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Treatment of Uncertainty in Long-Term Planning

1 Introduction

The problem that the long-term planner is faced with solving is an inherently uncertain one because it addresses the future. In making use of software which implements generation expansion planning (GEP), transmission expansion planning (TEP), or co-optimized expansion planning (CEP), it is necessary to make many assumptions on what that future will be. Examples of attributes characterizing the future about which the planner must make assumptions include:

- Cost of money (discount rate)
- The rate at which technology investment cost will change (maturation rate)
- Fuel costs forecast
- Demand forecast
- Plant retirement dates and salvage value
- Policy changes (e.g., changes in production tax credit and/or renewable portfolio)
- Renewable resources (e.g., wind and solar resources)
- Distributed generation growth

In these notes, we describe different ways to represent uncertainty and different ways to model it within optimization models such as GEP, TEP, and CEP.

2 Representing uncertainty

One can represent uncertainty by identifying the range within which one may reasonably expect each attribute to lie. For example, we could specify the price of natural gas in one of the following ways:

<u>Time-independent</u>:

- Point value: For all years, it will be $4.5/MMBTU;

- Range: For all years, it lies between \$3/MMBTU and \$6/MMBTU;
- Distribution: For all years, it is normally distributed with an expected value of \$4.5/MMBTU and a standard deviation of \$0.5/MMBTU; as shown in Figure 1 below, this means it will fall within the $\mu\pm3\sigma=4.5\pm1.5=(3,6)$ with probability 0.997, i.e., there is only a 0.003 probability of finding it outside the range of (3,6).

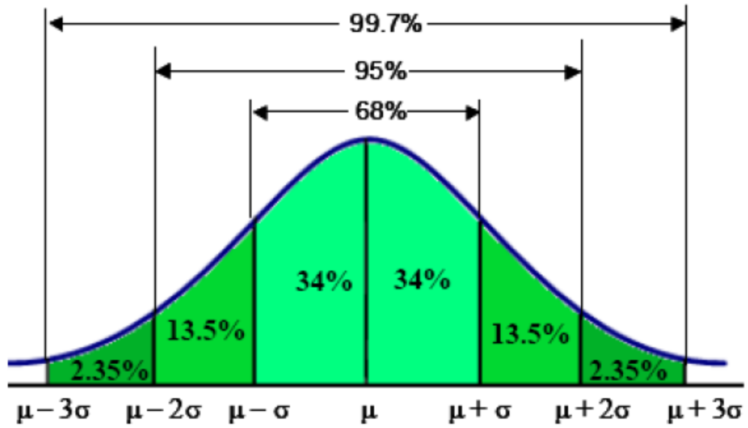


Figure 1: Confidence intervals for a normally distributed variable

<u>Time-dependent</u>:

- Point value: The year 1 value will be \$4.5/MMBTU and will grow at 2% per year.
- Range: The year 1 value will fall within a range of \$3/MMBTU to \$6/MMBTU, with the central value of \$4.5/MMBTU growing at 2% per year; the lower bound growing at 1% per year and each the upper bound growing at 3% per year.
- Distribution: The year 1 expected value will be \$4.5/MMBTU with a \$0.5 standard deviation, the expected value will grow 2% per year and the standard deviation will grow 5% per year. A plot of this uncertainty would appear as in Figure 2. One observes in this figure how (a) the expected price will increase with time, and (b) the uncertainty will also increase with time.

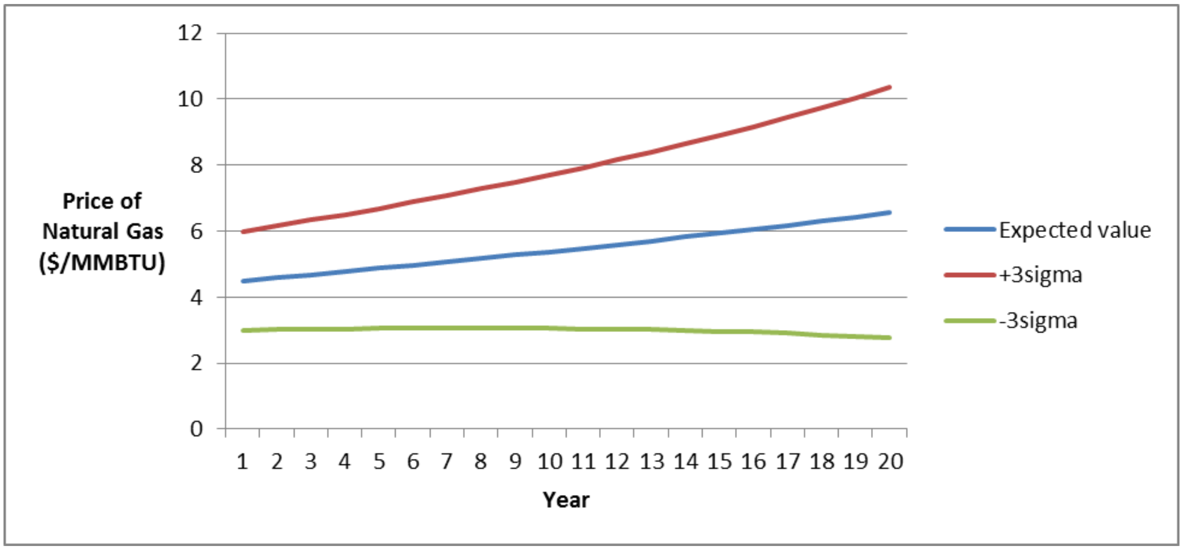


Figure 2: Specification of uncertainty in natural gas price

Aside: We may also apply advanced forecasting techniques to provide future estimates of expected value and uncertainty. Some forecasting methods that are commonly used for this purpose include regression, time series forecasting (ARIMA models and exponential smoothing models), or neural networks and other machine learning methods. These are worthy topics of study for uncertainty representation, but we do not have time to address them.

3 Two classes of uncertainty

We may group uncertainty into two different classes.

- **Global uncertainties** are those for which different values produce significantly different expansion planning results. Examples of global uncertainties are those related to the implementation of emissions policies, very large changes in demand growth, public rejection of a certain type of resource (nuclear) and its consequential unavailability, or an innovation that results in dramatic change in a technology's investment costs. A set of realizations on global uncertainties are appropriately thought of as a *future* (some literature will use the term *scenario* with this). It is often difficult to forecast global uncertainties because they may have occurred rarely or never, so that there is no historical information that can be used in making statistical inferences about their future occurrence.

- **Local uncertainties** can be parameterized by probability distributions or uncertainty sets based on historical data. Examples of local uncertainties include small variations in near-term load growth, investment costs, and fuel prices.

Figure 3 illustrates conceptualization of a single uncertainty in terms of being represented globally and locally.

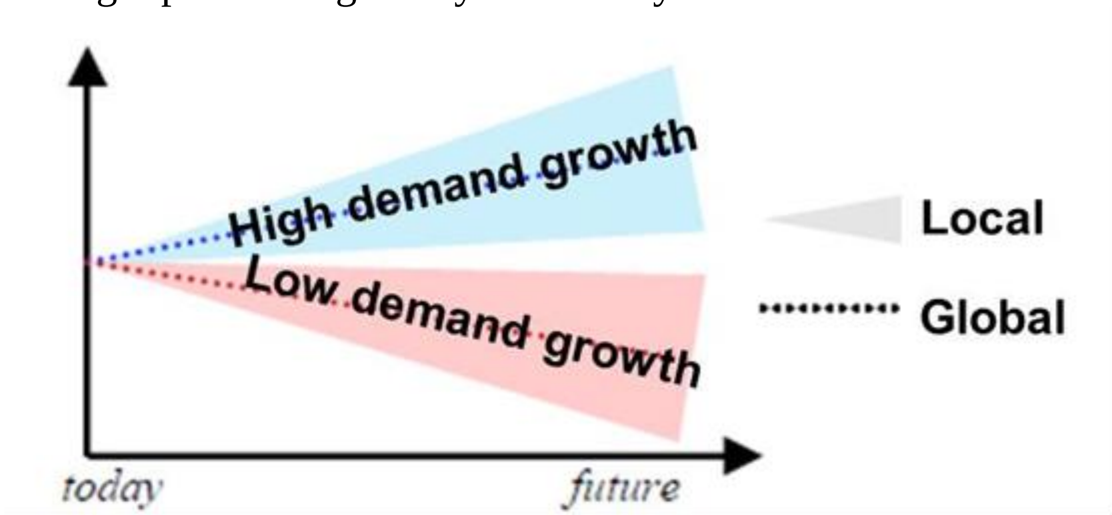


Figure 3: Conceptualization of a single uncertainty characterized globally and locally

Figure 4 illustrates conceptualization of multiple uncertainties in terms of being represented globally and locally. Each large red arrow represents a different set of realizations on several global uncertainties, i.e., they are different futures. The grey cones represent local uncertainty within each future. The pie charts are generation portfolios corresponding to the GEP solution resulting from consideration of the given uncertainties.

