In the below notes, text in this color will not be covered in class, though you may find it useful to review.

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Module PE.PAS.U21.5

Multiarea reliability analysis

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U21.1 Introduction

Modules U19 and U20 have addressed reliability analysis of the generation system assuming that the transmission system is perfectly reliable. Ultimately, we would like to be able to address the reliability of the generation and the transmission system together. An incremental step taken in that direction is the so-called multiarea reliability problem, addressed in this module.

In the multiarea reliability problem, we view the electric power system as comprised of multiple *areas* of generation, with the transmission within each area being perfectly reliable. However, the transmission interconnecting the various areas has non-zero failure probability. Figure U21.1 illustrates the situation.

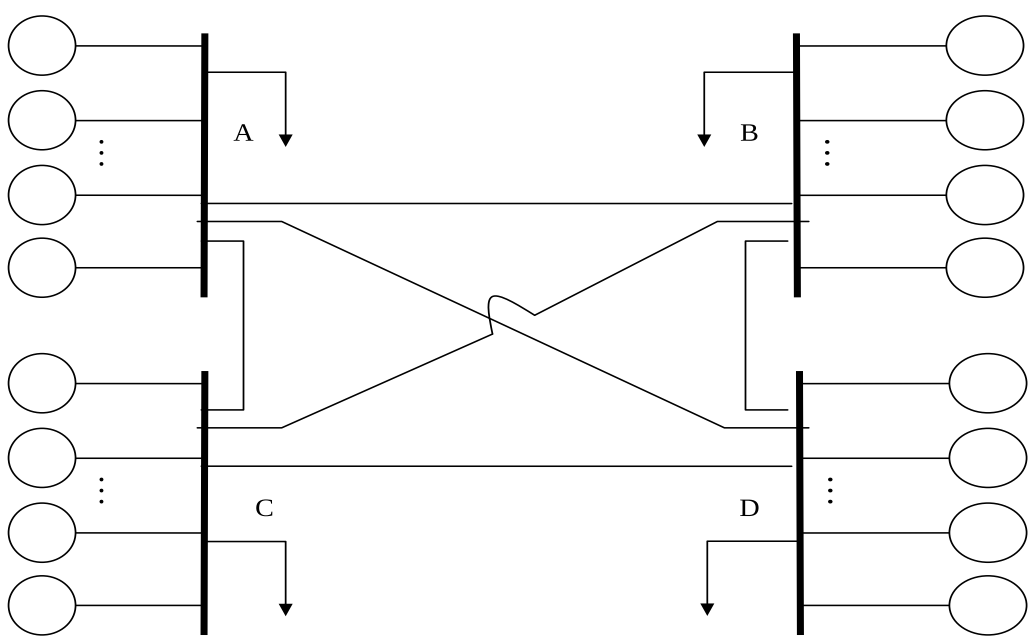


Fig. U21.1: Illustration of multiarea model

This problem has applicability whenever one or more generation units may be grouped together *physically* and *contractually*, and each group has the obligation of providing assistance to neighboring groups when needed, if capacity to do so exists.

There are four main issues embedded in the last statement, as described in what follows:

- Physical grouping: Transmission within each group must be assumed perfectly reliable so that supply of load may be performed by any generator within the group with equal reliability, given the generator is in service.
- Contractual grouping: The generators within each group operate under the same contract or set of contracts, i.e., they are dispatched together to meet load obligations.
- Neighboring groups: Group B is a neighboring group to group A if there is available transmission capacity for power delivery from group B to group A.
- Assistance: Each group is obligated to provide assistance to neighboring groups, if reserves exist, in the event the group is not able to serve its load from its own generation resources.

There were several publications on specialized studies in the past to perform multi-area reliability analysis for the region of the Mid-continent Independent System Operator (MISO), including [1, 2, 3]. For example, in [3], the following excerpts are worth reviewing, which includes Fig. U21-1a:

> *"The computing tool used for the calculation of the reliability indices in this study is the MARS program developed by General Electric International Inc. MARS uses a sequential Monte Carlo simulation technique to calculate the reliability indices of a generation system that is made up of a number of interconnected areas. Generating units and an hourly load profile are assigned to each area. MARS performs a*

chronological hourly simulation of the interconnected system, comparing the hourly load in each area to the total available generation in the area taking into account the random outages of thermal generating units, availability of interconnection tie lines and the energy limited nature of hydro and wind resources. If an area's available generation, including assistance from other areas, is less than its load, the area is in a loss of load state for that hour and statistics required to compute the reliability indices will be collected. This process is continued for all of the hours in a sample year."

…

In MARS, a generation system can be modeled as a number of interconnected areas. Each area is composed of one or several individual generating systems which can be represented as a single bus system as shown in Figure 1. The areas are defined by the limiting interfaces that may exist throughout the transmission system. The program assumes that there are no transmission limits within an area. Any generating units assigned to an area can, therefore, serve any load associated with that area. For this study, the MAPP interconnected system is modeled as five areas consisting of Manitoba Hydro (MHEB), North Dakota, Western Minnesota, and Northern South Dakota (NDAK), Central-Northern Minnesota and Western Wisconsin (MINN), Western South Dakota and Western Nebraska (WNB), and Eastern Nebraska, Southeastern South Dakota, Iowa, and Southern Minnesota (ENB/IA). A simplified diagram of the MAPP system for resource adequacy evaluation in MARS for this study is shown…"

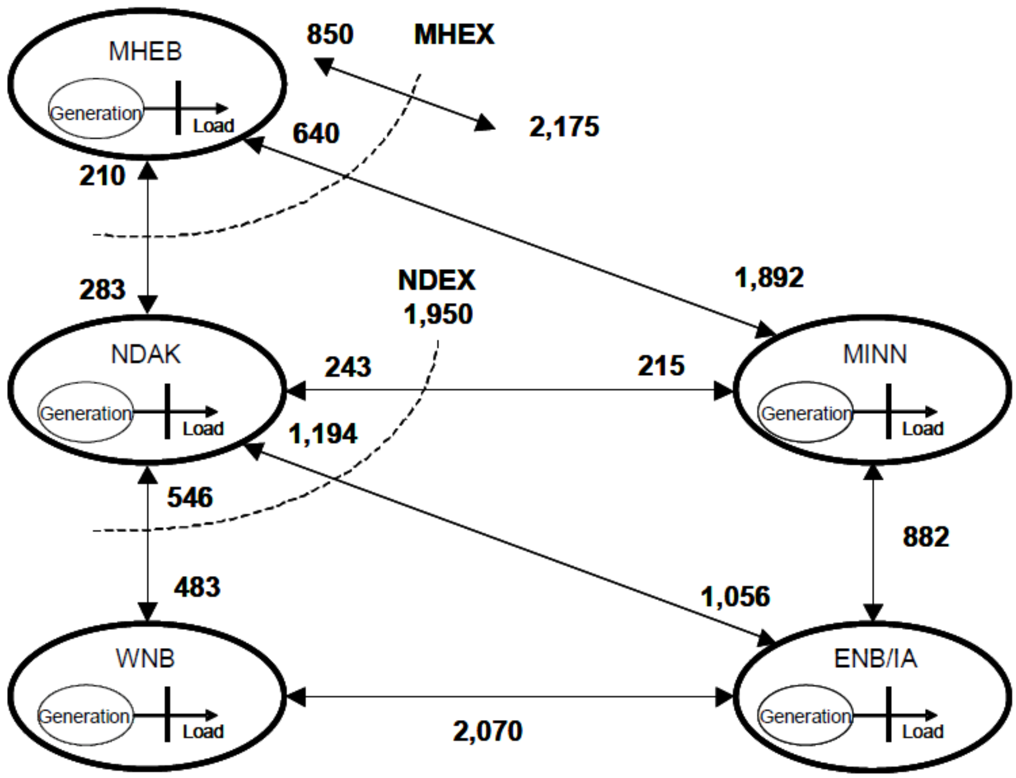
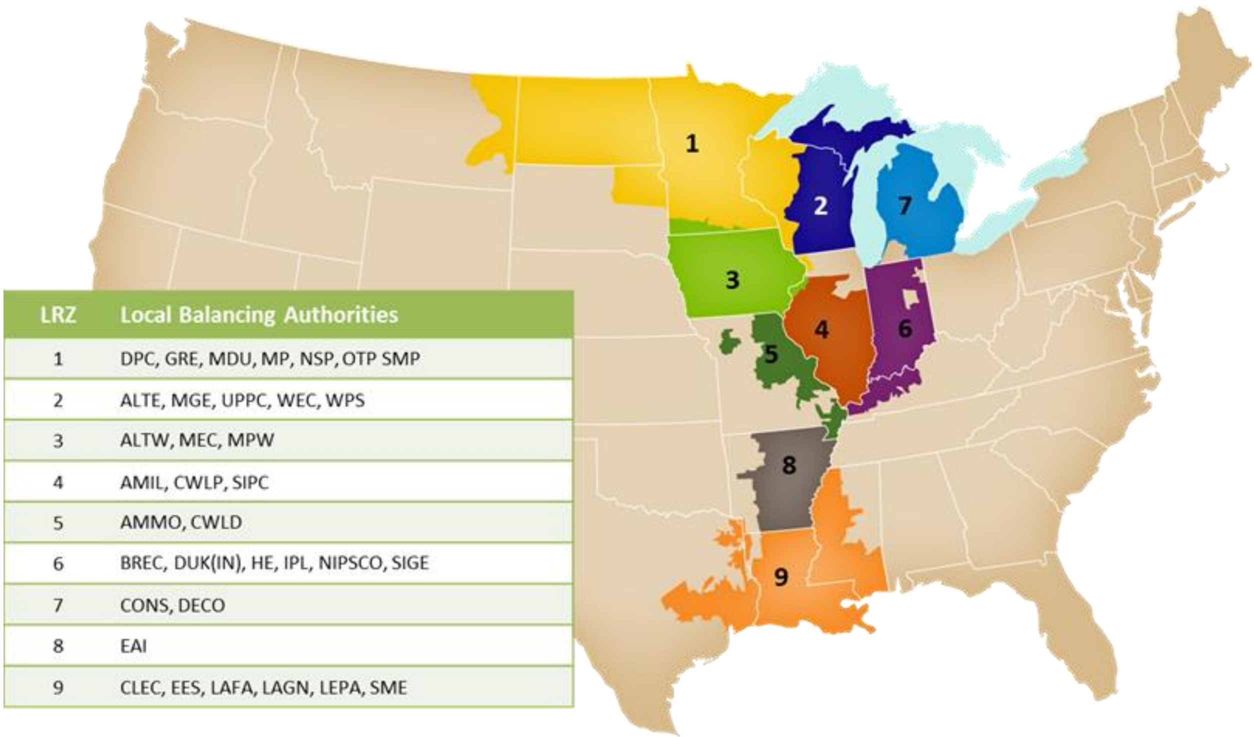


Fig. U21.1-a

The above mentioned studies were specialized ones performed once every several years. In addition, MISO includes multi-area reliability analysis in its annual Midwest Transmission Expansion Planning (MTEP) studies. For example, Figure U21.1-b below, taken from the MTEP-2015 report [4], illustrates how the MISO system is divided into "local resource zones," which is a multi-area characterization of the MISO power grid.



LRZ	Local Balancing Authorities
1	DPC, GRE, MDU, MP, NSP, OTP SMP
2	ALTE, MGE, UPPC, WEC, WPS
3	ALTW, MEC, MPW
4	AMIL, CWLP, SIPC
5	AMMO, CWLD
6	BREC, DUK(IN), HE, IPL, NIPSCO, SIGE
7	CONS, DECO
8	EAI
9	CLEC, EES, LAFA, LAGN, LEPA, SME

Fig. U21.1-b

From the MISO 2015 Loss of Load Expectation study [5] we read the following:

> *"MISO utilizes a program developed by General Electric called Multi-Area Reliability Simulation (MARS) to calculate the LOLE for the applicable planning year. GE MARS uses a sequential Monte Carlo simulation to model a generation system and assess the system's reliability based on any number of interconnected areas. GE MARS calculates the annual LOLE for the MISO system and each Local Resource Zone (LRZ) by stepping through the year chronologically and taking into account generation, load, load modifying and energy efficiency resources, equipment forced outages, planned and maintenance outages, load forecast uncertainty and external support."*

We observe two things here:

- MISO uses a multiarea reliability analysis in their annual planning processes. We will study this problem in these notes.

- The model they use is a "sequential Monte Carlo simulation." The term "sequential" means that the operating conditions are treated sequentially through time, i.e., the simulation is chronological. The term "Monte Carlo" means that uncertainty is represented by re-running the sequential simulation many times, selecting values for uncertainty variables based on a "random draw" of possible values for each variable, where the draw is from a probability distribution appropriate for the particular variable corresponding to the uncertainty. We will not address this particular approach to multiarea reliability evaluation in these notes but instead will address a method that builds on our convolution approach.

We should not assume that the GE MARS tool (originally introduced in [6]) is the only multiarea reliability analysis tool available. Table U21.1 below summarizes some other tools. Reference [7] provides an excellent summary of such tools.

