Analytical Development of Loadability Characteristics for EHV and UHV Transmission Lines

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Abstract
This paper contains the development of an analytical basis for the St. Clair line loadability curve and presents the extension of its use into the EHV and UHV transmission area. A brief historical background describes the origin and pertinent aspects of the St. Clair curve including the fact that the old curve, originally intended for transmission voltages up to 330-kV, is derived empirically based upon practical considerations and experience. In order to extend the usefulness of such line loadability characteristics into the EHV and UHV range, a simplified representation of the system, which incorporates flexibility to include both line and system parameters, is utilized to compute maximum line loadability subject to assumed system performance criteria. It is shown that, for a reasonable and consistent set of assumptions, with regard to system parameters and performance criteria, EHV and UHV transmission line loadability characteristics are nearly identical to the original St. Clair curve. The paper further illustrates the relative influence of these assumptions on the derived characteristics. In particular, the electrical strength of the sending- and receiving-end systems is found to have an increasingly important influence on the loadability of transmission lines as the voltage class increases. The analytical approach to determination of transmission line loadability curves enables the user to examine specific situations and assumptions and to avoid possible misinterpretation of generalized conceptual guides—particularly in the EHV/UHV range where system parameters can have a significant impact on loadability.

Introduction
The transmission line power-transfer capability curves, also known as "St. Clair curves," have been a valuable tool for planning engineers ever since their publication in 1953. These curves, having been extrapolated for use with longer lines, are generally accepted in the industry as a convenient reference for estimating the maximum loading limits on transmission lines.

The widespread use of these curves warrants the extension of their development for application into the UHV area. This, unlike the development of existing curves, cannot be done by using "judgment based upon practical considerations and experience," for no such experience yet exists. While the conception of existing curves and their proven validity over many years clearly demonstrate the genius of their author, it should be stressed that at higher voltage classes the loading limits depend not only on the transmission line itself, but also, to a growing degree, on the strength of terminal systems. This new element, the system strength, becomes especially important when considering UHV lines.

Since the expression "line capability" -- as traditionally used in the past -- is easily confused with physical properties of a line (such as thermal capability), a modified expression "line loadability" is used throughout this paper to describe the load carrying ability of a transmission line operating under a specified set of performance criteria.

Historical Background
The original St. Clair curves of 1953, presented in Figure 1 below, show the loadability of transmission lines in terms of their surge impedance loading (SIL) for line lengths of up to 400 miles. It is interesting to note how these curves, and in particular the "Heavy Loading" curve B, came about. It had been a well established fact, even long before 1953, that a conventional 60-Hz line approaching 300 miles in length has a loadability of about 1.0 SIL. Lines of that length were known to operate with very little or no reactive power supplied from either end, owing to the equalization of stored inductive and capacitive energy that oscillates between the magnetic and electric fields of the line. This 300-mile rating of 1.0 SIL was taken by St. Clair as one of the two bench-mark points he used in establishing his line loadability characteristic.

The other bench-mark point on the curve B in Figure 1 is the 50-mile line length at which the thermal limits, more than any other factor, were responsible for setting a ceiling of 3.0 SIL on the line loadability. It appeared at first that the entire curve, above and below 300 miles, could have been constructed on the basis of a constant kW-mile product, but, if such were the case, then the 50-mile line loadability would need be equal to 5.0 SIL, which was deemed impossible not only from a current and loss standpoint but also from the standpoint of reasonable amount of power to be concentrated in a single circuit, with due regard to service and reliability. Thus, for lines shorter than 300 miles, this kW-mile product was progressively reduced and "the extent of this reduction was a matter of judgment based upon practical considerations and experience."

In 1967, the Planning Department of the American Electric Power Service Corporation -- faced with a growing need for similar curves applicable to lines of voltage classes higher than 345-kV and longer than 400 miles -- modified the St. Clair's curve, as shown in Figure 2. This figure, just like the original curve, was arrived at through practical considerations, rather than through a rigorous analytical derivation. This extended curve has been widely accepted and used in various industry reports.

In order to extend the transmission line loadability concept to future EHV and UHV applications it is necessary to (1) develop an analytical basis for deriving such characteristics including appropriate criteria and assumptions, (2) de-
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monstrate ages model ability, flows ed in and was limiting factors concepts receiving a w 1.5-J0: included in the this monitored at the basic shown in Figure 1.5.

In this part of the paper, attention is focused on the development of the model and computational procedures for establishing loadability characteristics, on limiting factors and assumptions, and on verification of the analytical approach. The extension of this analytical approach to the development of EHV and UHV transmission loadability characteristics will be treated separately in the next section.

Model

The basic analytical model used in this study of the transmission line loadability is shown in Figure 3. It is comprised of a variable-length line which is modeled by a positive-sequence equivalent-R circuit, shunt and series compensation, and a positive-sequence equivalent system representation at the sending and receiving ends of the line. Series compensation, although not studied here, was included in the model for the sake of generality. The real and reactive power flows were monitored at the terminals, as indicated in Figure 3, and were expressed in per-unit of the SIL of the line studied.

![Figure 3. Mathematical model developed for line loadability study](image)

\[
X_1, X_2 \quad \text{- equivalent sending- and receiving-end system positive-sequence reactances (includes reactance value of generators, transformers, and any associated transmission)}
\]

\[
N \quad \text{- percent series compensation}
\]

\[
N_s, H_s \quad \text{- percent shunt compensation at the line sending- and receiving-end, respectively}
\]

\[
IE_{S1}, IE_{2}/\theta_0 \quad \text{- specified voltage quantities}
\]

\[
(IE_{RI})_L \quad \text{- limiting value of receiving-end line-voltage (uniquely defines line-voltage-drop criterion)}
\]

\[
(\theta_1)_L \quad \text{- limiting value of sending-end system-voltage angle (uniquely defines steady-state-stability criterion)}
\]

For a suitable control of the line voltage drop and the angular displacement across the entire network, a reference point was chosen at \( E_2 \) (with its magnitude and angle given), and the magnitude of \( E_1 \) was specified. Then, a maximum permissible value of the voltage drop across the line element was introduced by specifying the magnitude of \( E_2 \) as a desired voltage solution at the line receiving-end. To start the solution procedure, the angle of \( E_2 \) was increased until either \( IE_{RI} \) matched the desired value, \( IE_{RI} \) or the angle \( \theta_1 \) reached its allowed limit, \( (\theta_1)_L \), whichever came first. These limits are discussed in more detail later. The remaining voltage magnitudes and angles were found based on the outcome of this test.