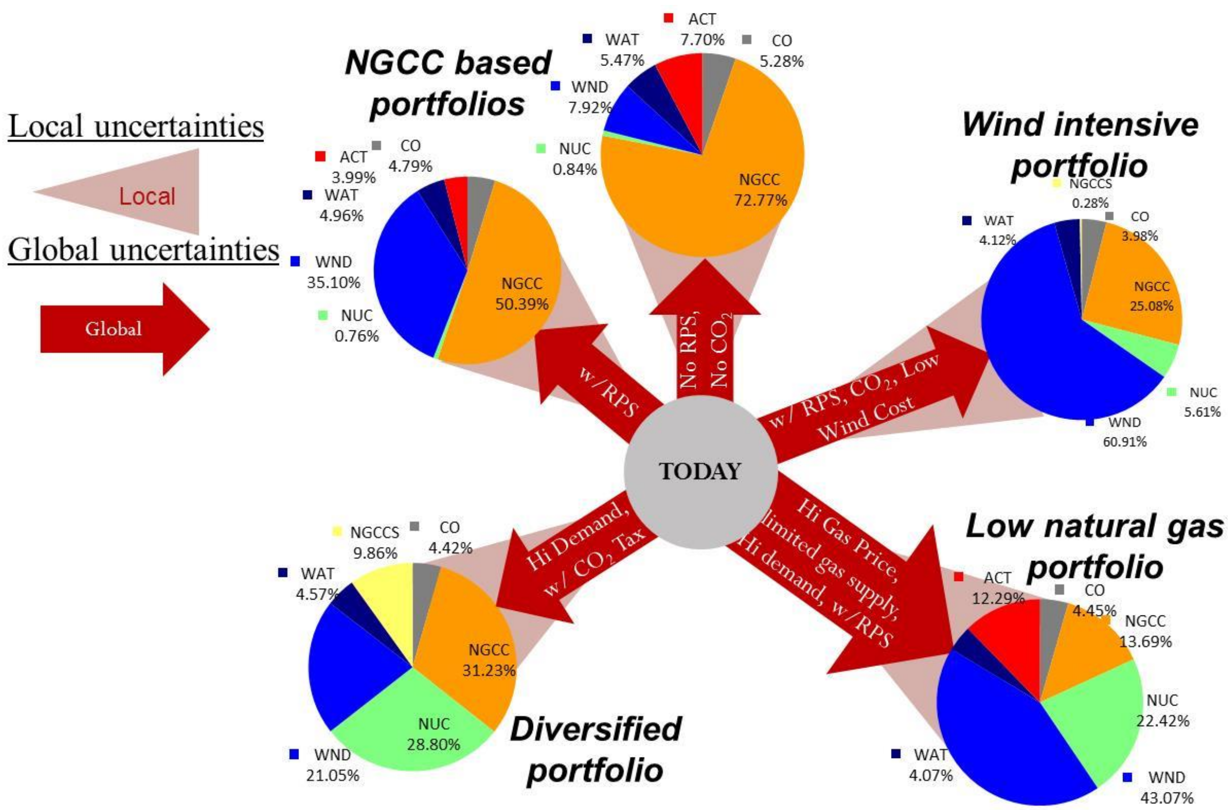


Figure 4: Conceptualization of multiple uncertainties characterized globally and locally

4 Methods of handling uncertainty within optimization

There are at least five ways of handling uncertainty within expansion planning optimization.

- Scenario analysis
- Monte Carlo simulation
- Stochastic programming
- Adaptation: core approach
- Robust optimization

We will describe each of these in the following sections.

5 Scenario analysis

In the simplest of scenario analyses, each uncertain attribute may take on two or more point values. A scenario is defined as a set of realizations on each uncertain attribute. An example from a 2008 study done by MISO is illustrative[1]. This example was taken from [1]. Table 1 shows an uncertainty matrix which provides six point values (low, med/low, reference, med/high, and high) for each of several uncertainties. The uncertainties are classified into capital costs, load, fuel prices, environmental allowance cost, economic variables, and siting limitations.

Table 1: Uncertainty matrix

Uncertainty	Unit	Low	Mid/Low	Reference	Mid/High	High
Alternative Capital Costs						
Coal	($/KW)	1,653		1,835		2,019
CT	($/KW)	545		605		665
CC	($/KW)	774		859		945
IGCC	($/KW)	1,901		2,111		2,323
Nuclear	($/KW)	2,245		2,493		2,743
Wind	($/KW)	1,720		1,910		2,101
CC w/Sequestration	($/KW)	1,003		1,114		1,226
IGCC w/Sequestration	($/KW)	2,475		2,748		3,023
Demand Response	($/KW)					
Storage	($/KW)					
Load						
Demand Growth Rate	%	PowerBase -25%		PowerBase		PowerBase +25%
Energy Growth Rate	%	PowerBase -25%		PowerBase		PowerBase +25%
Energy Efficiency	%	None		None		3% over 10 Years
Fuel Prices						
Gas	($/MMBtu)	Reference -20%		2007 w/4% Growth	Ref + 20%	Reference + 50%
Oil	($/MMBtu)	Reference -10%		2007 w/4% Growth		Reference + 10%
Coal	($/MMBtu)	Reference -10%		2007 w/2% Growth		Reference + 10%
Uranium	($/MMBtu)	Reference -10%		2007 w/2% Growth		Reference + 10%
Environmental Allowance Cost						
SO2	($/ton)	Reference -25%		PowerBase		Reference +25%
NOx	($/ton)	Reference -25%		PowerBase		Reference +25%
CO2	($/ton)	0		0	7	25
Hg	($/ton)	Reference -25%		PowerBase		Reference +25%
Economic Variables						
Wind Credit	$	0		19		19
Discount Rate	%	5		8		10
Inflation Rate	%	2		3		5
Uneconomic Coal Retirement		As Scheduled		As Scheduled		Forced Retirement
Siting Limitations						
Limited Transmission		No Limitations		No Limitations		Limitation
Delayed Lead Time on Coal/IGCC		No Delay		No Delay		5 Year Delay
Nuclear Siting Limitation		Existing & Allowed		Existing & Allowed		No Limitation
CT & CC Siting Limitation		No Limitations		No Limitation		Limited to Urban Areas

Five different scenarios were created by selecting specific values for the various uncertainties. The five different scenarios were named Reference, DOE 20% Wind Mandate, DOE 30% Wind Mandate, Environmental, and Regulatory Limitation. The specific choices of each uncertain variable for each scenario is listed in the scenario matrix of Table 2 where the entries are L (low), R (reference), M (not sure), and H (high).

[1] This was a part of the so-called Joint-Coordinated System Plan (JCSP) studies. Many other analyses were done for the JCSP studies than what are shown here, and certainly, since then, MISO has evolved this procedure in many other studies.

Table 2: Scenario Matrix

Uncertainty Matrix	Capital Investments										Load			Fuel				Effluent $				Econ Variab.				Siting			
Futures	CC	CT	Coal	IGCC	Wind	Nuclear	CC w/Sequestration	IGCC w/sequestration	Demand Response	Storage	Demand growth Rate	Energy Growth Rate	Energy Efficiency	Gas	Coal	Oil	Uranium	SO2	NOx	CO2	Hg	Wind Credit	Inflation	Discount Rate	Coal Uneconomic	Limited Transmission	Delayed Lead Time on Coal/IGCC	Nuclear Siting Limitation	CC & CT Siting Limitation
Reference	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R	R
DOE 20% Wind Mandate	R	R	R	R	R	R	R	R	R	M	R	R	R	R	R	R	R	R	R	R	R	L	R	R	M	R	R	R	R
DOE 30% Wind Mandate	R	R	R	R	R	R	R	R	R	H	R	R	R	R	R	R	R	R	R	R	R	L	R	R	H	R	R	R	R
Midwest ISO Additional Futures Being Modeled for MTEP 2009																													
Environmental	H	H	R	R	R	R	H	H	H	H	L	L	H	H	L	H	R	R	R	H	H	R	H	R	H	R	R	H	R
Regulatory Limitation	H	H	R	R	R	R	R	R	H	R	R	R	H	M	R	H	R	R	M	M	R	R	R	R	R	H	H	R	H

A generation expansion plan was created, for each Eastern Interconnection region (see Figure 5), and for each scenario, using a 15% planning reserve margin.