There was a good comparison of GE-MARS and PJM's PRISM performed by PJM [8]; we provide the executive summary of this report in Fig. U21.1c (spread out over the three pages). Reference [8] is an excellent resource on generation adequacy evaluation; it is strongly recommended that you get a copy and review it. I have posted it on the EE 552 course website.

Table U21.1: Available G&T Reliability Evaluation Products

Developer	Multiarea reliability	G&T Adequacy	Distribution	Sub-stations	Operations
BC Hydro	MCGSR	MECORE		RISK_A	
General Reliablty	GENREL	TRANSREL	DISREL	SUBREL	
EPRI		TRELLS CREAM	DRIVE		
PTI	MAREL	TPLAN, LARA		SRA	
ABB		NETREL	RELINET		
ABB			Performance Advantage		
GE	MARS				
Astrape	SERVM				
Powertech		CRUSE			
Milsoft			Windmil		
CYME			CYMDIST		
OTI			ETAP		
PG&E			DREAM		

Executive Summary

For many years, PJM has conducted its annual resource adequacy modeling and Installed Reserve Margin (IRM) studies using their in-house, two-area model: the Probabilistic Reliability Index Model (PRISM) program. By comparison, PJM's neighboring control areas (most notably the New York Independent System Operator (NYISO), the Independent System Operator of New England (ISO-NE) and the Midwest Independent System Operator (MISO)) utilize a multiple-area program, the Multi-Area Reliability Simulation (MARS) that was developed by General Electric International, Inc. (GEII).

Resource adequacy models apply both deterministic and probabilistic methods with varying degrees of data requirements and various input / output. By nature of its inherent complexity, multi-area assessments involve more operational specific and rigorous data requirements than does a two-area model. Multi-area assessments also require more time, computing horsepower and resources to evaluate and interpret the results.

The PJM Capacity Market is a $6+ billion annual enterprise. As such, the determination, characterization and quantification of resource adequacy is important to all PJM Stakeholders. Over the past four decades, resource adequacy modeling has typically involved the modeling of interconnected areas. Such programs can employ either a one- or two-area modeling approach or more expansive multiple-area modeling.

Several members on the PJM Reserve Adequacy Analysis Subcommittee (RAAS) have long encouraged PJM to adopt MARS in place of PRISM while other members believe that PRISM already fills the bill with respect to modeling resource adequacy.

Despite the fact that both PRISM and MARS have been known and proven in the electric industry for decades, there is a lingering question: Which is the better model for resource assessments?

Evaluation of this question began years ago – at the RAAS and its predecessor groups. By early 2010, it was agreed that a detailed comparison was in order. The genesis of the report began with a simple comparison table (similar to that of Table 1). Subsequent feedback from the RAAS participants thereby prompted more comprehensive evaluation and the opportunity to document the history and evolution of resource adequacy modeling.

PJM has used both PRISM and MARS since 2005, taking advantage of the complementary features offered by both. This "blended" approach using both programs is regarded by PJM Staff as more beneficial than using just one of the programs exclusively. Realizing that there is somewhat more intensive and rigorous database requirements for MARS, PJM has relied more heavily on PRISM for its resource modeling needs. This decision enables PJM to fully utilize their existing staff resources, structures and tools to achieve the best technical results at the least cost.

This evaluation summarizes the models, calculations and output with detailed comparison of various attributes, strengths and weaknesses. Cost estimates were also developed to provide a comparative framework for needed resources.

The purpose of this report is to deliver an objective technical evaluation – not to endorse one modeling method over another (per unanimous RAAS directive). In some cases, PRISM and MARS provide complementary information that enhances the overall reliability study. This report is designed to stimulate technical discussion and lay the groundwork for possible further study and future action items.

The intent of this evaluation is to provide an objective analysis and technical comparison of both PRISM and MARS. This comparative analysis attempts to evaluate the consistency of: A) the models, B) the data and calculations and C) the standard output. Key findings are summarized as follows:

A. Comparing the Models:

PRISM

- PRISM is a two-area simulation model.
- PRISM uses a probabilistic distribution to model load.

- PRISM uses probabilistic distributions for capacity modeling (as does MARS).
- PRISM uses deterministic distributions for transmission system modeling (as does MARS).
- PRISM uses statistical input data requirements. This is a distinct advantage as PRISM has relatively few statistical parameters that incorporate the full model.
- PRISM uses a probabilistic forecast load model.
- PRISM features a comparatively fast solution time; this advantage thereby allows more time for additional sensitivity cases to more fully assess system impacts.
- Running PRISM requires no additional PJM staff to complete a resource adequacy assessment.
- Unlike MARS, PRISM cannot perform Hourly assessments or determine metrics regarding either Loss-of-Load Hours (LOLH) or Expected Unserved Energy (EUE).

MARS

- MARS is a multiple-area model; this is a tremendous advantage.
- MARS uses a deterministic load distribution.
- MARS uses probabilistic distributions for capacity modeling (as does PRISM).
- MARS uses deterministic distributions for transmission system modeling (as does PRISM).
- MARS can perform hourly calculations and include more direct Operational parameters.
- Compared to PRISM, data collection and maintenance of inputs for MARS is relatively time-intensive.
- Being a Monte Carlo simulator, MARS requires longer solution times.
- MARS is ably supports and continuously refined and updated by GE technical staff.
- MARS has gained industry-wide acceptance and usage throughout North America. MARS will therefore, be used to comply with any forthcoming North American Reliability Corporation (NERC) Planning Committee (NERC-PC) reporting for new metrics, LOLH and EUE.

B. Comparing the Data & Calculations:

The significance of the mantra *"It is all about the data!"* cannot be overlooked in Loss of Load Expectation (LOLE) assessment work: Having high-quality data allows enables greater confidence, correct interpretation of reported results, appropriate decision making and high accuracy of final reported values. PJM's database management conforms to standards and processes governed by third-party audits and assurance that Best Practices are used in the underlying data systems.

- PJM uses the same main graphical user interface (GUI) and underlying database for both PRISM and MARS. Both programs are included in the Applications for Reliability Calculations (ARC) process used by the PJM staff. Many data relationships are established to assist automating a consistent model between both tools. (See Table 3 for further details.)
- MARS contains more data input categories than does PRISM (see Appendix F). This allows MARS more flexibility and the potential to perform assessments that PRISM cannot perform.

But it also requires increased coordination among PJM Staff and more time to maintain and update the data.

- While PJM Staff is able to perform most Adequacy work in-house, increased MARS efforts would likely increase member technical representatives' responsibilities and efforts. This dynamic can be witnessed by comparing MARS assessment work that is done by neighboring RTOs, ISOs and EROs.

C. Comparing the Output:

- MARS contains several output summaries not found in PRISM, including: load level, Emergency Operating Procedure level, and Interface flows.
- PRISM uses a database schema both for its input and output results. This allows a mapping of the relationships between these data and is defined in an OLAP metadata process that allows assessment and reporting of several complex summaries.
- Both PRISM and MARS offer many detailed outputs to perform LOLE assessments.

Fig. U21.1c: Executive Summary from [8]

U21.2 Multiarea reliability failure states

The simplest situation to consider is a 2-area case; we begin from there. Denote the areas as A and B. Consider initially that there is no tie between the two areas such that they operate in an isolated fashion. Then we generate the capacity outage table for each area, and given the load level in each area, easily identify the capacity outage states for which no load is lost and for which load is lost. Denote the success and fail states for the two areas as A_S, A_F, and B_S, B_F, respectively. Figure U21.2 illustrates the different states, where we assume that areas A and B are comprised of 11 and 13 units, respectively, with each unit having 1 MW capacity.

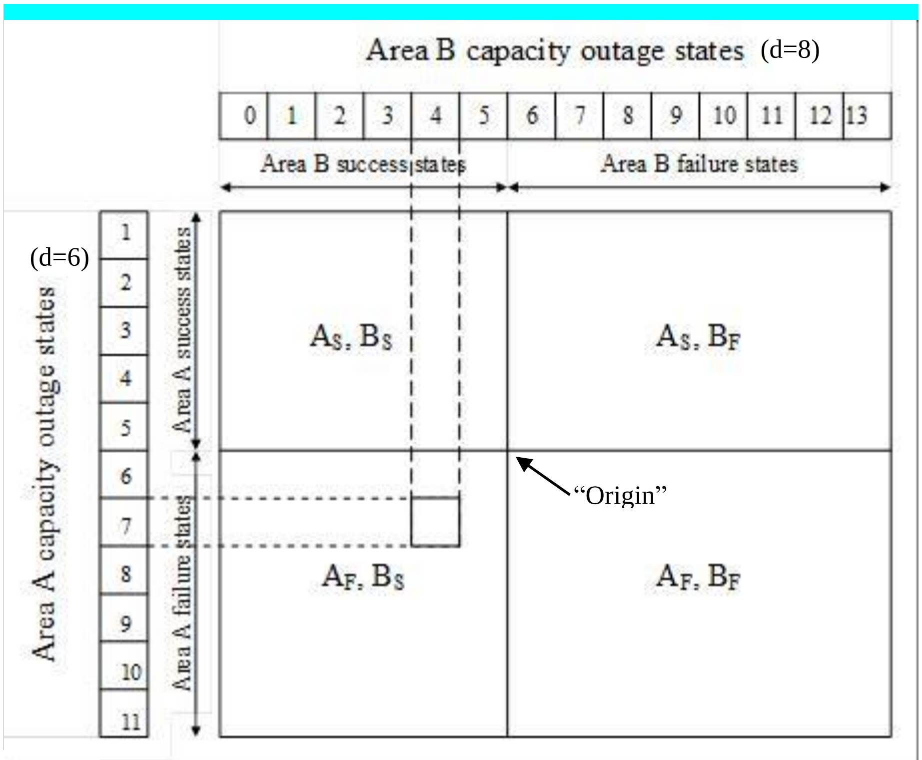


Fig. U21.2: Classification of 2-area capacity states without tie line

Areas A and B loads are 6 and 8, respectively. Failure states are identified as those for which $C>C_T$-d (implying that the capacity outage C is greater than the reserve C_T-d, or, from d>C_T-C, that available generation C_T-C is less than the load d).

- For Area A, this would be states for which C>11-6=5, i.e., states with capacity outage of 6, 7, 8, 9, 10, and 11.
- For Area B, this would be states for which C>13-8=5, i.e., states with capacity outage of 6, 7, 8, 9, 10, 11, 12, and 13.

Note that we assume a state for which the capacity outage equals the reserve is a success state. An example is, for Area A, C=5, then available generation is 11-5=6 MW, which equals the load. It may be prudent in some cases to define this state as a failure state.

Consider adding a transmission circuit having infinite capacity, and assume that each area will provide additional power to the other area only insofar as it does not cause loss of load for itself. This increases success states by the areas between dashed lines & staircases.

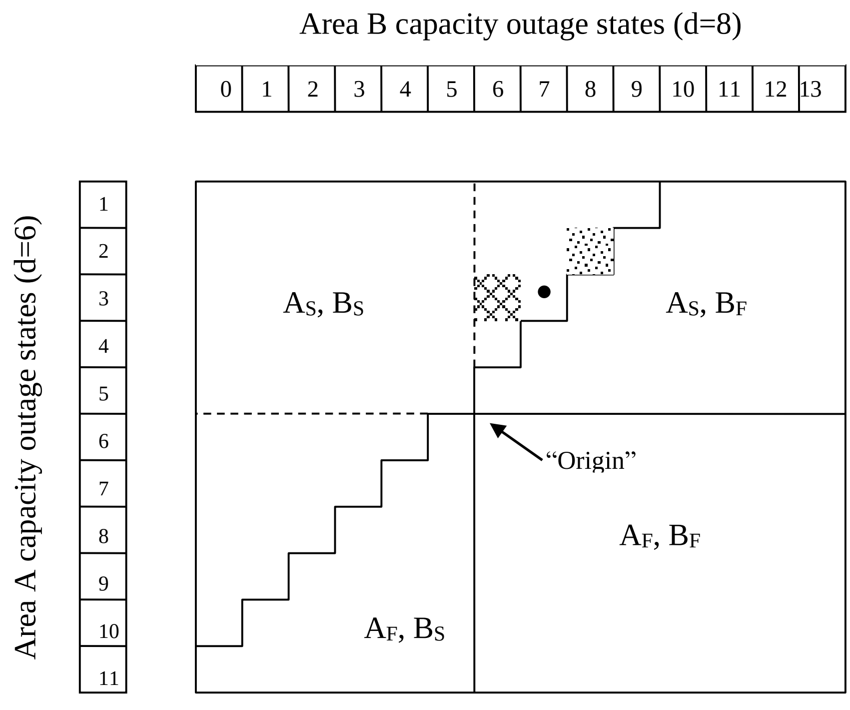


Fig. U21.3: Classification of 2-area capacity states with infinite interarea transmission capacity

We observe in Fig. U21.3 the hatched state corresponding to Area A capacity outage of 3 and Area B capacity outage of 6, which, for the case of no transmission, is a failure state, since the Area B available generation is 13-6=7 MW, not enough to supply Area B's 8 MW of load.

However, with transmission, the hatched state is a success state. Let's see why.