With all voltages known, the sending- and receiving-end power flows were then calculated. The entire procedure, which was computerized, is illustrated in the flowchart shown in Figure 4.
Limiting Factors and Assumptions

Of all limiting factors that normally set a ceiling on how much power can be carried by a particular transmission line, three major line loading limitations were considered here:

1. Thermal limitation
2. Line-voltage-drop limitation
3. Steady-state-stability limitation

The thermal limitation can be qualified as a line design -- rather than operating -- problem. It is basically a problem of proper conductor selection, once its current-carrying requirements and ambient operating conditions are known. The thermal limitation is critical primarily in case of lower voltage lines of 50 miles or less.

At the EHV, and even more so at the UHV transmission level, environmental considerations such as corona discharges and field effects dictate line design characteristics which result in very high thermal capabilities. The thermal capability derived from typical bundle-conductor arrangements at EHV and UHV operating levels generally exceeds, by a significant margin, network requirements for transfer of power through a given line. In such cases line terminating equipment, wave traps, and substation design provide a more restrictive thermal limit than the line itself.

Consequently, from the viewpoint of determining line loadability, thermal capability is significant only for very short lines at 138-kV and below. Thus, for EHV and UHV transmission lines, the only practical limitations to line loadability are provided by line-voltage-drop and by steady-state-stability considerations.

In the initial stages of this work, it became apparent that some of the existing literature on the subject assume a flat voltage profile over the entire line length, and some do not even mention the concept of a voltage-drop criterion altogether in their analysis of the line loadability. It will be shown here, that the voltage-drop limitation is a very important one; in fact, for moderate-length EHV lines of up to 200 miles it is the controlling factor on line loadability. Thus, it needs to be considered on an equal basis with the steady-state-stability limitation.

The voltage-drop limitation across the line was set at 5% maximum. Line loadings at more severe voltage drops could be investigated but, it is the considered judgment of the authors that this value (5%) adequately represents the condition of a line carrying heavy, but permissible, loads without encountering unusual operating problems.

In contrast with the line-voltage-drop limitation, the steady-state-stability limitation has been discussed quite extensively in the technical literature. However, one important point is rarely made or given proper emphasis; that is, the stability limitations should take the complete system into account, not just the line itself. This has been a common oversight which, for the lower voltage lines generally considered in the past, has not led to significant misinterpretations concerning line loadability. This is because at lower voltage levels, say 345-kV and below, the line impedance comprises a major portion of the total equivalent reactance from source to load -- provided this line is long enough (over 200 miles) in the first place, to be limited by stability rather than voltage-drop considerations.

At higher voltage classes such as 765-kV and above, the typical levels of equivalent system reactance at the sending- and receiving-end of a line become a significant factor which cannot be ignored in determining line loadability as limited by stability considerations.

The steady-state-stability limitation is defined in terms of the desired margin between the maximum power transfer ability of the system (P_max) and the operating level (P_rated):  

\[ \% \text{ Stability Margin} = \frac{P_{\text{max}} - P_{\text{rated}}}{P_{\text{max}}} \times 100 \]

This margin is chosen so as to provide for stable system operating performance following a variety of credible contingencies which may cause steady state and/or transient increases in a given line loading. Such changes in loading may be caused by line switching operations, by changes in generation-dispatch, and by transient disturbances such as temporary faults or loss of generation.

The amount of margin which is desirable in a given situation is dependent on many factors. For the general application of developing conceptual guides to line loadability, the level of margin becomes a matter of judgment which reflects the on-going philosophy of a particular system with regard to planning criteria and desired level of operating reliability. The authors believe that a steady-state-stability margin of 30-35% is a reasonable level for typical heavy line loading situations. As shown in Figure 5, this corresponds to about 44-40° angular displacement across the system; i.e., the complete system from source to load, including the line under study together with the equivalent reactance of the sending- and receiving-end systems.

![Figure 5. Steady-state stability margin](image)

Test Case and Verification with St. Clair Curve

The original line loadability curve was published in 1953, which is also when the world’s first 345-kV line went into operation on the AEP system -- initially operated at 330-kV. Since such a line was included by St. Clair in his early loadability chart as shown in Figure 1, it was used here to demonstrate the validity of the analytical method.

The system strength at each end of the line, for a heavy loading condition, was based on the 50-kA fault duty which is representative of a well-developed system. This, at the 345-kV level, corresponds to a 3-phase fault equivalent of about 30,000 MVA.

The system operating criteria were set at the levels established in the previous section on "Limiting Factors and Assumptions"; namely, a line-voltage-drop limitation of 5% and a steady-state-stability margin of 35%. Accordingly, the separate computer runs were carried out; each with only one constraint at a time, holding throughout the full range of line lengths between 50 and 600 miles. The result is shown in Figure 6.

The basic line-loadability curve, as shown in Figure 6, is derived from the two loadability-limiting curves intersecting at a line length of about 200 miles. To the left, there is a "Region of Line-Voltage-Drop Limitation," where the voltage-drop criterion is more restrictive than the stability limit. To
the right, the situation reverses with stability being the limiting factor; this is a "Region of Steady-State-Stability Limitation". The point of intersection of these two curves is not fixed at any particular line length, and it moves to the left for higher voltage classes. In fact, at UHV levels with similar voltage-drop and stability criteria, it can even drift below the 100-mile mark, meaning that the loadability of an UHV line of 100 miles, or more, will be limited primarily by the steady-state-stability constraint.

Note that, if the line voltage drop were held at a constant value for all line lengths of up to 600 miles, then the system angular displacement would well exceed its allowed maximum value set by the stability criterion, even to the point of complete elimination of the stability margin.

Now, let the line voltage drop change freely with the line length, so that this line always operates at its very margin of stability. Such a condition, on the other hand, will clearly result in excessive line voltage drops, again, well over their allowed maximum value.