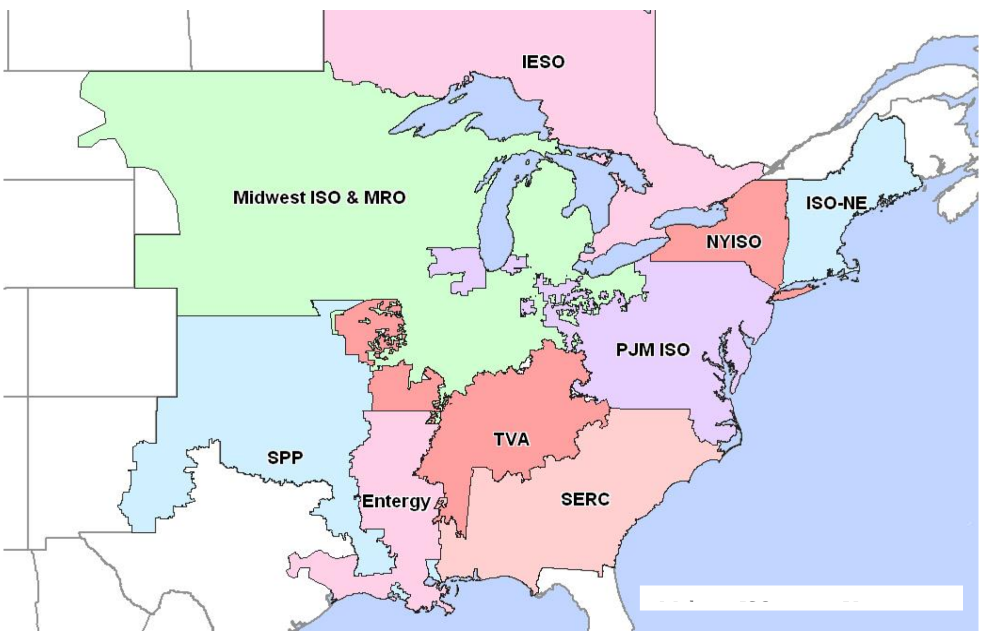


Figure 5: Eastern Interconnection Regions used in Study

Two transmission designs were developed, one under the reference scenario and one under the DOE 20% wind mandate scenario. They are illustrated below in Figure 6 and Figure 7.

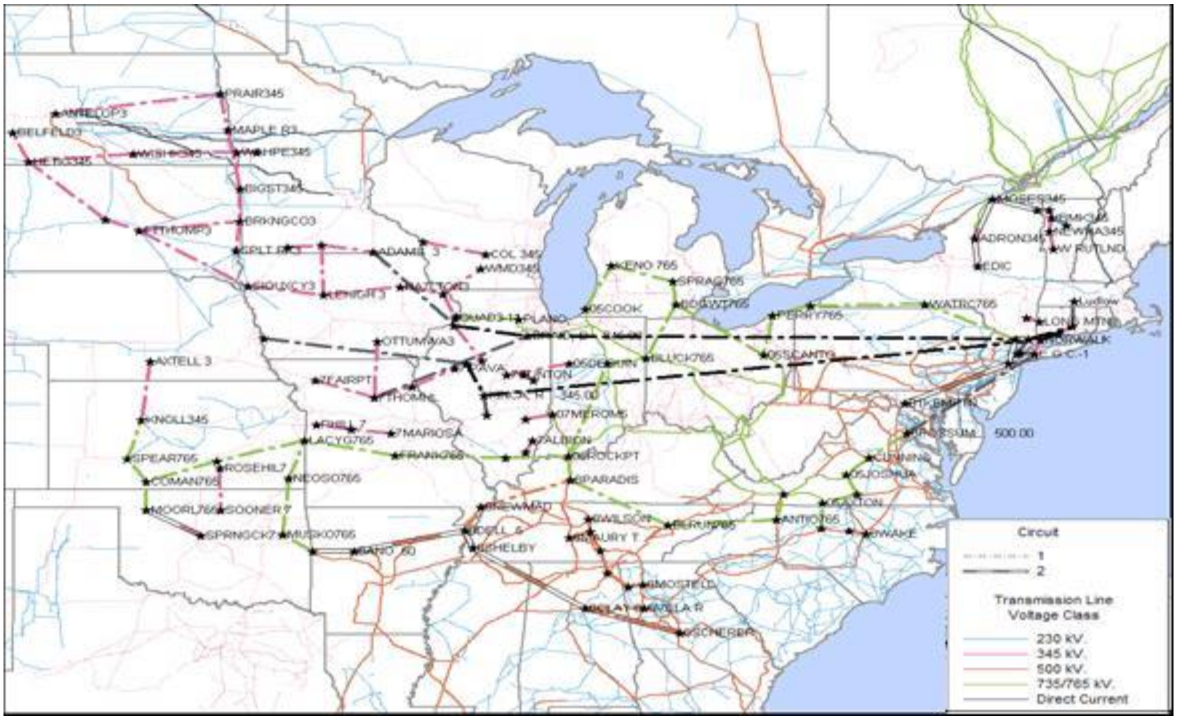


Figure 6: Transmission design created for Reference Scenario

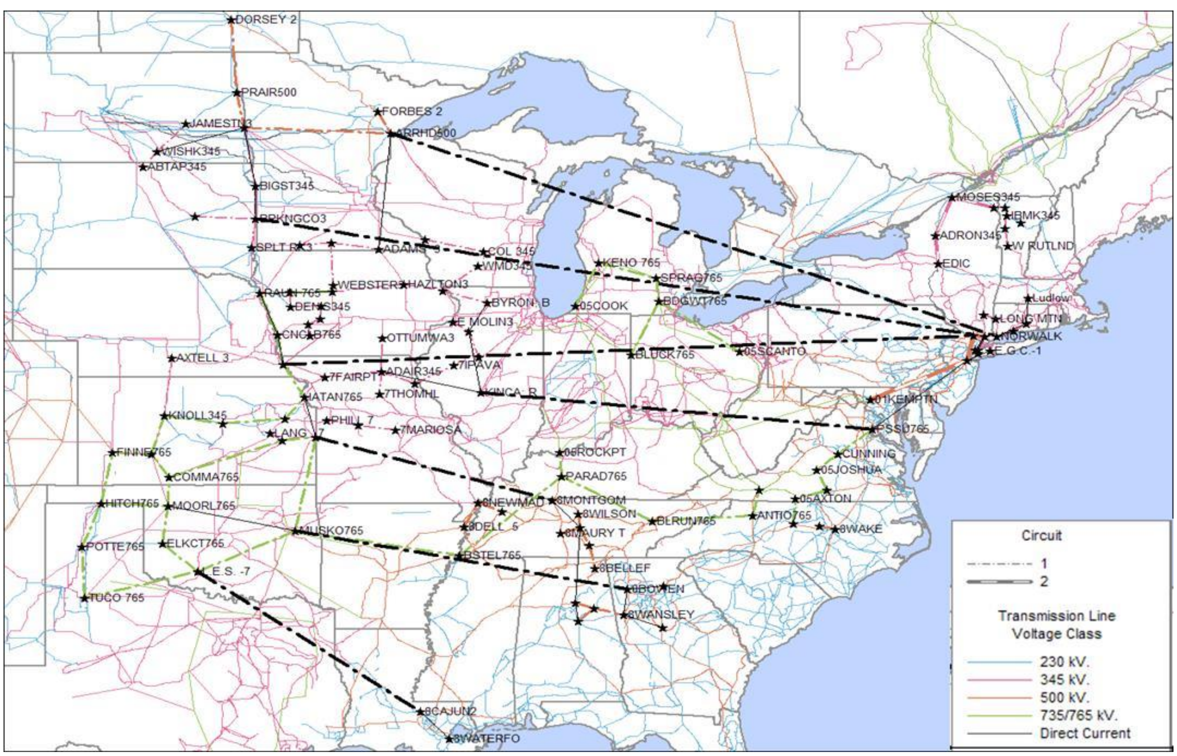


Figure 7: Transmission design created for 20% DOE scenario

A robustness testing was performed by evaluating each of the two transmission designs under various scenarios. They were looking for the transmission plan that performs best under the various scenarios.