Since Area B has capacity outage of 6 MW, it has only 13-6=7 MW of generation available to supply a load of 8 MW. But since Area A has capacity outage of 3 MW, it has 11-3=8 MW to supply its load of 6 MW and therefore has 2 MW of reserve. If Area A supplies Area B with 1 MW, then area B has 7+1=8 MW of generation and is therefore no longer a failed state according to our criteria. In this case, Area A generation will be 6+1=7 MW, and with capacity outage of 3 MW, leaves 11-7-3=1 MW reserve.

A similar argument applies for the state just right of the hatched state (with the single dot in it), but in this case, Area B has capacity outage of 7 MW and therefore only 13-7=6 MW of generation to supply a load of 8 MW. Therefore, Area A must supply 2 MW to Area B, leaving Area A with no reserve. States having any more capacity outage in either Area A or Area B result in a failed state.

The dotted state above and right of the single dot state has Area B with an increased capacity outage of 8 MW and therefore only 13-8=5 MW of generation to supply a load of 8 MW. In this case, the capacity outage of Area A is only 2 MW, leaving Area A with 11-2=9 MW of generation to supply 6 MW of load. Therefore, Area A has 3 MW of reserve, which it can supply to Area B to prevent loss of load, making this a success state.

Comparison of Fig. U21.3 with Fig. U21.2 indicates the effect of increasing the number of success states that interarea transmission can have. One notes that infinite capacity transmission is only able to increase the number of success states insofar as available generation will allow.

Note in Fig. U21.3 that the "boundary" between success and failed states is a climbing staircase to the right. The significance of this is that, *with infinite transmission capacity*,

- *above the "origin," every decrease in Area A capacity outage (a step up) results in an additional MW being available to supply Area B (a step to the right)*;
- *below the "origin," every decrease in Area B capacity outage (a step to the left) results in an additional MW being available to supply Area A (a step down)*.

Finally, consider that the transmission interconnecting the two areas has finite capacity of 2 MW, and that capacity is only used when one area is in need of assistance from the other area (i.e., transmission is not used simply for economic purposes, so that the full transmission capacity is always available to provide reliability backup). Fig. U20.4 illustrates the resulting capacity states.

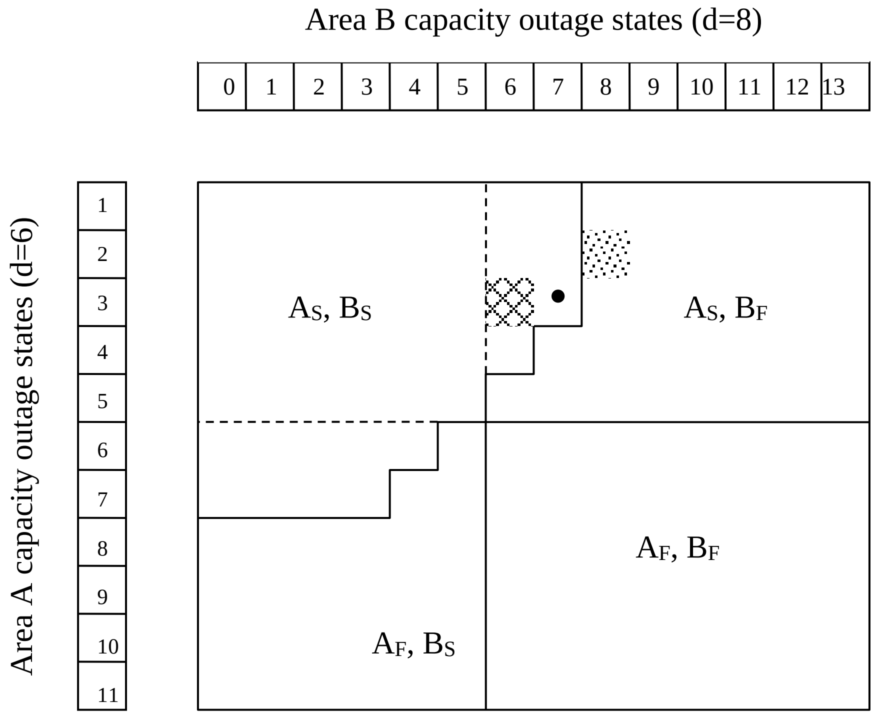


Fig. U21.4: Classification of 2-area capacity states with 2 MW interarea transmission capacity

Consider the hatched and single-dot states in Fig. U21.4. As before, we see that the effect of transmission is to turn both of these states into success states. However, notice that the dotted state, which was a success for the case of infinite transmission, is now a failure state. The reason is that, although Area A does have available generation to supply the additional 3 MW needed by Area B, the transmission capacity limits that supply to 2 MW, and Area B experiences loss of load.

Notice from Fig. U21.4 that the "boundary" between success and failure states is the same as in Fig. U21.3 in the middle of the diagram (i.e., for $3<C_A<7$ and $4<C_B<7$). The difference between the two boundaries, towards the edges of the diagram, is due to the limiting effect that transmission has on the ability to provide assistance from one area to another.

U21.3 Evaluation approaches for 2 area system

Section U21.2 only addresses the effect of transmission on the number of states that are failures vs. the number that are successes. However, we said nothing about the actual probability of these states. Once we get the probability of the states and their classification (success or failure), then we can compute the desired failure probability (loss of load probability in this case) simply as the summation of the probabilities of all failure states. There are two approaches: the all-failure states approach and the equivalent assisting unit approach. In both approaches, we assume that the transmission is limited, but perfectly reliable.

U21.3.1 All-failure states approach

One simple approach, at least conceptually, that is applicable to operating reserve evaluation when there is little uncertainty in the load, is the all-failure states approach, as follows:

1. Compute the capacity outage table for each area, lumping identical capacity outage states together. This provides the probabilities of each state for each area.
2. Identify the failure states F. Then

$$LOLP = \sum_{k,j \in F} p_{kj} \qquad \text{(U21.1)}$$

   where $p_{kj}=p_k p_j$, $k \in A$, $j \in B$, i.e., the probability of state kj is the product of the probability of state k in Area A and the probability of state j in Area A. We are assuming here that the Areas A and B states are independent.

If we want to account for the possibility of transmission failure, then we need to repeat the above algorithm for every distinct value of transmission line capacity. In this case, (U21.1) becomes

$$LOLP_i = \sum_{k,j \in F_i} p_{kj} \qquad \text{(U21.2)}$$

where we see that the failure states, denoted by F_i, are a function of the transmission line capacity i, as they should be. Then, the total LOLP is computed as

$$LOLP = \sum_i p_{Ti} LOLP_i \tag{U21.3}$$

where each transmission line capacity has a probability of p_{Ti}. In the simplest case, consisting of a single transmission line interconnecting the two areas, the interconnecting transmission line would have capacity possibilities of "full" (corresponding to "up") and "zero" (corresponding to "down").

This approach can be quite computationally intense, however, due to the need to compute the probabilities of all failure states of both areas (which has an upper bound of $N_A \times N_B$, where N_A and N_B are the number of capacity outage states in Areas A and B, respectively).

U21.3.2 Equivalent assisting unit approach

An alternative approach, called the equivalent assisting unit (EAU) approach, is described in this section. We draw heavily from reference [9] in describing this approach using some background from [10].

In the EAU approach, the benefits of the interconnection between the two systems are represented by an equivalent multi-state unit which describes the potential ability of one area to accommodate capacity deficiencies in the other area.

Here, we denote area A as the *assisted* area and area B as the *assisting* area. Some specifics of this method follow:

- The capacity assistance level for a particular outage state in Area B is given by the minimum of the transmission capacity and the available area reserve at that outage state.
- All capacity assistance levels greater than or equal to the transmission capacity are replaced by one assistance level which is equal to the tie capacity.

The resulting capacity assistance table can be converted into a capacity model of an equivalent multi-state unit which is added to the existing capacity model of Area A. Reliability indices may then be computed using the methods of Module U19 (for capacity evaluation) or the methods of Module U20 (for operating reserve evaluation).

<u>Example 1</u>: An example adapted from [9] will clarify. Consider the system data for a 2-area system as given in Table U21.2.

Table U21.2: System data for example [1]

<table>
<tr><th>Area</th><th>Number of units</th><th>Unit capacity (MW)</th><th>FOR per unit</th><th>Installed capacity (MW)</th><th>Load (MW)</th></tr>
<tr><td rowspan="2">A</td><td>5</td><td>10</td><td>0.02</td><td rowspan="2">75</td><td rowspan="2">50</td></tr>
<tr><td>1</td><td>25</td><td>0.02</td></tr>
<tr><td rowspan="2">B</td><td>4</td><td>10</td><td>0.02</td><td rowspan="2">60</td><td rowspan="2">40</td></tr>
<tr><td>1</td><td>20</td><td>0.02</td></tr>
</table>

There is one transmission line interconnecting the two areas; it has capacity of 10 MW and is perfectly reliable (FOR=0).

The capacity outage table for both areas is given in Table U21.2. Probabilities less than 10^{-8} can be neglected in this table.

Table U21.2: Capacity outage tables for example [9]

Area A				Area B			
State j	Cap out	State prob	Cum prob	State j	Cap out	State prob	Cum prob
1	0	.8858424	1.0	1	0	.9039208	1.0
2	10	.0903921	.1141576	2	10	.0737894	.096079
3	20	.0036895	.0237656	3	20	.0207062	.0222898
4	25	.0180784	.0200761	4	30	.0015366	.0015835
5	30	.0000753	.0019977	5	40	.0000463	.0000469
6	35	.0018447	.0019224	6	50	.0000006	.0000006
7	40	.0000008	.0000776	7	60	.0000000	.0000000
8	45	.0000753	.0000769				
9	50	.0000000	.0000016				
10	55	.0000015	.0000016				
11	65	.0000000	.0000000				
12	75	.0000000	.0000000				

These states look the same to Area A; they all provide 0 MW of assistance, so merge them.

Note that "Cum prob" gives probability that capacity outage is greater than or equal to the corresponding value. (This differs from what we called $F_Y(y)$ in module 20, where there it was probability that capacity outage is greater than the corresponding value.)

Area B has a reserve of 20 MW; this is the maximum assistance it can provide at this load level (assuming infinite transmission capacity). Therefore, any capacity outage of 20 MW or greater will have the same influence on the available capacity, as far as area A is concerned, limiting the assistance to zero. As a result, we merge all Area B capacity outage states greater than or equal to 20 MW into one state, accumulating the probabilities. Table U21.3 shows the Area B EAU capacity outage table.

Table U21.3: EAU capacity outage table for Area B [9]

Cap out (MW)	State prob
0	.9039208
10	.0737894
20	.0222898

These states look the same to Area A; they both provide 10 MW of assistance, so merge them.

In Table U21.3, the first 2 capacity outage states (0, 10 MW) have state probabilities corresponding to the Area B state probabilities of Table U21. 2.

The last state probability in Table U21.3 (20 MW) has a state probability corresponding to the Area B *cumulative probability* of Table U21.2. This is because, as previously stated, all Area B states having capacity outage of 20 MW or greater have the same effect on Area A, since the Area B reserve is 20 MW and therefore will not be able to assist Area A if capacity outage is 20 or greater.

Now recall that the transmission has capacity of 10 MW; we see that the assistance available from Area B to Area A is 10 MW regardless of whether the Area B capacity outage is 0 MW constrained by transmission, or 10 MW constrained by transmission as well as generation.

As a result, we merge the 0 MW capacity outage state with the 10 MW capacity outage state. The result of this merging is effectively a 2- state unit, as indicated in Table U21.4.

Table U21.4: Transmission-constrained EAU capacity outage table

Cap out (MW)	State prob
0	.9777102
20	.0222898

This table implies capacity outage state of 0 MW makes 20 MW of capacity available to Area A, but transmission limits it to 10 MW.

One problem with Table U21.4 is, however, that it suggests an equivalent unit of 20 MW capacity. This is inconsistent with the fact that maximum assistance from Area B is 10 MW due to transmission limitation. Therefore we change the bottom capacity outage value in Table U21.4 from 20 MW to 10 MW. Table U21.5 shows this change.

Table U21.5: Transmission-constrained EAU capacity outage table with adjustment for transmission capacity

Cap out (MW)	State prob
0	.9777102
10	.0222898

The transmission-constrained EAU capacity outage table of Table U21.5 is now convolved into the Area A capacity outage table of Table U21.2, giving an equivalent Area A installed capacity of 75+10=85 MW. The result is given in Table U21.6 (probabilities smaller than 10^{-8} have been truncated).

The load of Area A is 50 MW and therefore loss of load occurs when the capacity outage in Area A is greater than the reserve of 85-50=35 MW. The cumulative probability for a capacity outage of 35 MW is read from Table U21.6 as LOLP=.0023270.

Table U21.6: Area A modified capacity outage probability table

State	Cap out (MW)	State prob	Cum prob
1	0	.8660972	1.0
2	10	.1081225	.1339028
3	20	.0056221	.0257804
4	25	.0176755	.0201583
5	30	.0001559	.0024829
6	35	.0022066	.0023270
7	40	.0000024	.0001204
8	45	.0001147	.0001180
9	50	.0000000	.0000033
10	55	.0000032	.0000032
11	60	.0000000	.00000005
12	65	.00000005	.00000005
13	75	.00000000	.0000000

Example 2: One can repeat this analysis for a transmission line having capacity of 15 MW (instead of 10 MW). One would expect, with increased transmission capacity, the influence of assistance to be greater and thus LOLP to be smaller.