Thus, in order to keep within both previously established limits, a single, composite characteristic is drawn joining the lower sections of the two curves, in the region where the two curves meet, this new characteristic smooths out the irregularity of intersection by departing slightly from both curves.

The resulting loadability curve, derived analytically, is shown in Figure 7 in comparison with the "old" curve. It is interesting to point out how close these two curves are over most of their length.

Up to this point, no mention has been made of the reasoning behind a choice of the receiving-end system-voltage, E2. An earlier discussion revealed the angular value of E2 as a reference position and, accordingly, it was assigned a value of zero. The magnitude of E2 was arrived at empirically, and in such a manner as to realize loadings in the "Region of Voltage-Drop Limitation" consistent with those of St. Clair. The value so chosen results in magnitude of E2 slightly higher than 0.95 pu -- the receiving-end line-voltage under the 5% voltage drop constraint. This would be somewhat typical of heavy loading conditions where the receiving-end system is capable of providing some voltage support during contingency situations for which the maximum real power transfer is needed. In the "Region of Steady-State-Stability Limitation," E2 is of minimal consequence on line loadability, as that is primarily a function of angular displacement across the complete system, specified here in terms of the stability criterion. The value of E2 may be increased or decreased so that a greater or lesser degree of reactive support by the receiving-end system may be reflected in the resultant line loadability characteristics.

Figure 6. New line-loadability curve derived analytically

Figure 7. Comparison between "analytical" and "old" curves

EXTENSION OF ANALYTICAL APPROACH TO EHV/UHV TRANSMISSION

To study UHV transmission loadabilities, the latest available sources were consulted for the line and terminal system data. The line constants were calculated based on projected 1100-kv and 1500-kv configurations given in Reference (4) and modified by our own research. The terminal system reactance was obtained assuming a fault duty of 50-kA. This is consistent with the previously established value for 345-kv and is representative of a well developed EHV/UHV system. The line-voltage drop criterion was set at 5% and the steady-state-stability margin at 30%. The receiving-end system voltage, \( V_{E2} \), was determined according to the guidelines established in the preceding section, Table 1, below, conveniently summarizes these parameters for all voltage classes studied.

<table>
<thead>
<tr>
<th>TABLE 1</th>
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<tbody>
<tr>
<td><strong>NORMAL VOLTAGE CLASS (kV)</strong></td>
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<tr>
<td><strong>%</strong></td>
</tr>
<tr>
<td><strong>%</strong></td>
</tr>
<tr>
<td><strong>NOMINAL RATING</strong></td>
</tr>
<tr>
<td>345</td>
</tr>
<tr>
<td>765</td>
</tr>
<tr>
<td>1000</td>
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<tr>
<td>1500</td>
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* System strength corresponding to half-kA fault duty

Unlike the 345-kv or 765-kv line parameters, UHV line data is still tentative because both the choice of voltage level and optimum line design are not finalized. This uncertainty about the line constants, however, is not very critical in determining the line loadability -- expressed per-unit of rated SIL -- especially at UHV levels. The reason lies in the fact that for a lossless line, it can be shown that the line loadability -- or the receiving-end power -- in terms of SIL of that line, \( S_p/SIL \), is not dependent on the line constants, but rather is a function of the line length and its terminal voltages. This concept is discussed further in the Appendix.

Since the resistance of the EHV/UHV lines is much smaller than their 60-Hz reactance, such lines closely approximate a lossless line from the standpoint of loadability analysis. Therefore, the loadabilities in per-unit of SIL of these lines are practically independent of their respective line constants and, as a result, of their corresponding voltage classes. This is illustrated in Figure 8 which shows three loadability curves for 765, 1100 and 1500-kv system very close to each other. Consequently, all three loadability curves are combined into one average, generalized curve which closely resembles the original St. Clair characteristic.

This strong agreement with St. Clair further confirms the validity of the mathematical model used, not to mention the useful results established in the
process. It is reassuring to know that one single curve can be applied to all voltage classes in the EHV/UHV range. Obviously, a general transmission loading curve will not cover the complete range of possible applications; nonetheless, it can provide a reasonable basis for any preliminary estimates of the amount of power that can be transferred over a well-designed transmission system.

Any departures from the assumed performance criteria and system parameters -- which, for convenience, are clearly enumerated on the EHV/UHV loadability chart shown in Figure 8 -- must not be ignored and, depending on their extent, they should properly be accounted for in the line loadability estimates. To illustrate this, the effect of some of the variations in these assumed parameters such as terminal system strength, shunt compensation, line-voltage-drop criterion and stability margin, are investigated in the next section.

EFFECTS OF SYSTEM PARAMETERS ON LINE LOADABILITY

Effect of Terminal System Impedance

Earlier, attention was called to the significance of terminal system impedance, stressing its increasing importance at higher voltage classes, 765-kV and above. A closer look at this leads to some interesting conclusions.

The effect of system impedance was investigated at 765, 1100, and 1500-kV transmission levels under the heavy loading criteria of a maximum line-voltage-drop of 5% and a steady-state-stability margin of 30%. The terminal system strength was varied between a "weak" system of 12.5-kA and a "strong" one of 75-kA fault duty.

Figures 9a-c show three sets of curves, one for each of the three voltage classes, representing the line loadability in per-unit of SIL at various system impedances, corresponding to the range of system strengths. At 1500-kV these curves are spaced further apart, particularly for shorter lines, than at 765-kV. This is equivalent to saying that, at higher voltages, the system impedance becomes a stronger controlling factor on line loadability, and that a similar increase in system strength (e.g. from 12.5-kA to 75-kA) produces a faster growth in line loadability at 1500-kV than at 765-kV.

This effect should be kept in mind in planning UHV transmission systems; that is, in the early stage of implementation when the system is not well developed, the utilization of the inherent loadability of the line may be limited to a very significant degree by the system. It should be observed that this effect is much smaller for long-distance lines, making those lines nearly insensitive to any but the most dramatic changes in the terminal system impedance.

Figure 8. EHV/UHV line loadability curves
Effect of Shunt Compensation

Because of the naturally large charging current of EHV and UHV lines at 765-kV and above, it is usually necessary to employ some degree of shunt compensation to control steady state overvoltages particularly during light-load periods. Although it helps to alleviate the voltage problems on a line in an effective and economical manner, shunt compensation generally impairs the maximum load-carrying ability of the line it compensates. The degree of that impairment depends on many factors with two -- voltage class and terminal system impedance -- clearly standing out. Figures 10a-c show the effect of line shunt compensation in light of these two parameters.

