Updated: 3/21/2024

Transmission Line Design Information

In these notes, I would like to provide you with some background information on AC transmission lines.

1. AC Transmission Line Impedance Parameters

AC transmission is implemented entirely as 3-phase systems. Initial planning studies typically only consider balanced, steady-state operation. This simplifies modeling efforts greatly in that only the positive sequence, per-phase transmission line representation is necessary.

Essential transmission line electrical data for balanced, steady-state operation includes:

- Line reactance
- Line resistance
- Line charging susceptance
- Current rating (ampacity)
- Surge impedance loading

Figures 1a and 1b below illustrate a *distributed parameter model* of a transmission line where $z=r+jx$ is the series impedance per unit length (ohms/unit length), and $y=jb$ is the shunt admittance per unit length (mhos/unit length).

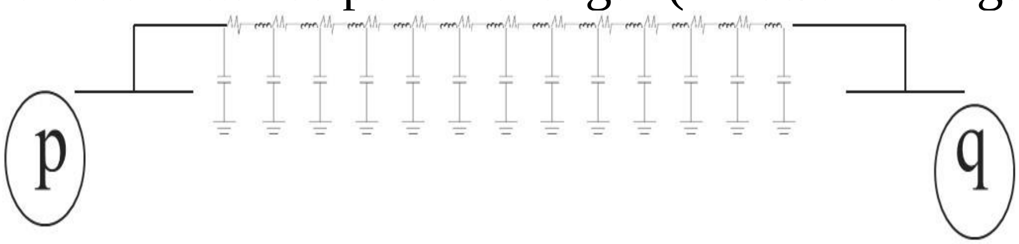


Fig. 1a: Distributed parameter model - conceptual view

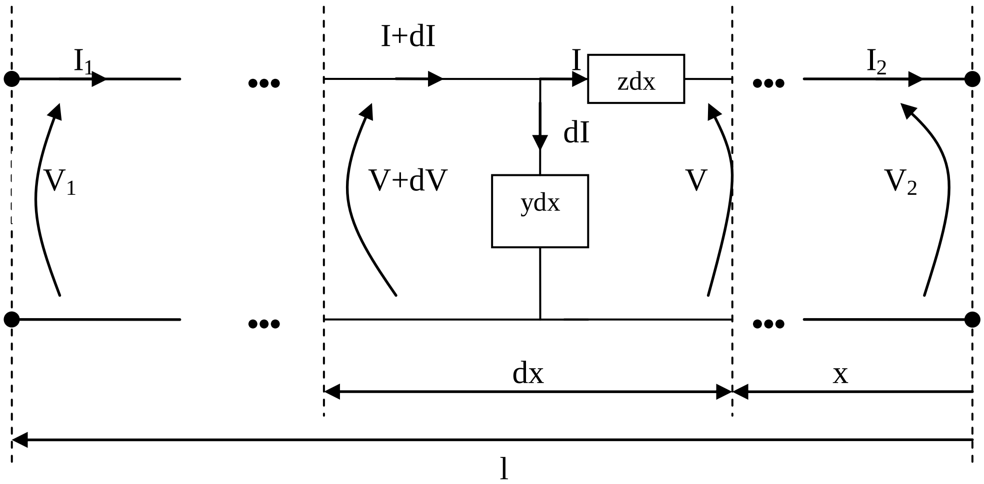


Fig. 1b: Distributed parameter model – analytic view

I have notes posted under the lecture for 9/13, at http://home.engineering.iastate.edu/~jdm/EE456/ee456schedule.htm, (called "TerminalRelations") that derive the following model relating voltages & currents at either end of a line.

$$I(l) = I_1 = I_2 \cosh \gamma l + \frac{V_2}{Z_C} \sinh \gamma l \qquad \text{(1a)}$$

$$V(l) = V_1 = V_2 \cosh \gamma l + Z_C I_2 \sinh \gamma l \qquad \text{(1b)}$$

where

- l is the line length,
- γ is the propagation constant, in general a complex number, given by

$$\gamma = \sqrt{zy}$$ with units of 1/(unit length), (1c)

where z and y are the per-unit length impedance and admittance, respectively, as defined previously.

- Z_C is the characteristic impedance, also known as the surge impedance, given by

γ is the propagation constant and is complex, so we can write γ=α+jβ where α is the attenuation constant and β is the phase constant. If β is expressed in rad/m, then the wavelength is given by λ=2π/β meters. We also have that the wave's velocity of propagation, v, is close to the speed of light, 3×10^8m/sec, so that the wavelength is computed as λ=v/f where f is the frequency. Thus, we have λ=3×10^8/60=5,000,000m=5000km=3106miles.

A typical value of Z_C is 400 ohms for a single three-phase overhead line, but lower values can be found for highly bundled designs.

$$\boldsymbol{Z_C} = \sqrt{\frac{\boldsymbol{z}}{\boldsymbol{y}}} \quad \text{with units of ohms.} \qquad (1d)$$

And *cosh* and *sinh* are the hyperbolic cosine and sine functions, respectively, given by:

$$\cosh x = \frac{e^x + e^{-x}}{2}; \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

Those same notes ("TerminalRelations") show that equations (1a, 1b) may be represented using the following pi-equivalent line model

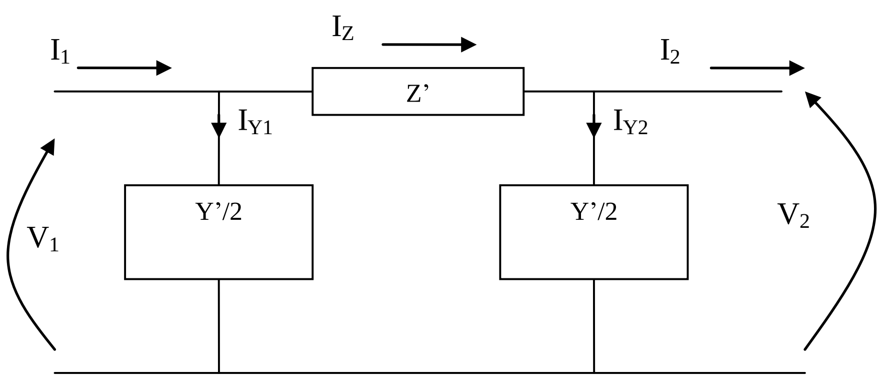


Fig. 2

where

$$Z' = Z \frac{\sinh \gamma l}{\gamma l} \qquad (2a)$$

$$Y' = Y \frac{\tanh(\gamma l / 2)}{\gamma l / 2} \qquad (2b)$$

and $Z=zl$, $Y=yl$.

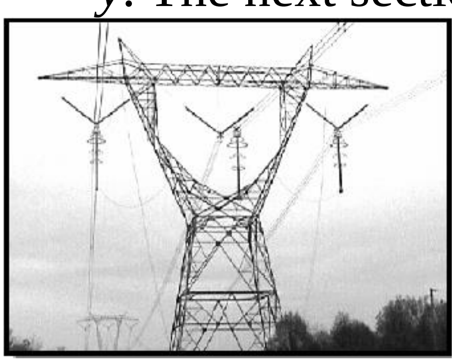
Two comments are necessary here:

1. Equations (2a, 2b) show that the impedance and admittance of a transmission line are not just the impedance per unit length and admittance per unit length multiplied by the line length, $Z=zl$ and $Y=yl$, respectively, but they are these values corrected by the factors

$$\frac{\sinh \gamma l}{\gamma l} \qquad \frac{\tanh(\gamma l/2)}{\gamma l/2}$$

   It is of interest to note that these two factors approach 1.0 (the first from above and the second from below) as γl becomes small. This fact has an important implication in that short lines (less than ~100 miles) are usually well approximated by $Z=zl$ and $Y=yl$, but longer lines are not and need to be multiplied by the "correction factors" listed above. The "correction" enables the lumped parameter model to exhibit the same characteristics as the distributed parameter device.

2. We may obtain all of what we need if we have z and y. The next section will describe how to obtain them.



2. Obtaining per-unit length parameters

In the 9/6 and 9/8 notes at

http://home.engineering.iastate.edu/~jdm/EE456/ee456schedule.htm

I have derived expressions to compute per-unit length inductance and per-unit length capacitance of a transmission line given its geometry. These expressions are:

μ_0 is the permeability of free space, given by $4\pi\times10^{-7}$ Henry/meter.

Inductance (h/m): $l_a = \frac{\mu_0}{2\pi}\ln\frac{D_m}{R_b}$

$d^{(1)}_{ab}$ is distance between phases a and b under configuration (1). There are 3 configurations (1), (2), and (3) due to the common practice of transposition where phase positions are interchanged every few miles.

- D_m is the GMD between phase positions:

$$D_m \equiv \left(d_{ab}^{(1)} d_{ab}^{(2)} d_{ab}^{(3)}\right)^{1/3}$$

- R_b is the GMR of the bundle

$$\begin{aligned} R_b &= \left(r' d_{12}\right)^{1/2}, && \text{for 2 conductor bundle} \\ &= \left(r' d_{12} d_{13}\right)^{1/3}, && \text{for 3 conductor bundle} \\ &= \left(r' d_{12} d_{13} d_{14}\right)^{1/4}, && \text{for 4 conductor bundle} \\ &= \left(r' d_{12} d_{13} d_{14} d_{15} d_{16}\right)^{1/6}, && \text{for 6 conductor bundle} \end{aligned}$$

The effects of bundling are to increase R_b. This tends to decrease inductance and therefore inductive reactance of the line.

d_{12} is the distance between conductors 1 and 2 in a bundle. Bundling is the practice of using multiple conductors in each phase.

Capacitance (f/m): $\bar{c}_a = \frac{2\pi\varepsilon}{\ln(D_m / R_b^c)}$

- D_m is the same as above.
- R_b^c is Capacitive GMR for the bundle:

$$\begin{aligned} R_b^c &= \left(r d_{12}\right)^{1/2}, && \text{for 2 conductor bundle} \\ &= \left(r d_{12} d_{13}\right)^{1/3}, && \text{for 3 conductor bundle} \\ &= \left(r d_{12} d_{13} d_{14}\right)^{1/4}, && \text{for 4 conductor bundle} \\ &= \left(r d_{12} d_{13} d_{14} d_{15} d_{16}\right)^{1/6}, && \text{for 6 conductor bundle} \end{aligned}$$

The effects of bundling are to increase R_b^c. This tends to increase capacitance and therefore capacitive susceptance of the line.

In the above, r is the radius of a single conductor, and r' is the Geometric Mean Radius (GMR) of an individual conductor, given by

$$r' = re^{-\frac{\mu_r}{4}} = r \times 0.7788 \tag{3}$$

It is the radius of an equivalent hollow cylindrical conductor that would have the same flux linkages as the solid conductor of radius r. (According to Ampere's Law $\oint_{\Gamma} \underline{\boldsymbol{H}} \bullet \underline{\boldsymbol{dl}} = \boldsymbol{i}_{EN}$, the magnetic field is zero if the closed contour Γ encloses no current. Therefore, a solid conductor has flux within the conductor whereas a hollow conductor has no flux within the conductor.)

<u>2.1 Inductive reactance</u>

The per-phase inductive reactance in Ω/m of a non-bundled transmission line is $2\pi f l_a$, where $l_a = \frac{\mu_0}{2\pi} \ln \frac{D_m}{R_b}$ Ω/m. Therefore, we can express the reactance in Ω/mile as

$$X_L = 2\pi f l_a \frac{1609 \text{ meters}}{1 \text{ mile}} = 2\pi f \left(\frac{\mu_0}{2\pi} \ln \frac{D_m}{R_b} \right) \frac{1609 \text{ meters}}{1 \text{ mile}}$$

$$= f \left(\mu_0 \ln \frac{D_m}{R_b} \right) \frac{1609 \text{ meters}}{1 \text{ mile}} = 2.022 \times 10^{-3} f \ln \frac{D_m}{R_b} \quad \Omega/\text{mile} \tag{4}$$

Let's expand the logarithm to get

$$X_L = \underbrace{2.022\times10^{-3} f \ln\frac{1}{R_b}}_{X_a} + \underbrace{2.022\times10^{-3} f \ln D_m}_{X_d} \quad \Omega/\text{mile} \qquad (5)$$

where f=60 Hz. The first term is called the inductive reactance *at 1-foot spacing*, because it expresses equation (4) with $D_m=1$ foot.

Note: to get X_a, you need only to know R_b, which means you need only know the conductor used and the bundling. But you do *not* need to know the geometry of the phase positions.

But what is X_d? This is called the inductive reactance spacing factor. Note that it depends only on D_m, which is the GMD between phase positions. So you can get X_d by knowing only the distance between phases, i.e., you need not know anything about the conductor or the bundling.