The new EAU capacity outage table is identical to that of Table U21.3, with the exception of the last capacity outage value, as given in Table U21.7.

Table U21.7: EAU capacity outage table for Area B with 15 MW transmission capacity

Cap out (MW)	State prob
0	.9039208
10	.0737894
15	.0222898

In comparing Table U21.7 to Tables U21.3 and U21.4, we observe:

- The 0 and 10 MW capacity outage states of Table U21.7 remain distinct since they have different effects on Area A. With 0 MW capacity outage, Area A receives 15 MW of assistance (limited by transmission). With 10 MW capacity outage, Area A receives 10 MW of assistance (limited by generation reserve).
- The largest capacity outage state is now 15, instead of 10 (as in Table U21.4), since the transmission capacity is 15.

Convolution of the Area B capacity outage data of Table U21.7 with the capacity outage data of Area A given in Table U21.2 results in Table U21.8, where installed capacity is 75+15=90. The load of Area A is 50; therefore loss of load occurs when the capacity outage in Area A equals or exceeds a reserve=90-50=40. The cumulative probability for a capacity outage of 40 is read from Table U21.8 as LOLP=.00066504, lower than the LOLP=.0023270 obtained for the case of transmission capacity=10.

It is interesting to compare the state probability for a capacity outage of 35 MW in the two cases. The 10 MW transmission capacity case yields .0022066 (Table U21.6) whereas the 15 MW transmission capacity case yields .0030837. It may be surprising to find the 35 MW outage capacity state probability is *higher* for the 15 MW transmission case whereas the LOLP is *lower*. In fact, individual state probabilities may go up or go down as we change unit capacities in a problem of this sort.

We can be sure, however, that (for a given load level), whenever we increase the installed capacity of a unit, the number of states identified as failure (loss of load states) will decrease. In this case, we increased the installed capacity of the equivalent unit from 10 to 15 MW and therefore provided that we need not include the 35 MW capacity outage state in our LOLP calculation.

Table U21.8: Area A modified capacity outage probability table

State	Cap out (MW)	State prob	Cum prob
1	0	.8007300	1.0
2	10	.1470700	.1992700
3	15	.0197450	.0521960
3	20	.0100050	.0324500
4	25	.0183560	.0224450
5	30	.0000340	.0004089
6	35	.0030837	.0037487
7	40	.0004092	.00066504
8	45	.0002059	.00025579
9	50	.0000418	.00004993
10	55	.0000069	.00000875
11	60	.0000017	.00000182
12	65	.0000001	.00000014
13	75	.00000003	.00000003

The below summarizes the steps taken in the above examples:

1. Develop the capacity outage table for both areas.
2. Develop the EAU capacity outage table by merging all capacity assisting area outage states for which the available assistance provided to the assisted area is the same. This can be done in the following 2-step process.
   a. *Effect of assisting area reserve*: Merge all assisting area capacity outage states having 0 MW assistance capability. These states are those for which the assisting area capacity outage equals or exceeds the assisting area reserve. The new state has state probability equal to the sum of all merged states, which is the cumulative probability of the capacity outage state equal to or just greater than the assisting area reserve.
   b. *Effect of transmission capacity*: Merge all capacity outage states having assistance capability equal to the transmission capacity. These states include those for which the assisting area reserve exceeds the capacity outage by the transmission capacity (or, one can say, the capacity outage is less than or equal to the reserve less transmission capacity). The new state has state probability equal to sum of all merged states.
3. *Denoting reserve by P_R and transmission capacity as C_{Tr}, decrease all non-zero capacity outage values by an amount equal to P_R-C_{Tr}. This will force the maximum capacity outage to be equal to the transmission line capacity.*
4. Convolve in the EAU capacity outage table with the assisted area capacity outage table.
5. Compute the LOLP for the assisted area as the cumulative probability corresponding to the capacity outage state equal to or just greater than the reserve.

Figure U21.5 illustrates the various assisting area states to be merged in step 2, where the numbers simply enumerate the states in order of increasing capacity outage but do not correspond to any particular capacity outage values. Note that there may be no states in the "not merged" category, as in the 10 MW transmission capacity example, there may be 1 state, as in the 15 MW transmission capacity example, or there may be several states.

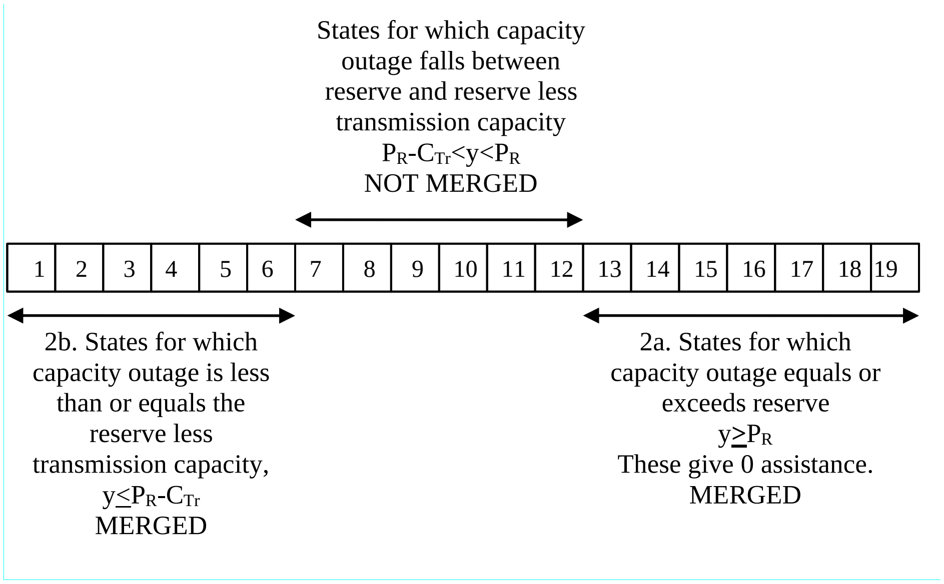


Fig. U21.5: Illustration of merged and not merged states

U21.4 Accounting for transmission reliability

In Section U21.3.2, we assumed that the transmission was perfectly reliable and developed a capacity outage table for a fictitious unit that, as far as the assisted area was concerned, was probabilistically equivalent to the assisting area. However, we assumed that the transmission interconnecting the two areas was perfectly reliable. This of course is not the case, so in this section, we show how to account for transmission unreliability.

The approach is tedious, but conceptually straightforward. The idea is to just compute the LOLP for each transmission capacity state as if there exists transmission of that capacity that is perfectly reliable. Then the composite LOLP is the weighted sum of these individual LOLP's where the weights are the transmission capacity state probabilities. Two examples will illustrate.

Example 3: Consider the example with a 10 MW capacity transmission line, except now assume it has an outage probability of .00815217 so that its availability is 1–.00815217=.99184783.

There are 2 transmission capacity states: 0 and 10 MW with probabilities of .00815217 and .99184783, respectively.

The LOLP for the 0 MW case is obtained based on single (unassisted) analysis of area A, which comes from Table U21.2. Here, we see that the cumulative probability of the 75-50=25 MW capacity outage state is .0200761.

The LOLP for the 10 MW case is obtained based on Example 1 where we found the LOLP to be .0023270. Therefore,

$$LOLP = .00815217 \times .0200761 + .99184783 \times .0023270 = .0024716937$$

The LOLP is a little larger than the case with perfect transmission and a great deal smaller than the case with no transmission at all.

Example 4: Consider now the case of the two areas connected by 2 tie lines on different right-of-ways, one of which is 10 MW capacity and the other is 5 MW capacity. The outage probabilities of each line are identical to the outage probability used in the previous example, i.e., .00815217 and .99184783.

Because the lines are on different right-of-ways, they may not fail in a dependent or common mode fashion, so the line capacities and corresponding probabilities are given by as in Table U21.9.

Table U21.9: Transmission line capacity probabilities

Capacity	Probability
0	.00006646
5	.00808571
10	.00808571
15	.98376212

We have already found the LOLP for the 0, 10, and 15 MW capacity cases, and they were .0200761, .0023270, and .00066563, respectively. Therefore we need only find the LOLP for the 5 MW case.

Following step 2-a, we require the Area B capacity outage table with all states having Area B capacity outage equal to or exceeding the reserve, as given in Table U21.3, repeated below for convenience.

Cap out (MW)	State prob
0	.9039208
10	.0737894
20	.0222898

Following step 2-b, we need to merge the states for which the Area B capacity outage is less than or equal to the Area B reserve less the transmission capacity, which in this case, is 20-5=15. So we merge the two top states in the above table, resulting in the following capacity outage table, identical to Table U21.4.

Cap out (MW)	State prob
0	.9777102
20	.0222898

Now, however, we need to adjust the maximum capacity outage value from 20 to 5 MW, to reflect that we have a probability of .9777102 of having 5 MW assistance and .0222898 of having 0 MW assistance, resulting in the EAU capacity outage data of Table U21.10.

Table U21.10: EAU capacity outage data for example

Cap out (MW)	State prob
0	.9777102
5	.0222898

This capacity outage table is convolved into that of Area A (given by Table U21.2), resulting in Table U21.11.

Table U21.11: Area A modified capacity outage probability table

State	Cap out (MW)	State prob	Cum prob
1	0	.8661000	1.0
2	5	.0197450	.1339000
3	10	.0883770	.1141600
4	15	.0020148	.0257800
5	20	.0036073	.0237650
6	25	.0177580	.0201580
7	30	.0004766	.0024006
8	35	.0018053	.0019240
9	40	.0000419	.0001187
10	45	.0000736	.0000768
11	50	.0000017	.0000032
12	55	.0000015	.0000015
13	60	.00000003	.00000003

The installed capacity following convolution of the 5 MW EAU 75+5=80. The load of Area A is 50; therefore loss of load occurs when the capacity outage in Area A equals or exceeds a reserve=80-50=30. The cumulative probability for a capacity outage of 30 is read from Table U21.11 as LOLP=.0024006.

The composite LOLP is then given by:

$$LOLP=.0200761\times.00006646+.0024006\times.00808571$$

$$+.0023270\times.00808571+.00066563\times.98376212=.00069438$$

U21.5 Effect of contractual agreements

The section is adapted from [9].

Consider the situation where Areas A and B agree that Area B will provide firm capacity to Area A of z MW (of course, at a price). This means that Area B is guaranteeing that Area A receive z MW of capacity. The guarantee may come with or without conditions on transmission.

U21.5.1 Without conditions on transmission

If the guarantee is made without conditions on transmission, then it means that the capacity is perfectly reliable. From Area A's point of view, this simply appears as an increase in its installed capacity by an amount equal to z.

<u>Example 5</u>: Consider Example 1, where we had a perfectly reliable transmission line of capacity 10 MW. We found that the Area B EAU had a probability of delivering at 0 MW capacity outage of .9777102, and that there is 1-.9777103=.0222898 probability of delivering at 10 MW capacity outage (see Table U21.5), implying that there is about a 2.2% chance that Area B cannot deliver the assistance (disregarding transmission unreliability).

When the Area B EAU capacity outage table was convolved into the capacity outage table of Area A, Table U21.6 resulted. Then, with an Area A load of 50 MW, loss of load occurs when the capacity outage in Area A is greater than the reserve of 85-50=35 MW, and the cumulative probability for a capacity outage of 35 MW is read from Table U21.6 as LOLP=.0023270.

However, in the case that Area B is willing to take all of the risk and guarantee the 10 MW of capacity, then Area A uses the original capacity outage table of Table U21.2 (without the Area B EAU capacity outage table convolved in), and simply increases the installed capacity from 75 MW to 85 MW. Again, loss of load occurs when the capacity outage in Area A is greater than the reserve of 85-50=35 MW, and the cumulative probability for a capacity outage of 35 MW is read from Table U21.2 as LOLP=.0019224. Note the improvement from the LOLP=.0023270 when we account for Area B unreliability. Of course, the contract does not change the unreliability of Area B; it simply requires that Area B take the risk by, for example, cutting its own load or paying penalties to Area A in the event it not be able to deliver the 10 MW. Whether Area B wants to sign such a contract depends on how much Area A is willing to pay for the additional capacity. Note that, without any assistance capacity, Area A's LOLP, evaluated at the capacity outage of 25 MW is .0200761, so the assistance capacity provides an order of magnitude improvement in LOLP.

U21.5.2 With conditions on transmission

If the guarantee is made contingent upon there being sufficient transmission, then it means that Area B is only guaranteeing that it will always have reserve equal to at least the contracted capacity. The Area B EAU can then be formed as a two-state capacity outage table having probability 1.0 capacity outage of 0 and probability 0 of capacity outage of the contracted capacity. Then we account for the transmission unreliability as in Section U21.4, where we

1. compute the LOLP for each transmission capacity state as if there exists transmission of that capacity that is perfectly reliable, and
2. Compute the composite LOLP as the weighted sum of the individual LOLP's where the weights are the transmission capacity state probabilities.

