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## Exam 1, EE 553, Fall 2012, Dr. McCalley

 Close books, closed notes, calculator permitted, 75 minute time limit. Students are permitted to bring their $\mathrm{HW} \# 3$ to the exam and use it.1. (14 pts) (This problem is identical to prob 11.3 in the $W \& W$ text; if you worked it in advance and brought the solution with you, you may turn in that as your solution to this part of the exam.) Consider the three-bus network shown in the figure below where $\mathrm{x}_{12}=0.2 \mathrm{pu}, \mathrm{x}_{13}=0.4 \mathrm{pu}$ and $\mathrm{x}_{23}=0.25 \mathrm{pu}$. The $[\mathrm{X}]$-matrix for this system is also given below:


$$
[X]=\left[\begin{array}{ccc}
0.2118 & 0.1177 & 0 \\
0.1177 & 0.1765 & 0 \\
0 & 0 & 0
\end{array}\right]
$$

Use a 100-MVA base. The base loads and generations are as follows:

| Bus | Load <br> (MW) | Gen <br> (MW) | Min Gen <br> (MW) | Max Gen <br> (MW) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 100 | 150 | 50 | 250 |
| 2 | 300 | 180 | 60 | 250 |
| 3 | 100 | 170 | 60 | 300 |

a. Find base power flows on the transmission lines.
b. Calculate the generation shift factors for line 1-2. Calculate the shift in generation on bus 1 and 2 so as to force the flow on line 1-2 to zero MW. Assume for economic reasons that any shifts from base conditions are more expensive for shifts at the generator on bus 1 that for shifts on bus 2 , and that the generator on bus 3 can be shifted without any penalty.

## Solution:

a)

Problem i1.3, continued

2. (12 pts) Consider the four-bus, five-line power system shown below. All indicated values are admittances given in per-unit.

a. Develop the Y-bus matrix (i.e., the admittance matrix).

Solution:
$Y_{\text {bus }}=\left[\begin{array}{cccc}3-j 9 & -2+j 6 & -1+j 3 & 0 \\ -2+j 6 & 3.666-j 11 & -0.666+j 2 & -1+j 3 \\ -1+j 3 & -0.666+j 2 & 3.666-j 11 & -2+j 6 \\ 0 & -1+j 3 & -2+j 6 & 3-j 8.8\end{array}\right]$
b. Develop the B' matrix used in the DC power flow calculation. Assume bus 1 is the reference.

## Solution:

With resistance neglected, and ignoring shunt terms, we have:

$$
Y_{\text {bus }}=\left[\begin{array}{cccc}
-j 9 & j 6 & j 3 & 0 \\
j 6 & -j 11 & j 2 & j 3 \\
j 3 & j 2 & -j 11 & j 6 \\
0 & j 3 & j 6 & -j 9
\end{array}\right]
$$

Dropping the j , and multiplying by -1 , we have:

$$
\left[\begin{array}{cccc}
9 & -6 & -3 & 0 \\
-6 & 11 & -2 & -3 \\
-3 & -2 & 11 & -6 \\
0 & -3 & -6 & 9
\end{array}\right]
$$

Since the reference is bus 1 , we eliminate row 1 and column 1 , resulting in

$$
B^{\prime}=\left[\begin{array}{ccc}
11 & -2 & -3 \\
-2 & 11 & -6 \\
-3 & -6 & 9
\end{array}\right]
$$

3. ( 20 pts) A 6000 bus system has 900 generators, all of which are modeled in a power flow program with specified terminal voltage and reactive limits, i.e., they are voltage-controlled buses, and there are no other voltage-controlled buses besides these 900 generator buses.
a. At the beginning of the Newton-Raphson solution procedure (initialization of iteration 1):
i. How many type PV buses are there in the power flow model?
ii. How many type PQ buses are there in the power flow model?
iii. What is the number of power flow equations used?
iv. How many bus voltage magnitudes are unknown?
v. How many bus voltage angles are unknown?
b. After one iteration, the reactive power of two of the type PV buses is computed to exceed its upper reactive limit. In initializing for iteration 2, relative to iteration 1 (+ is increase, - is decrease):
i. What is the change in the number of type PV buses in the power flow model?
ii. What is the change in the number of type PQ buses in the power flow model?
iii. What is the change in the number of power flow equations used?
iv. What is the change in the number of bus voltage magnitudes that are unknown?
v. What is the change in the number of bus voltage angles that are unknown?