The effect of line shunt compensation on line loadability is typically small, and it gets smaller as the voltage class increases. In fact, a 1500-kV line connecting two well-developed systems will exhibit almost no change in its loadability for a full range of shunt compensation between 0% and 100% (Figure 10c). Effects of any practical significance can be observed only when the terminal system strength is very low. At lower voltage classes, such as 1100-kV or 765-kV, this effect is more pronounced. However, at voltages any lower than that, shunt compensation is generally not required. As before, this effect will be markedly greater in the case of high rather than low terminal system impedances, and especially on moderate-length lines of about 200-300 miles as shown in Figure 10a.

The reason that line shunt compensation impairs the loadability of the line it compensates is clear. It suffices to say that a highly compensated line approaches a simple series inductive impedance, and as such, it tends to experience intolerable voltage drops from the viewpoint of line-voltage-drop criterion unless the line loading is reduced. Consequently, shunt-compensation works in a way of extending the line-length for which its loadability is still limited by the voltage drop constraint. In case of "weak" terminal systems this extension encompasses lines even as long as 600 miles.

Effect of Voltage-Drop and Stability Criteria

In discussing the influence of either the line-voltage-drop or the steady-state-stability criteria on the line loadability it should be recalled that, except for unusual conditions of system strength, a portion of the loadability characteristic is determined by one criterion and the remainder is determined by the other. The influence of changing each criterion will be discussed and illustrated separately.

Line-Voltage-Drop Criterion:

Figure 11 shows the effect of the line-voltage-drop criterion on line loadability at 1100-kV level. Choice of this level is representative of the 765-1500 kV range.

The effect of the line-voltage-drop criterion on loadability of short-to-moderate length lines follows the law of diminishing returns. An unusually low voltage-drop criterion (e.g., 3% or less) will severely penalize line loadability. As the permissible voltage drop increases, line loadability improves rapidly at first, and then more gradually to a point -- in this case 6% and above -- where maximum...
loadability is dictated not by the line voltage drop allowed but, rather, by the stability criterion. At that point, no matter how high the voltage-drop criterion is, line loadability cannot be further improved except by allowing a smaller stability margin.

The loadability of long lines is generally restricted by stability margin rather than the voltage-drop constraint, and the actual voltage drops on such lines are only minimal and well within the generally accepted limits.

Steady-State-Stability Criterion:

Figure 12 shows the effect of varying the margin of stability on line loadability. Again, for the purpose of discussion, 1100-kV was chosen to illustrate the point. The need for an "adequate" steady-state-stability margin is established elsewhere in this paper. The definition of this adequacy will vary from one utility to another, or even within the same system for different levels of transmission voltage.

The variation in line loadability appears to be related in a linear fashion to the change in stability margin; that is, equal steps of the stability-margin reduction bring about almost equal-sized increments in line loadability. This, however, only holds true where the stability criterion is a controlling factor, which usually implies longer lines. Short lines benefit much less from the stability margin reduction unless unusually high line-voltage drops are allowed. Thus, for short lines, a key to the improvement in line loadability lies in choosing an acceptable mix of both the line-voltage-drop and the steady-state-stability criteria.

![Diagram showing the effect of steady-state-stability criterion on line loadability](image)

Figure 12. Effect of steady-state-stability criterion on line loadability

The foregoing discussion has highlighted the most influential effects on transmission line loading limits. Some of these effects, especially terminal system strength, are more significant than others; accordingly, substantial care must be exercised in applying the loadability curves developed here. This suggests the necessity of resorting to the analytical method in order to assure best possible results if such are required by the nature of problem at hand.

From a planning viewpoint, the proper knowledge of all parameters involved -- present, future, and also their timing -- is essential to a proper understanding and interpretation of transmission loading limits at the various stages of system development. This work on EHV/UHV transmission loadability has been carried out in the interest of enhancing such understanding through the development of an analytical tool for studying transmission line loadability. The authors are indebted to their predecessor, Harry St. Clair, who first introduced these concepts.

**SUMMARY AND CONCLUSIONS**

The transmission line loadability concept was introduced by St. Clair in 1953 as a means of depicting the maximum load-carrying ability of a line as a function of its length. He observed that, when loadability is expressed in per-unit of SIL, a single curve could be applied to a range of voltage classes up to 330-kV. The maximum permissible loading implied by the St. Clair curve was based upon practical considerations and experience of its author with transmission lines already designed and in service at that time.

However, to extend such loadability concepts into the EHV/UHV range, where system parameters play an increasingly important role, an analytical approach is required. In this paper, a mathematical model is recommended which accounts for the characteristics of the line and a simplified equivalent of the sending- and receiving-end systems. Maximum loadability is then computed based on assumed system performance criteria as expressed in terms of allowable line-voltage-drop and/or system stability limits. It is shown that for reasonable assumptions with regard to system parameters and performance criteria, the analytical approach yields a loadability curve for 345-kV lines which closely resembles the original St. Clair curve, thus verifying the viability of the recommended analytical approach.

Using the analytical approach as recommended and verified in this paper, the authors conclude the following:

1. The concept of transmission line loadability -- expressed in per-unit of SIL as a function of line length -- can be extended into the EHV/UHV range. For a reasonable and consistent set of assumptions with regard to system parameters and performance criteria, EHV and UHV transmission line loadability curves are nearly identical to the original St. Clair curve.

2. As the voltage class increases, assumptions regarding the electrical strength of the system play an increasingly important role in determining loadability characteristics. Accordingly, utilization of the inherent loadability of UHV lines will depend significantly on the electrical strength of the terminating systems and will increase as the UHV system develops from the initial stages of its implementation to ultimate maturity.

3. Shunt line compensation, which is generally required for over-voltage control purposes at 755-kV and above, has a decreasing effect on loadability as the voltage class increases from 755-kV to UHV levels.

4. Variations in system performance criteria confirm the expected result; namely, that the line-voltage-drop criterion has a primary influence on the loadability of short lines, whereas the stability criterion has a primary influence on long lines.