An example will illustrate.

Example 6: Now consider the case where Area B guarantees only the reserve of 10 MW but not the transmission capacity. It will have to cut its own load or pay a penalty if it does not have the capacity, but Area A accepts the risk brought on by unreliability in transmission capacity. The question is, in this case, what reliability level does Area A see?

Consider Example 3, with a 10 MW capacity transmission line, and an outage probability of .00815217 so that its availability is 1–.00815217=.99184783.

Therefore there are 2 transmission capacity states: 0 and 10 MW with probabilities of .00815217 and .99184783, respectively.

The LOLP for the 0 MW case is obtained based on single (unassisted) analysis of area A, which comes from Table U21.2. Here, we see that the cumulative probability of the 75-50=25 MW capacity outage state is .0200761.

The LOLP for the 10 MW case is obtained similarly to Example 1 where we used Table U21.5 as the Area B EAU capacity outage table for repeated below for convenience.

Cap out (MW)	State prob
0	.9777102
10	.0222898

Now, however, Area B is guaranteeing the reserve, therefore we will use the following capacity outage table:

Cap out (MW)	State prob
0	1.0
10	0.0

So now we convolve in this Area B EAU capacity outage table to the Area A capacity outage data of Table U21.2. This is equivalent to increasing the installed capacity of Area A by 10 MW. The resulting LOLP is read from Table U21.2 as .0019224 (corresponding to capacity outage of 35 MW). Therefore

LOLP=.00815217×.0200761+.99184783×.0019224=.002070392.

U21.6 Evaluation approach for three-area system

We have so far described and illustrated reliability analysis for two-area systems, the most basic of the multi-area situations, and one with wide applicability. However, one would be interested in knowing whether the concepts have more general applicability. In this section, we extend our approaches to the three-area situation.

U21.6.1 Radial interconnected three area systems

Figure U21.6 illustrates three areas interconnected radially.

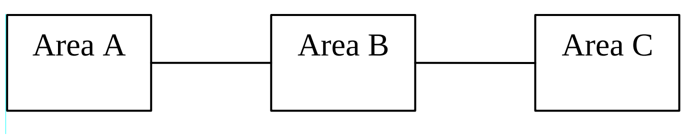


Fig. U21.6: Three areas interconnected radially

There are 2 situations of interest, described in what follows. In both cases, we assume perfectly reliable but capacitated transmission.

- Assistance to Area B: Here, we apply the two-area case twice. The steps are as follows:

1. Obtain the EAU capacity outage tables for the assisting areas A and C.
2. Convolve the Area B EAU capacity outage table with the Area A capacity outage table. Denote the new capacity outage table as A'.
3. Convolve the Area C EAU capacity outage table with the Area A' capacity outage table. Denote the new capacity outage table as A''.
4. The LOLP is obtained by reading from the capacity outage table A'' the cumulative probability corresponding to the capacity outage of installed capacity less the load.

Note that the order in which one convolves in the EAU capacity outage table does not matter in this case, i.e., one could either convolve in the Area B EAU and then the Area C EAU or one could convolve in the Area C EAU and then the Area B EAU. The answer would be the same in either case.

- <u>Assistance to Area A</u>: We again apply the two-area case twice.

1. Obtain the EAU capacity outage table for the assisting area C.
2. Convolve the Area C EAU capacity outage table with the Area B capacity outage table. Denote the new capacity outage table as B'.
3. Obtain the EAU capacity outage table for the assisting area B from the capacity outage table B'.
4. Convolve the EAU capacity outage table for Area B with the Area A capacity outage table. Denote the new capacity outage table as A'.
5. The LOLP is obtained by reading from the capacity outage table A' the cumulative probability corresponding to the capacity outage of installed capacity less the load.

We do not address the situation of assistance to Area C since this is just like the case of assistance to Area A.

In either of the above cases, if transmission is not perfectly reliable, then all possible transmission states must be identified and the method repeated for each state. The composite LOLP is then the weighted average of LOLPs for all transmission states where the weights are the transmission state probabilities.

The difficulty of this approach for the case of unreliable transmission is that there could be several transmission states. Reference to Fig. U21.6 reveals that, minimally, there would be 4 states (assuming 2-state models for both the A-B and the B-C transmission, implying AB and BC have only 1 transmission circuit each). These 4 states would be (AB up, BC up), (AB up, BC down), (AB down, BC up), (AB down, BC down). LOLP would therefore need to be computed 4 times, one for each of these states.

There could be more states depending on how many transmission circuits comprise the AB and BC connections.

U21.6.2 Networked interconnected three area systems

Figure U21.7 illustrates three networked interconnected areas.

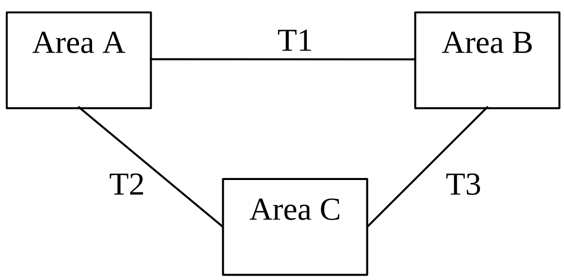


Fig. U21.6: Three networked interconnected areas

This situation is quite difficult to handle using our present techniques because of the following reasons:

1. Two transmission paths: Each area can assist another area over 2 possible paths, the direct connection to the assisted area and the connection through the third area to the assisted area. This presents two basic problems.

   a. Controlled flows: Here, we assume that each area may specify the amount of assistance flowing over a particular path. Although this is the simplest case, we see that it is probabilistically complex, as the amount of assistance over a transmission path depends not only on the reserve in the assisting area and the path's transmission capacity but also the extent to which the other area is using that path.
   b. Uncontrolled flows: Here, we must recognize that, unless special flow-control devices (FACTS devices) are available, it is not possible to assign a particular amount of assistance to a specific transmission path since Kirchoff's laws dictate that any assistance from one area to another will actually divide and flow along both paths. This is called loop flow. As a result, any assistance will utilize transmission capacity in all three paths.
2. Contractual agreements:
   a. On reserve: There are numerous possible agreements that bear on the problem. If only one area is deficit, then that deficit area gets as much assistance as it needs, (up to what is available of course) from the other two areas. However, the issue is not so clear if there are two deficit areas. For example, if Area A and Area C are both supply-deficit, how do they share the Area B assistance? Area A may have priority over Area C such that Area C only receives assistance when Area A's needs are met. Or Areas A and C may share Area B's assistance according to some specified proportion.
   b. On transmission: Transmission agreements need to be consistent with reserve agreements so that transmission contracts do not constrain assistance levels beyond that of the reserve agreements. This is generally possible if the transmission and generation are owned by the same organization, but if not, it can be quite complicated.

We will study a new method in the next section which addresses some of these issues.

U21.7 Multiarea analysis by network flows

The material in this section is adapted from [11] and [12]. Some publications illustrating the approach are in [13, 14, 15].

An area of systems engineering has grown from the numerous systems that can be thought of in terms of physical movement within a network. Such systems include

- Transportation systems, e.g., bus, rail, airlines, shipping.
- Communication systems, e.g., telephone and internet.
- Energy systems, e.g., electricity, gas, coal, and water.

One approach for analysis of such systems is generally referred to as "network flows." We will find network flow theory to be useful in multiarea reliability analysis.

U21.7.1 Preliminaries: some graph-theoretic definitions and concepts

The essential notion on which a network flow problem is based is the graph. We define some related notation below.

<u>Graph</u>: G(V,Γ) consists of a set of elements called nodes, denoted V, and a set of pairs of elements called arc (or branches), denoted Γ. G can be a *directed* graph, where flow on each arc may only occur in one direction, or G can be an *undirected* graph. G may also have both directed and undirected arcs. A particular node is denoted V_i. Each arc is denoted (i,j) if it is directed or [i,j] if it is undirected. A graph is another name for a network.

<u>Flow</u>: With each arc (i,j) or [i,j], we associate a weight f(i,j) or f[i,j] which is called the flow of arc (i,j) or [i,j].

<u>Capacity</u>: With each arc we associate another weight c(i,j) or c[i,j] which is called the capacity of branch (i,j) or [i,j]. It represents the maximum flow that the branch can carry.

Source node: Each graph has a source node, denoted *s*, which produces all the flow that is flowing through the network.

Sink node: Each graph has a sink node, denoted *t*, which consumes all the flow that is flowing through the network.

Flow pattern: The flow pattern is a set of flows associated with the branches in a graph and is denoted F.

Feasible flow pattern: Define $f_{s,t}$ as the total flow between s and t. Then a flow pattern F is said to be feasible if it satisfies:

Directed graph:

$$\underbrace{f(i,V)}_{\text{flow out of node i}} - \underbrace{f(V,i)}_{\text{flow into node i}} = \begin{cases} f_{s,t} & \text{if } i = s \\ 0 & \text{if } i \neq s,t \\ -f_{s,t} & \text{if } i = t \end{cases} \qquad \text{(U21.4)}$$

$$c(i,j) \geq f(i,j) \geq 0 \quad \forall\ i \text{ and } j$$

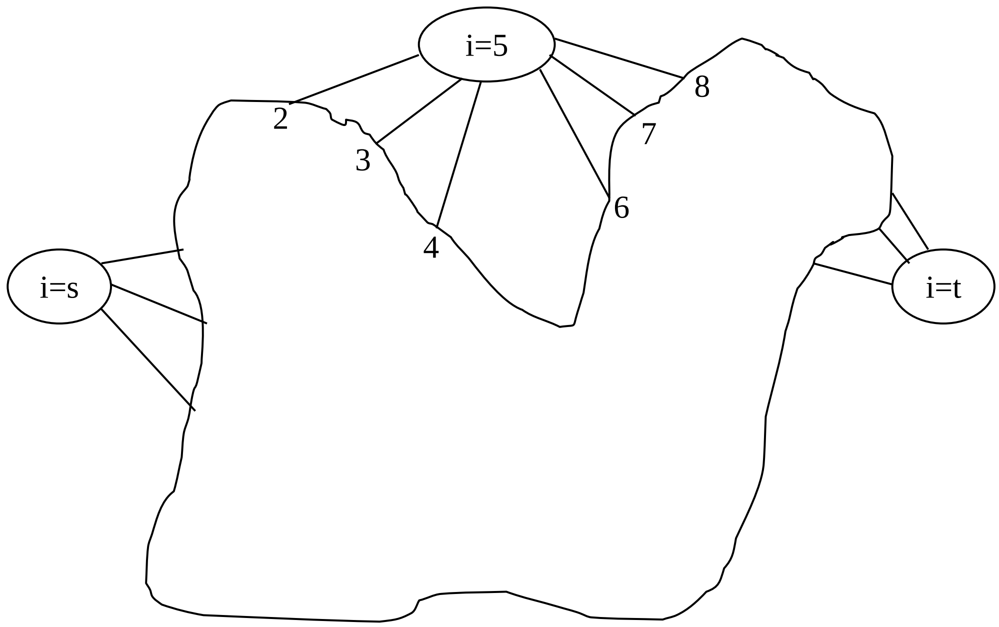
Undirected graph:

$$\underbrace{f[i,V]}_{\text{flow out of node i}} - \underbrace{f[V,i]}_{\text{flow into node i}} = \begin{cases} f_{s,t} & \text{if } i = s \\ 0 & \text{if } i \neq s,t \\ -f_{s,t} & \text{if } i = t \end{cases} \qquad \text{(U21.5)}$$

$$c[i,j] \geq f[i,j] \geq -c[i,j] \quad \forall\ i \text{ and } j$$

In the above, V represents all nodes in the graph. Therefore, the above represents the conservation of flow at each node. For example, in the below figure, we may have

f[5,V]-f[V,5]=
[f(5,6)+f(5,7)+f(5,8)] - [f(2,5)+f(3,5)+f(4,5)]
= [1 + 1 + 4] - [2 + 3 + 1] = 0



Sets of branches: Let A and B be sets of nodes contained in V, i.e., A⊆V and B⊆V. Denote the set of all branches which are incident out of (connected from) a node in A and incident into (connected to) a node in B. That is,

$$(A,B) = \{(i,j) \in \Gamma \,|\, V_i \in A,\ V_j \in B\} \qquad \text{(U21.6)}$$

Set theoretic complement: Denoting a subset V_1 of nodes of G(V,Γ), the set theoretic complement of V_1 in V is denoted by $\overline{V}_1$ and defined by $V_1 \cup \overline{V}_1 = V$.

Cut: Combining the last two definitions, we define that for any V_1⊆V, the set of branches identified by $(V_1,\overline{V}_1)$ is a *cut*. In this definition, remember that V_1 and $\overline{V}_1$ represent sets of nodes, not sets of branches, and that $(V_1,\overline{V}_1)$ therefore represents a certain group of branches that connects a node in the node-set V_1 with a node in the complementary node-set $\overline{V}_1$. Figure U21.7a illustrates a cut, where V_1={1,2,3,4} and $\overline{V}_1$={5,6}, such that $(V_1,\overline{V}_1)$={[4,5], [3,5], [4,6]}.

