NAME____

Exam 2, EE 553, Fall 2012, Dr. McCalley

Closed books, Closed notes, calculator permitted, 75-minute time limit.
Answer Question 6 on the exam paper. Otherwise, do all work on separate paper.
Turn in that paper together with this exam.

1. (23 pts) Two interconnected identical control areas having a single generating unit in each are characterized by the following information:

   Machine base: 300 MVA
   Frequency sensitivity of load: D=1.0 pu on nominal base
   Nominal load: 250 MVA
   Droop setting R=0.05 pu on machine base

   a. Compute the frequency sensitivity D and the droop setting R, all on a 500 MVA base.

      Solution:

      $$D = (1.0)\frac{250}{500} = 0.5 \qquad R = (0.05)\frac{500}{300} = 0.083$$

   b. For this part of the problem, assume that the machines in both control areas are on governor control but the secondary AGC loop is disabled.

      i. The following equation can be used to obtain steady-state frequency: $\Delta\omega_\infty = -\Delta P_L \Big/ \left(\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2\right)$. Compute the steady-state frequency in Hz in control area 1 following a step load increase of 20 MW in control area 1.

      Solution:

      $$\Delta\omega_\infty = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2} = \frac{-0.04}{\frac{1}{0.083} + \frac{1}{0.083} + 0.5 + 0.5} = \frac{-0.04}{25} = -0.0016\,pu$$

      $$\Delta f = -0.0016 \times 60 = -0.096 Hz$$

      $$f_{\infty 1} = 60 - 0.096 = 59.904 Hz$$

      ii. Compute the new steady-state generation level, in MW, for the machine in control area 1 and the machine in control area 2.

      Solution:

      $$\Delta P_{m1\infty} = \frac{-\Delta\omega_\infty}{R_1} = \frac{+0.0016}{0.083} = 0.0192\,pu$$

      In MW, $\Delta P_{m1\infty} = (500)(0.0192) = 9.6 MW$

      The machine in control area 2 is the same size with the same droop, therefore $\Delta P_{m2\infty} = \Delta P_{m1\infty} = 9.6$ MW.

      iii. Why is the total generation change in the interconnection not equal to the increase in load of ΔP_L=20MW?

      Solution:

      $\Delta P_{m1\infty} + \Delta P_{m2\infty} = 9.6 + 9.6 = 19.2$ which is less than 20, so no it is not equal because the load is sensitive to the frequency decline and has decreased by 0.8 MW.

iv. The change in tie-line flow $\Delta P_{tie}=\Delta P_{12}$ is -0.02 pu on a 500 MW base. Compute the area control error for both control areas if the frequency bias for both control areas are B=12.5 pu MW/pu freq. Note that $ACE_i=-\Delta P_{int,i}-\beta_i\Delta\omega_\infty$ where $\Delta P_{int,i}$ is the change in net export for control area i.

Solution:
In pu, ACE_1=-[-0.02]-12.5[-0.0016]=0.04
In MW, ACE_1=(0.04)(500)=20.0
In pu, ACE_2=-[0.02]-12.5[-0.0016]=0.

c. Now generalize the above results for a two-area system by completing the following table with either – (for decrease), + (for increase), or 0 (no change). All cells should be filled. The $\Delta\omega$ and ΔP_{12} values are steady-state values following governor action but before secondary control action.

<table>
<tr><th rowspan="2">$\Delta\omega$</th><th rowspan="2">ΔP_{12}</th><th rowspan="2" colspan="2">Load change</th><th colspan="2">Resulting secondary control action</th></tr>
<tr><th>ΔP_{gen1}</th><th>ΔP_{gen2}</th></tr>
<tr><td>-</td><td>-</td><td>ΔP_{L1}</td><td>+</td><td rowspan="2">+</td><td rowspan="2">0</td></tr>
<tr><td></td><td></td><td>ΔP_{L2}</td><td>0</td></tr>
<tr><td>+</td><td>+</td><td>ΔP_{L1}</td><td>-</td><td rowspan="2">-</td><td rowspan="2">0</td></tr>
<tr><td></td><td></td><td>ΔP_{L2}</td><td>0</td></tr>
<tr><td>-</td><td>+</td><td>ΔP_{L1}</td><td>0</td><td rowspan="2">0</td><td rowspan="2">+</td></tr>
<tr><td></td><td></td><td>ΔP_{L2}</td><td>+</td></tr>
<tr><td>+</td><td>-</td><td>ΔP_{L1}</td><td>0</td><td rowspan="2">0</td><td rowspan="2">-</td></tr>
<tr><td></td><td></td><td>ΔP_{L2}</td><td>-</td></tr>
</table>

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2. (8 pts) In the Laplace domain, the change in mechanical power into a generation unit is given by

$$\Delta P_m(s) = -\frac{-\Delta P_L(s)}{R(1+sT_4)(1+sT_G)(Ms+D)+1}$$

Derive the steady-state value of ΔP_m for a step-change in load of "L", i.e., $\Delta P_L(t)=Lu(t)$ where u(t) is the unit step function occurring at t=0.

