})}{\partial \theta_k} = -|V_j||V_k|\left(G_{jk}\cos(\theta_j - \theta_k) + B_{jk}\sin(\theta_j - \theta_k)\right) \quad \text{(T7.49)}$$

$$J_{jk}^{PV,new} = |V_k|\frac{\partial P_j(\underline{x})}{\partial |V_k|} = |V_k||V_j|\left(G_{jk}\cos(\theta_j - \theta_k) + B_{jk}\sin(\theta_j - \theta_k)\right) \quad \text{(T7.51)}$$

Thus, we see that

$$J_{jk}^{P\theta} = J_{jk}^{QV,new} \quad \text{and} \quad J_{jk}^{Q\theta} = -J_{jk}^{PV,new}$$

Therefore, the benefit is in terms of computation (only have to compute one element for each pair of the above) and in storage (only have to store one element for each pair of the above).

3. A transformer with an off-nominal tap ratio t connects two buses as shown in Fig. 1. The transformer admittance is y. An equivalent representation of the Fig. 1 transformer, used for power flow calculations, is shown in Fig. 2. For each model, give the elements of the admittance matrix $\underline{Y}$, where $\underline{I}=\underline{Y}\underline{V}$, $\underline{I}=[\underline{I}_1\ \ \underline{I}_2]^T$, and $\underline{V}=[\underline{V}_1\ \ \underline{V}_2]^T$. From your results, express the admittances of Fig. 2 in terms of tap t and transformer admittance y. Doing so will allow us to model the off-nominal turns transformer of Fig. 1 in the standard π-equivalent model of Fig. 2 so that we can represent the tap changer in our power flow algorithm.

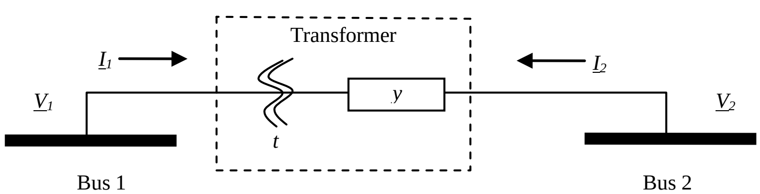


Fig. 1

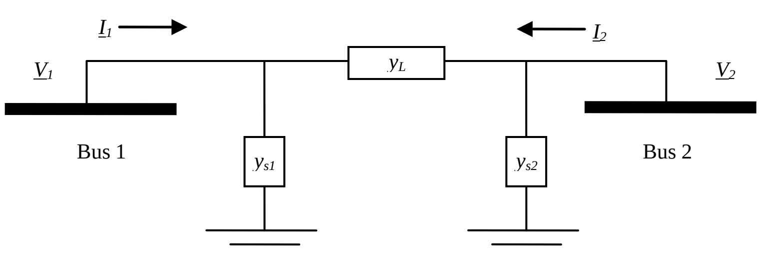


Fig. 2

Solution:

From Fig. 1,

$$\frac{\underline{I}_1}{t} = \left[\underline{V}_1 t - \underline{V}_2\right] y \Rightarrow \underline{I}_1 = t^2 y \underline{V}_1 - ty\underline{V}_2$$

$$\underline{I}_2 = \left[\underline{V}_2 - t\underline{V}_1\right] y \Rightarrow \underline{I}_2 = y\underline{V}_2 - ty\underline{V}_1$$

In matrix form, we have

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} t^2 y & -ty \\ -ty & y \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix}$$

From Fig. 2,

$$\underline{I}_1 = \left[\underline{V}_1 - \underline{V}_2\right] y_L + \underline{V}_1 y_{s1} \Rightarrow \underline{I}_1 = \underline{V}_1 \left(y_L + y_{s1}\right) - \underline{V}_2 y_L$$

$$\underline{I}_2 = \left[\underline{V}_2 - \underline{V}_1\right] y_L + \underline{V}_2 y_{s2} \Rightarrow \underline{I}_2 = \underline{V}_2 \left(y_L + y_{sq}\right) - \underline{V}_1 y_L$$