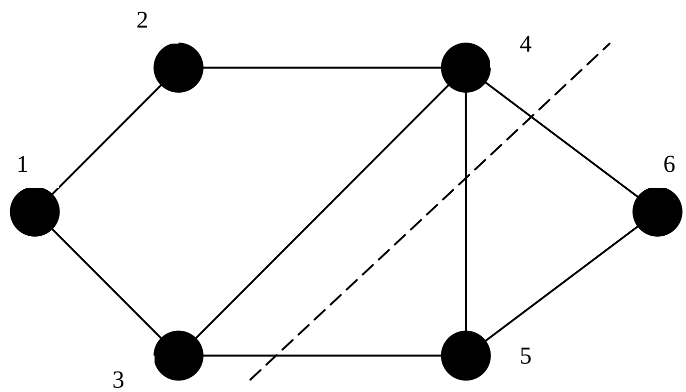


Fig. U21.7a: Illustration of a cut

<u>s-t cut</u>[1]: An s-t cut is a cut $(V_1, \overline{V}_1)$ where s$\in$V1 and t$\in \overline{V}_1$. We will denote the Kth s-t cut as $A_{s,t}^{K}$.

<u>Capacity of a cut</u>: The capacity of a cut, denoted by $c(V_1, \overline{V}_1)$ or $c[V_1, \overline{V}_1]$, is the maximum total flow that may cross the cut when connected in the graph. It is given by:

Directed graph:

$$c(V_1, \overline{V}_1) = \sum_{(i,j)\in(V_1,\overline{V}_1)} c(i, j) \tag{U21.7}$$

Undirected graph:

$$c[V_1, \overline{V}_1] = \sum_{[i,j]\in[V_1,\overline{V}_1]} c[i, j] \tag{U21.8}$$

<u>Minimal cut</u>: The minimal cut is the cut with the smallest capacity.

<u>MaxFlow-MinCut theorem</u>: This theorem, developed by Ford and Fulkerson [16], is the basis for determining the maximal flow from source to sink within a network. In words, the theorem says that *the maximal flow from source to sink in any network is equal to the capacity of the minimal s-t cut*. Mathematically, the theorem is:

---

[1] This definition of an s-t cut is similar to the definition (given in Module U15) of a cutset, which was, “A cutset K is a set of components whose failure results in system failure. The removal of the corresponding set of blocks in the logic diagram interrupts the continuity between the input and output of the diagram [1]. Removal of all components in any cutset ‘disconnects’ the ‘input’ from the ‘output’ in the logical diagram.”

$$\max_F [f_{s,t}] = \min_K [c(A_{s,t}^K)]$$

This theorem enables a max-flow calculation.

We use an analogy to gain physical intuition related to this theorem. Imagine a sequence of piping stages from an originating pool of water, identified as "s", to a destination pool of water, identified as "t". The stages are in series, but each stage has several pipes in parallel; each pipe has its own unique capacity. An s-t cut is an interruption of all pipes in a given stage. The minimal s-t cut is the s-t cut which interrupts the least capacity. The maximum flow through the system is the capacity of the minimal s-t cut. Fig. U21.7b illustrates where we observe that stage 2 is the minimal s-t cut, and it is the "bottleneck" that determines the maximum flow through the system.

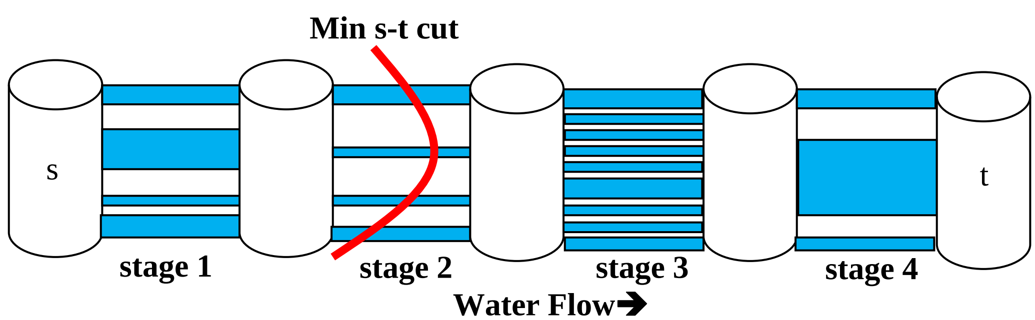


Fig. U21.7b: Illustration of minimal s-t cut

We now provide a preview of how we will use our ability to compute a max-flow calculation:

Problem set-up: All of the arcs connected to node s represent generation, all of the arcs connected to node t represent load, and what is flowing from s to t is power. ***A "state" is a capacity designation for all arcs***. There are two observations at this point:

- We normally associate generation and load with nodes; in this case, we are associating them with arcs (or branches).
- The "s" and "t" nodes do not correspond to any physical node; they are simply endpoints of the generation and load arcs, respectively.

Objective: Identify all "success" and "failure" states (and their probabilities).

Fact 1: We will see that it is easy to compute the max flow.

Fact 2: The max flow is a "success" state if the flows in the load arcs are at their capacities.

A part of the idea: Pick the highest capacity state and compute a max flow (note the max flow will not necessarily use all capacity of all arcs). If this is a success state, then the capacity state corresponding to the max flow is a "success" state, and all states ***between*** this capacity state and the highest capacity state are "success" states.

Between…? What does this mean?

A (capacity) state S is ***between*** two states S_L and S_H if

- S is not S_L and S is not S_H.
- Each arc in S has capacity greater than or equal to its corresponding arc in S_L
- Each arc in S has capacity less than or equal to its corresponding arc in S_H

Let's now go back to learning how to compute a max flow. We first see an example that illustrates the MaxFlow-MinCut theorem.

Example 7: Determine the maximal flow of the network in Fig. U21.8.

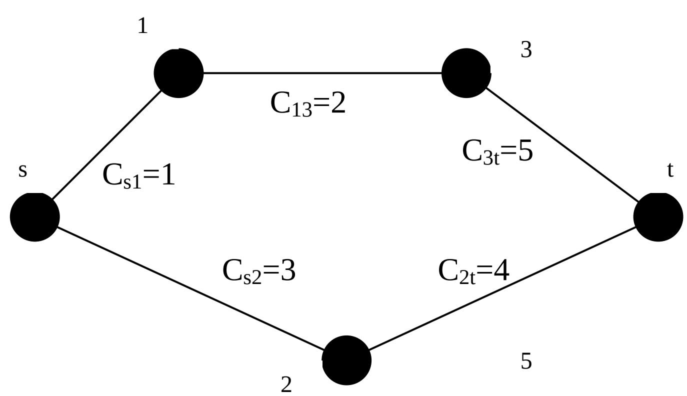


Fig. U21.8: Example to illustrate maximal flow calculation

The s-t cuts for the network of Fig. U21.8, their node sets, their complementary node-sets, and their capacities, are listed in Table U21.12.

Table U21.12: Summary of s-t cuts for example

s-t cut $A_{s,t}^{K}$	Node-set V_1	Node-set $\overline{V}_1$	s-t cut-capacity $c(A_{s,t}^{K}) = c(V_1, \overline{V}_1)$
$A_{s,t}^{1}$	(s)	(1,2,3,t)	4
$A_{s,t}^{2}$	(s,1)	(2,3,t)	5
$A_{s,t}^{3}$	(s,1,3)	(2,t)	8
$A_{s,t}^{4}$	(s,1,2)	(3,t)	6
$A_{s,t}^{5}$	(s,1,2,3)	(t)	9
$A_{s,t}^{6}$	(s,2)	(1,3,t)	5

From Table U21.12, we see that the minimal cut, and therefore the maximal flow, is 4.

One observation is that this approach adheres to Kirchhoff's first law (sum of flows into a node must be zero), otherwise known as flow conservation, but not Kirchhoff's second law (KVL - sum of voltages around a closed loop must be zero). One must be aware of this approximation when applying network flow theory to power grids. Solutions obtained this way satisfy necessary, but not sufficient conditions that the load will be supplied. Sufficient conditions would need to compute flows using a power flow to impose KVL. The issue is our modeling of lines.

- Whereas our network flow formulation models lines as "pipes" so that, except for nodal power balance, flows on lines are independent of each other,
- the power flow accounts for the effects of impedances.

We need to be able to articulate an algorithm for identifying the maximal flow. We present such an algorithm in what follows. But first, we need three more definitions.

<u>Path</u>: A sequence of branches starting at the source node and ending at the sink node such that no node is visited more than once.

<u>Forward and backwards arcs</u>: A directed arc (i,j) in a path is a forward arc if in traversing from s to t, i comes before j; otherwise it is a backwards arc in the path.

<u>Flow augmentation path</u>: For a given flow pattern F, a flow augmentation path is a path (i.e., from source to sink) for which there exists unused capacity.

The max-flow (also called Ford-Fulkerson) algorithm follows:

1. *Initialization*: Initialize the graph with a feasible flow (capacity restrictions and flow conservation must be satisfied). One flow that is always feasible is 0 flow on all branches.
2. *Labeling*: Use the labeling routine to find a flow augmentation path (i.e., a path (from s to t) for which flow may be increased). The labeling routine is:
    a. Starting with s, node j can be labeled if a positive flow can be sent from s to j. If no node can be labeled, proceed to step 5.
    b. Find a node to label. From node j, any node i can be labeled if:
        - the j to i arc is a forward arc and flow in this arc is less than its capacity.

    - the j to i arc is a backward arc and flow in this arc is greater than zero.

  c. Three things may happen at this point.
    - A node i is found such that i≠t. Repeat step b.
    - No node i can be labeled. This means that no augmentation path can be found through node j. Proceed to step 4.
    - Node i is found such that i=t. We have found an augmentation path and should proceed to step 3.

3. *Augmentation:*
   a. Identify the maximal flow increase δ that can be sent along the augmentation path identified in step 2.
   b. Augment flow on all arcs in the augmentation path by δ. Forward arc flows are increased. Backward arc flows are decreased. Undirected arc flows are increased if the flow augmentation is in the same direction as the original flow. Undirected arc flows are decreased if the flow augmentation is in the opposite direction as the original flow.
4. *Repeat*: Go to step 2.
5. *Stop*: The maximal flow is the flow out of the source node (or into the sink node) resulting from the last augmentation path found.

We repeat Example 7 but this time we use the algorithm.

<u>Example 8</u>: Figure U21.9 shows the initialized graph of Fig. U21.8. Numbers in parentheses indicate (capacity, flow). All arcs are undirected.

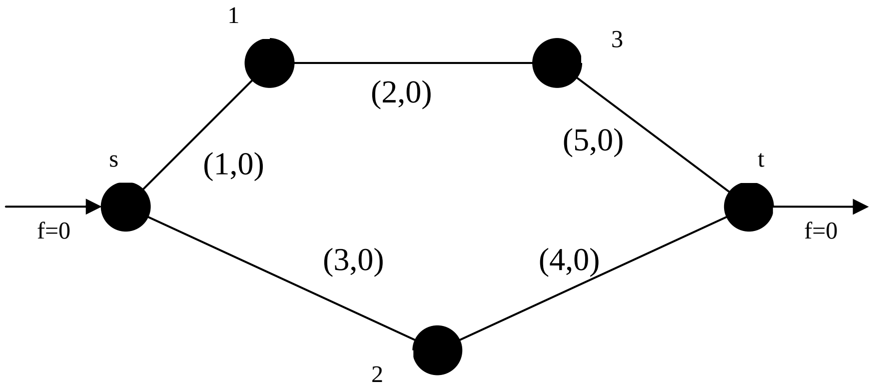


Fig. U21.9: Example to illustrate maximal flow calculation

Application of step 2 (labeling) to the network of Fig. U21.9 results in the augmentation path illustrated in Fig. U21.10.

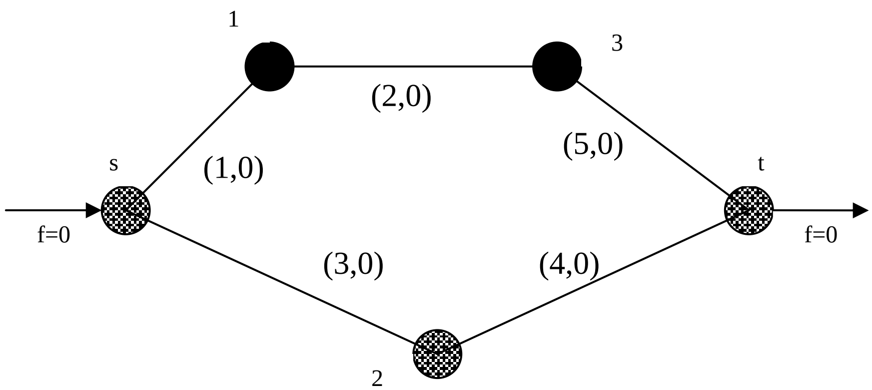


Fig. U21.10: Results of first step 2 iteration

Application of step 3 (augmentation) to the network of Fig. U21.10 results in the network of Fig. U21.11.