Solution:

Using $\Delta P_L(s)=\dfrac{L}{s}$, and applying the final value theorem, we have:

$$\lim_{t\to\infty} \Delta P_m(t) = \lim_{s\to 0} s\Delta P_m(s)$$

$$= \lim_{s\to 0} s \frac{L/s}{R(1+sT_4)(1+sT_G)(Ms+D)+1}$$

$$= \lim_{s\to 0} \frac{L}{R(1+sT_4)(1+sT_G)(Ms+D)+1}$$

$$= \frac{L}{RD+1}$$

3. (16 pts) The following optimization problem is solved by branch and bound:

maximize z=5x1+ 8x2

subject to

x1+x2≤ 6,   5x1+9x2 ≤ 45,   x1 ≥ 0,   x2 ≥ 0, x1 and x2 integer

a. The solution to the first step is z*=41.25, x1*=2.25, x2*=3.75. Write down four problems you could solve in the second step of the branch and bound algorithm.
b. For each one of the below situations, assume that the only thing you know is that one feasible solution is z*=39, x1*=3, x2*=3.
    i. Someone tells you that they have identified a certain node in the branch and bound algorithm for which the corresponding linear program provides a solution of z*=41, x1*=1.8, x2*=4. Would you branch further from this node? Why or why not?
    ii. Someone tells you that they have identified a certain node in the branch and bound algorithm for which the corresponding linear program is infeasible. Would you branch further from this node? Why or why not?
    iii. Someone tells you that they have identified a certain node in the branch and bound algorithm for which the corresponding linear program provides a solution of z*=37, x1*=1, x2*=4. Would you branch further from this node? Why or why not?
    iv. Someone tells you that they have identified a certain node in the branch and bound algorithm for which the corresponding linear program provides a solution of z*=40, x1*=0, x2*=5. Would you branch further from this node? Why or why not?
c. **Solution:**

a.

PROBLEM 1

maximize z=5x1+ 8x2

subject to

x1+x2≤ 6,   5x1+9x2 ≤ 45,   x1 ≥ 0,   x2 ≥ 0, x1 and x2 integer

x1≥3

PROBLEM 2

maximize z=5x1+ 8x2

subject to
$x1+x2 \le 6, \quad 5x1+9x2 \le 45, \quad x1 \ge 0, \quad x2 \ge 0$, x1 and x2 integer
$x1 \le 2$

PROBLEM 3
maximize z=5x1+ 8x2
subject to
$x1+x2 \le 6, \quad 5x1+9x2 \le 45, \quad x1 \ge 0, \quad x2 \ge 0$, x1 and x2 integer
$x2 \ge 4$

PROBLEM 4
maximize z=5x1+ 8x2
subject to
$x1+x2 \le 6, \quad 5x1+9x2 \le 45, \quad x1 \ge 0, \quad x2 \ge 0$, x1 and x2 integer
$x2 \le 3$

b.
i. Yes, we should branch further because the objective, 41, is better than the best bound obtained so far, 39.
ii. No, we should not branch further; because this solution is infeasible, it cannot be made feasible by adding constraints, and so further branching will just yield more infeasible linear programs.
iii. No, we should not branch further, for two reasons (either of which is enough to justify the answer). First, the given solution is feasible, therefore further branching (adding constraints), cannot improve it and remain feasible. Second, the objective, 37, is not as good as our best, 39, so further branching (adding constraints), cannot do better than 37 and so cannot do better than 39.
iv. No, we should not branch further because the given solution is feasible, therefore further branching (adding constraints) cannot improve it and remain feasible.

4. (16 pts) A two-unit system is dispatched so that P_{g1}=220 MW and P_{g2}=66.6 MW. The cost-rate functions for the two units are given as

$$C_1(P_{g1}) = 70 + 8.2P_{g1} + 0.01P_{g1}^2, \qquad C_2(P_{g2}) = 90 + 9.5P_{g2} + 0.02P_{g2}^2$$

An approximate loss expression is given as $P_L(P_{G2}) = 0.0002(P_{G2} - 50)^2$.