<u>2.2 Capacitive reactance</u>

Similar thinking for capacitive reactance leads to

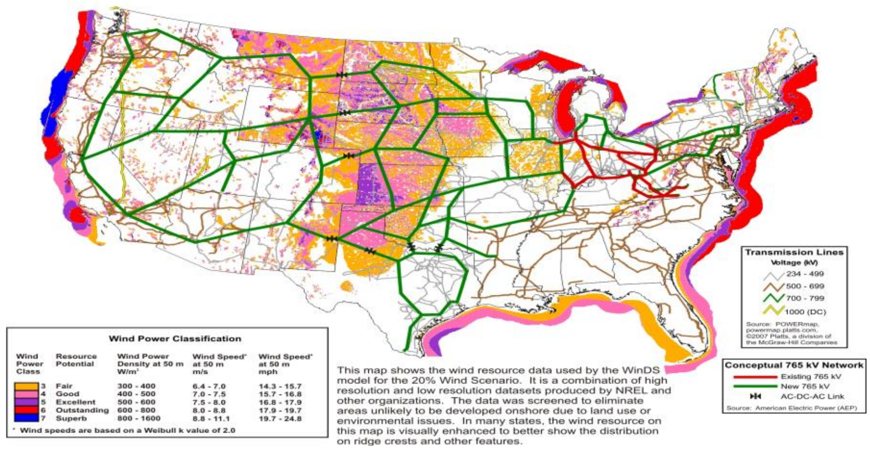
$$X_C = \underbrace{\frac{1}{f}\times1.779\times10^{6}\ln\left(\frac{1}{R_b^c}\right)}_{X'_a} + \underbrace{\frac{1}{f}\times1.779\times10^{6}\ln(D_m)}_{X'_d} \quad \Omega\text{ - mile}$$

X'_a is the capacitive reactance at 1 foot spacing, and X'_d is the capacitive reactance spacing factor. Note the units are ohms-mile, instead of ohms/mile, so that when we invert, we will get mhos/mile, as desired.

> A circular mil is a unit of area, equal to the area of a circle with a diameter 1mil (1 mil=0.001in=0.0254 mm). It corresponds to approximately 5.067×10^{-4} mm^2. ... 1000 circular mil equals 0.5067 mm^2. The area of a circle of 1 mil is $\pi r^2 = \pi(d/2)^2$, or $\pi(10\text{-}3\text{in}/2)^2 = 7.854\text{x}10\text{-}7$ in^2.

3. **Example**

Let's compute the X_L and X_C for a 765 kV AC line, single circuit, with a 6-conductor bundle per phase, using conductor type Tern (795 kcmil). AEP considered a similar design a few years ago when they proposed a 765kV transmission overlay for the nation, shown below.



The bundles have 2.5' (30") diameter, and the phases are separated by 45', as shown in Fig. 3. Assume the line is lossless.

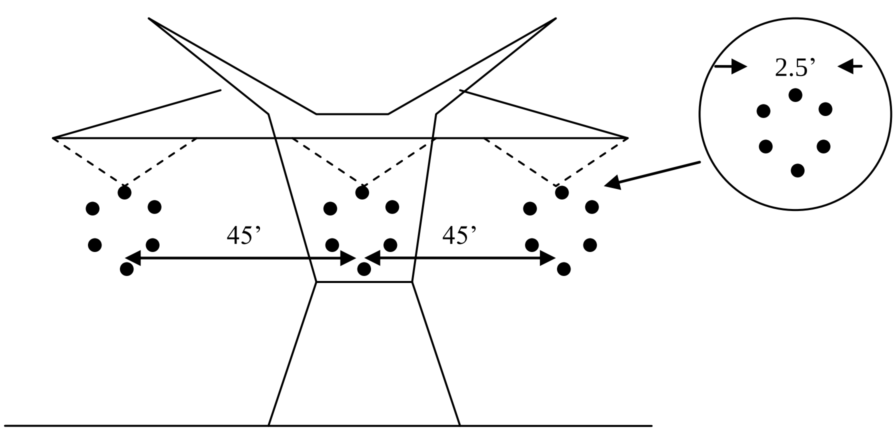


Fig. 3

We will use tables from [1] (available in pdf [2]), which I have copied out and placed on the website. Noting the below table (obtained from [3] and placed on the website), this example focuses on line geometry AEP 3.

TABLE 2 LINE GEOMETRIES

Company/Country	Nominal Voltage (kV)	No. of Sub-conductors	Conductor Diameter (cm)	Phase Spacing (m)	Min. Conductor Heights* (m)
Hydro-Québec 1	735	4	3.50	15.3	15.3
Hydro-Québec 2	735	4	3.56	12.8	14.1
AEP 1	765	4	2.96	13.7	12.2
AEP 2	765	4	3.52	13.7	12.2/13.7
AEP 3	765	6	2.70	13.7	13.7
NYPA	765	4	3.52	15.2	15.5
Eskom	765	6	2.86	15.8	15.0
FURNAS	765	4	3.20	14.3	13
EDELCA 1 &2	765	4	3.33	15.0	14.7
EDELCA 3	765	4	3.33	13.2	13.7
KEPCO	765	6	3.042	See Note 1	19/28
POWERGRID	765	4	3.50	15.4	15
RUSSIA 1	750	5	2.24	17.5	12
RUSSIA 2	750	4	2.91	19	12
RUSSIA 3	1150	8	2.75	21.5-25	17.5
TEPCO	1000	8	3.42/3.84**	See Note 1	25/35

* Minimum heights in areas frequented by people including agricultural areas.
** Larger conductor used in populated areas; smaller conductor used in mountainous areas.
1. Double-circuit low reactance line

The tables show data for 24” and 36” 6-conductor bundles, but not 30”, and so we must interpolate.

Get per-unit length inductive reactance:

From Table 3.3.1, we find

Table 3.3.1 (Cont.)

INDUCTIVE REACTANCE (X_a) OF BUNDLED CONDUCTORS AT 60 HZ IN OHMS PER MILE FOR ONE-FOOT SPACING

Code	(kcmil) Al	(sq mm) Tot	Strand	6-Cond Bundle Diam 24	36	54	8-Cond Bundle Diam 27	40	60	12-Cond Bundle Diam 33	50	75	16-Cond Bundle Diam 40	60	90
Aluminum-Conductor-Steel-Reinforced (ACSR)															
—	2776.	1521.	84/19	0.019	-0.022	-0.063	-0.003	-0.045	-0.088	-0.035	-0.079	-0.124	-0.057	-0.015	-0.151
Joree	2515.	1344.	76/19	0.020	-0.021	-0.062	-0.002	-0.044	-0.087	-0.032	-0.079	-0.124	-0.058	-0.104	-0.150
Thrasher	2312.	1235.	76/19	0.021	-0.020	-0.061	-0.001	-0.043	-0.086	-0.032	-0.078	-0.123	-0.058	-0.104	-0.150
Kiwi	2167.	1146.	72/7	0.022	-0.019	-0.060	-0.001	-0.042	-0.085	-0.032	-0.078	-0.123	-0.057	-0.104	-0.150
Bluebird	2156.	1181.	84/19	0.021	-0.020	-0.061	-0.001	-0.043	-0.086	-0.032	-0.078	-0.123	-0.058	-0.104	-0.150
Chukar	1781.	976.	84/19	0.023	-0.018	-0.059	0.000	-0.041	-0.084	-0.031	-0.077	-0.122	-0.057	-0.103	-0.149
Falcon	1590.	908.	54/19	0.024	-0.017	-0.058	0.001	-0.041	-0.084	-0.031	-0.077	-0.122	-0.057	-0.103	-0.149
Plover	1431.	817.	54/19	0.025	-0.016	-0.057	0.002	-0.040	-0.083	-0.030	-0.076	-0.121	-0.056	-0.102	-0.149
Bobolink	1431.	775.	45/7	0.026	-0.015	-0.056	0.002	-0.039	-0.083	-0.030	-0.076	-0.121	-0.056	-0.102	-0.148
Pheasant	1272.	726.	54/19	0.026	-0.015	-0.056	0.002	-0.039	-0.082	-0.030	-0.076	-0.121	-0.056	-0.102	-0.148
Bittern	1272.	689.	45/7	0.027	-0.014	-0.055	0.003	-0.039	-0.082	-0.029	-0.075	-0.120	-0.056	-0.102	-0.148
Finch	1114.	636.	54/19	0.027	-0.014	-0.055	0.003	-0.038	-0.081	-0.029	-0.075	-0.120	-0.055	-0.102	-0.148
Bluejay	1113.	603.	45/7	0.028	-0.013	-0.054	0.004	-0.038	-0.081	-0.028	-0.075	-0.120	-0.055	-0.101	-0.147
Ortolan	1034.	560.	45/7	0.029	-0.012	-0.053	0.005	-0.037	-0.080	-0.028	-0.074	-0.119	-0.055	-0.101	-0.147
Cardinal	954.	546.	54/7	0.029	-0.012	-0.053	0.005	-0.037	-0.080	-0.028	-0.074	-0.119	-0.055	-0.101	-0.147
Rail	954.	517.	45/7	0.030	-0.011	-0.052	0.005	-0.036	-0.079	-0.028	-0.074	-0.119	-0.054	-0.101	-0.147
Drake	795.	469.	26/7	0.030	-0.011	-0.052	0.006	-0.036	-0.079	-0.027	-0.074	-0.119	-0.054	-0.100	-0.146
Condor	796.	456.	54/7	0.031	-0.010	-0.051	0.006	-0.036	-0.079	-0.027	-0.073	-0.118	-0.054	-0.100	-0.145
Cuckoo	795.	455.	24/7	0.031	-0.010	-0.051	0.006	-0.036	-0.079	-0.027	-0.073	-0.118	-0.054	-0.100	-0.146
Tern	795.	431.	45/7	0.031	-0.010	-0.051	0.007	-0.035	-0.078	-0.027	-0.073	-0.118	-0.054	-0.100	-0.146

24" bundle: 0.031

36" bundle: -0.010

30" bundle: interpolation results in X_a=0.0205.

As you add more conductors to the bundle it looks increasingly like a circle, which increases Rb from r' (single conductor). Increasing Rb of the bundle lowers the inductance. The reason for this is that, as b gets large, there is less and less flux interior to the "circle" and therefore less flux linking the conductors. When b is very large, there is no flux interior to the circle, which is the case of the hollow conductor. See my EE 456 notes.

From Table 3.3.12, we find

Table 3.3.12

INDUCTIVE REACTANCE SPACING FACTOR, X_d, AT 60 HZ (OHMS PER MILE)

This means 45.0, 46.0, etc.

ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
45	0.4619	0.4622	0.4624	0.4627	0.4630	0.4632	0.4635	0.4638	0.4640	0.4643
46	0.4646	0.4648	0.4651	0.4654	0.4656	0.4659	0.4661	0.4664	0.4667	0.4669
47	0.4672	0.4674	0.4677	0.4680	0.4682	0.4685	0.4687	0.4690	0.4692	0.4695
48	0.4697	0.4700	0.4702	0.4705	0.4707	0.4710	0.4712	0.4715	0.4717	0.4720
49	0.4722	0.4725	0.4727	0.4730	0.4732	0.4735	0.4737	0.4740	0.4742	0.4744

45' phase spacing: X_d=0.4619

And so $X_L=X_a+X_d$=0.0205+0.4619=0.4929 ohms/mile.

Now get per-unit length capacitive reactance.

From Table 3.3.2, we find

Table 3.3.2 (Cont.)