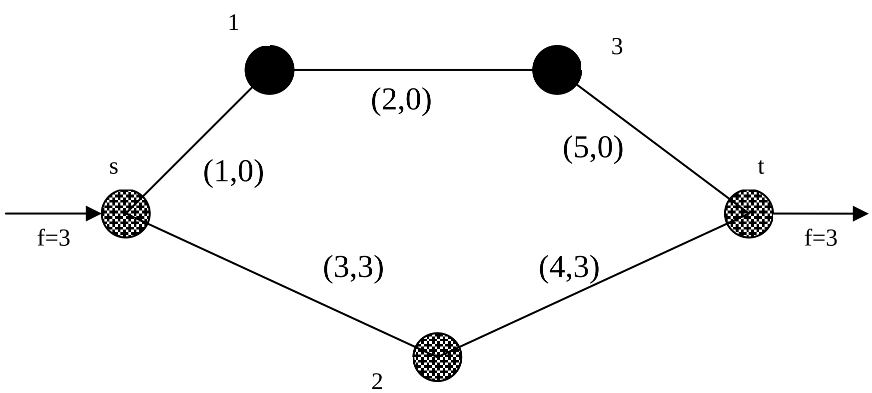


Fig. U21.11: Results of first step 3 iteration

We now apply step 2 again (labeling), this time to the network of Fig. U21.11, resulting in the augmentation path of Fig. U21.12.

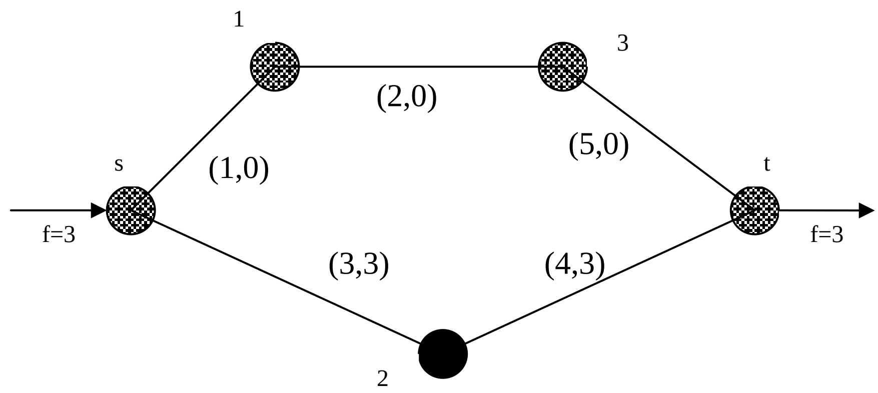


Fig. U21.12: Results of second step 2 iteration

We now apply step 3 (augmentation) again, this time to the network of Fig. U21.12, resulting in the network of Fig. U21.13.

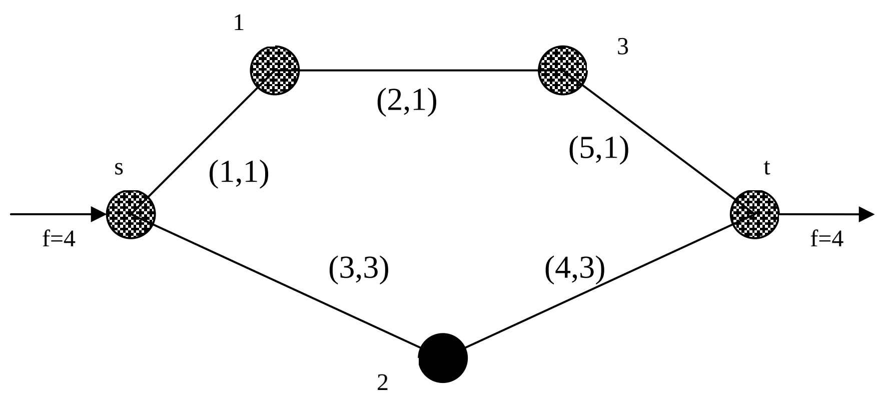


Fig. U21.13: Results of second step 3 iteration

When we try to apply step 2 again, we find that, beginning with the source node s, we are unable to label any other node since all arcs leaving s are at capacity. So we go to step 5, where we terminate the algorithm, with the maximal flow recognized as 4.

Example 9: Figure U21.14 shows another example. Use the algorithm to determine the maximal flow for this network for the case of:

a. All arcs are undirected. The correct answer is 16. Try it!

b. All arcs are undirected except for [1,2] which may have flow only in the direction 1 to 2. The correct answer is 15.

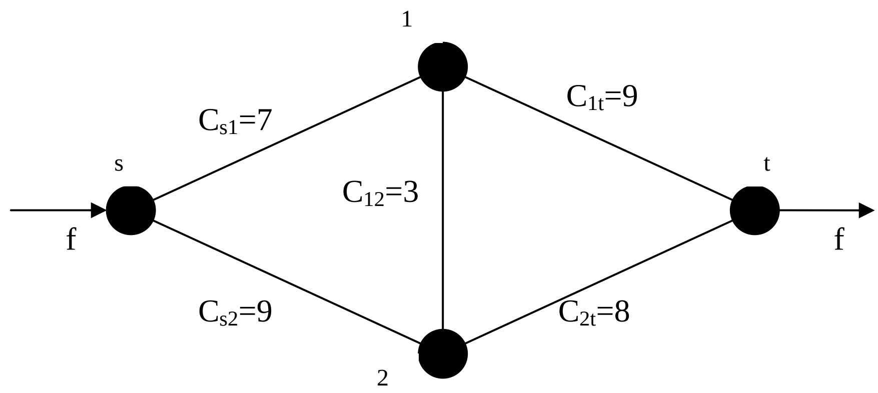


Fig. U21.14: Example to illustrate maximal flow algorithm

Below is the answer to part (b). This is taken from [12]. Notice that the nomenclature on the diagrams is (flow, capacity) instead of (capacity, flow) as we have used in previous examples.

The two problems, (a) and (b), have different answers because of the last step below (called "Fig. 14") where, with a directed arc in the center, we may reduce the flow on (1,2) to only 0, and thus take only capacity of the (1,n) arc to 7. With undirected arc in the center, we can push the flow on (1,2) to -1 (thus flowing +1 from node 2 to node 1) and thus take advantage of one more unit of capacity in (1,n) where it goes to 8.

The flow network is shown below:

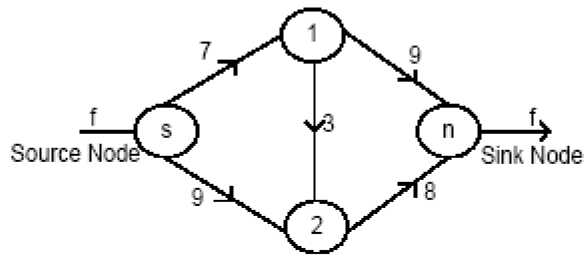


Fig. 5

Initial flows

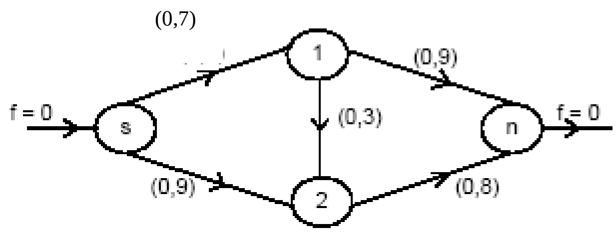


Fig. 6

Flows and augmentation paths:

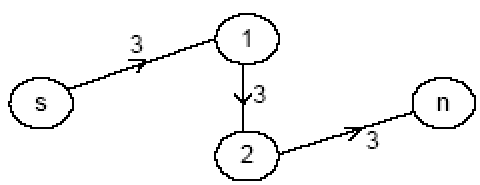


Fig. 7

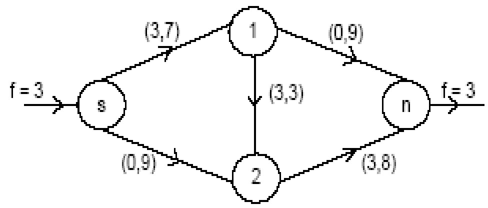


Fig. 8

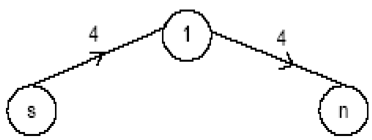


Fig. 9

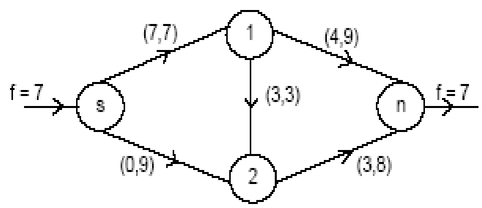


Fig. 10

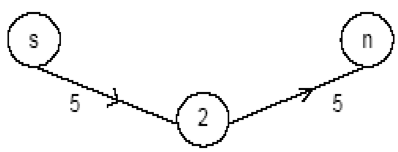


Fig. 11

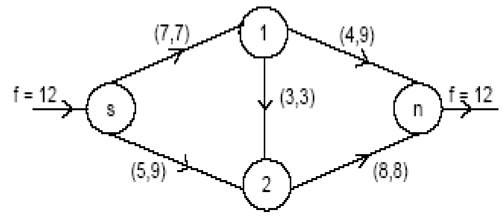


Fig. 12

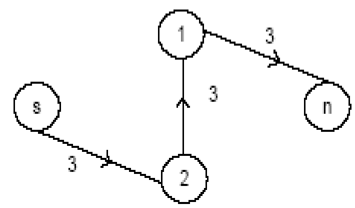


Fig. 13

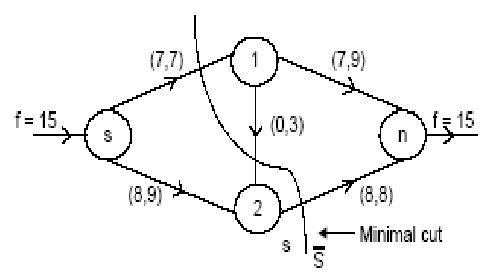


Fig. 14

Observe in the “Fig. 14” above that the capacity of the minimal cut is 7+0+8=15 and not 7+3+8=18. The implication is that the capacity of a directed arc contributes non-zero capacity only if it has flow.

We think about this in 2 other ways:

- A directed arc has two different capacities: 0 and c, so if the flow on a directed arc is zero, then its capacity is considered to be zero.
- If we repeat the problem without the arc (1,2), the result of Fig. 14 above is the same.

U21.7.2 Representation of multiarea system as a network flow problem

KEY IDEAS FOR THIS SECTION:

- A power system with components (area generation, transmission between areas, and area load) can be represented as a network flow problem like we have studied above. Area generation, inter-area transmission, and areas loads are represented with ***arcs*** having capacity and flow.
- We will utilize nomenclature where x_j represents the capacity designation of component j, x_j=1,…,N, e.g., x_j=1 designates zero capacity, and x_j=N designates the maximum capacity.
- Recall ***a "state" is a capacity designation for all arcs***. A system state is represented by a vector

$$\underline{x}=(x_1\ x_2\ \ldots x_N)$$

- We may identify whether a system state is a failure or success state by running a max flow on it.
- We may obtain probabilities of a component's capacity, and (under an independence assumption), we may obtain probabilities of a system state.

The material in this subsection is adapted from [12].

Consider again the networked three area interconnected system illustrated in Fig. U21.6, repeated here for convenience, where we have changed the area designation from A, B, C to 1, 2, 3.

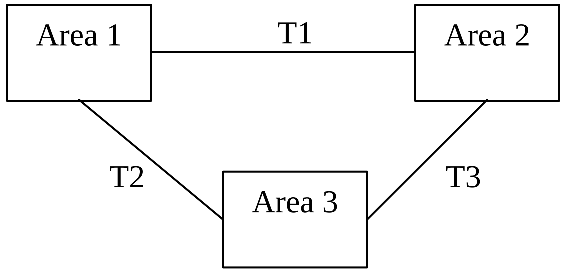


Fig. U21.15: Three networked interconnected areas

Assume the following data for this multiarea system:

Area 1: installed capacity = 500 MW, Load=400 MW.

Area 2 installed capacity=600 MW, Load=500 MW.

Area 3 installed capacity=500 MW, Load=400 MW.

Transmission capacity T1=T2=T3=100 MW.

Problem: Determine whether the system with all components up is a loss of load state or not. This means that we want to determine whether a particular system state is a failure state or not.

How can we represent this as a network flow problem?

Represent all possible generation as originating from the source node and all possible load at the sink node. The other elements are represented as follows:

- Generation arc: A directed arc from source node s to node i, with capacity c(s,i) represents a particular discrete capacity state for area i (these capacity states can be obtained from the capacity outage table for area i). Of course, these capacity states have their corresponding probabilities.
- Transmission arc: An undirected arc between nodes i and j having capacity c(i,j) represents a particular discrete capacity state of the transmission between areas i and j. The transmission states also have their corresponding probabilities.
- Load arcs: A directed arc between node i and the sink node t represents the load in area i. We will assume these loads to be fixed.

Note that generation and load are represented using directed arcs and transmission is represented using undirected arcs. This is a result of the fact that generation and load flow is in one direction only, whereas transmission flow can be in either direction.