5. The analytical approach to determination of transmission line loadability curves enables the user to examine specific situations and assumptions and to avoid possible misinterpretation of generalized conceptual guides -- particularly in the EHV/UHV range where system parameters can have a significant impact on loadability.

Transmission line loadability as set forth in this paper is intended primarily as a conceptual tool for the planning engineer where design aspects of the transmission line and system can be related to the maximum load-carrying ability of the line within the constraints imposed by practical system performance considerations. Such loadability characteristics, and especially their extension to the EHV/UHV range, must be carefully viewed within the context of the simplified system representation and the chosen system performance criteria. As helpful as these loadability characteristics may be in achieving an understanding of how system performance considerations influence maximum line loading, they cannot be viewed as a substitute for detailed analysis and simulation required in assessing alternative plans for network expansion. Detailed planning studies are generally required to account for the actual structure of the network which may be more complex than the simplified two-system equivalent used here, to properly represent the influence of voltage control sources distributed throughout the network, and to assess the performance of the system during contingency situations.

**APPENDIX**

This appendix is devoted to a discussion of the fact that transmission line loadability, when expressed in per-unit of surge impedance loading (SIL), is nearly independent of the line characteristics -- particularly for transmission lines in the EHV/UHV range. The result is that for a consistent set of assumptions with regard to system parameters and performance criteria, the loadability characteristics -- expressed in per-unit of SIL -- are close enough to be represented by a single characteristic which is independent of voltage level.
The concept of surge impedance loading (SIL) and the surge impedance (SI) from which it is derived, both well known in the utility industry, are very useful quantities in line loadability analysis.

SI is defined as the characteristic impedance \( Z_0 \) for the special case of an assumed lossless line. Thus, SI is a real number and is independent of frequency. If a line with surge impedance SI is terminated with a load whose ohmic value is also equal SI, then, at exactly the nominal voltage, this line is said to be loaded to 1.0 SIL. In the per-unit system, the nominal voltage is 1.0 p.u. by definition and, therefore, SI and SIL become reciprocals of each other.

Shown below is a line, modeled by an equivalent-mi carrying a load \( S_R \) which is assumed to be the maximum allowable value under a given set of system parameters and performance criteria.

\[
\begin{align*}
\frac{I_s}{E_s} &= Z_o \frac{E_s}{E_r} \\
\frac{Y}{Z} &= \frac{Z_o \sinh \gamma L}{Z_o} \\
\frac{I}{I_r} &= \frac{S_R + jQ_R}{E_r} \\
&= \frac{E_s - E_r}{2} \frac{E_s - 2E_r - ZYE_R}{2Z} \\
&= \frac{Z_o \sinh \gamma L}{2Z} \frac{2E_s - (Z_o \sinh \gamma L) (\frac{Z_o - \tanh \gamma L}{Z_o})}{2Z} \\
&= \frac{S_R}{SIL} = \frac{1}{SIL} \left( \frac{E_s - E_r}{Z} \right) \left( \frac{E_s - 2E_r - ZYE_R}{2Z} \right) \left( \frac{2E_s - (Z_o \sinh \gamma L) (\frac{Z_o - \tanh \gamma L}{Z_o})}{2Z} \right) \left( \frac{S_R}{SIL} \right) = \frac{I}{I_r} \frac{S_R}{SIL}
\end{align*}
\]

where:
- \( Z_0 = \sqrt{\frac{E_o}{P}} \), characteristic impedance
- \( \gamma = \alpha + j \beta = \sqrt{Z_o P} \), propagation constant
- \( \alpha = \) attenuation constant
- \( \beta = \) phase constant
- \( Z_p, Y_p \) = per-mile line constants (series impedance, shunt admittance)
- \( E_S, E_R \) = sending- and receiving-end voltages (complex)
- \( I_S, I_R \) = sending- and receiving-end currents (complex)

The complex power drawn from the line at its receiving-end is:

\[ S_R = P_R + jQ_R = E_R \cdot I_R \]

\[ \text{where,} \quad I_R = \frac{E_S - E_R}{2} \quad \text{and} \quad E_R = \frac{2E_S - 2E_R - ZYE_R}{2Z} \]

When all quantities are in per-unit, the receiving-end power may be expressed as follows:

\[ \frac{S_R}{SIL} = \frac{I}{I_r} \frac{S_R}{SIL} = \left( \frac{E_s - E_r}{Z} \right) \left( \frac{E_s - 2E_r - ZYE_R}{2Z} \right) \left( \frac{2E_s - (Z_o \sinh \gamma L) (\frac{Z_o - \tanh \gamma L}{Z_o})}{2Z} \right) \left( \frac{S_R}{SIL} \right) = \frac{I}{I_r} \frac{S_R}{SIL} \]

\[ \text{For a given set of terminal voltages, } E_S \text{ and } E_R \text{ (in per-unit of nominal voltage), and a line length, } L, \text{ the ratio of power transferred over a transmission line to its } \text{SIL}, (S_R/SIL), \text{ is a function of the propagation constant, } \gamma, \text{ and the ratio of surge impedance to characteristic impedance, } Z_o/Z_e. \text{ Because the series resistance, } R, \text{ of EHV/UHV transmission lines is much smaller than their inductive reactance, } X, \text{ the attenuation constant, } \alpha, \text{ is very small, and the propagation constant, } \gamma, \text{ approaches the phase constant } j\beta \text{ also, } Z_o^* \text{ approaches SI. Thus, equation (2) reduces to:} \]

\[ \frac{S_R}{SIL} = \frac{1}{SIL} \left( \frac{E_s - E_r}{E_r} \right) \left( \frac{E_s - 2E_r - ZYE_R}{2Z} \right) \left( \frac{2E_s - (Z_o \sinh \gamma L) (\frac{Z_o - \tanh \gamma L}{Z_o})}{2Z} \right) \left( \frac{S_R}{SIL} \right) = \frac{I}{I_r} \frac{S_R}{SIL} \]

\[ \text{but} \quad \beta = \omega \sqrt{\frac{L}{C}} = \frac{\omega}{\nu} \]

\[ \text{where,} \quad \nu = 3 \cdot 10^8 \text{ m/sec, speed of light} \]

Therefore, assuming that a line is lossless, the ratio of the line loading to its SIL is independent of the line electrical parameters. It depends exclusively on the line length and the terminal voltages.