In matrix form, we have

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} y_L + y_{s1} & -y_L \\ -y_L & y_L + y_{s2} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix}$$

To model the off-nominal turns transformer of Fig. 1 in the standard y-equivalent model of Fig. 2 (so that we can represent the tap changer in our power flow studies), we require:

- Elements in row 1, column 2, and in row 2, column 1, to be equal:

  $$-ty = -y_L \Rightarrow y_L = ty \qquad \text{(eq *)}$$

- Elements in row 1, column 1 to be equal:

  $$y_L + y_{s1} = t^2 y \Rightarrow y_{s1} = t^2 y - y_L$$

  But by (eq *), we have:

  $$y_{s1} = t^2 y - ty = t(t-1)y$$

- Elements in row 2, column 2 to be equal:

  $$y_L + y_{s2} = y \Rightarrow y_{s2} = y - y_L$$

  Again, by (eq *), we have:

  $$y_{s2} = y - ty = (1-t)y$$

In summary, when the program data indicates a non-unity tap transformer, the program should model it as a standard pi-equivalent circuit (as in Fig. 2) with numerical values according to:

$$y_L = ty$$

$$y_{s1} = t(t-1)y$$

$$y_{s2} = (1-t)y$$

4. Consider that the transformer of Fig. 1 is a tap-changing-under-load (TCUL) transformer, regulating bus 1, and that buses 1 and 2 are interconnected to a larger system. Bus 2 is a type PQ bus. Which parameters associated with the bus 1 to bus 2 subsystem would be included in the state vector $\Delta\underline{x}$ used to solve the equation $\underline{J}\Delta\underline{x}=-\underline{f}(\underline{x})$ for one iteration of the Newton-Raphson power follow solution? That is, what are the unknowns for this subsystem? Note that the matrices $\underline{J}$ and $\underline{f}(\underline{x})$ are the Jacobian and the mismatch vector, respectively.

   Solution: Since the TCUL transformer is regulating bus 1, the voltage magnitude at bus 1 is fixed and not a variable. This regulation is performed by modifying the tap, *t*, and so it is a variable, along with both voltage angles and the bus voltage magnitude at bus 2. Therefore the variables are:
   V_2, t, θ_1, and θ_2.

5. Consider the two-bus system of Fig. 3, where the tap *t* is used to regulate the voltage at bus 2 to be equal to 1.0. pu.

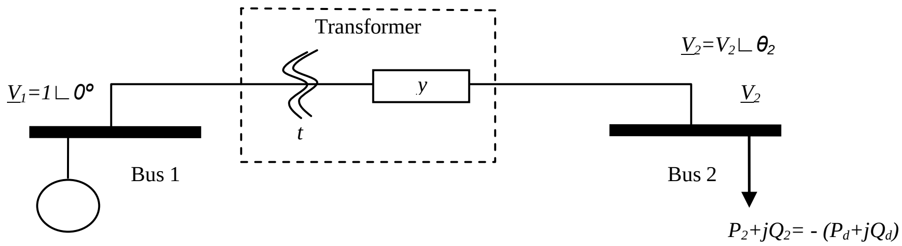
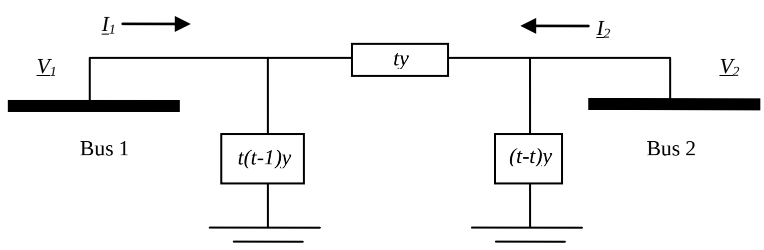


Fig. 3

To solve this system by Newton-Raphson, we need to develop the equation $\underline{J}\Delta\underline{x}=-\underline{f}(\underline{x})$. For this system, specify $\underline{J}$, $\Delta\underline{x}$, and $\underline{f}(\underline{x})$ in terms of the unknowns in the problem and the admittance $y=|y|\angle\gamma$.