How would you redispatch the two units in order to lower the overall production costs? Show computations to justify your answer.

<u>Solution</u>:

You would lower P_{g1} and raise P_{g2} since these actions will decrease the product $L_1IC(Pg1)$ and increase the product $L_2(IC_2)$. Two sets of computations are provided below to justify this answer.

$$\frac{\partial C_1(P_{g1})}{\partial P_{g1}} = 8.2 + 0.02P_{g1} = 8.2 + 0.02*220 = 12.6; \quad \frac{\partial C_2(P_{g2})}{\partial P_{g2}} = 9.5 + 0.04P_{g2} = 9.5 + .04*66.6 = 12.164$$

$$L_1 = 1$$

$$L_2 = \frac{1}{\left[1 - \frac{\partial}{\partial P_{G2}} 0.0002(P_{G2} - 50)^2\right]} = \frac{1}{[1 - 0.0004(P_{G2} - 210)]} = \frac{1}{[1 - 0.0004(66.6 - 50)]} = 1.0067$$

The optimality criterion is $L_1 \dfrac{\partial C_1(P_{g1})}{\partial P_{g1}} = L_2 \dfrac{\partial C_2(P_{g2})}{\partial P_{g2}} \Rightarrow 1(12.6) = ?1.0067 * 12.164 = 12.2453$

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$$\frac{\partial C_1(P_{g1})}{\partial P_{g1}} = 8.2 + 0.02P_{g1} = 8.2 + 0.02 * 215 = 12.5; \quad \frac{\partial C_2(P_{g2})}{\partial P_{g2}} = 9.5 + 0.04P_{g2} = 9.5 + .04 * 71.6 = 12.36$$

$$L_1 = 1$$

$$L_2 = \frac{1}{\left[1 - \frac{\partial}{\partial P_{G2}} 0.0002(P_{G2} - 50)^2\right]} = \frac{1}{[1 - 0.0004(P_{G2} - 210)]} = \frac{1}{[1 - 0.0004(71.6 - 50)]} = 1.0087$$

The optimality criterion is $L_1 \dfrac{\partial C_1(P_{g1})}{\partial P_{g1}} = L_2 \dfrac{\partial C_2(P_{g2})}{\partial P_{g2}} \Rightarrow 1(12.5) = ?1.0087 * 12.36 = 12.352$

5. (24 pts) Define the following for a two-generator, three bus, three line system. Buses 1 and 2 have generation but no load. Bus 3 has load only.

P_i^0	pre-contingency injection at bus i
P_i^k	post-contingency injection at bus i for outage of circuit k.
P_i^{min}	minimum injection at bus i
P_i^{max}	maximum injection at bus i
$a_{\ell,i}$	the shift factor for circuit ℓ, given injection at bus i, when all circuits are in service. It is used to obtain the flow on circuit ℓ given injections at all buses i according to $f_\ell^0 = \sum_{i=1}^{3} a_{l,i} P_i$
$d_{\ell,k}$	the line outage distribution factor for circuit ℓ, given outage of circuit k.
$\alpha_{\ell,k,i}$	the effective shift factor, given by $a_{\ell,i} + d_{\ell,k} a_{k,i}$. It is used to obtain the flow on circuit ℓ following outage of circuit k and given injections at all buses i according to $f_\ell^k = \sum_{i=1}^{3} \alpha_{l,k,i} P_i$
f_l^{CMax}	the maximum continuous flow on circuit ℓ
f_l^{EMax}	the maximum emergency flow on circuit ℓ

$C_1 = f_1(P_1^0)$ and $C_2 = f_2(P_2^0)$ are cost-rate functions for generators at buses 1 and 2, respectively.

K_i The 10 minute maximum change in P_i, i=1,2.

Using only the nomenclature given above,

a. Provide a formulation for the optimal power flow (OPF).

Solution:

min $C_1 + C_2 = f_1(P_1^0) + f_2(P_2^0)$

subject to

$P_1 + P_2 = P_3$

$P_i^{min} \le P_i^0 \le P_i^{max}$ for i=1,2,3.

$$-f_l^{C\max} \le \sum_{i=1}^{3} a_{l,i} P_i^0 \le f_l^{C\max} \quad \text{for } \ell = 1,2,3$$

b. Provide a formulation for the preventive-security-constrained OPF.