In actuality, lines are not truly lossless, and the ratio \( S_R/SIL \) does depend, to a small degree, on the line constants and thus on the line voltage class. This dependence, however, is quite small for lines 345-kv and up, and is proportional to the resistance of the line. Since all these lines have \( X/R \approx 10 \) or higher, their loadabilities at a given length, for all practial purposes, are constant if expressed in per-unit of their respective SIL. This is the main premise behind normalizing the line loading curves for all EHV/UHV transmission lines, and showing them as a single graph applicable to all voltage classes within the range considered here.

REFERENCES


Raymond D. Dunlop (M'50) was born on August 4, 1938 in Saco, Maine. He received the B.S.E.E. degree from the University of Maine, Orono, in 1960, and the M.S. and Ph. D. degrees in electrical engineering from Illinois Institute of Technology, Chicago, in 1963 and 1965, respectively. He was employed in 1960 by the Central Illinois Electric and Gas Company, Rockford, Illinois. In 1961, he received the Power Systems Fellowship at the Illinois Institute of Technology and worked for the Department of Electrical Engineering as an instructor from 1962 to 1966. Since 1966, he has been employed by the American Electric Power Service Corporation, New York, N.Y., where he presently holds the position of Head, Technical and Special Studies Section in the Bulk Transmission Planning Division. His responsibilities include analysis and simulation of bulk power systems for evaluation of dynamic performance, steady-state and transient over-voltages, transmission line characteristics, and his particular area of interest is in assessment of the electromagnetic dynamic performance of large interconnected power systems.

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From 1937 to 1948 he was an electrical engineer in the construction and maintenance of various power plants in eastern and central Europe. He joined the American Electric Power Service Corporation, New York, N.Y., in 1948 and had held several positions in Operating, Generation Planning, and Bulk Transmission Planning Divisions. In 1974, upon his retirement, he was named a Consulting Engineer to the System Planning Department, and served in this capacity until 1977.

**Discussion**

N. B. Johnsen (Tucson Gas & Electric Co., Tucson, AZ): The authors are to be complimented for presenting a paper which so clearly describes the capabilities of EHV and UHV transmission systems. It is comforting to know that the more rigorous approach taken by the authors confirms the original St. Clair curves which the industry has used extensively over the past 25 years.

Will the authors please comment on the application of this approach to series compensated transmission lines? Do you plan to prepare a set of St. Clair curves for series compensated lines?

Manuscript received February 24, 1978.

Harry B. Smith (C. T. Main, Engineers, Boston, MA): The authors have made an excellent contribution to the collection of tools for the system planner. Perhaps because I am an old fossil of the Harry St. Clair age, I was particularly gratified to find that a thorough analytical approach yielded a generalized transmission line capability curve that is, for practical purposes, identical with the old St. Clair curve (re Figure 1 on the paper).

I was somewhat disappointed in that the authors make no mention of the effect on line capability of intermediate switching stations.

This subject was investigated by Butler, Fiedler and Saline of the General Electric Co., and reported to the industry in a paper presented at an AIEE district meeting in Toledo, Ohio, October 1952. At the time, the study concerned a 230 kV transmission system and, among other things, yielded data for a set of curves of optimum number of intermediate switching stations for a double circuit transmission line. From these curves, it is possible to obtain a de-rating factor to apply to capability numbers read directly from the generalized capability curve, in cases where actual number of intermediate switching stations is less than optimum.

While it is acknowledged that the above is not a precise type of calculation, it has been found that results check reasonably well with the results of detailed system stability studies and with actual system performance.

As the authors imply, the generalized capability curve is a system planners tool, to be used with discretion and good judgement, and answers obtained should eventually be checked by detailed load flow and stability tests. It has been found that intelligent use of the generalized capability curve enables the system planner to quickly make the correct choice of transmission voltage level.

Manuscript received February 21, 1978.

S. Linke (Cornell University, Ithaca, NY): In this paper the authors have presented some interesting and worthwhile additions to the literature of surge-impedance loading (SIL) and transmission-line capability/loadability, with emphasis on applications to EHV and UHV transmission. The comments on the well-known St. Clair curves are particularly welcome in that the impacts of constant steady-state stability and constant line-voltage-drop on the genesis of the curves are clearly set forth. The introduction of the term "loadability curve" is also useful. Apparently, the authors may have been unfamiliar with my recent paper [1] in which the theoretical basis for loadability curves is presented in detail, together with several convenient generalized equations that may be used to generate families of curves, for a wide variety of design conditions, without requiring complex digital computer program for the solution of a network for each condition. This discussion will demonstrate that the expressions derived in my paper, when modified by the authors' constant steady-state stability and constant line-voltage-drop limitations, will provide sufficiently accurate reproductions of the St. Clair curves. Some general comments will also be made on the procedures that the authors have used to prove the validity of the St. Clair curves, and a question will be raised on the choice of sending and receiving-end impedances as they relate to system strength and steady-state-stability criteria.

It is important to realize that the solution of a transmission-line network in great detail, for a wide variety of conditions, is not necessary in order to prove the validity of the St. Clair curves. In the strictest sense, going through such an exercise is not an "analytical" solution. It is simply a verification of the fundamental behavior of the lines. Therefore, it should not be surprising that the authors achieved such an exact correspondence with the results of earlier published work. In fact, their digital-computer solution only provides a much more complete, and undoubtedly more accurate, rendition of the network-analyzer solutions obtained by Clarke and Crazy [2] in 1941. Using the results of these early solutions, Mr. St. Clair later developed and published [3] the very useful curves that have become standards for the industry.