CAPACITIVE REACTANCE (X'_a) OF BUNDLED CONDUCTORS AT 60 HZ IN MEGOHM-MILES FOR ONE-FOOT SPACING

Code	(kcmil) Al	(sq mm) Tot	Strand	6-Cond Bundle Diam 24	36	54	8-Cond Bundle Diam 27	40	60	12-Cond Bundle Diam 33	50	75	16-Cond Bundle Diam 40	60	90
Aluminum-Conductor-Steel-Reinforced (ACSR)															
-	2776.	1521.	84/19	0.0034	-0.0066	-0.0166	-0.0016	-0.0117	-0.0223	-0.0086	-0.0199	-0.0309	-0.0147	-0.0260	-0.0372
Joree	2515.	1344.	76/19	0.0037	-0.0063	-0.0163	-0.0013	-0.0115	-0.0220	-0.0085	-0.0198	-0.0308	-0.0146	-0.0259	-0.0371
Thrasher	2312.	1235.	76/19	0.0039	-0.0061	-0.0161	-0.0012	-0.0114	-0.0219	-0.0084	-0.0197	-0.0307	-0.0145	-0.0258	-0.0371
Kiwi	2167.	1146.	72/7	0.0041	-0.0059	-0.0159	-0.0010	-0.0112	-0.0217	-0.0083	-0.0196	-0.0306	-0.0145	-0.0257	-0.0370
Bluebird	2156.	1181.	84/19	0.0040	-0.0060	-0.0160	-0.0011	-0.0113	-0.0218	-0.0083	-0.0196	-0.0306	-0.0145	-0.0257	-0.0370
Chukar	1781.	976.	84/19	0.0045	-0.0055	-0.0155	-0.0007	-0.0109	-0.0214	-0.0081	-0.0194	-0.0304	-0.0143	-0.0256	-0.0368
Falcon	1590.	908.	54/19	0.0047	-0.0053	-0.0153	-0.0006	-0.0108	-0.0213	-0.0080	-0.0193	-0.0303	-0.0142	-0.0255	-0.0368
Plover	1431.	817.	54/19	0.0050	-0.0051	-0.0151	-0.0004	-0.0106	-0.0211	-0.0079	-0.0192	-0.0302	-0.0141	-0.0254	-0.0367
Bobolink	1431.	775.	45/7	0.0051	-0.0049	-0.0149	-0.0003	-0.0105	-0.0210	-0.0078	-0.0191	-0.0301	-0.0141	-0.0254	-0.0366
Pheasant	1272.	726.	54/19	0.0052	-0.0048	-0.0148	-0.0002	-0.0104	-0.0209	-0.0077	-0.0190	-0.0300	-0.0140	-0.0253	-0.0366
Bittern	1272.	689.	45/7	0.0054	-0.0046	-0.0146	-0.0001	-0.0103	-0.0208	-0.0077	-0.0190	-0.0300	-0.0140	-0.0252	-0.0365
Finch	1114.	636.	54/19	0.0056	-0.0044	-0.0144	0.0001	-0.0101	-0.0206	-0.0076	-0.0189	-0.0299	-0.0139	-0.0252	-0.0364
Bluejay	1113.	603.	45/7	0.0057	-0.0043	-0.0143	0.0002	-0.0100	-0.0205	-0.0075	-0.0188	-0.0298	-0.0139	-0.0251	-0.0364
Ortolan	1034.	560.	45/7	0.0059	-0.0041	-0.0141	0.0003	-0.0099	-0.0204	-0.0074	-0.0187	-0.0297	-0.0138	-0.0251	-0.0363
Cardinal	954.	546.	54/7	0.0060	-0.0040	-0.0141	0.0004	-0.0098	-0.0203	-0.0074	-0.0187	-0.0297	-0.0138	-0.0250	-0.0363
Rail	954.	517.	45/7	0.0061	-0.0039	-0.0139	0.0004	-0.0097	-0.0203	-0.0073	-0.0186	-0.0296	-0.0137	-0.0250	-0.0362
Drake	795.	469.	26/7	0.0063	-0.0037	-0.0137	0.0006	-0.0096	-0.0201	-0.0072	-0.0185	-0.0295	-0.0136	-0.0249	-0.0362
Condor	796.	456.	54/7	0.0064	-0.0036	-0.0136	0.0007	-0.0095	-0.0200	-0.0072	-0.0184	-0.0295	-0.0136	-0.0249	-0.0361
Cuckoo	795.	455.	24/7	0.0064	-0.0036	-0.0136	0.0007	-0.0095	-0.0200	-0.0072	-0.0184	-0.0295	-0.0136	-0.0249	-0.0361
Tern	795.	431.	45/7	0.0065	-0.0035	-0.0135	0.0008	-0.0094	-0.0199	-0.0071	-0.0184	-0.0294	-0.0136	-0.0248	-0.0361

24" bundle: 0.065

36" bundle: -0.0036

30" bundle: interpolation results in X'_a=0.0343.

From Table 3.3.13, we find

Table 3.3.13

SHUNT CAPACITIVE REACTANCE SPACING FACTOR, X'_d, AT 60 HZ (MEGOHM-MILES)

ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
45	0.1128	0.1128	0.1129	0.1130	0.1130	0.1131	0.1132	0.1132	0.1133	0.1134
46	0.1134	0.1135	0.1136	0.1136	0.1137	0.1138	0.1138	0.1139	0.1139	0.1140
47	0.1141	0.1141	0.1142	0.1143	0.1143	0.1144	0.1144	0.1145	0.1146	0.1146
48	0.1147	0.1148	0.1148	0.1149	0.1149	0.1150	0.1151	0.1151	0.1152	0.1152
49	0.1153	0.1154	0.1154	0.1155	0.1155	0.1156	0.1157	0.1157	0.1158	0.1158

45' phase spacing: X'_d=0.1128

And so $X_C=X'_a+X'_d$=0.0343+0.1128=0.1471Mohms-mile. Note the units of X_C are ohms-mile×10^6 [so that $B_C=1/X_C$ has units of 1/(ohms-mile×10^6)=Mhos×10^{-6}/mile, i.e., $B_C=1/(0.1471\times10^6)=6.7981\times10^{-6}$ Mhos/mile.

So, for the 6 bdl, 765 kV circuit,
z=jX_L=j0.4929 Ohms/mile,
And y=1/-jX_C=1/-j(0.1471×10^6)=j6.7981×10^{-6} Mhos/mile

Now compute the propagation constant, γ,

$$\gamma=\sqrt{zy}=\sqrt{j0.4929\times\ j6.7981\times10^{-6}}$$

$$=\sqrt{-3.3508\times10^{-6}}=\ j0.0018\ /\ mile$$

Recalling (2a, 2b)

$$Z'=Z\frac{\sinh\gamma l}{\gamma l}\qquad(2a)$$

$$Y'=Y\frac{\tanh(\gamma l/2)}{\gamma l/2}\qquad(2b)$$

Let's do two calculations:

- The circuit is 100 miles in length. Then *l*=100, and

$$Z=\ j.4724 ohms/mile*100miles=\ j47.24\ ohms$$

$$Y=\ j6.986\times10^{-6}mhos/mile*100miles=\ j0.0006986mhos$$

$$\gamma l=\frac{j0.0018}{mile}(100miles)=\ j0.18$$

Convert *Z* and *Y* to per-unit, V_b=765kV, S_b=100 MVA

Z_b=$(765\times10^3)^2/100\times10^6$=5852.3ohms,
Y_b=1/5852.3=0.00017087mhos
Z_{pu}=j47.24/5852.3=j0.0081pu,
Y_{pu}=j0.0006986/.00017087=j4.0885pu

$$Z'=Z\frac{\sinh\gamma l}{\gamma l}=j0.0081\frac{\sinh(j.18)}{j.18}=j0.0081\frac{j.179}{j.18}=j0.00806$$

$$Y'=Y\frac{\tanh(\gamma l/2)}{\gamma l/2}=j4.0885\frac{\tanh(j.18/2)}{j.18/2}=j4.0885\frac{j0.0902}{j.09}=j4.0976$$

The propagation constant γ of an electromagnetic wave is a measure of the change undergone by the amplitude of the wave as it propagates in a given direction.
γ is in general complex, so that γ=α+jβ. For a lossless transmission line, γ=jβ.

β, the phase constant, determines the wavelength, given by λ=2π/β. For the example, we obtain λ=2π/0.0018=3463miles which means it requires 3463 miles to complete 2π radians of the wave (NY to SF=2906miles)

- The circuit is 500 miles in length. Then l=500, and
  $Z = j.4724 ohms/mile * 500 miles = j236.2\ ohms$
- $Y = j6.986\times10^{-6} mhos/mile * 500 miles = j0.0035 mhos$

  $\gamma l = \frac{j0.0018}{mile}(500 miles) = j0.90$

Convert Z and Y to per-unit, V_b=765kV, S_b=100 MVA

Z_{pu}=j236.2/5852.3=j0.0404pu,

Y_{pu}=j0.0035/.00017087=j20.4834pu

$$Z' = Z\frac{\sinh \gamma l}{\gamma l} = j.0404\frac{\sinh(j.90)}{j.90} = j.0404\frac{j.7833}{j.90} = j.0352$$

$$Y' = Y\frac{\tanh(\gamma l/2)}{\gamma l/2} = j20.4834\frac{\tanh(j.90/2)}{j.90/2} = j20.4834\frac{j0.4831}{j.45} = j21.99$$

We observe that Z_{pu} and Y_{pu} are accurate for 100 mile-long lines, but not for 500 mile-long lines.

It is of interest to calculate the surge impedance for this circuit. From eq. (1d), we have

$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{j.4724}{j6.9686\times10^{-6}}} = 260.3647\text{ohms}$$

A line terminated in Z_C has a very special character with respect to reactive power: the amount of reactive power consumed by the series X is exactly compensated by the reactive power supplied by the shunt Y, for every inch of the line. In addition, such a line appears to the source as an infinitely long line; it produces no reflections.

Then the surge impedance loading is given by

$$P_{SIL} = \frac{V_{LL}^2}{Z_C} = \frac{\left(765\times10^3\right)^2}{260.3647} = 2.2477e+009$$

The SIL for this circuit is 2247 MW. We estimate line loadability from the Fig. 4 St. Clair curves as a function of line length (we further discuss these curves later).

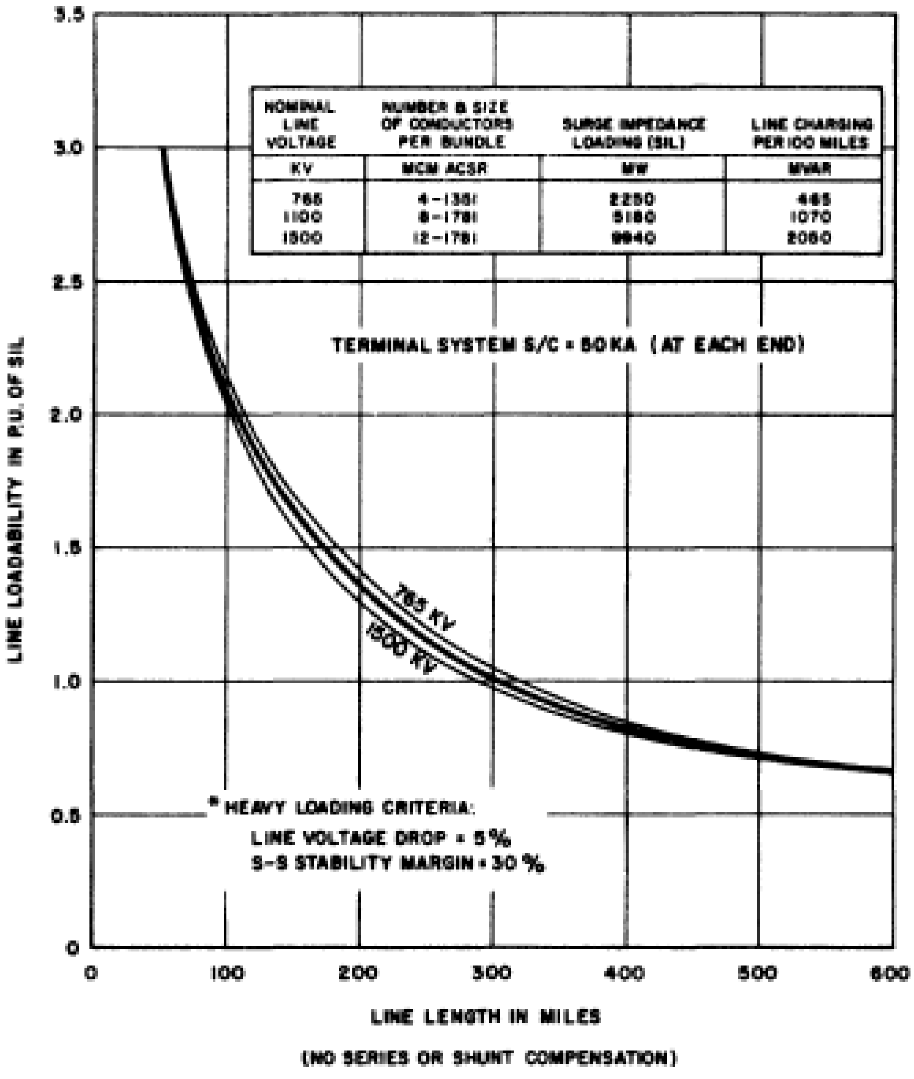


Fig. 4

100 mile long line: P_{max}=2.1(2247)=4719 MW.

500 mile long line: P_{max}=0.75(2247)=1685 MW.

4. Conductor ampacity (thermal ratings)

A conductor expands when heated, and this expansion causes it to sag. Conductor surface temperatures are a function of the following:

a) Conductor material properties
b) Conductor diameter
c) Conductor surface conditions
d) Ambient weather conditions
e) Conductor electrical current

IEEE Standard 738-2006 (IEEE Standard for Calculating Current–Temperature Relationship of Bare Overhead Conductors) [4] provides an analytic model for computing conductor temperature based on the above influences.

In addition, this same model is used to compute the conductor current necessary to cause a "maximum allowable conductor temperature" under "assumed conditions."

- Maximum allowable conductor temperature: This temperature is normally selected so as to limit either conductor loss of strength due to the annealing of aluminum or to maintain adequate ground clearance, as required by the National Electric Safety Code. This temperature varies widely according to conductor type and engineering practice and judgment [4], with 100 °C being not uncommon.

There is strong interest today to invest in dynamic line ratings (DLRs) where ambient weather conditions and/or conductor temp are measured and the associated limit is computed. This results in the constantly changing evaluation of line ampacity.

- Assumed conditions: It is good practice to select "conservative" weather conditions such as 0.6 m/s to 1.2 m/s wind speed (2ft/sec-4ft/sec), 30 °C to 45 °C (86°F-113°F) for summer conditions.

Given this information, the corresponding conductor current (I) that produced the maximum allowable conductor temperature under these weather conditions can be found from the steady-state heat balance equation [4].

For example, the Tern conductor used in the 6 bundle 765kV line (see example above) is computed to have an ampacity of about 860 amperes at 75 °C conductor temperature, 25 °C ambient temperature, and 2 ft/sec wind speed. At 6 conductors per phase, this allows for 6×860=5160 amperes, which would correspond to a power transfer of $\sqrt{3}$ * 765000 * 5160=6837 MVA.