Four scenarios were used for the robustness testing: Reference, Scenario 2 (20% Wind), Scenario 3 (30% wind), and Scenario 4 (Environmental). The scenario for which the design was developed was not used in the robustness testing.

To evaluate a design under a particular scenario, a set of performance measures were identified, as follows:

- Long-term cost
- Short-term cost
- Benefit/Cost ratio
- Reliability
- Environmental Impacts (carbon emissions)
- Land use criteria
- Local economic impacts
- National security criteria
- Others

Each performance measure was scored on a basis of 1-10 (with the higher score being better) and then a total score was computed as the sum of individual scores. Figure 8 shows the result for the transmission design performed under the reference scenario. Figure 9 shows the result for the transmission design performed under scenario 2. The results indicate that the scenario designed under the reference scenario is more robust to the different futures.

MISO has certainly evolved the approach used in this 2008 study, but the basic approach of identifying scenarios, each as a particular choice of a global uncertain parameter, is still a foundational part of their MTEP procedure. We return to this approach in Section 8 where we will compare the latest MISO-MTEP scenario analysis approach to a recently developed optimization approach.

Transmission Design for Reference Scenario

Attributes	Ranking Under Scenario 2	Ranking Under Scenario 3	Ranking Under Scenario 4
Short –term cost	10	5	7
Long-term cost	9	10	4
Reserve Margin	6	7	8
Investor Impacts	6	7	3
Environmental	5	7	10
Local economic	5	5	10
National Security	8	9	10
	49	50	52

Figure 8: Scoring for Transmission Design Performed Under Reference Scenario

Test Portfolio based on ***Scenario 2*** conditions

Attributes	Ranking Under Scenario 1	Ranking Under Scenario 3	Ranking Under Scenario 4
Short –term cost	4	5	3
Long-term cost	9	2	4
Reserve Margin	6	7	4
Investor Impacts	2	7	3
Environmental	5	7	5
Local economic	5	5	4
National Security	7	4	10
	38	38	34

Figure 9: Scoring for Transmission Design Performed Under Scenario 2

6 Monte Carlo Simulation

One method of modeling parameter uncertainty is to represent each uncertain parameter x_1, x_2,… with its numerical distribution. Then repeatedly draw values from each distribution, and for each draw, make the desired computation using those values. If the parameter values are drawn as a function of their probabilities, as indicated by the distribution, then the computed reliability indices will also form a distribution, from which we may compute their statistics, e.g., mean and variance. The process is illustrated in Figure 10, where the loop must be implemented many times before the output converges to a steady-state distribution.

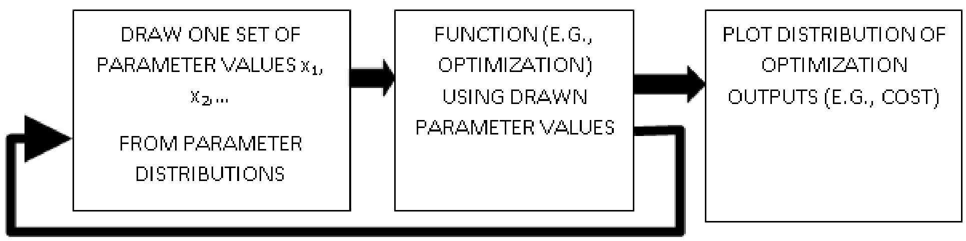


Figure 10: Monte Carlo Simulation

The draws (left-hand box in Figure 10) can be made by discretizing the probability density function (PDF) of each uncertain parameter, with each interval of each PDF assigned to an interval on [0,1] in proportion to its probability (area under the PDF curve for the discrete interval). Then a random draw on [0,1], which is then converted to the uncertain parameter value through the assignment, reflects the desired PDF of the uncertain parameter. Figure 11 illustrates the process, where the uncertain parameter is load, assumed to be normally distributed about an expected value.

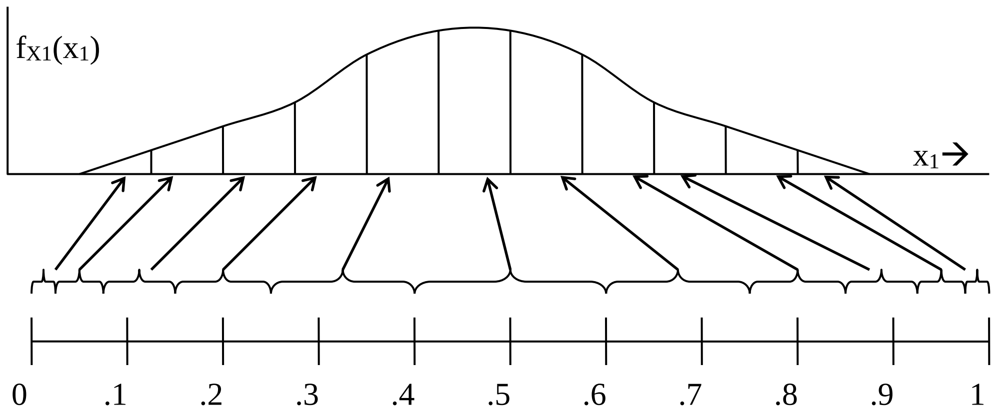


Figure 11: Drawing Parameter Values According to a PDF

This process is called Monte Carlo simulation (MCS) and is almost always an available option for making complex computations involving uncertain parameters. An advantage to MCS is that it is conceptually simple to implement.

A disadvantage is that it can be computationally intensive if

- the function (second box in Figure 10) is computationally intensive, because the function must be executed a large number of times to establish enough data to converge to a statistically valid output sample.
- the number of uncertain parameters is large;

It can be especially computationally intensive if both are true, i.e., if the function is computationally intensive and there are a large number of uncertain parameters.

A particularly useful approach is called "Guided MCS." There is a rich literature associated with application of Guided MCS to the development of operating rules, i.e., the rules associated with security-economy decision-making in real-time operations; a representative sample of this literature is [2, 3, 4, 5, 6]. This application is illustrated in Figure 12.

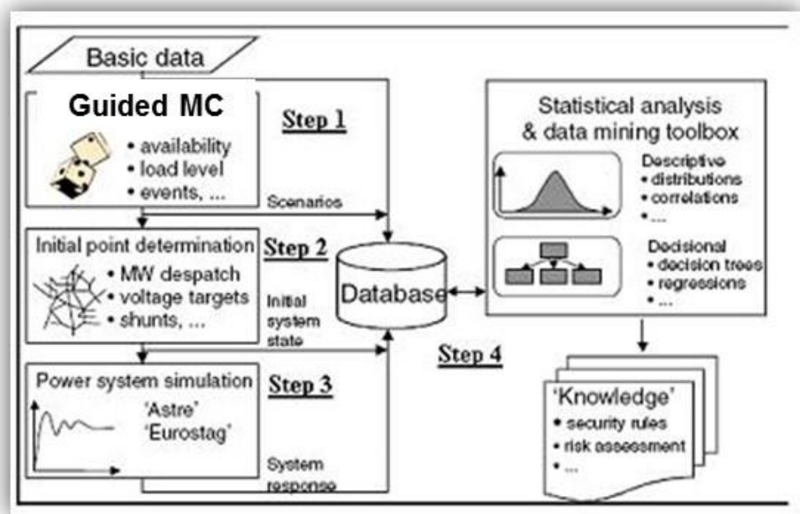


Figure 12: Guided Monte Carlo Simulation

This particular application of Guided MCS for developing operating rules is not an expansion planning application. It is presented here because it is a method of treating uncertainty that could be applied to expansion planning if there is information about the investment solution that could be used to weight the uncertainty space.

As observed in Figure 12, there are two main steps to Guided MCS: (A) Database Generation and (B) Statistical analysis. These steps are further broken down into sub-steps as indicated below.

1. Database Generation
    1. Guided MCS
    2. Optimal power flow
    3. Contingency analysis
2. Statistical analysis
    4. Estimate reliability indices (LOLE, LOLP, risk, …)
    5. Perform statistical analysis on output data to develop the operating rules.

Our interest is the use of step 1 to “guide” the MCS; the implication here is that we will use insight to focus simulations on the part of the decision space of most interest. In the case of generating operating rules, this part of the decision space is the boundary (based on reliability criteria) between acceptable and unacceptable operating conditions. This is illustrated in Figure 13.