Any solution for transmission-line loadability, as a function of line length, depends upon the combination of the power-angle equations of the line with the classical solution of the transmission-line Heaviside Equations, as demonstrated in my paper. A similar combined equation, in complex form, is given in equation (3) of the Appendix of the authors' paper. Although this expression is based upon the same considerations outlined in my paper, it cannot be used to develop the loadability curves while in its present form. In their derivation the authors have assumed a per-unit value of 1 for SIL, so that SIL becomes equal to SIL, a condition that can only be true at one particular line.
length, \( l \) (equal to 300 miles for the example used in the authors' paper). Consequently, their Equation (3) gives the incorrect impression that the loadability is a function of the voltage level of the line because of the factor \( E_2 \), whereas in actuality the theoretical loadability is virtually independent of the line-voltage level, as the authors themselves point out in the discussion of their Figure 8. Subsequently in Figure 9, and their related discussion, the authors correctly state that various voltage levels will affect the actual line loadability due to the influence of specific terminal impedances used for each voltage level. If the factor, \( E_2 \) is deleted, curves of the sending-end transfer function are evidently correct, but the sending-end and receiving-end terminals, and the importance of var transfer will also become evident. These procedures were followed in my paper and will now be demonstrated for the conditions assumed in the authors' paper.

In my paper, power loadability is defined as the ratio of sending-end power, \( P_s \), to SIL. For a lossless line, and a quantity which could be termed "var loadability" is defined as the ratio of sending-end vars, \( Q_s \), to SIL for the same line. These definitions result in the following two expressions:

\[
k = \frac{P_s}{SIL} = m \sin \delta_s / \sin \beta = m \sin \delta_s / \sin(0.116) \tag{1}
\]

\[
q = \frac{Q_s}{SIL} = m^2 / \tan \beta - m \cos \delta_s / \sin \beta / l
= (m / \sin(0.116)) [m \cos(0.116) - \cos \delta_s] \tag{2}
\]

where: \( \delta_s \) = power angle between \( E_s \) and \( E_a \) in degrees

\( m = |E_s| / |E_a| \)

\( l \) = length of line in miles

\( \beta = (\alpha L C)^{1/2} = \alpha / c = (377 \times 180)/(186,282 \times n) = 0.116 \) degrees per mile.

In the authors' paper, a constant voltage drop of 5 percent was assumed between \( E_s \) and \( E_a \), so that the factor, \( m \), would be equal to 1.05. Also, they chose a benchmark of SIL = 1 at 300 miles to conform with the original St. Clair curve. It should be recalled that for SIL = 1, the var transfer, \( Q_s \), will be zero. Solving Equation (1) for \( \delta_s \) at SIL = 1 results in an angle of 32.9°. Based on their assumed values of terminal reactances at the sending and receiving ends of the line, the authors' digital-computer solution apparently determined that the overall angle between their input voltage, \( E_s \), and their terminal voltage, \( E_a \), would be within the 44° (35%) stability margin that was to be maintained. Thus, holding \( \delta_s \) constant at 32.9°, Equation (1) can be solved for \( k \) for line lengths between 50 and 600 miles. Figure A shows the result of this computation as a set of dots superimposed upon the constant steady-state-stability-margin curve of Figure 6 of the authors' paper. The close correspondence is obvious.

To solve for the condition of constant line-voltage drop, it is necessary to obtain values of \( \delta_s \) as the line length is varied. Since the line is assumed to be lossless, the var loss for any particular length, \( l \), of the line is given by:

\[ Q_l = E_l / X_c \]

But the voltage drop along the line, \( E_l \), is assumed to be constant for all lengths, and the reactance of the line, \( X_c \), varies directly with the length of the line. Consequently, \( Q_l \) is inversely proportional to the line length. If a benchmark of 3 per-unit SIL at 50 miles is chosen, substitution into Equation (1) results in an angle, \( \delta_s \), of 22.6°. Placing this angle in Equation (2) at 50 miles provides the corresponding value of \( q \) at \( m = 1.05 \). Since the var values vary inversely with distance, the value of \( q \) at 100 miles will be one-half of the value at 50 miles. Substitution of this value of \( q \) into Equation (2) for 100 miles provides the new value of \( \delta_s \), which can then be used in Equation (1) to find the corresponding value of \( k \) at 100 miles. Similar calculations for greater distances result in the complete loadability curve at constant line-voltage drop. This calculation at the benchmark of \( k = 3 \) at 50 miles was found to match the upper portion of the original St. Clair Curve A (Normal Loading). Use of an assumed benchmark of \( k = 4 \) at 50 miles was found to produce a curve that was very close to St. Clair Curve B (Heavy Load) that also matched the corresponding curve of the authors' Figure 6. The plot of Figure A also displays the result of this latter computation as a series of dots superimposed upon the authors' curve. Again the close correspondence is clearly shown.

Dr. Kanu R. Shah has suggested [4] that the factors \( k \) and \( q \) could be combined into the form: \( s = k + jq \) to give total loadability in terms of the complex power transfer of the line. Computations based on this combined expression would provide data equal to that which could be obtained from a corrected version of the authors' Equation (3). The results of such a computation for the test conditions of Figure A are plotted in Figure B as a series of dots superimposed on the St. Clair curve of the authors' Figure 7. Note the pronounced effect of the reactive-power component at both ends of the line, a condition that is also displayed graphically in the original Clarke and Cray paper [2]. Since these var effects are not shown in the curves of the authors' paper, it must be concluded that their calculation of loadability is based on the ratio of active power to SIL, rather than the ratio of complex power to SIL, as implied in the authors' Equation (3).

It would be helpful to have the authors discuss their use of short-circuit reactances as the terminal impedances at the sending and receiving ends of the line. While it is true that these very low impedances are indicative of system strength, nevertheless short-circuit impedances are more appropriate for transient-stability than for steady-state-stability considerations. Table I of the authors' paper indicates that the largest terminal reactance that was used was 0.033 p.u. for the 345 kV case. The corresponding line reactance for 50 miles was 0.32 p.u. so that for greater distances the terminal reactances are essentially negligible. At higher voltage levels, the terminal reactances are even smaller. For steady-state-stability studies Kimbark [5] suggests use of the reciprocal of the short-circuit-ratio for the equivalent reactance of turboalternators, with 0.8 p.u. (on a circuit Mva base) being a typical value. Clarke and Cray [2] used a sending-end reactance of 0.4 p.u. and a receiving-end reactance of 0.25 p.u. (based on SIL) and reactances of 0.16 p.u. and 0.1 p.u., respectively, (on a circuit Mva base) in their studies. It would appear that the authors' use of very low terminal reactances in their study would give overly optimistic results for steady-state-stability investigations.
This discussion is in no way meant to detract from the value of the authors' paper but rather to clarify and enhance their findings. It is clear that the combination of their paper with mine provides a set of very useful tools for the transmission-line design engineer.