Recall the SIL for this line was 2247 MW. Figure 4 indicates the short-line power handling capability of this circuit should be about 3(2247)=6741 MW. (Note that Fig. 4 shows the power limit does not exceed this value.)

➔Short-line limitations are thermal-constrained.

When considering relatively long lines, you will not need to be too concerned about ampacity. Limitations of an amount equal to the SIL or lower will be more appropriate to use for these long lines.

5.0 St. Clair Curves

Figure 4 is a well-known curve that should be considered as a planning guide and not an exact relationship. But as a planning guide, it is very useful. You should have some understanding of how this curve is developed. Refer to [5], a predecessor paper [6], a summary [7], and an extension (for voltage instability) in [8] for more details.

This curve represents three different types of limits:

- Short-line limitation at approximately 3 times SIL
- Medium-line limitation corresponding to a limit of a 5% voltage drop across the line;
- A long-line limitation corresponding to a limit of a 44 degree angular separation across the line.

This curve was developed based on the circuit in Fig. 5.

Given: R, X, B, X_1, X_2, θ_1, $|E_2|$, $|E_S|$
Find: $|E_1|$, θ_s, $|E_R|$, θ_R

Write 2 KCL equations at the nodes corresponding to E_S and E_R; then separate these into real & imaginary parts, giving 4 equations to find 4 unknowns.

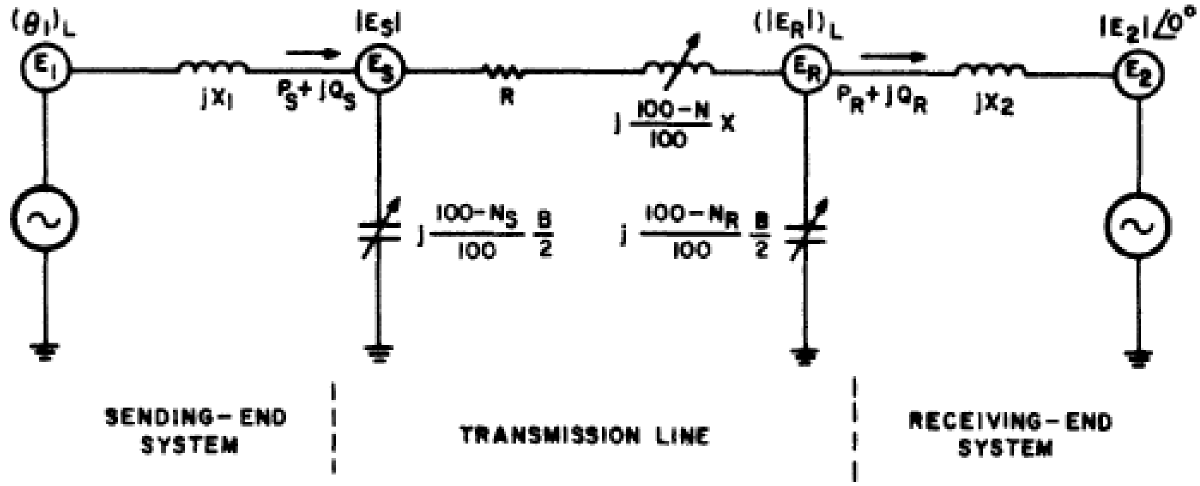


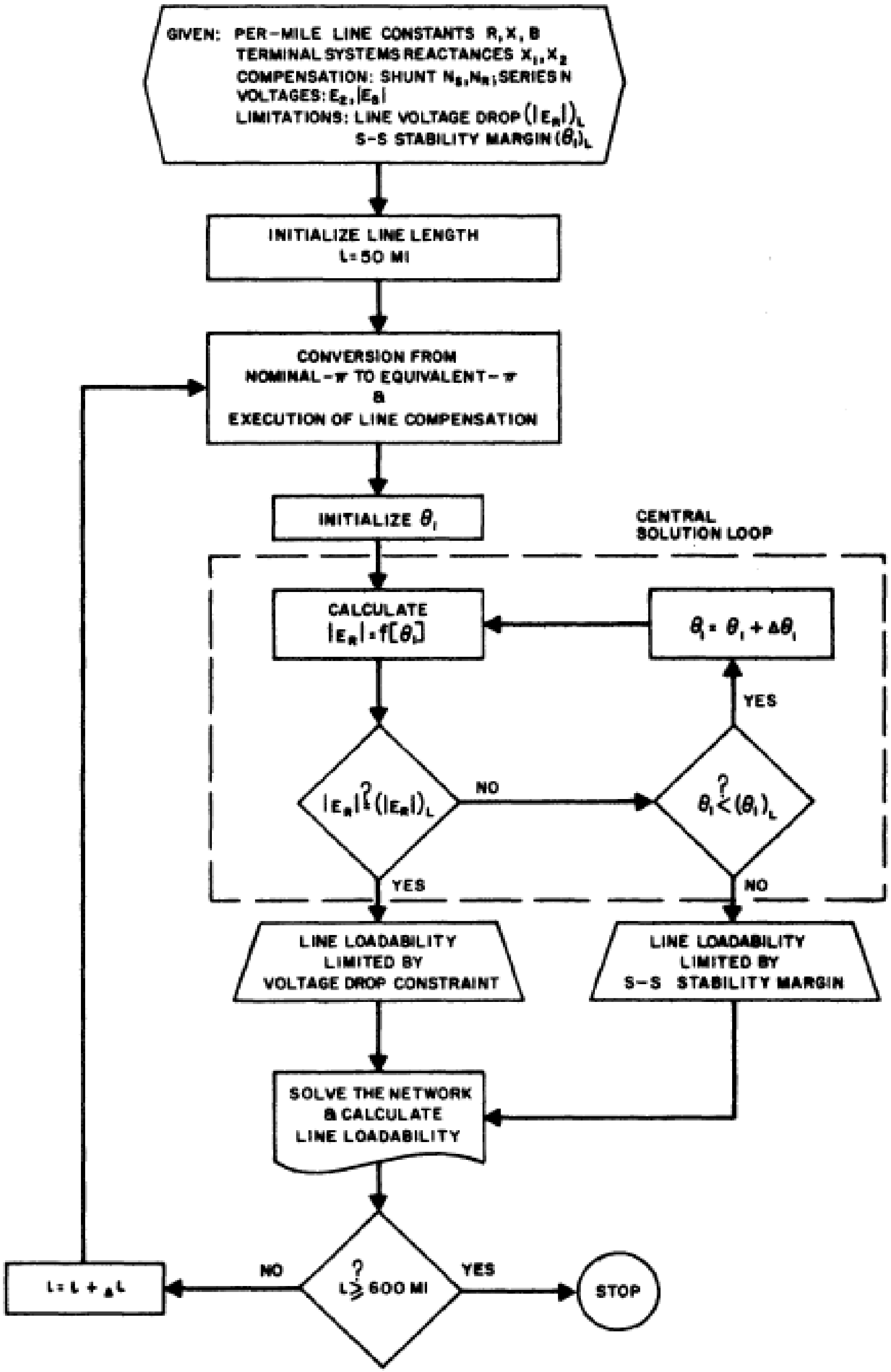
Figure 3. Mathematical model developed for line loadability study

R - positive-sequence resistance*
X - positive-sequence inductive reactance*
B - positive-sequence capacitive susceptance*

* corrected for long-line effect

Fig. 5

This circuit was analyzed using the following algorithm, Fig. 6. Observe the presence of the voltage source E_2, which is used to represent reactive resources associated with the receiving end of the transmission line. The reactances X_1 and X_2 represent the transmission system at the sending and receiving ends, respectively.



GIVEN: PER-MILE LINE CONSTANTS R, X, B
TERMINAL SYSTEMS REACTANCES X1, X2
COMPENSATION: SHUNT NS, NR; SERIES N
VOLTAGES: E2, |ES|
LIMITATIONS: LINE VOLTAGE DROP (|ER|)L
S-S STABILITY MARGIN (θ1)L
INITIALIZE LINE LENGTH
L=50 MI
CONVERSION FROM
NOMINAL-π TO EQUIVALENT-π
&
EXECUTION OF LINE COMPENSATION
INITIALIZE θ1
CENTRAL
SOLUTION LOOP
CALCULATE
|ER| = f[θ1]
θ1 = θ1 + Δθ1
YES
NO
YES
NO
LINE LOADABILITY
LIMITED BY
VOLTAGE DROP CONSTRAINT
LINE LOADABILITY
LIMITED BY
S-S STABILITY MARGIN
SOLVE THE NETWORK
& CALCULATE
LINE LOADABILITY
L = L + ΔL
NO
L ≥ 600 MI
YES
STOP

Fig. 6

The key calculation performed in the algorithm is represented by block having the statement

CALCULATE

$|E_R|=f(\theta_1)$

Referring to the circuit diagram, this problem is posed as:

Given: R, X, B, X_1, X_2, θ_1, $|E_2|$, $|E_S|$

Find: $|E_1|$, θ_s, $|E_R|$, θ_R

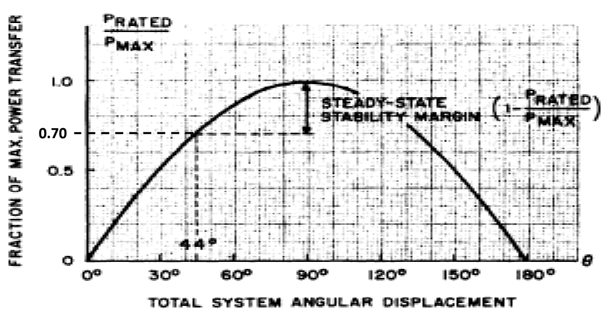
Although the paper does not say much about how it makes this calculation, one can write two KCL equations at the two nodes corresponding to E_S and E_R, and then separate these into real and imaginary parts, giving 4 equations to find 4 unknowns (note that the angle of E_2 is assumed to be the reference angle and thus is 0 degrees).

The result of this analysis for a particular line design (bundle and phase geometry) is shown in Fig. 7, where we observe two curves corresponding to

- Constant steady-state stability margin curve of 30% (angle is θ_1, which is from node E_1 to node E_2).

This value is computed based on

$$\%\text{StabilityMargin} = \frac{P_{\max} - P_{rated}}{P_{\max}} \times 100\%$$



Here, P_{max} is the ampacity of the line, and P_{rated} is the allowable flow on the line.

- Constant line voltage drop curve of 5%, given by

$$\%\text{VoltageDrop} = \frac{E_s - E_r}{E_s} \times 100\%$$

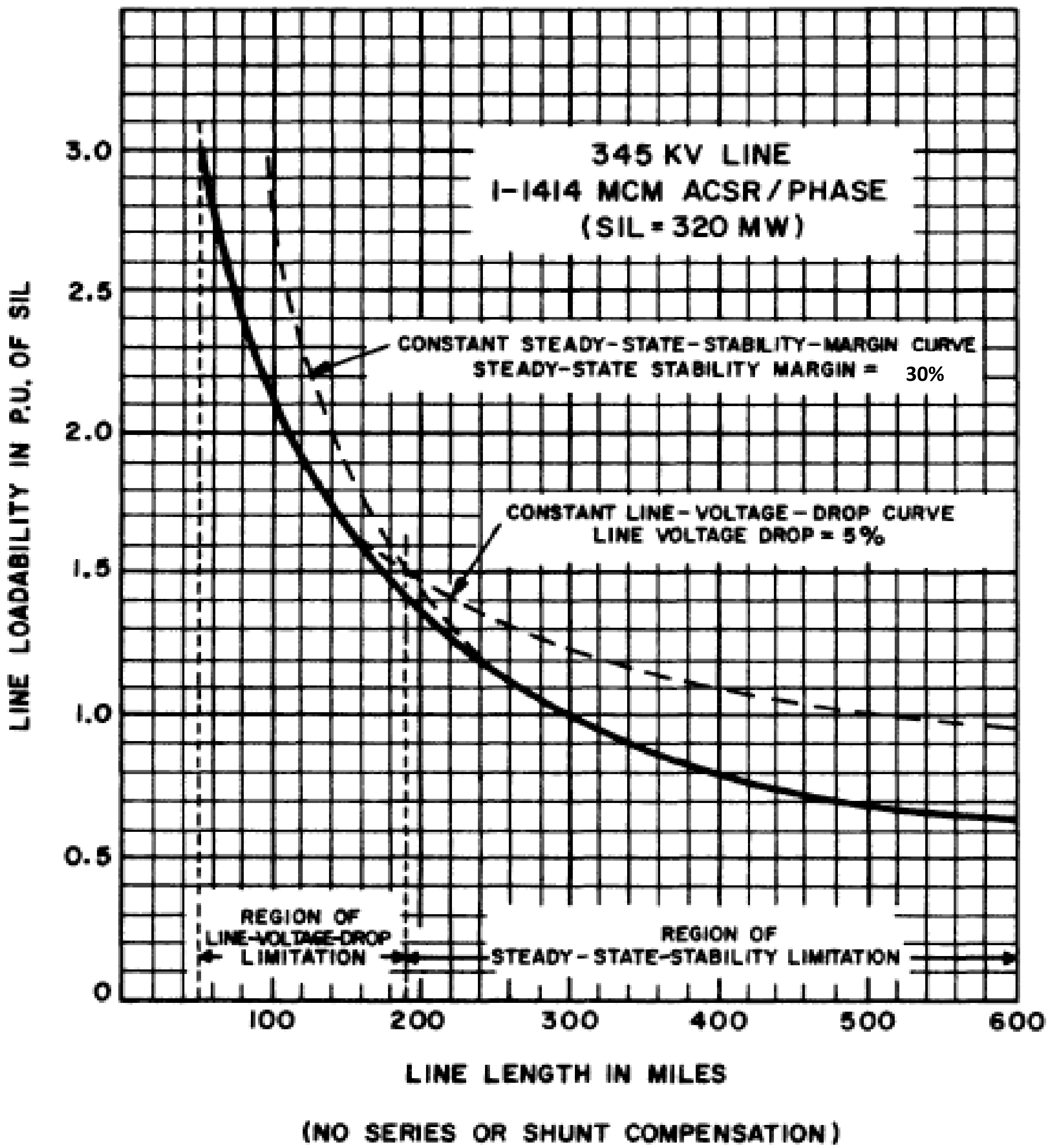


Fig. 7

In Fig. 7, the dark solid curve is the composite of the two limitations associated with steady-state stability and voltage drop. The 3.0 pu SIL value which limits the curve at short distances is associated with the conductor's thermal limit.