Solution:

$$\min \; C_1 + C_2 = f_1(P_1^0) + f_2(P_2^0)$$

subject to

$P_1+P_2=P_3$

$$P_i^{\min} \le P_i^0 \le P_i^{\max} \quad \text{for i=1,2,3.}$$

$$-f_l^{C\max} \le \sum_{i=1}^{3} a_{l,i} P_i^0 \le f_l^{C\max} \quad \text{for } \ell = 1,2,3$$

$$-f_l^{E\max} \le \sum_{i=1}^{3} \alpha_{l,1,i} P_i^0 \le f_l^{E\max} \quad \text{for } \ell = 2,3$$

$$-f_l^{E\max} \le \sum_{i=1}^{3} \alpha_{l,2,i} P_i^0 \le f_l^{E\max} \quad \text{for } \ell = 1,3$$

$$-f_l^{E\max} \le \sum_{i=1}^{3} \alpha_{l,3,i} P_i^0 \le f_l^{E\max} \quad \text{for } \ell = 2,3$$

c. Provide a formulation for the corrective-security-constrained OPF, where we assume corrective actions must be limited to how much the controls can move in 10 minutes.

Solution:

$$\min \; C_1 + C_2 = f_1(P_1^0) + f_2(P_2^0)$$

subject to

$P_1+P_2=P_3$

$$P_i^{\min} \le P_i^0 \le P_i^{\max} \quad \text{for i=1,2,3.}$$

$$P_i^{\min} \le P_i^1 \le P_i^{\max} \quad \text{for i=1,2,3.}$$

$$P_i^{\min} \le P_i^2 \le P_i^{\max} \quad \text{for i=1,2,3.}$$

$$P_i^{\min} \le P_i^3 \le P_i^{\max} \quad \text{for i=1,2,3.}$$

$$-f_l^{C\max} \le \sum_{i=1}^{3} a_{l,i} P_i^0 \le f_l^{C\max} \quad \text{for } \ell = 1,2,3$$

$$-f_l^{E\max} \le \sum_{i=1}^{3} \alpha_{l,1,i} P_i^1 \le f_l^{E\max} \quad \text{for } \ell = 2,3$$

$$-f_l^{E\max} \le \sum_{i=1}^{3} \alpha_{l,2,i} P_i^2 \le f_l^{E\max} \quad \text{for } \ell = 1,3$$

$$-f_l^{E\max} \le \sum_{i=1}^{3} \alpha_{l,3,i} P_i^3 \le f_l^{E\max} \quad \text{for } \ell = 2,3$$

$$-K_i \le P_i^1 - P_i^0 \le K_i, \text{ i=1,2.}$$

$$-K_i \le P_i^2 - P_i^0 \le K_i \text{ i=1,2.}$$

$$-K_i \le P_i^3 - P_i^0 \le K_i \text{ i=1,2.}$$

6. (13 pts) True-False:
    a. _F_ If load for a certain system consisted of 95% induction motors, it would make sense to set the frequency sensitivity coefficient D=0 in AGC analysis.
    b. _T_ A control area that always increases its generation in response to its ACE when the interconnection steady-state frequency is below 60 Hz will have a good CPS1 score.
    c. _F_ A reasonable range for heat rates in units of MBTU/MWhr, is 2.0-7.0.
    d. _T_ The lower the heat rate, the more efficient the unit.
    e. _T_ A generator cost function, in $/hr, is convex if its slope is non-decreasing with increasing generation.
    f. _F_ Modeling of combined cycle units in our LPOPF formulation is not possible because their cost functions are convex.
    g. _F_ Modeling of combined cycle units in our LPOPF formulation is not possible because piecewise linear approximations cannot be used for nonconvex cost curves.
    h. _T_ The exact cost curve for any steam unit with multiple valves is nonconvex.
    i. _T_ Assuming identical cost curve representation, running a fixed demand LPOPF with infinite transmission capacity results in the same dispatch obtained from a traditional economic dispatch calculation where transmission is not represented.
    j. _T_ A lossless LPOPF formulation with unconstrained transmission will always result in all identical nodal prices throughout the network.
    k. _T_ In an LPOPF formulation, the nodal price for a bus is the Lagrange multiplier corresponding to the equality constraint on the bus's MW injection.
    l. _F_ One efficient method of solving the 24-hour unit commitment problem that yields a very good but approximate solution when start-up costs are high is to solve a security-constrained preventive-corrective optimal power flow with all units connected at each individual hour, and then for that hour, shut down the units that are at their lower limits.
    m. _F_ The day-ahead markets for most ISOs use only the security-constrained unit commitment in obtaining the locational marginal prices for each hour of the given 24 hour period.