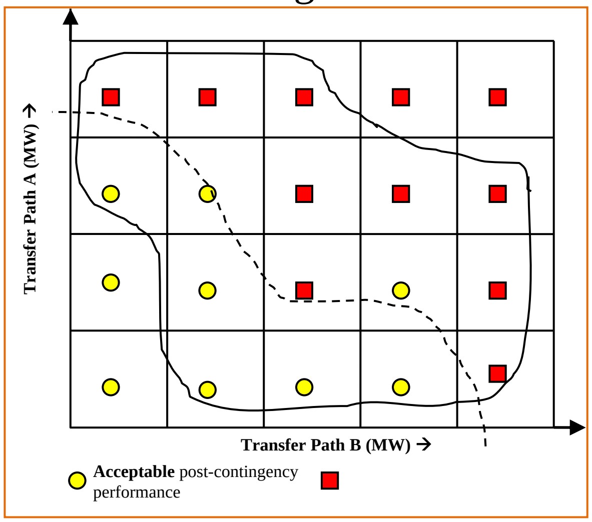
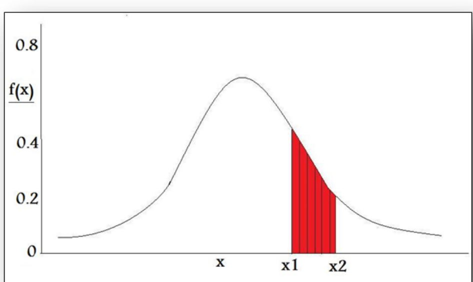
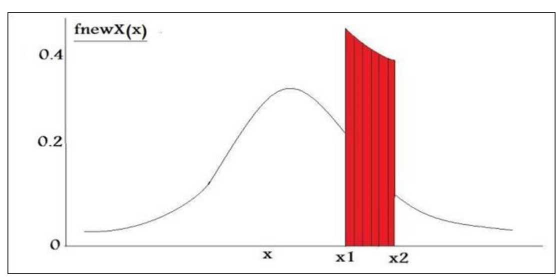


Figure 13: Illustration of boundary between acceptable and unacceptable conditions

The "guiding" part of the MCS is also referred to in the literature as importance sampling. The idea in importance sampling is that the selection of operating points is done based on a revised distribution, where the revision is made so as to bias the selection towards the desired conditions. This idea is illustrated in Figure 14 below.



Original distribution $f_X(x)$ and region S of desired bias



Revised distribution $f_{newX}(x)$ showing bias in region S

$$fnew_{\underline{X}}(\underline{x}) = \begin{cases} p_1 f_{1\,\underline{X}}(\underline{x}) \text{ for } x \in S \\ p_2 f_{2\,\underline{X}}(\underline{x}) \text{ for } x \notin S \end{cases}$$

where $p_1+p_2=1$.
For example, if $p_1=0.75$, then 75% of the points are from S.

Figure 14: Guided MCS (importance sampling)

This could be applied to expansion planning by biasing the selection of uncertainty *realizations* (more general term than “conditions”) to focus more heavily on those realizations that motivate investments.

7 Stochastic Programming -

These notes are adapted from notes developed by J. Beasley of Brunel University, West London [7].

Stochastic programming can be separated into two distinct classes of problems: those with probabilistic constraints and problems with recourse.

7.1 Chance-constrained programming

Problems with probabilistic constraints are those that are posed with constraints that must be met with a certain probability. An example is provided below.

$$
\begin{aligned}
\text{max} \quad & f(\underline{x}) = 3x_1 + x_2 \\
\text{s.t.} \quad & \mathrm{x}_1 + x_2 = 16 \\
& \Pr\left[a_1 x_1 + a_2 x_2 \le 4\right] \ge \gamma
\end{aligned}
$$

where a_1 and a_2 are uncertain and described by distributions; γ is a probability level chosen by the decision-maker to be acceptable to the particular situation to which the problem applies.

This problem containing probabilistic constraints has been described as a chance-constrained optimization (CCO) problem, and its solution is referred to as chance-constrained programming; there is a rich literature related to it. Interestingly, up until 2012 there were only a few CCO applications to expansion planning in the literature, including a 2012 paper [8], but one of the best was a 2009 paper by Kit Po Wong’s group [9] (Dr. Wong passed away in 2018). One can enter titles of these papers into scholar.google.com to identify related papers published since then.

One solves this problem by choosing values of x_1 and x_2 such that the objective function is minimized, the deterministic constraint is satisfied, and the probability that the inequality is satisfied is greater than γ. A conceptual approach for solving this problem is as follows:

1. Identify the $\{x_1, x_2\}$ space that satisfies the equality constraint; call this space S_1.
2. Identify the $\{x_1, x_2\}$ space that satisfies the probability constraint; call this space S_2.
3. The solution is $\{x_1,x_2\}^*$ contained in $S_1 \cap S_2$ that minimizes $f(\underline{x})$.

Most solution approaches involve transforming the chance constraints into deterministic ones and then applying an appropriate solver accounting for structure and convexity of the problem.

7.2 Recourse problems

Recourse problems are so-called because they enable *recourse* following a decision. What is recourse?

An internet definition indicates it is

> "the act of resorting to a person, course of action, etc., in difficulty or danger."

A less formal equivalent of this is that recourse is an

> "act" that you take, once you have made some decision to get yourself in trouble.

There are two "steps" here: a decision and then a recourse action.
This very well characterizes recourse-oriented stochastic programs, or recourse problems. Over the past few years, reference to a "stochastic program" without further specification usually implies a recourse problem.

We adapt two examples from Beasley [7].

7.2.1 Example 1: Single stage SP recourse problem

We desire to make a decision now (period t=1) about the amount of capacity we need in year 5 (period t=2).

We assume that this capacity is going to cost $2000/kW.

We assume that the growth in peak load (including needed reserves), which drives the need for this capacity, is stochastic. We adopt a simple representation of the demand uncertainty by assuming the increase in peak load will be either

- Low: 500 MW with probability 0.6 or
- High: 700 MW with probability 0.4.

We have to make a decision now (in period t=1) on how much capacity to build because it will take us 5 years to build the new capacity. Thus, we need to decide before the demand is actually known.

We may represent this situation as a tree-like structure as indicated in Figure 15.

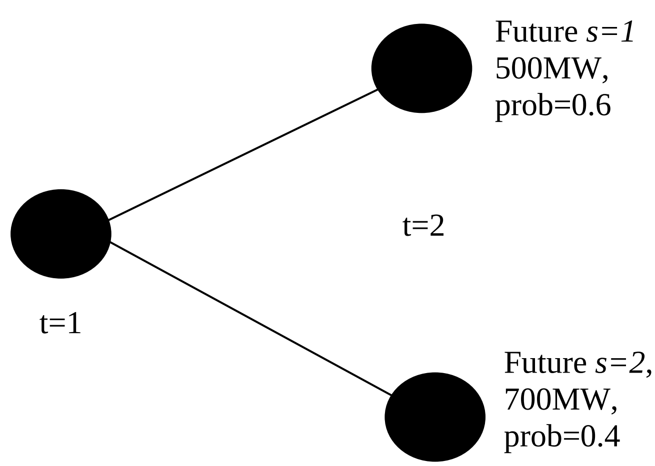


Figure 15: Illustration of decision problem

It is clear we will build no less than 500MW; no more than 700MW.

But do we build 500MW? 550MW? 600MW? 650MW? 700MW?

Let's consider that we build 500MW at t=1. This decision will be a good one if the t=2 demand for capacity is indeed 500MW.
However, if we build 500MW at t=1 but the t=2 demand for capacity is 700MW, then we will have to take recourse and add 200MW at time 2 in order to meet that demand. For example, we can purchase (at a cost of $3000/kw) 200MW of capacity from our capacity-rich neighbor, or we can pay some large loads to shut down during peak conditions.

We will assume in this simple model that we can *buy* capacity at t=2 but we cannot *sell* capacity at t=2. This assumption is to keep things simple; we could easily relieve this assumption.

We observe that, in this model:

- We **decide to build** at t=1
- We **observe** the realization of the uncertainties at t=2
- We **employ recourse**, a further decision, depending upon the realization observed.

Let's set up an analytic model to reflect this situation. To do so, we will refer to the two different realizations of the future demand for capacity (i.e., 500 or 700 MW) as "futures" or "scenarios."

Define

- t,s as denoting the time period and the future;
- x_1 is the amount of capacity we decide to build at period $t=1$. We might call these the "build" variables.
- C_s is the required capacity corresponding to future s (assume the number of futures is S, i.e., $s=1,2,\ldots,S$).
- $y_{2,s}$ is the amount of capacity we will need to purchase at $t=2$ when the value of the demand is realized. We might call these the "recourse" variables.