## Solution:

a.
i. 899
ii. 5100
iii. 11099
iv. 5100
v. 5999
b. Lkj
i. -2
ii. +2
iii. +2
iv. $\quad+2$
v. 0
4. (18 pts) A two-bus power system is interconnected by a transmission line having series admittance of $\mathrm{Y}_{12}=0.304-\mathrm{j} 01.88 \mathrm{pu}$ and total line charging of $\mathrm{Y}_{\text {charging }}=\mathrm{j} 0.064 \mathrm{pu}$. Bus 2 is a load bus with specified demand as $P_{D}$ pu and $Q_{D}$ pu. Bus 1 is a generator bus with specified terminal voltage magnitude of $\left|V_{1}\right|$ pu. We desire to solve the power flow problem for this system.
a. Form the Y-bus for this system.
b. Identify the variables in the solution vector $\underline{x}$.
c. Write down the mismatch equation(s) that are required in the solution procedure. Express each equation symbolically (no numbers). Denote each equation by $g_{i}, i=1, \ldots$
d. Write down the Jacobian matrix $\underline{J}$ to be used in the solution procedure. Indicate the elements in the matrix using partial derivative notation; you do not need to differentiate anything or provide any numerical values.
e. Write down the update formula for this system in terms of $\mathbf{J}$ and the solution vector.
f. A generator is now brought on-line at bus 2 so that it becomes a type PV bus. Repeat part d.

## Solution:

a. $\quad Y_{\text {bus }}=\left[\begin{array}{cc}.304-j 1.35 & -.304+j 1.88 \\ -.304+j 1.88 & .304+j 1.35\end{array}\right]=\left[\begin{array}{cc}1.875 \angle-30.7^{\circ} & 1.9 \angle 99.18^{\circ} \\ 1.9 \angle 99.18^{\circ} & 1.875 \angle-30.7^{\circ}\end{array}\right]$
b. $\quad$ solution $=\underline{x}=\left[\begin{array}{c}\theta_{2} \\ \left|V_{2}\right|\end{array}\right]$

$$
g_{1}=P_{2, s c h}-P_{2}(\underline{x})
$$

c. $\quad g_{2}=Q_{2, s c h}-Q_{2}(\underline{x})$
d. $\underline{J}=\left[\begin{array}{ll}\frac{\partial g_{1}}{\partial \theta_{2}} & \frac{\partial g_{1}}{\partial\left|V_{2}\right|} \\ \frac{\partial g_{2}}{\partial \theta_{2}} & \frac{\partial g_{2}}{\partial\left|V_{2}\right|}\end{array}\right]$
e. $\quad \underline{J} \Delta \underline{x}=\underline{J}\left[\begin{array}{c}\Delta \theta_{2} \\ \Delta\left|V_{2}\right|\end{array}\right]=\left[\begin{array}{c}\Delta P_{2} \\ \Delta Q_{2}\end{array}\right]$
f. $\quad \underline{J}=\left[\frac{\partial g_{1}}{\partial \theta_{2}}\right]$
5. ( 15 pts ) A 2-bus power system is interconnected by a transmission line having series admittance of $\mathrm{y}_{12}=-\mathrm{j} 10$ pu. Bus 2 is a load bus. Bus 1 is a generator bus. A SCADA system presents the following measurements to the state estimator: $z_{1}=V_{1}=1.02 ; z_{2}=V_{2}=1.0 ; z_{3}=P_{12}=2.0$.
The standard deviations of error on the meters are $0.05,0.05$, and 0.1 , respectively.
a. Identify the elements for the state vector x for this problem.