The paper being discussed [5], in addition to 345 kV, also applies its approach to higher voltage transmission, 765 kV, 1100 kV, and 1500 kV (Unfortunately, for some reason, 500 kV was not included). For these various transmission voltages, it presents a table of data that can be used in the circuit of Fig. 5 and the algorithm of Fig. 6. This table is copied out below.

NOMINAL VOLTAGE CLASS (kV)	SYSTEM STRENGTH AT EACH TERMINAL*		LINE CHARACTERISTICS**		
	$MVA_{3\phi}$	$\%X = \frac{100}{MVA_{3\phi}} \cdot 100\%$	Z (%/mi)	Y (%/mi)	SIL (MW)
345	30,000	.333	.00571 + j.06432	j.6604	320
765	66,000	.151	.00033 + j.00918	j4.659	2250
1100[6]	95,000	.105	.00007 + j.00398	j10.69	5180
1500[6]	130,000	.077	.00003 + j.00207	j20.51	9940

* SYSTEM STRENGTH CORRESPONDING TO 50 KA FAULT DUTY

** POSITIVE-SEQUENCE CHARACTERISTICS (ON 100 MVA BASE)

The "system strength at each terminal"[1] is quantified by the fault duty at that terminal, assumed in both cases to be

---

[1] The fault duty or short circuit current at a bus provides an indication of the network's voltage "stiffness" or "strength" at that bus. The higher a bus's short circuit current, the lower the impedance between that bus and current sources (generators), the less the variation in voltage magnitude will occur for a given change in network conditions.

50 kA. Using this, we can get the fault duty in MVA according to

$$MVA_{3\phi} = \sqrt{3} \times V_{LL,nom} \times 50E3$$

Then the corresponding reactance may be computed by

$$X_{pu} = \frac{V_{pu}^2}{MVA_{pu}}$$

This pu reactance is computed at each terminal and used to represent the sending and receiving end impedances X_1 and X_2 respectively (see Fig. 5).

This can be shown as follows:

$S_{3\varphi}=3V_{LN}^2/X$.

Writing all S, V, and X quantities as products of their pu values and their base quantities, we get

$S_{3\varphi,base}S_{pu}=3[(V_{pu}V_{LN,base})^2/(X_{pu}X_{base})$

Rearranging,

$S_{3\varphi,base}S_{pu}=[3V_{LN,base}{}^2/X_{base}][(V_{pu})^2/X_{pu}]$

And we see that

$S_{3\varphi,base}=3V_{LN,base}{}^2/X_{base}$ and

$S_{pu}=(V_{pu})^2/X_{pu}$

➔$X_{pu}=V_{pu}^2/S_{pu}$.

We will assume that V_{pu}=1, and with a 100 MVA base, the last equation results in

$$X_{pu} = \frac{1}{MVA_{3\phi}/100} = \frac{100}{MVA_{3\phi}}$$

For example, let's consider the 765 kV circuit, then we obtain

$$MVA_{3\varphi} = \sqrt{3} \times V_{LL,nom} \times 50000$$

$$= \sqrt{3} \times 765000 \times 50000 = 6.6251E10 \quad \text{volt-amperes}$$

which is 66,251 MVA.

Observe the table above gives 66,000 MVA.

Then, X_{pu}=100/66,000=0.00151pu
which is 0.151%, as given in the table.

The table also provides line impedance and susceptance, which can be useful for rough calculations. Note that the values are given in % per mile, which are 100 times the values given in pu per mile.

Finally, the table provides the surge impedance loading (SIL) of the transmission lines at the four different voltage levels, as

320, 2250, 5180, and 9940 MW for

345, 765, 1100, and 1500 kV,

respectively.

Recall what determines SIL:

$$P_{SIL} = \frac{V_{LL}^2}{Z_C} \qquad Z_C = \sqrt{\frac{z}{y}} = \sqrt{X_L X_C}$$

$$X_L = \underbrace{2.022\times10^{-3} f \ln\frac{1}{R_b}}_{X_a} + \underbrace{2.022\times10^{-3} f \ln D_m}_{\mathrm{X_d}} \;\; \Omega/\text{mile}$$

$$X_C = \underbrace{\frac{1}{f}\times1.779\times10^{6} \ln\left(\frac{1}{R_b^c}\right)}_{X'_a} + \underbrace{\frac{1}{f}\times1.779\times10^{6} \ln(D_m)}_{X'_d} \;\; \Omega\text{-mile}$$

D_m is the GMD between phase positions:

$$\boldsymbol{D_m \equiv \left(d_{ab}^{(1)} d_{ab}^{(2)} d_{ab}^{(3)}\right)^{1/3}}$$

R_b is the GMR of the bundle

$$\begin{aligned} R_b &= \left(r' d_{12}\right)^{1/2}, && \text{for 2 conductorbundle} \\ &= \left(r' d_{12} d_{13}\right)^{1/3}, && \text{for 3 conductorbundle} \\ &= \left(r' d_{12} d_{13} d_{14}\right)^{1/4}, && \text{for 4 conductorbundle} \\ &= \left(r' d_{12} d_{13} d_{14} d_{15} d_{16}\right)^{1/4}, && \text{for 6 conductorbundle} \end{aligned}$$

$$r' = re^{-\frac{\mu_r}{4}}$$

$\boldsymbol{R_b^c}$ is <u>Capacitive</u> GMR for the bundle:

$$\begin{aligned} R_b^c &= \left(r d_{12}\right)^{1/2}, && \text{for 2 conductorbundle} \\ &= \left(r d_{12} d_{13}\right)^{1/3}, && \text{for 3 conductorbundle} \\ &= \left(r d_{12} d_{13} d_{14}\right)^{1/4}, && \text{for 4 conductorbundle} \\ &= \left(r d_{12} d_{13} d_{14} d_{15} d_{16}\right)^{1/6}, && \text{for 6 conductorbundle} \end{aligned}$$

So in conclusion, we observe that SIL is determined by

- Phase positions (which determines D_m)
- Choice of conductor (which determines r and r' and influences R_b and R_b^c)
- Bundling (which influences R_b and R_b^c).

We refer to data which determines SIL as "line constants." (Although SIL is also influenced by voltage level, the line loadability limit, P_{rated}/P_{SIL}, is not.)

Reference [5] makes a startling claim (italics added):

> "Unlike the 345-kV or 765-kV line parameters, UHV line data is still tentative because both the choice of voltage level and optimum line design are not finalized. This uncertainty about the line constants, however, is not very critical in determining the line loadability -- expressed in per-unit of rated SIL – especially at UHV levels. The reason lies in the fact that *for a lossless line*, it can be shown that the line loadability -- or the receiving-end power -- in terms of SIL of that line, $S_{Receiving}$/SIL, is not dependent on the line constants, but rather is a function of the line length and its terminal voltages. This concept is discussed further in the Appendix."

Translation: 1100 and 1500 kV transmission have never been built and so we are really just guessing in regards to its line constants…

…but it does not matter, because P_{rated}/P_{SIL} is almost independent of line constants but rather depends on just the line length and terminal voltages.

The paper's appendix derives this result for a lossless line:

$$\frac{S_{\text{Receiving}}}{P_{SIL}} = j\frac{\left(\frac{E_S}{E_R}\right)^* - \cos\beta L}{\sin\beta L}\left|E_R\right|^2$$

where $S_{Receiving}=P_{Receiving}+jQ_{Receiving}$ (a complex number), $\beta=\omega/\upsilon$ and ω is $2\pi f$ (f=60Hz), and υ is approximately the speed of light (3E8m/sec).

The paper justifies the "lossless line" requirement:

> "Since the resistance of the EHV/UHV lines is much smaller than their 60-Hz reactance, such lines closely approximate a lossless line from the standpoint of loadability analysis. Therefore, the loadabilities in per-unit of SIL of these lines are practically independent of their respective line constants and, as a result, of their corresponding voltage classes."

The paper develops the St. Clair curves for a 765 kV, 1100 kV, and a 1500 kV transmission line, and I have replicated it in Fig. 8 below. Observe that the 3 curves are almost identical. The paper further states (italics added):

> "It is reassuring to know that one single curve can be applied to all voltage classes in the EHV/UHV range. Obviously, a general transmission loading curve will not cover the

complete range of possible applications; *nonetheless, it can provide a reasonable basis for any preliminary estimates of the amount of power that can be transferred over a well-designed transmission system*."

Or… it can provide a reasonable basis for a preliminary estimate of the transmission system voltage level necessary to achieve a given power transfer level.

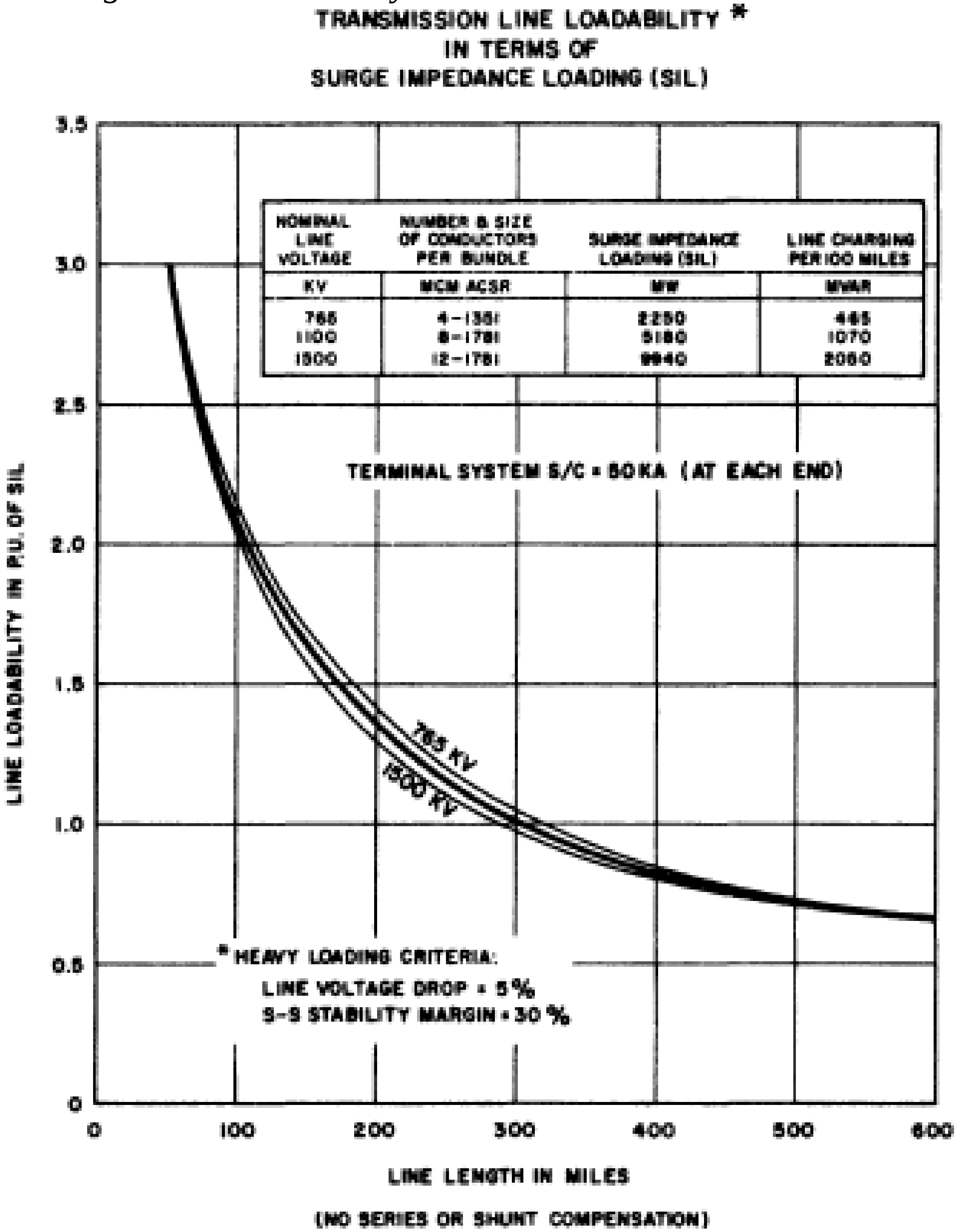


Fig. 8

A final statement made in the paper is worth pointing out (italics added):

> "Any departures from the *assumed performance criteria* and *system parameters* -- which, for convenience, are clearly enumerated on the EHV/UHV loadability chart shown in Figure 8 -- must not be ignored and, depending on their extent, they should properly be accounted for in the line loadability estimates. To illustrate this, the effect of some of the variations in these assumed parameters such as terminal system strength, shunt compensation, line-voltage-drop criterion and stability margin, are investigated in the next section."