We can write a constraint to ensure that the capacity requirement is always met:

$$x_1 + y_{2s} \geq C_s \qquad s = 1, 2, \ldots S$$

Observe that the amount of capacity we have in period *t=2* may exceed the requirement. That is, we are ***not*** requiring

$$x_1 + y_{2s} = C_s \qquad s = 1, 2, \ldots S$$

because the equality sign would require either that we allow capacity sales (enabling $y_{2s}<0$), or our solution would always be $x_1=500MW$ since otherwise, it would be impossible to satisfy the equality if we overbuild (i.e., choosing to build x_1 and then learning in period *t=2* that the required capacity is less than x_1).

We desire our objective to minimize total *expected* cost, given by

$$2 \times 10^6 x_1 + \sum_{s=1}^{S} \Pr_s \times 3 \times 10^6 y_{2s}$$

We have already argued that $y_{2,s}<0$ is not allowed. We will also impose the same for x_1, i.e., $x_1<0$ is not allowed, meaning we cannot elect to retire capacity in period *t=1* (again, this is for simplicity and could be lifted if desired).

We can now write down an optimization problem which achieves our objective, as follows:

$$\min \quad 2 \times 10^6 x_1 + \sum_{s=1}^{S} \Pr_s \times 3 \times 10^6 y_{2s}$$

subject to

$$x_1 + y_{2s} \geq C_s \qquad s = 1, \ldots, S$$

$$x_1 \geq 0$$

$$y_{2s} \geq 0 \qquad s = 1, \ldots, S$$

What will solving this optimization problem give us?

- A value for x_1, which is the amount of capacity we should decide to build now.

- Values for y_{2s}, $s=1,\ldots,S$; this provides us with the optimal recourse decisions for all possible futures given that we choose to build x_1 now. Only one of these values will be relevant once the actual capacity requirement is known; the other values will be irrelevant.

It is important to observe here that the uncertainty is characterized using a discrete distribution (i.e., a probability mass function) instead of a continuous distribution (i.e., a probability density function). This is typical; if one desires to make use of continuous distributions, the computations become more intensive.

Five comments about terminology:

- Both sets of variables x_1 and y_{2s} are *decision variables* in the sense used within the optimization literature.
- The variable x_1, previously referred to as "build" variables, is also referred to as a "*here and now*" decision variable.
- The variables y_{2s}, previously referred to as "recourse variables, are also referred to as "*wait and see*" decision variables.
- We refer to the problem presented here as a *single-stage problem* because there is only one set of variables x_1 corresponding to a decision under uncertainty (the variables y_{2s} correspond to decisions made only after the uncertainties of the problem are revealed and so do not correspond to decisions made under uncertainty).
- The SP recourse problem may also occur in a *multistage* form, which we address next.

7.2.2 Example 2: Two-stage SP recourse problem

Let's now consider that we have a third period $t=3$, in addition to our first two periods $t=1,2$. Here, period $t=1$ is "now," period $t=2$ is "year 5," and period $t=3$ is "year 10." We will retain all information used in Example 1 above, and to it we add information for period $t=3$. The problem is illustrated in Figure 16. Observe that t=2 probabilities are non-conditional, whereas the t=3 probabilities are conditional.

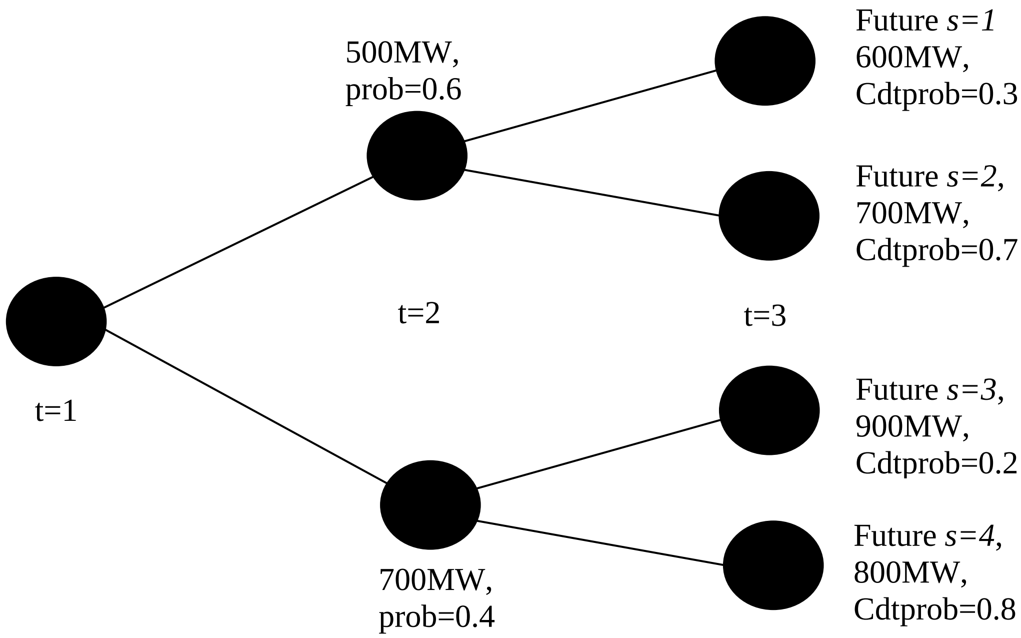


Figure 16: Illustration of decision problem

Here, we initially make a decision in period *t=1* of how much capacity to build in period *t=2*, where we know the capacity requirement will either be 500MW (prob=0.6) or 700MW (prob=0.2). Once the uncertainty in period *t=2* is revealed, we may make a recourse decision to purchase additional capacity in order to meet the capacity requirement in period *t=2*. All of this seems similar to the situation we had in Example 1.

But now, at period *t=2*, we have another decision to make, which is how much capacity to build in period *t=3*. This is a decision under uncertainty; once made, uncertainty in period *t=3* is revealed, and we may make a recourse decision to purchase additional capacity.

To summarize then, as we move left to right across the tree of Fig. 11, we encounter the following decision problems:

- In the *t=1* period, we decide how much capacity to build for the *t=2* period. This is x_1, as in Example 1.
- In the *t=2* period, the *t=2* uncertainty is revealed.

- In the *t=2* period, we make the recourse decision of how much capacity to purchase in order to satisfy capacity requirements of period *t=2*. These are the $y_{2,s}$ variables, as in Example 1. However, these variables may change, depending on the ultimate future we encounter, and there are four such futures. Therefore, we have $y_{2,1}$, $y_{2,2}$, $y_{2,3}$, $y_{2,4}$. ***Note carefully***! By defining these variables across all four futures, we are recognizing that the best recourse decision at the *t=2* period may differ depending on what happens during the *t=3* period.
- In the *t=2* period, we decide how much capacity to build for the *t=3* period. This would be x_2, but there are four possible futures for *t=2, s=1, 2, 3, 4*. Therefore we have $x_{2,1}$, $x_{2,2}$, $x_{2,3}$, $x_{2,4}$. ***Note carefully***! By defining these variables across all four futures, we are recognizing that the best decision at the *t=2* period may differ depending on what happens during the *t=3* period.
- In the *t=3* period, the *t=3* uncertainty is revealed.
- In the *t=3* period, we make recourse decision of how much capacity to purchase in order to satisfy capacity requirements of period *t=3*. These are the $y_{3,s}$ variables, and we will have four of them, i.e., $y_{3,1}$, $y_{3,2}$, $y_{3,3}$, $y_{3,4}$.

We assume the cost to build in period *t=1* is the same as the cost to build in period *t=2*. We also assume the cost to buy capacity in period *t=2* is the same as the cost to buy capacity in period *t=3*.

We first consider period *t=2*, requiring that what we build in period t=1 plus capacity we buy via recourse during period t=2 must equal or exceed the required capacity in period t=2, i.e.,

$$x_1 + y_{2s} \geq C_s \qquad s = 1, 2, \ldots S$$

These constraints will be:

$$x_1 + y_{2s} \geq 500 \qquad s = 1, 2$$

$$x_1 + y_{2s} \geq 700 \qquad s = 3, 4$$

At the *t=2* period, we may have excess capacity given by

$$x_1 + y_{2s} - 500 \qquad s = 1,2$$

$$x_1 + y_{2s} - 700 \qquad s = 3,4$$

And then at the *t=2* period, we will make a decision to build additional capacity, and then at the *t=3* period, we will learn the capacity requirement and subsequently take a recourse decision to purchase additional capacity. Thus, we will require that:

Excess Capacity+Capacity built+Capacity Purchased>=CapRequired

Writing in terms of our defined nomenclature, we have

$$x_1 + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \geq 600 \qquad s = 1$$

$$x_1 + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \geq 700 \qquad s = 2$$

$$x_1 + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \geq 900 \qquad s = 3$$

$$x_1 + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \geq 800 \qquad s = 4$$

We might think we are done with constraints; however, we need to reconsider our build&recourse variables at the *t=2* period; these are:

$$x_{2,1}, x_{2,2}, x_{2,3}, x_{2,4}$$

$$y_{2,1}, y_{2,2}, y_{2,3}, y_{2,4}$$

The question we must ask is this: When we are at period *t=2*, how will we know what is going to happen at period *t=3*? The answer is that we will not know! We can only distinguish between variables if their past is different; we cannot distinguish between variables that have a different future but a common past!