REFERENCES


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R. D. Dunlop, R. Gutman, and P. P. Marchenko: The authors wish to thank the discussors for their comments and interest in this paper.

In reply to the comments of Mr. Smith regarding the effect of intermediate switching stations, this subject has been treated at most in an indirect manner in the paper. The central purpose of this paper was to provide an analytical basis for loadability characteristics of simple point-to-point transmission lines, and to demonstrate that system parameters and the choice of performance criteria can play an increasingly important role as the use of such generalized characteristics is extended to the UHV range. In the context of this paper, the impact of intermediate switching stations could be evaluated indirectly in terms of their effect on the strength of the sending- and/or receiving-end systems. This would in turn affect the voltage drop and/or stability performance of the transmission line being studied. This subject has been discussed extensively in the technical literature including references to this paper. Generally, the use of intermediate switching stations has been justified primarily on the basis of enhanced stability margins and improved reliability for parallel transmission lines associated with remote generating stations or load centers.

Mr. Smith's favorable comments on the practical usefulness of curves presented in this paper are appreciated. It was the authors' objective to show the importance of considering all pertinent factors in evaluating transmission line loadabilities through prudent application of generalized loadability characteristics to any specific problem.

At Mr. Johnsen's request, the authors welcome the opportunity to comment on the application of the approach developed in this paper to series-compensated transmission lines. The effect of series compensation was included in the formulation of the model (Figure 3), but was not illustrated in the results. The approach described in this paper is limited only to distributed series-capacitor compensation. No attempt has been made to evaluate the individual merits of various discrete series-compensation schemes on line loadability.

In simplified terms, series compensation reduces the apparent electrical line length and, thus, increases the limits of power transfer. A direct application of the loadability curves by means of a simple reduction in line length equal to the degree of series compensation installed, however, is not recommended by the authors. This is because series compensation affects the series reactance component of the line and not the shunt line charging component and, thus, does not influence voltage and stability performance in a similar manner.

Figure 1 shows the effect of series compensation on the loadability of an 1100-kV transmission line. Compensation was varied between zero (no compensation) and 75% for lines connected to a well developed system capable of delivering 50-kA fault duty. To illustrate the effect of series compensation at lower system strength, two curves for 75% and no compensation are also shown when system fault duty is reduced to 12.5-kA. For short lines, the curves are truncated at a line loadability of 3.5 SIL, as this may approach the thermal limit of the

![Figure B. Comparison of curves of authors' Figure 7 with points obtained by use of complex form of the generalized equations.](image)

![Figure I. Effect of series compensation on line loadability.](image)
conductors used. The results demonstrate the performance aspects of series compensation for the levels indicated. The technical and economic feasibility for such a range of application at any particular length of line is another matter which will not be discussed here.

The effect of series compensation on UHV line loadability is especially significant when lines are long and terminal systems are well developed. For example, compensating a 400-mile line at 50% could nearly double the line loadability. Clearly, the stronger the system, the lower is its portion of overall reactance and, thus, the more pronounced is the reduction of that total reactance for every percent of line compensation. One point worth noting is that every additional percent of series compensation yields increasingly greater improvement in line loadability; e.g., increasing line compensation from 25% to 50% brings a larger increment in loadability than that between 0% and 25%. In fact, to achieve a greater economy of installed line compensation, one might be tempted to increase it beyond what is necessary—losing sight of a number of the complicating technical problems which are commensurate with an increasing degree of series compensation.

Mr. Linke bases his discussion on his recent paper on the related subject. Although the work reported on in this paper was essentially completed when Mr. Linke’s paper was presented, the authors did not choose to include it as a reference because we concluded that his approach was founded on the basis of inadequate and misleading assumptions. It does not seem at all correct to define power limits of transmission lines solely in terms of the maximum stability limit. It is especially misleading when this maximum stability limit is defined as the angular displacement in the line itself and not that across the entire system under study—a point which was so emphatically made in this paper and yet totally ignored in Mr. Linke’s analysis. Precisely, as Mr. Linke points out, his expressions need to be “modified by the authors’ constant steady-state-stability and constant line-voltage-drop limitations [to] provide sufficiently accurate reproductions of the St. Clair curves.”

Mr. Linke, also, appears to have misunderstood the purpose of the Appendix, evidently assuming that it gives a derivation of the principal equations used in the development of loadability curves. The purpose of the Appendix, which is clearly stated in its first paragraph, is to discuss the fact that the EHV/UHV transmission line loadability, when expressed in per-unit of surge impedance loading (SIL), is nearly independent of the line characteristics. It is only a conceptual analysis appended to the paper to show the general formulation of ideas, and it was not used in the actual development of loadability curves—the exact derivation being much too involved to be shown here. Further, in Equation (3), Mr. Linke confuses the voltage class with per-unit voltage value, which is the reason why he is left with the “incorrect impression that the loadability is a function of the voltage level.”

More confusion still, results from his statement, “...the authors have assumed a per-unit value of 1 for SI, so that SI becomes equal to SIL, a condition that can only be true at one particular line length...” This assertion is nowhere made or implied in this paper. Rather, it is explicitly stated that, in the per-unit system, SI and SIL are reciprocals of each other. Surely, Mr. Linke must have forgotten the relation between these two quantities which is,

$$\text{SIL} = |E|_{\text{low}} = 1$$

where

$$\text{SI} = \sqrt{L}, \quad \text{surge impedance in per-unit}$$

and which, obviously, is independent of the length of line.

Lastly, to answer Mr. Linke’s question regarding the use of short-circuit reactances as the terminal impedances, the authors agree with the discussor that the fault duty levels, from which these reactances are derived, are quite indicative of system strength, the fact stated in this paper. Furthermore, the authors’ definition of stability margin does include certain aspects of transient stability (see section “Limiting Factors and Assumptions”) and, as such, it was only reasonable to use the fault duty levels in describing the system strength for the purpose of this study.

In closing the authors would like to stress once again the importance of all the underlying assumptions made in arriving at the generalized EHV/UHV transmission loadability characteristics, and remind the reader that such characteristics should not be construed as a substitute for detailed planning studies.

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