Note from Fig. 8 the "assumed performance criteria":

- Line voltage drop = 5%
- S-S stability margin = 30%

and the "system parameters":

- Terminal system S/C – 50 kA (each end)
- No series or shunt compensation →

No series or shunt compensation means distance is an uncompensated distance. If you use series or shunt compensation, voltage (particularly w/ shunt) and stability (particularly w/ series) constraints will be partly alleviated. The model, Fig. 5 above uses N (series) and N_S, N_R (shunts at sending/receiving ends, respectively) to allow for compensation.

The paper provides sensitivity studies on both the performance criteria and some system parameters.

Finally, observe that Fig. 8 also provides a table with

- Nominal voltage
- Number and size of conductors per bundle
- Surge impedance loading
- Line charging per 100 miles

These are "line constant" data! Why do they give them to us?

Although P_{rated}/P_{SIL} is independent of the "line constant" data, P_{rated} is not. To get P_{rated} from the St. Clair curve, we must know P_{SIL}, and P_{SIL} very much depends on the "line constant" data.

6.0 Resistance

I have posted on the website tables from reference [7] that provide resistance in ohms per mile for a number of common conductors and provided a section of those tables below.

Table 3.3.3

CHARACTERISTICS OF MULTILAYER ALUMINUM-CONDUCTOR-STEEL-REINFORCED (ACSR)

Code	Cross Section (kcmil) Al	Cross Section (sq mm) Al	Cross Section (sq mm) Tot	Stranding Aluminum	Stranding Steel	Diameter Cond (in.)	Diameter Core (in.)	Layers	wt (lbs per 1000 ft)	STRG (Kips)	DC 25 C	Resistance (Ohms/Mile) AC at 60 HZ 25 C	50 C	75 C	100 C	GMR (ft)	Reactance 1 ft Rad. 60 HZ X_a (Ohm/Mile)	X_a' (Megohm-Mile)
—	2776.	1407.	1521.	84x.1818	19x.1091	2.000	.546	4	3219	81.6	.0338	.0395	.0421	.0452	.0482	.0667	.329	.0736
Joree	2515.	1274.	1344.	76x.1819	19x.0849	1.880	.425	4	2749	61.7	.0365	.0418	.0450	.0482	.0516	.0621	.337	.0755
Thrasher	2312.	1171.	1235.	76x.1744	19x.0814	1.802	.407	4	2526	57.3	.0397	.0446	.0482	.0518	.0554	.0595	.342	.0767
Kiwi	2167.	1098.	1146.	72x.1735	7x.1157	1.735	.347	4	2303	49.8	.0424	.0473	.0511	.0550	.0589	.0570	.348	.0778
Bluebird	2156.	1092.	1181.	84x.1602	19x.0961	1.762	.480	4	2511	60.3	.0426	.0466	.0505	.0544	.0584	.0588	.344	.0774
Chukar	1781.	902.	976.	84x.1456	19x.0874	1.602	.437	4	2074	51.0	.0516	.0549	.0598	.0646	.0695	.0534	.355	.0802

A DC value is given, at 25°C, which is just $\rho l/A$, where ρ is the electrical resistivity in ohm-meters, l is the conductor length in meters, and A is the conductor cross-sectional area in meters^2.

The tables also provide 4 AC values, corresponding to 4 different operating temperatures (25, 50, 75, and 100°C). These values are all higher than the DC value because of the skin effect, which causes a non-uniform current density to exist such that it is greater at the conductor's surface than at the conductor's interior. This reduces the effective cross-sectional area of the conductor[2].

Resistance also increases with temperature because temperature changes the level of electron mobility within the material.

7.0 Recent MISO information

MISO has put together some excellent information on transmission loadability. The information is in the form of two sets of slides:

- Transmission Line Ratings Workshop, Typical Industry Practices, Jan 15, 2021 [9].
- Discussion of Legacy, 765 kV, and HVDC Bulk Transmission [10].

I have posted these slides to the course website, and I encourage you to review them on your own. I extract just a few of them here as they very well complement the material we have presented in this document.

We observe in Fig. 9 [10] that MISO is identifying three categories of limits: thermal, safe, and absolute.

[2] Loss studies may model AC resistance as a function of current, where ambient conditions (wind speed, direction, and solar radiation) are assumed.

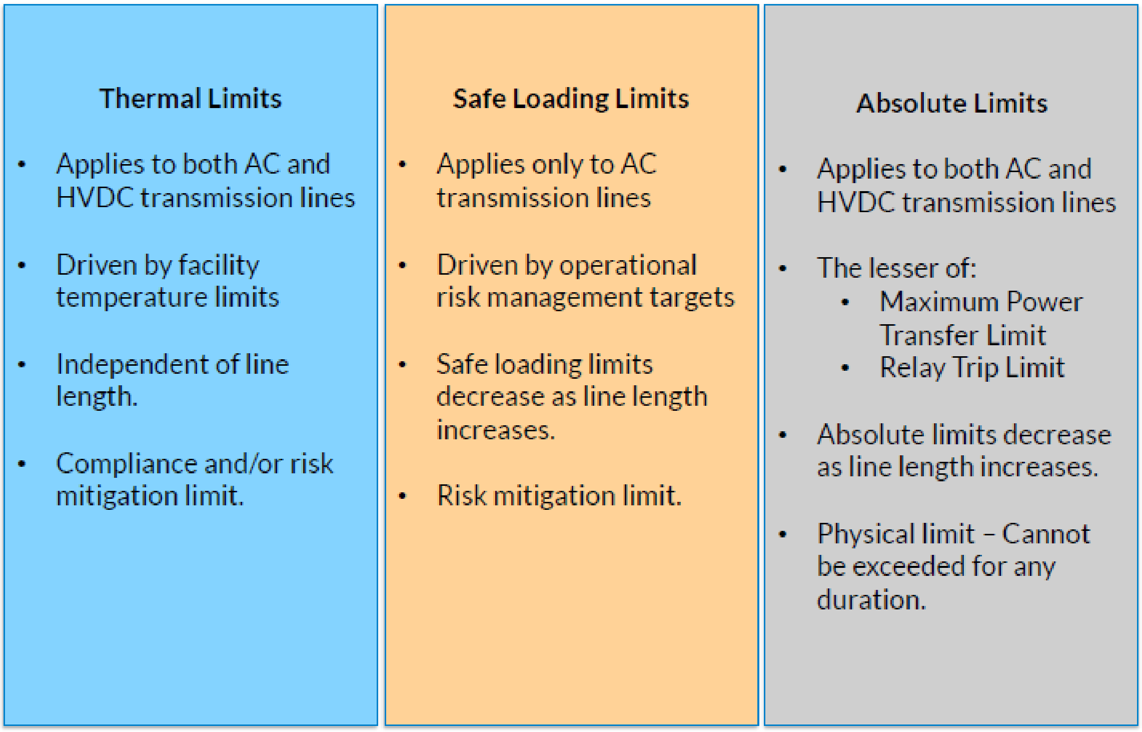


Fig. 9

Two of these limits are already familiar to us:

- Thermal limits: We have referred to this in these notes as ampacity, per above Section 4.0. It is shown on the St. Clair diagrams as 3.0×SIL.
- Safe loading limits: We have referred to this in these notes as the St. Clair limits. Whereas thermal limits are independent of line length, safe loading limits decrease as line length increases. These limits are driven by “operational risk management targets” such as the 5% voltage drop and the 44° maximum angle separation.

The third limit indicated in Fig. 9 is the "absolute limit." We have not identified this limit in these notes, and so we explain it here, for AC lines.

Fig. 9 indicates that the absolute limit is the lesser of the maximum power transfer limit (MPTL) and the relay trip limit. It is not common for a relay to be set low enough so that it can trip as a result of a load current (relays trip on fault currents). Nonetheless, it is possible, and so MISO has included it in its analysis. I will not say more about the relay trip limit, but more information is provided in [9].

Perhaps the more interesting limit is that identified as the MPTL. The underlying/basic concept of MPTL is simple: it is the maximum power that can flow across a line without the angular separation across the line exceeding 90°. This is described in Figs. 10a and 10b below [9].

The power flow in per unit through a transmission branch connected to Bus A at the source terminal and Bus B at the receiving terminal can be approximated by the following formula:

Power Flow = $[|V_S||V_R|\sin(\delta)] / |X_L|$
where V_S = Voltage at Bus A in per unit
V_R = Voltage at Bus B in per unit
X_L = Series reactance of line in per unit
δ = Angle by which V_S leads V_R in radians

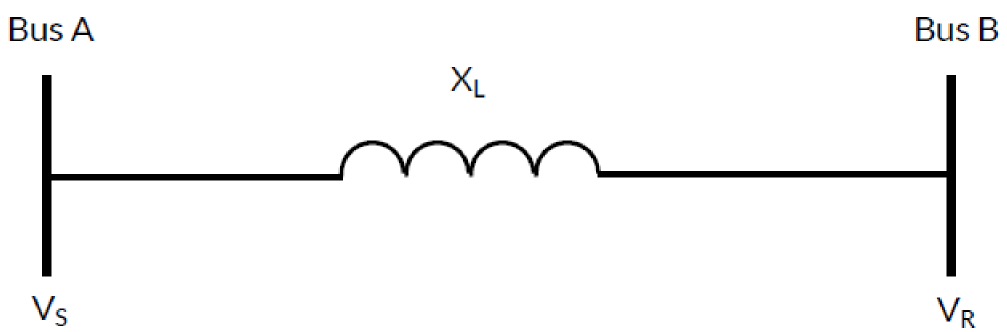


Fig. 10a

Since the maximum value of the sine function is 1.0 and occurs when the angle is 90°, the maximum power flow through a transmission branch occurs when the source voltage leads the receiving voltage by 90° and is equal to the following:

Maximum Power Flow = $|V_S||V_R| / |X_L|$
where V_S = Voltage at Bus A in per unit
V_R = Voltage at Bus B in per unit
X_L = Series reactance of line in per unit

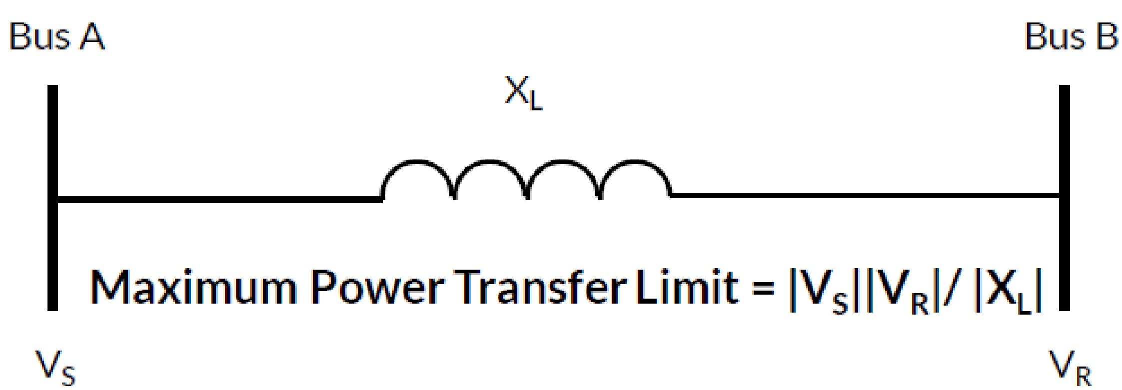


Fig 10b

MISO refers to the MPTL when computed as in Figs. 10a and 10b as $MPTL_{branch}$.

MISO refines the underlying/basic concept shown in Figs. 10a and 10b by including the branch within a "system," where the influence of "system" is provided by including

- source impedances at sending and receiving buses (souce impedance computed at each respective terminal based on 1.0 pu voltage and an assumed fault duty to reflect system strength, i.e., relative to the terminal, the proximity and amount of generation);
- an impedance in parallel with the branch of interest.

These refinements are illustrated in Figs. 11a, 11b, and 11c [9].

Maximum Power Transfer Limit
With and Without Consideration of External System

- The transmission branch maximum power transfer limit shown on the previous slide is a true maximum power transfer limit for a transmission impedance branch, but not necessarily the most conservative maximum power transfer limit for a given transmission impedance branch.
- The most conservative maximum power transfer limit for a branch must consider the impact of the external system.
- The external system can be considered in developing a maximum power transfer limit by connecting the transmission branch to a two-bus equivalent network as shown on the following slide.