> In other words, a decision maker at t=2 can only make a single decision, s\he cannot make two separate decisions at t=2 depending on which t=3 future occurs.

This means that period t=2 variables originating from the 500MW node must be equal, i.e.,

$$x_{2,1}=x_{2,2}$$

$$y_{2,1}=y_{2,2}$$

and t=2 variables originating from the 700MW node must be equal, i.e.,

$$x_{2,3}=x_{2,4}$$

$$y_{2,3}=y_{2,4}$$

Why not define a single variable for each pair to start with? One answer is to clearly retain the expression of the nonanticipativity concept in the problem formulation, to remind us all of its necessity. Another answer is that stochastic programming problems are very "L-shaped" and as a result amendable to solution by decomposition methods where the nonanticipativity constraints are relaxed in the subproblem.

These are called the *non-anticipativity constraints*, implying that we cannot anticipate the future. This implies that futures with a common history must have the same set of decisions.

We can now formulate our objective function.

We have just one cost incurred with certainty, namely that associated with x_1. All other costs are probabilistic. Let's identify the probability of each future and the cost of each future, using total probabilities for each future. We also repeat our tree of Figure 16 below, for convenience.

Future	Total probability of each future	Cost
1	0.6×0.3=0.18	$2\times 10^6 x_{21} + 3\times 10^6 y_{21} + 3\times 10^6 y_{31}$
2	0.6×0.7=0.42	$2\times 10^6 x_{22} + 3\times 10^6 y_{22} + 3\times 10^6 y_{32}$
3	0.4×0.2=0.08	$2\times 10^6 x_{23} + 3\times 10^6 y_{23} + 3\times 10^6 y_{33}$
4	0.4×0.8=0.32	$2\times 10^6 x_{24} + 3\times 10^6 y_{24} + 3\times 10^6 y_{34}$

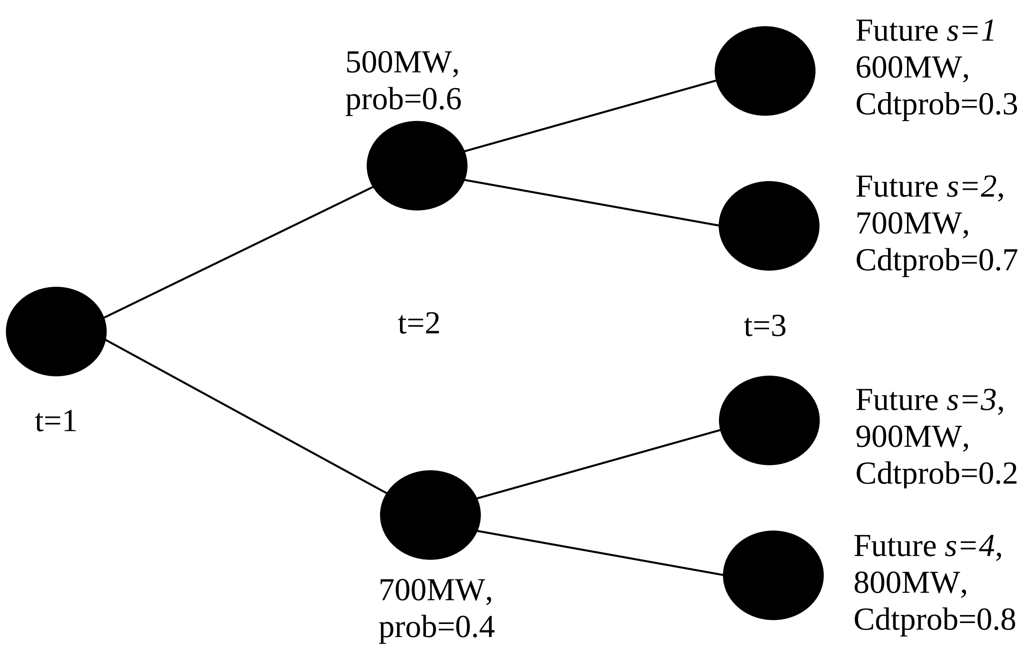


Figure 16: Illustration of decision problem

We can now write down our optimization problem. The objective is the cost of each future weighted by its probability, and we want to minimize it. The constraints are the need to satisfy the capacity requirements at the t=2 and t=3 periods, together with the non-anticipativity constraints. Thus,

$$\begin{aligned}
\min \quad & 2\times10^6 x_1 \\
& +0.18\left[2\times10^6 x_{21} + 3\times10^6 y_{21} + 3\times10^6 y_{31}\right] \\
& +0.42\left[2\times10^6 x_{22} + 3\times10^6 y_{22} + 3\times10^6 y_{32}\right] \\
& +0.08\left[2\times10^6 x_{23} + 3\times10^6 y_{23} + 3\times10^6 y_{33}\right] \\
& +0.32\left[2\times10^6 x_{24} + 3\times10^6 y_{24} + 3\times10^6 y_{34}\right]
\end{aligned}$$

Subject to

$$x_1 + y_{2s} \ge 500 \qquad s = 1,2$$

$$x_1 + y_{2s} \ge 700 \qquad s = 3,4$$

$$x_1 + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \ge 600 \qquad s = 1$$

$$x_1 + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \ge 700 \qquad s = 2$$

$$x_1 + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \ge 900 \qquad s = 3$$

$$x_1 + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \ge 800 \qquad s = 4$$

$$x_{2,1}=x_{2,2}$$

$$y_{2,1}=y_{2,2}$$

$$x_{2,3}=x_{2,4}$$

$$y_{2,3}=y_{2,4}$$

and all variables ≥ 0

Stochastic programming of this sort has been applied to electric power system investment planning. There are many papers on this topic; some good work was done by the group led by Ben Hobbs of Johns Hopkins University [10, 11, 12, 13].

8 Adaptation

Adaptation is an approach to design an investment strategy under uncertainty. The basic concept is illustrated in Figure 17.

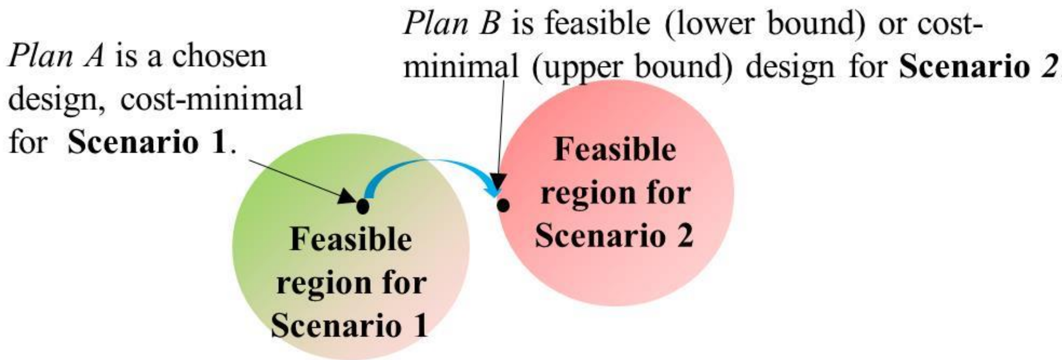


Figure 17: Illustration of adaptation cost

The adaptation cost of Plan A to Scenario 2 is the minimum cost to move Plan A to a feasible or optimal design, Plan B, in scenario 2. It measures the cost of our Plan A if scenario 2 happens.