## Solution:

States are $\underline{x}=\left[\begin{array}{lll}V_{1} & V_{2} & \theta_{2}\end{array}\right]^{T}$
b. Express $\underline{\mathrm{h}}(\underline{\mathrm{x}})$, the measurements as functions of the states using AC power flow expressions.

$$
\begin{aligned}
& h_{1}(\underline{x})=V_{1} \\
& h_{2}(\underline{x})=V_{2} \\
& h_{3}(\underline{x})=-10 V_{1} V_{2} \sin \theta_{2}
\end{aligned}
$$

c. The first iteration of the state estimator assumes an initial state $\underline{x}^{(0)}$ and solves $\underline{A} \Delta \underline{x}=\underline{b}$ using

$$
\underline{A}=\left.\underline{H}^{T}(\underline{x}) \underline{R}^{-1} \underline{H}(\underline{x})\right|_{\underline{x}^{(0)}}, \underline{b}=\underline{H}^{T}(\underline{x}) \underline{R}^{-1}[(\underline{z}-\underline{h}(\underline{x}))]_{\underline{x}^{(0)}}
$$

Express $\underline{H}, \underline{R}$, and $\underline{z}-\underline{h}(\underline{x})$ for this problem. You need not evaluate anything, just express in a fashion so it is clear that you know how to evaluate it.

## Solution:

$$
\begin{aligned}
& \underline{H}=\frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}=\left[\begin{array}{lll}
\frac{\partial h_{1}(\underline{x})}{\partial x_{1}} & \frac{\partial h_{1}(\underline{x})}{\partial x_{2}} & \frac{\partial h_{1}(\underline{x})}{\partial x_{3}} \\
\frac{\partial h_{2}(\underline{x})}{\partial x_{1}} & \frac{\partial h_{2}(\underline{x})}{\partial x_{2}} & \frac{\partial h_{2}(\underline{x})}{\partial x_{3}} \\
\frac{\partial h_{3}(\underline{x})}{\partial x_{1}} & \frac{\partial h_{3}(\underline{x})}{\partial x_{2}} & \frac{\partial h_{3}(\underline{x})}{\partial x_{3}}
\end{array}\right]=\left[\begin{array}{ccc}
\frac{\partial h_{1}(\underline{x})}{\partial V_{1}} & \frac{\partial h_{1}(\underline{x})}{\partial V_{2}} & \frac{\partial h_{1}(\underline{x})}{\partial \theta_{2}} \\
\frac{\partial h_{2}(\underline{x})}{\partial V_{1}} & \frac{\partial h_{2}(\underline{x})}{\partial V_{2}} & \frac{\partial h_{2}(\underline{x})}{\partial \theta_{2}} \\
\frac{\partial h_{3}(\underline{x})}{\partial V_{1}} & \frac{\partial h_{3}(\underline{x})}{\partial V_{2}} & \frac{\partial h_{3}(\underline{x})}{\partial \theta_{2}}
\end{array}\right] \\
& \underline{R}=\left[\begin{array}{cccc}
\sigma_{1}^{2} & 0 & \cdots & 0 \\
0 & \sigma_{2}^{2} & 0 & 0 \\
\vdots & 0 & \ddots & 0 \\
0 & 0 & 0 & \sigma_{m}^{2}
\end{array}\right]=\left[\begin{array}{ccc}
0.0025 & 0 & 0 \\
0 & 0.0025 & 0 \\
0 & 0 & 0.01
\end{array}\right]
\end{aligned}
$$

$$
\underline{z}-\underline{h}(\underline{x})=\left[\begin{array}{c}
1.02-V_{1} \\
1.0-V_{2} \\
2+10 V_{1} V_{2} \sin \theta_{2}
\end{array}\right]
$$

6. (21 pts) Answer the following questions:
a. Consider the full Newton power flow, the fast-decoupled power flow, and the DC power flow. For each of the following characteristics, rank the three methods as 1 (best), 2, and 3 (worst). Methods that are equal in one of the characteristics should get the same rank.