Fig. 11a

Maximum Power Transfer Through A Transmission Line
Example with External System Considered

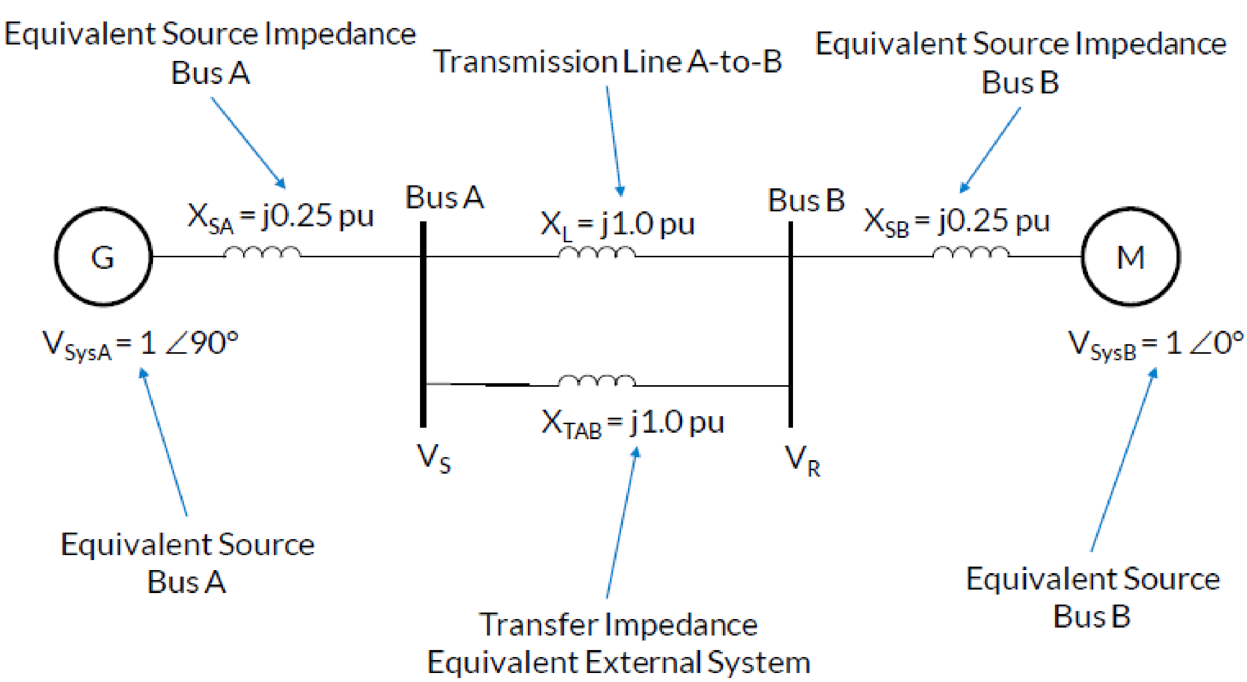


Fig. 11b

Maximum Power Flow Across System Further Limits Maximum Power Flow of Branch

- Considering the branch and the external system modeled on the previous slide, the maximum power transfer possible across the system would occur when the phase angles of the equivalent source voltages are displaced by 90°, and would be calculated as follows:

Max Power Across System

$= [|V_{SA}||V_{SB}|] / |[X_{SA} + X_L||X_{TAB} + X_{SB}]|$

$= [1.0 * 1.0] / [0.25 + 1.0||1.0 + 0.25]$

$= 1.0 / [0.25 + 0.5 + 0.25] = 1.0$ p.u.

- Since the line impedance is equal to the external system transfer impedance, the maximum power flow through the line occurs when there is maximum power flow across the system and would be equal to 50% of the maximum power flow across the system based on simple current division between the line and transfer impedance, which implies a maximum power transfer limit for the branch of 0.5 per unit.
- When the external system is ignored, the maximum power transfer limit of the branch is calculated as:
  - Max Power Transfer Limit = $|V_S||V_R| / |X_L|$ = (1.0)(1.0)/(1.0) = 1.0 p.u. (overstated by 100%)

Fig. 11c

Note that Fig. 11c computes the MPTF across the system (i.e., both the branch of interest as well as the "system" branch). But if we want to obtain the MPTF for just the branch of interest, then we need to compute only the power flowing across that branch. MISO identifies this as $MPTL_{BranchSystem}$, (read: MPTL of the branch with the system influence included). The difference between $MPTL_{Branch}$ and $MPTL_{BranchSystem}$ is indicated in Fig. 11d [9].

In the expression for $MPTL_{BranchSystem}$, the factor DF accounts for the fact that we only want the power flowing across "that branch" per the underlined sentence in the previous paragraph.

Two Maximum Power Transfer Limits for a Branch

- A transmission impedance branch has two maximum power transfer limits:
  - **$MPTL_{Branch}$** = The calculated limit when the external system is ignored.
  - **$MPTL_{BranchSystem}$** = The calculated limit when the external system is considered.
- The formulae for each type of maximum power transfer limit are as follows:
  - **$MPTL_{Branch}$** = $|V_S||V_R|/|X_L|$
  - **$MPTL_{BranchSystem}$** = $\{|V_{SA}||V_{SB}| / |[X_{SA} + X_{SB} + X_L||X_{TAB}]|\} * DF$

Where

$DF = 1.0$ if there is infinite external transfer impedance between Bus A and B

$DF = |X_{TAB} / [X_L + X_{TAB}]|$ if external transfer impedance is less than infinite

- $MPTL_{Branch} = MPTL_{BrnachSystem}$ when $X_{SA} = X_{SB} = 0$ (Infinite System Strength)

Fig. 11d

Fig. 11e [9] shows the relation between $MPTL_{Branch}$ and $MPTL_{BranchSystem}$ for values of equivalent source impedance ranging from 0% to 50% of X_L.

Plot of $MPTL_{Branch}$ vs. $MPTL_{BranchSystem}$

- **For $X_{SA} = X_{SB} = X_S$**
  - Blue Plot: Plot of $MPTL_{BranchSystem}$ as a percent of $MPTL_{Branch}$ assuming X_S varies from 0% to 50% of X_L with no external transfer impedance (i.e., infinite external transfer impedance)
  - Red Plot: Plot of $MPTL_{BranchSystem}$ as a percent of $MPTL_{Branch}$ assuming X_S varies from 0% to 50% of X_L with $X_{TAB} = X_L$

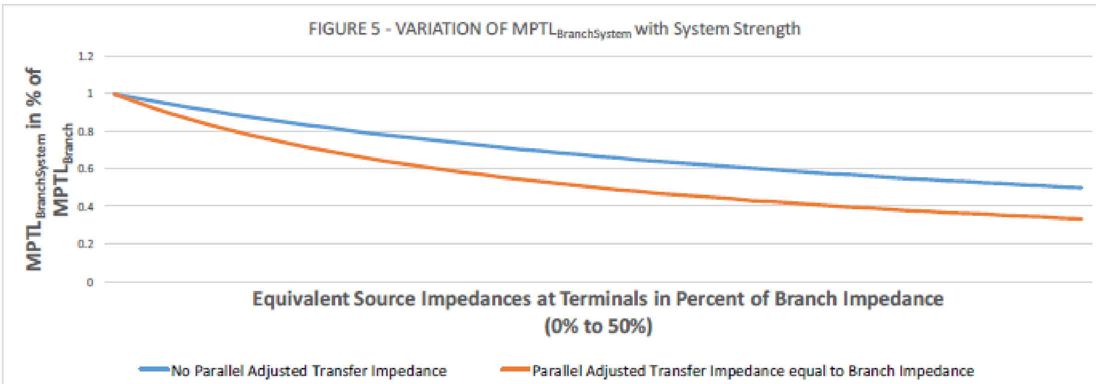


Fig. 11e

It is important to realize that the MPTL represents a very hard limit. It is absolutely essential that power transfer remain below it. How much below it? This question is addressed by MISO in Figs. 11f and 11g [9], where they indicate interest in St. Clair curves as a good way to address this. In [9], they address St. Clair curves (and many other rating-related issues) in some detail, and I strongly encourage you to go through these slides. Indeed, the final thought, as indicated in Fig. 11h [9], is to use St. Clair curves as MISO’s “safe loading limits.” Fig. 11i [10] compare thermal limits, safe loading limits, and MPTL for single circuit 345 kV and single circuit 765 kV designs.

The "So What" of Maximum Power Transfer Limits in a World of Ambient Adjusted Ratings and Dynamic Line Ratings

- While AARs and DLRs allow for reductions in production cost and increases in operating flexibility, they will tend to drive lower safety margins between thermal limits and maximum power transfer limits if the transmission line is long.
- Attempting to load a long line near or beyond a maximum power transfer limit typically will introduce angular stability issues, which could have adverse impacts on reliability.
- In the new world of renewables where power may travel longer distances on average and system strength will be lower on average, it is even more important to be aware of the existence of maximum power transfer limits.

Continued…

Fig. 11f

The "So What" of Maximum Power Transfer Limits in a World of Ambient Adjusted Ratings and Dynamic Line Ratings (continued)

It may be prudent to consider capping AARs and DLRs at some value to ensure operation never approaches maximum power transfer limits.

One such idea would be to use the St. Clair Curve which provides for a loadability limit based on the Surge Impedance Loading (SIL) of the line and the line length.

The St. Clair Curve incorporates a 30% steady state stability margin (maximum angular displacement of 44°) applied to the $MPTL_{Branch}$ to account for the impact of the external system.

Such a cap would be more restrictive for longer lines than shorter lines, which is appropriate.

Fig. 11g

Comparison of **Typical** EHV Line Limit Curves: Single Circuit 345 kV and 765 kV

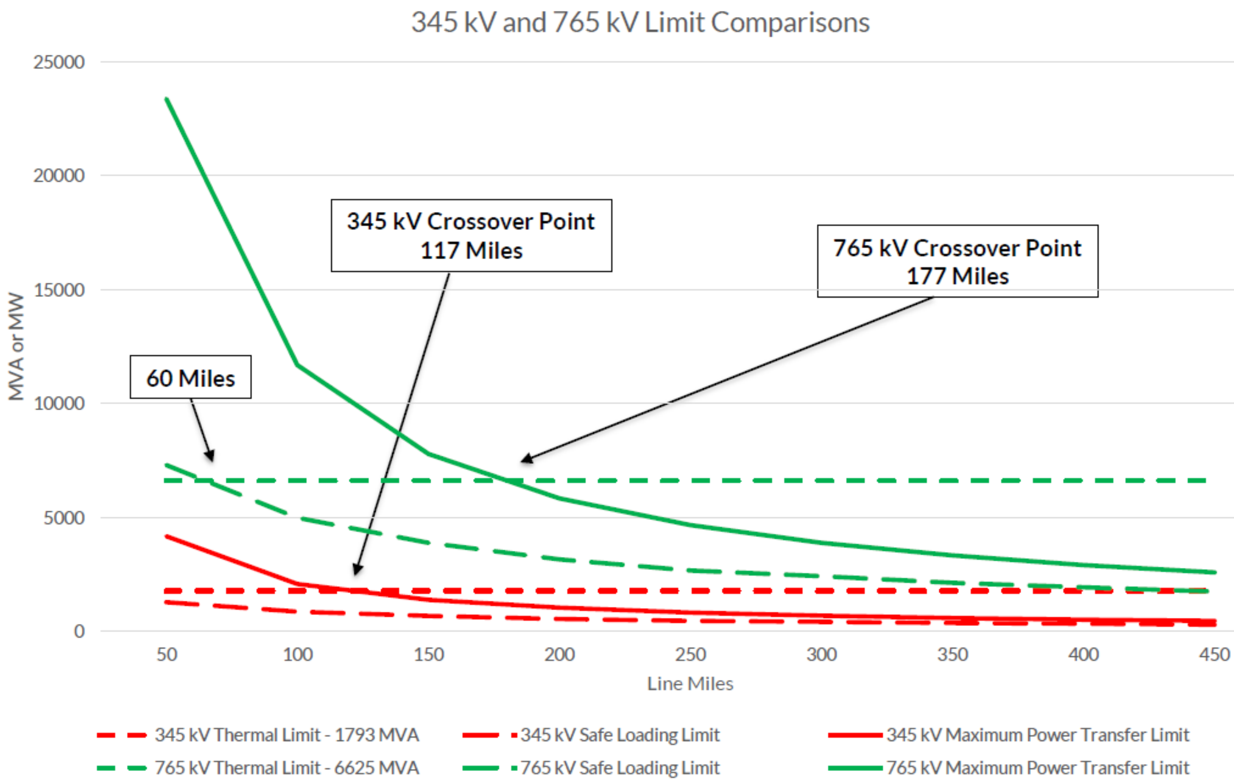


Fig. 11i

A summary statement for this section might be this:

- Short lines: w/o ARR or DLR they are always thermally-limited; w/ ARR or DLT, safe loading limits need to be considered.
- Med-to-long lines: They will most likely be limited to something less than the thermal limit. In such cases, safe-loading limits are good initial design proxies, but they may be too much or too little. The actual loadability will be a function of conditions, and power flow, voltage stability, and dynamic analysis should be performed to identify it with confidence.

8.0 Final comments on AC overhead transmission

In the US, HV AC is considered to include voltage levels 69, 115, 138, 161, and 230 kV.