This leads to an important new optimization problem, as follows:

Minimize:

$$\mathbf{CoreCosts}(\underline{\mathbf{x}}^{\mathbf{f}})+\boldsymbol{\beta}[\, \boldsymbol{\Sigma}_{\mathbf{i}}\ \mathbf{AdaptationCost}(\boldsymbol{\Delta}\underline{\mathbf{x}}_{\mathbf{i}})]$$

Subject to:

$$\textbf{Constraints for scenario i=1,…N: } \underline{\mathbf{g}}_{\mathbf{i}}(\underline{\mathbf{x}}^{\mathbf{f}}+\boldsymbol{\Delta}\underline{\mathbf{x}}_{\mathbf{i}})\underline{\boldsymbol{\leq}}\,\underline{\mathbf{b}}_{\mathbf{i}}$$

where:

- $\underline{\mathbf{x}}^{\mathbf{f}}$: Core investments, to be used by all scenarios i
- $\boldsymbol{\Delta}\underline{\mathbf{x}}_{\mathbf{i}}$: Additional investments needed to adapt to scenario i

This approach identifies an investment that is "core" in that the total "CoreCost" plus the cost of adapting it to the set of envisioned futures is minimum. The approach is illustrated in Figure 18.

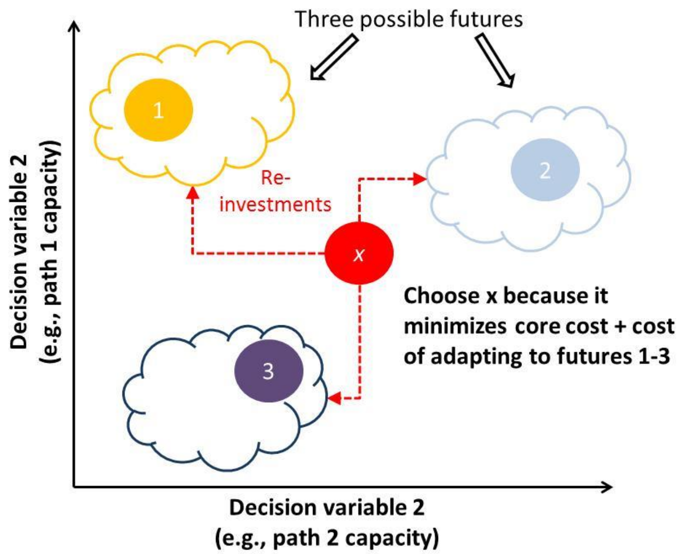


Figure 18: Illustration of adaptation approach

It is important to note that the "core" investments are not necessarily the same as the investments that are common to each scenario. This approach was applied to a GEP problem at the national level. Figure 19 shows the geographical scale of the problem addressed.

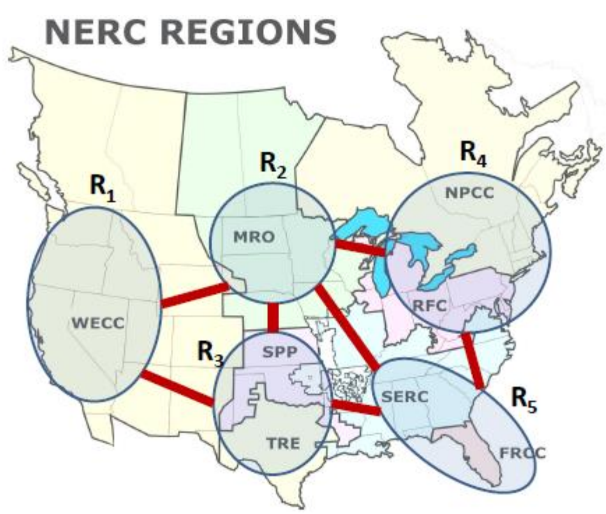


Figure 19: Geography of the problem addressed

Sixty-four scenarios were developed in terms of

- Gas price
- Gas production limits
- Demand
- National renewable portfolio standard

- CO_2 cap
- Wind plant investment cost

An aggregation approach was used to identify 10 scenarios that best represented the 64. These 10 scenarios are listed in Figure 20.

Cluster	Gas price GP	Gas production limits GPL	Demand D	RPS	CO_2^{cap}	Wind investment cost WC
Benchmark	L	No	L	No	No	H
1	L	No	L	No	No	L
2	L	No	L	Yes	No	H
3	L	No	L	Yes	Yes	L
4	L	No	H	No	Yes	H
5	L	Yes	L	Yes	Yes	H
6	H	No	L	Yes	Yes	L
7	H	No	H	No	Yes	L
8	H	Yes	L	No	No	L
9	H	Yes	L	Yes	Yes	H
10	H	Yes	H	Yes	No	H

Figure 20: Selected scenarios

The optimization problem was then solved for different values of β, and the results are plotted in Figure 21.

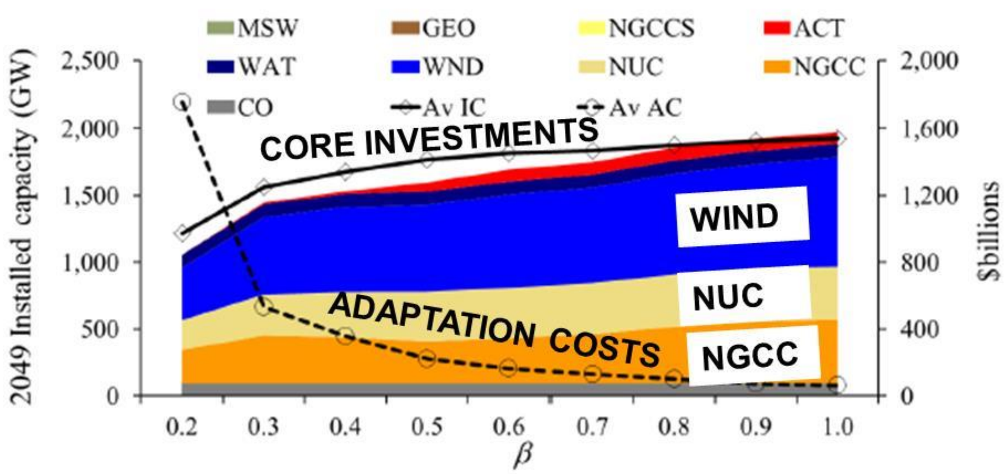


Figure 21: Adaptation solutions for different values of β

A single value of β was selected, and a complete solution was produced over a 40 year horizon. The total installed capacity of the solution is shown in Figure 22.

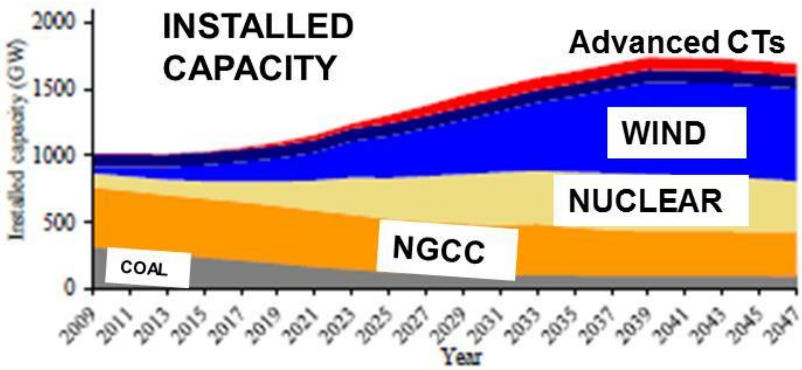
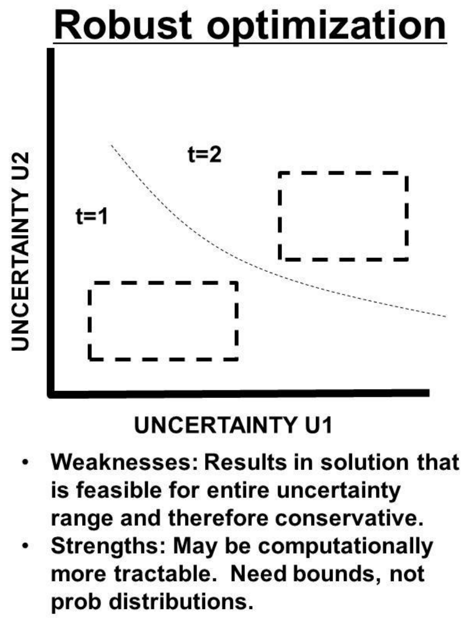


Figure 22: Total installed capacity over 40 years

The solution shown is considered to be adaptable, or "flexible". We observe that, with respect to the scenarios studied, adaptability means:

- Increase Advanced CTs
- Increase WIND
- Increase NUCLEAR
- Maintain NGCC
- Retire COAL

9 Robust optimization



10 Compare and contrast

It would be good to compare and contrast the various ways of handling uncertainty.

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