| Solution method | Computational Speed | Accuracy |
| :--- | :--- | :--- |
| Newton-Raphson |  |  |
| Fast decoupled |  |  |
| DC |  |  |

## Solution:

| Solution method | Computational Speed | Accuracy |
| :--- | :--- | :--- |
| Newton-Raphson | 3 | 1 |
| Fast decoupled | 2 | 1 |
| DC | 1 | 3 |

b. An engineer modifies a full Newton power flow method by zeroing the elements in the $\mathrm{J}^{\mathrm{P} \mathrm{\theta}}$ and $\mathrm{J}^{\mathrm{QV}}$ submatrices and using the resulting Jacobian matrix to update the solution at each iteration. Do you think this method will be faster than the full Newton method? Why or why not?

Solution: No, it would not be faster and in fact would probably not converge at all. The reason is that the elements of the $\mathrm{J}^{\mathrm{P} \theta}$ and $\mathrm{J}^{\mathrm{QV}}$ submatrices are the largest (in magnitude) elements of the Jacobian, as a result of the fact that the real power is highly sensitive to voltage angle, and reactive power is highly sensitive to voltage magnitude.
c. Identify the four contingency categories of the NERC disturbance-performance table, and for each one, characterize the event in terms of how many elements may be lost, and indicate whether demand interruption is allowed.

## Solution:

Category A: No elements lost, demand interruption is not allowed.
Category B: Loss of a single element, demand interruption is not allowed.
Category C: Loss of 2 or more elements, demand interruption is allowed but most be controlled.
CategoryD: Extreme event resulting in loss of 2 or more elements, demand interruption is allowed.
d. Which of the below situations are in the alert state?
i. The power system is operating with all elements in. There is just one "loss of one element" contingency which will result in second circuit to exceed its continuous rating but not its emergency rating.
ii. The power system is operating with all elements in. There is just one "loss of one element" contingency which will result in a second circuit to exceed its emergency rating.
iii. The power system is operating with all elements in. There is just one "loss of one element" contingency which will result in an operating condition such that a second "loss of one element" contingency, if it occurred following manual system adjustments subsequent to the first contingency, would result in a third circuit to exceed its emergency rating.

## Solution:

Situation (ii) and (iii) are in the alert state. Situation (i) is not.
e. Optimal ordering scheme \#1 numbered buses so that a lower numbered node has less or equal number of actual adjacent branches than any higher numbered node. In applying such a scheme to a 5000 bus network, which column of the Y-bus would tend to have the most non-zero elements, column 1, column 2000, or column 4000? Explain your choice.

## Solution:

Column 4000 would tend to have the most non-zero elements. Since column 4000 corresponds to a bus numbered higher than the other two buses, it must be connected to as many buses, and most likely more buses, than either of the buses corresponding to columns 0 and 2000. Since each time a bus is connected to another bus, it will create another non-zero element in the column corresponding to that bus, the higher numbered columns will tend to have the most non-zero elements.
f. You are given the generation shift factors $t_{b, k}$ and $t_{b, j}$ which are based on a reallocation policy of "bus \#1 compensates," i.e., the shift factors are computed using bus \#1 as a single slack bus. Write down an expression that shows how to use these shift factors to compute the change in flow on branch b for $\Delta \mathrm{P}_{\mathrm{k}}=+0.5 \mathrm{pu}$ and $\Delta \mathrm{P}_{\mathrm{j}}=-0.5 \mathrm{pu}$ ?

## Solution:

$$
\Delta P_{b}=t_{b, k} \Delta P_{k}+t_{b, j} \Delta P_{j}
$$

g. An AC state estimator based on minimization of least square error informs that the current condition is unobservable.
i. What led to this detection?

## Solution:

Singularity of the A matrix, or, equivalently, identification of the H matrix as having rank less than $n$, where $n$ is the number of columns of $H$.
ii. What the least expensive way of correcting the problem so that the condition becomes observable?

## Solution:

Add pseudo-measurements.