EHV is considered to include 345, 500, and 765 kV. There exists a great deal of 345 and 500 kV all over the country. The only 765 kV today in the US is in the Ohio and surrounding regions, owned by AEP, as indicated by Fig. 12 [11]. Transmission equipment designed to operate at 765 kV is sometimes referred to as an 800 kV voltage class. There also exists 800 kV-class transmission in Russia, South Africa, Brazil, Venezuela, South Korea, and Quebec.

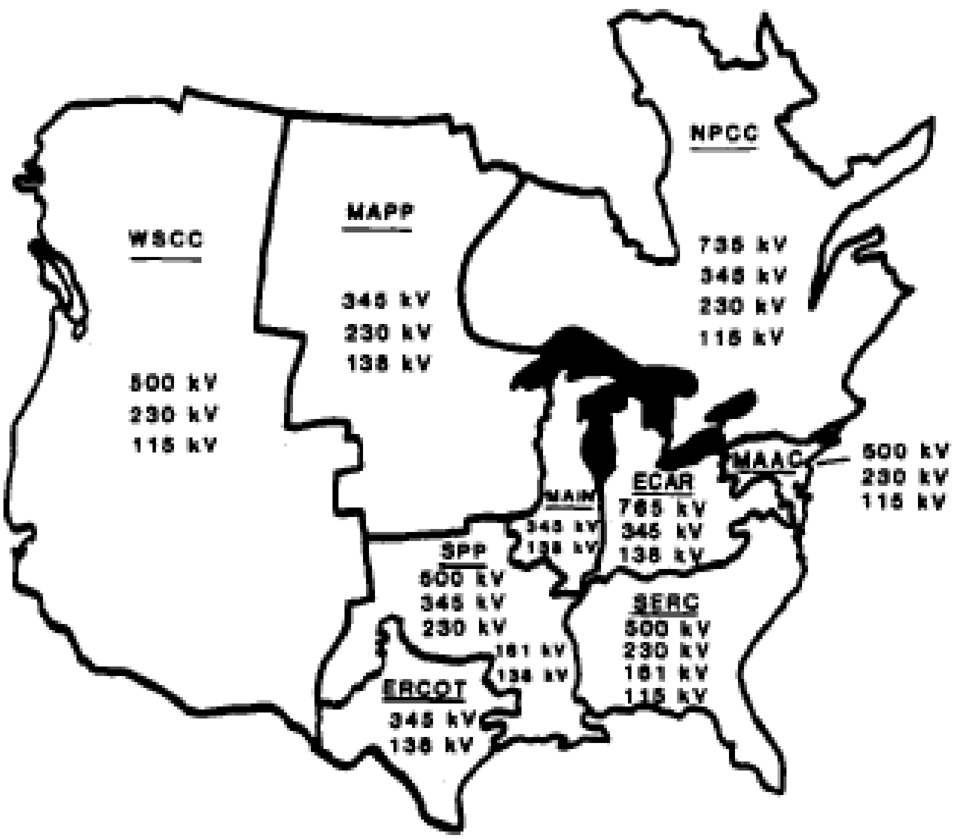


Fig. 12

Figure 13 shows ABB's deliveries of 800 kV voltage class autotransformers (AT) and generator step-up banks (GSUs) from 1965 to 2001 [12].

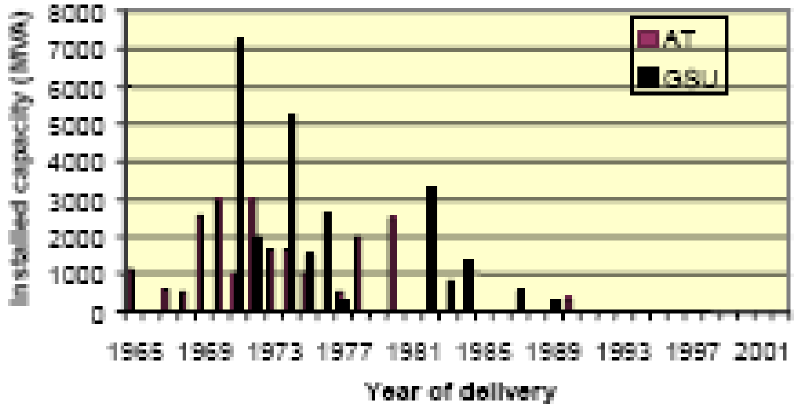


Fig. 13

It is clear from Fig. 10 there was a distinct decline in 765 kV AC investment beginning in early 1980s and reaching bottom in 1989. However, there has been renewed interest in 765 kV during the past few years, with recently completed projects in China & India. In addition, MISO on 3/15/24 put forward study results suggesting development of 765 kV transmission in its region, Fig. 14.

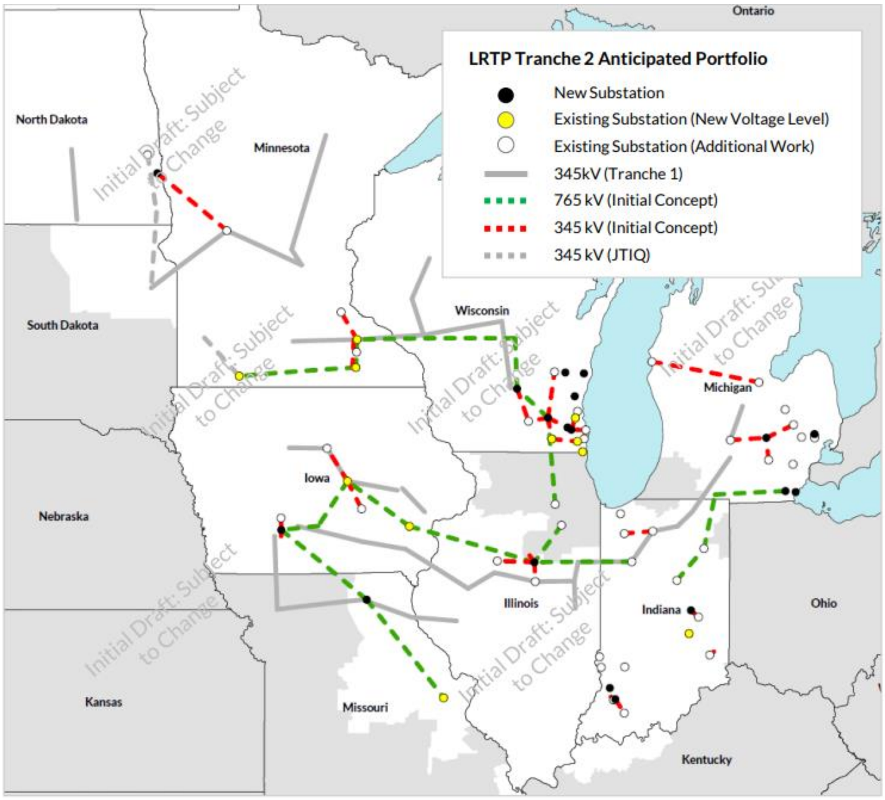


Fig. 14: MISO LRTP Tranche 2 proposal

UHV is considered to include 1000 kV and above. There is no UHV transmission in the US. There was 1200 kV UHV in neighboring countries to Russia [13], and in Japan, but the operational voltage of these lines were downgraded to 500kV. China completed a 1000 kV transmission project in 2009 [14].

9.0 General comments on underground transmission

Underground transmission has traditionally not been considered a viable option for long-distance transmission because it is significantly more expensive than overhead due to two main issues:

(a) It requires insulation with relatively high dielectric strength owing to the proximity of the phase conductors with the earth and with each other. This issue becomes more restrictive with higher voltage. Therefore the operational benefit to long distance transmission of increased voltage levels, loss reduction (due to lower current for a given power transfer capability), is, for underground transmission, offset by the significantly higher investment costs associated with the insulation.

(b) The ability to cool underground conductors as they are more heavily loaded is much more limited than overhead, since the underground conductors are enclosed and the overhead conductors are exposed to the air and wind.

Table 1 [15] provides a cost comparison of overhead and underground transmission for three different AC voltage ranges.

Table 1

Voltage Range	110 - 219kV	220 -362kV	363 -764kV
Mean MVA/circuit	200	600	1800
Mean Overhead Line Cost $/km/MVA	820	390	185
Mean Underground Cable Cost $/km/MVA	6100	4900	3700
Mean Cost Ratio	7	13	20
Spread	3.4 -16	5.1 - 21.1	14.6 -33.3

Although Table 1 is dated (1996), it makes the point that the underground cabling is significantly more expensive than overhead conductors. You can see why by comparing an overhead conductor in Fig. 15 (below left) to an underground cable (below right).

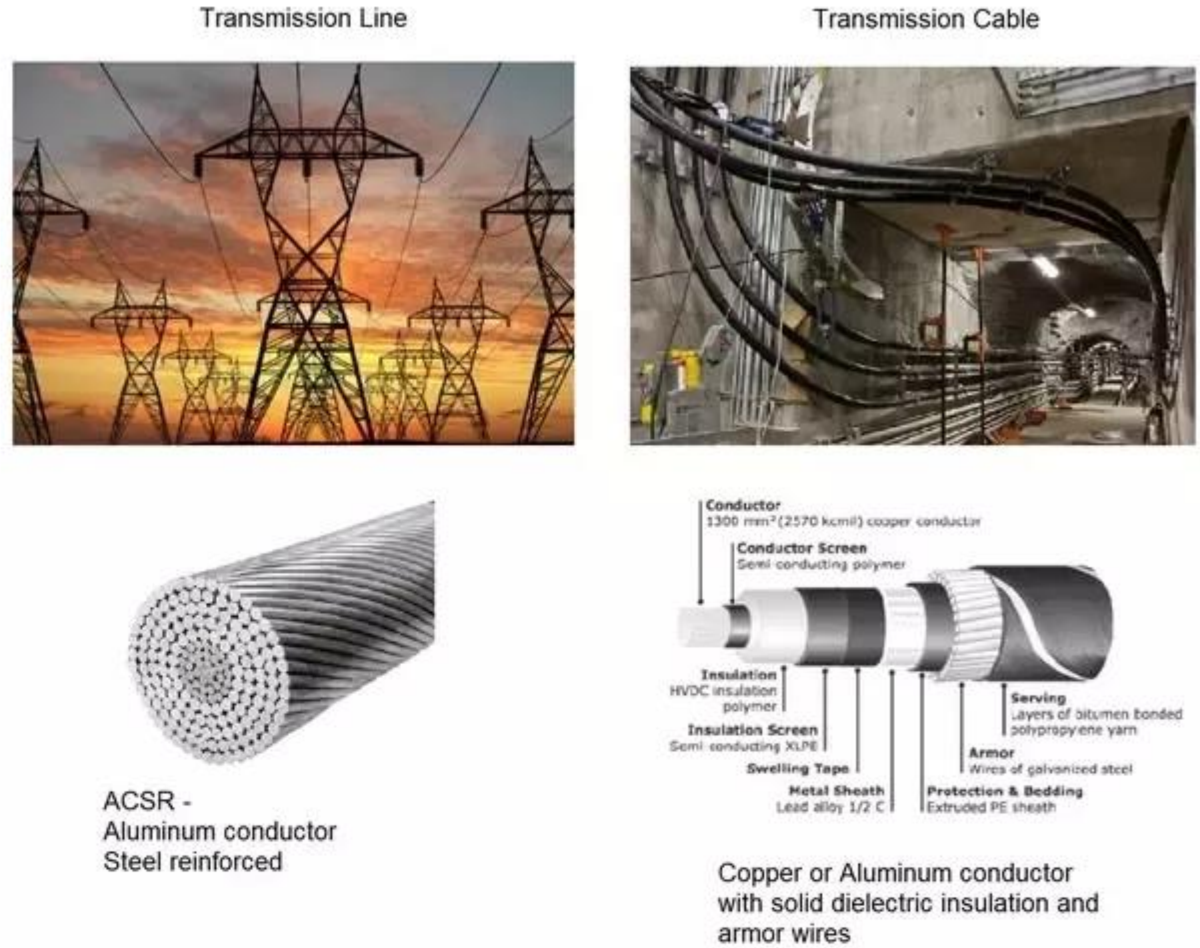


Fig. 15

Note, however, that this issue does not account for obtaining right-of-way. Because underground is not exposed like overhead, it requires less right-of-way. This fact, coupled with the fact that public resistance to overhead is much greater than underground, can bring overall installation costs of the two technologies closer together. This smaller difference may be justifiable, particularly if it is simply not possible to build an overhead line due to public resistance. Such has been the case in France now for several years.

Another issue for underground AC is the high charging currents generated because of the capacitive effect caused by the insulation shield and the conductor. These high charging currents make voltage regulation very difficult for long underground AC transmission, and so typically underground AC is not used beyond a certain length. Charging (capacitive) effects decrease power transfer capacity. This effect increases with voltage so that, for example, a 345kV cable reaches zero transfer capacity at about 43 miles, and a 500kV cable does so at about 30 miles [16]. These values assume no use of inductive compensation; guidelines for undersea AC transmission indicate it is limited to between 60 and 90 miles if inductive compensation is deployed at both ends [17].

One approach to taking advantage of benefits of underground transmission without incurring the high charging currents is to use HVDC. We will study this next.

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