Solution: From Problem 3, Fig. 3 can be redrawn as below:



The Y-bus is therefore:

$$\underline{Y}_{bus}=\begin{bmatrix}Y_{11} & Y_{12}\\ Y_{21} & Y_{22}\end{bmatrix}=\begin{bmatrix}|Y_{11}|\angle\gamma_{11} & |Y_{12}|\angle\gamma_{12}\\ |Y_{21}|\angle\gamma_{21} & |Y_{22}|\angle\gamma_{22}\end{bmatrix}=\begin{bmatrix}t^2y & -ty\\ -ty & y\end{bmatrix}=\begin{bmatrix}t^2|y|\angle\gamma & -t|y|\angle\gamma\\ -t|y|\angle\gamma & |y|\angle\gamma\end{bmatrix}$$

We have two unknowns: θ_2 and t, so that the solution vector is x=[θ_2 t]^T.
And so we need two equations.
These equations will be the real and reactive power flow equations at bus 2, which we write below.

$$P_2(\underline{x})=V_2\left[|Y_{21}|V_1\cos(\theta_2-\gamma_{21})+|Y_{22}|V_2\cos(-\gamma_{22})\right]$$

$$Q_2(\underline{x}) = V_2\left[|Y_{21}|V_1 \sin(\theta_2 - \gamma_{21}) + |Y_{22}|V_2 \sin(-\gamma_{22})\right]$$

With V_1 and V_2 specified to be 1.0, and with $|Y_{21}|=-t|y|$ and $|Y_{22}|=|y|$, we have:

$$P_2(\underline{x}) = -t\,|y|\cos(\theta_2 - \gamma) + |y|\cos(-\gamma)$$

$$Q_2(\underline{x}) = -t\,|y|\sin(\theta_2 - \gamma) + |y|\sin(-\gamma)$$

And so the equations to solve are, in vector form, as follows:

$$\underline{f}(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \end{bmatrix} = \begin{bmatrix} P_2(\underline{x}) - P_2 \\ Q_2(\underline{x}) - Q_2 \end{bmatrix} = \begin{bmatrix} -t\,|y|\cos(\theta_2 - \gamma) + |y|\cos(-\gamma) - P_2 \\ -t\,|y|\sin(\theta_2 - \gamma) + |y|\sin(-\gamma) - Q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

with the solution vector given as:

$$\underline{x} = \begin{bmatrix} \theta_2 \\ t \end{bmatrix}$$

Then the Jacobian is written as:

$$\underline{J} = \begin{bmatrix} \frac{\partial f_1}{\theta_2} & \frac{\partial f_1}{t} \\ \frac{\partial f_2}{\theta_2} & \frac{\partial f_2}{t} \end{bmatrix} = \begin{bmatrix} t\,|y|\sin(\theta_2 - \gamma) & -|y|\cos(\theta_2 - \gamma) \\ -t\,|y|\cos(\theta_2 - \gamma) & -|y|\sin(\theta_2 - \gamma) \end{bmatrix}$$

Define the mismatch vector as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} P_2(\underline{x}) - P_2 \\ Q_2(\underline{x}) - Q_2 \end{bmatrix}$$

Then we can write:

$$\underline{J}\Delta\underline{x} = -\begin{bmatrix} P_2(\underline{x}) - P_2 \\ Q_2(\underline{x}) - Q_2 \end{bmatrix}$$

Or

$$\underline{J}\underline{\begin{bmatrix} \Delta\theta_2 \\ \Delta t \end{bmatrix}} = -\begin{bmatrix} P_2(\underline{x}) - P_2 \\ Q_2(\underline{x}) - Q_2 \end{bmatrix}$$

And this will allow us to perform the Newton-Raphson method.