The particular state of interest for the three area system in Fig. U21.15 and the data provided (in terms of the generation capacity in each area, the transmission capacity in each area, and the load in each area) are represented using the network flow problem in Fig. U21.16, where values beside each arc represent (capacity, flow)/100.

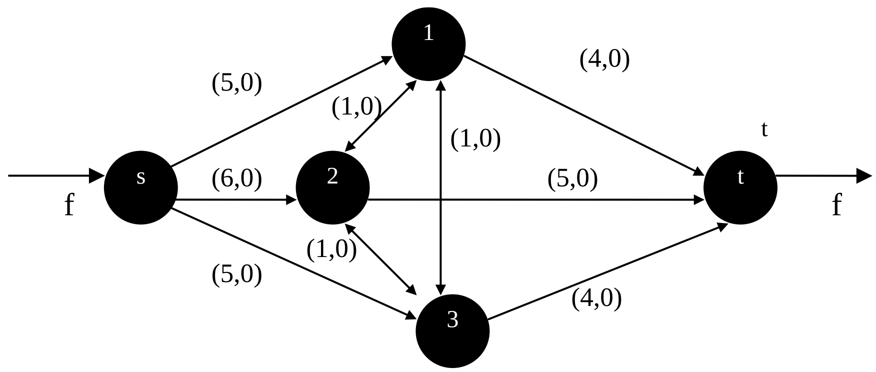


Fig. U21.16: Example to illustrate maximal flow algorithm

We can detect whether the state is failure or not by performing max-flow and then checking whether the max flow equals the sum of the load arc capacities. In this case, that would be 4+5+4=13.

Fig. U21.17 provides a sequence of max-flow algorithm steps for the network flow problem of Fig. U21.16. The dark lines indicate the augmentation path at each step.

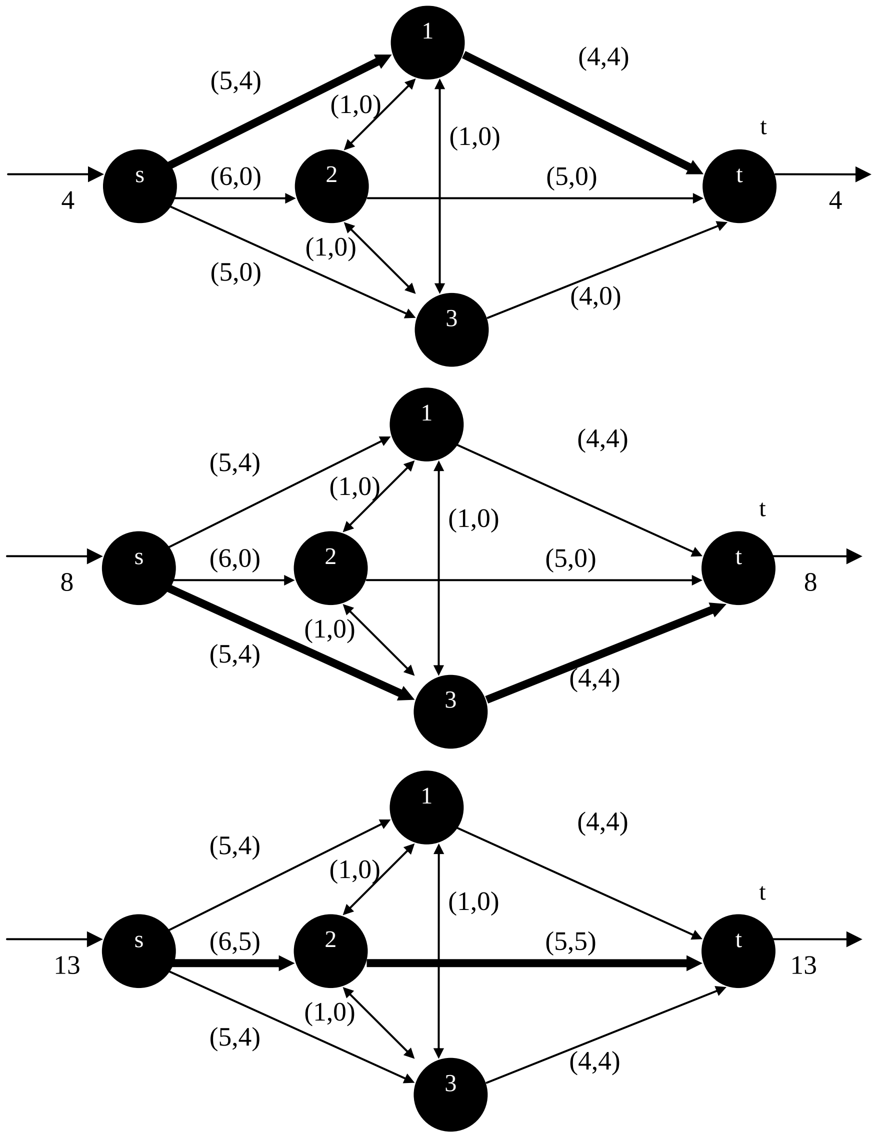


Fig. U21.17: Example to illustrate maximal flow algorithm

Notice that our first three augmentation paths were intentionally chosen to force the load of each area to be served by generation in that area, if possible.

However, the flow pattern for a particular max-flow problem is not unique (the max flow value itself is unique). This can be observed by repeating the above max-flow problem but choosing a different sequence of augmentation paths. Fig. U21.18 illustrates.

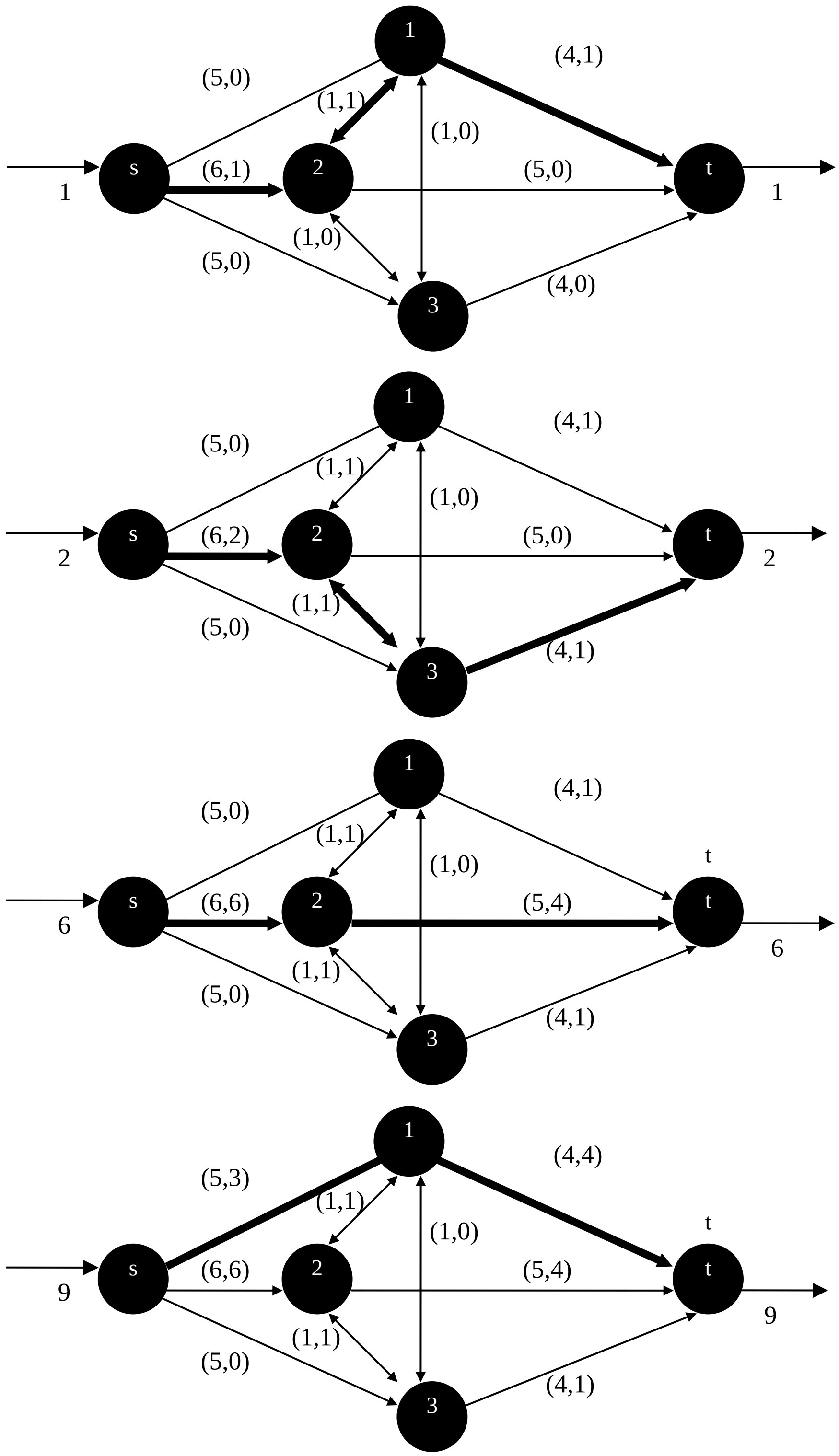


Fig. U21.18: Example to illustrate maximal flow algorithm

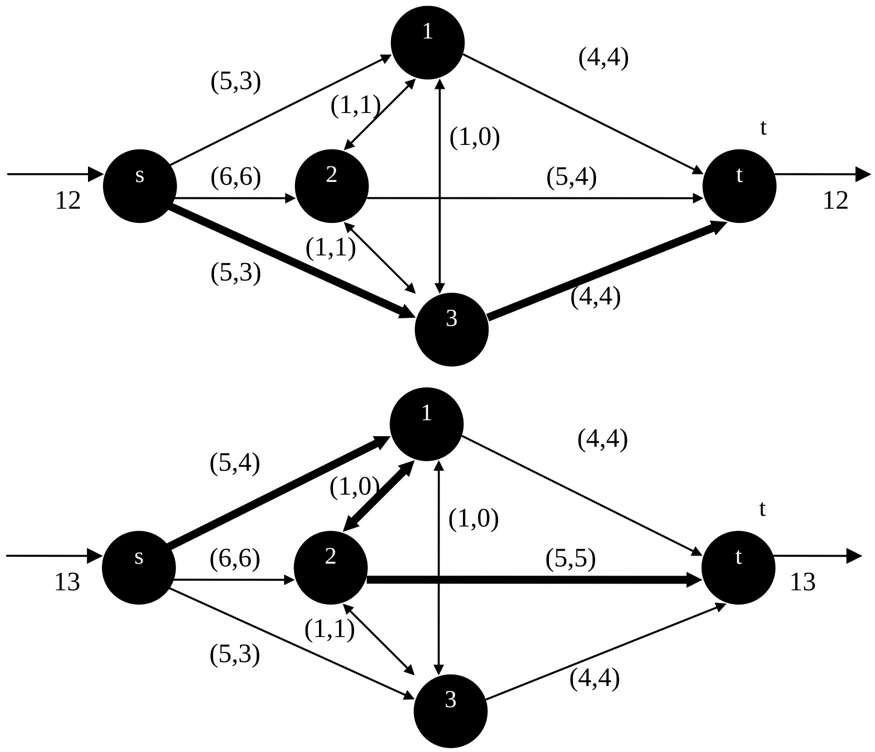


Fig. U21.18: (Continued from previous page)

In the case of Fig. U21.18, the max-flow is still 13 but the final flow pattern has Area 2 assisting Area 3.

In any case, we have determined that the system state corresponding to all components in service is not a failure state.

If each area has only a single 2-state unit (up or down) and if each transmission circuit between areas is represented as a 2-state component (up or down), then we have a total of 6 components, each with 2 possible states. Therefore the total number of (system) states to evaluate is 2^6=64 states.

To obtain the system LOLP, then, we must determine whether each state is a failure state or not. A straightforward enumeration approach would be to perform a max-flow calculation for each and every state and then add the probabilities corresponding to the states where the max-flow did not reach the total load. This is probably do-able for a 64 state system.

But let's consider a slightly more realistic situation where the installed capacity of each area is actually comprised of multiple units. We would then need to use our convolution technique to identify each capacity (or capacity outage) level for each area. In this case, the total number of system states can be very large, even for a three area system.

For example, consider characterizing our 3-area system using the data of Table U21.13. This data was generated using 100 MW generator units, each with availabilities of 0.8 (FOR=0.2). Note that it is a capacity table (rather than a capacity outage table). The data for each area in this table was generated by convolving the vector (0.8, 0.2) a number of times equal to the number of units in each area.

Table U21.13: Generation data for 3-area system

Area 1		Area 2		Area 3	
Cap	Prob	Cap	Prob	Cap	Prob
		600	.262140		
500	.32768	500	.393220	500	.32768
400	.40960	400	.245760	400	.40960
300	.20480	300	.081920	300	.20480
200	.05120	200	.015360	200	.05120
100	.00640	100	.001536	100	.00640
0	.00032	0	.000064	0	.00032

Each of the three transmission lines have availabilities of 0.99 (FOR=0.01).

Possible capacities for the various arcs in our network are given in Table U21.14. We have also identified each arc capacity with a number.

Table U21.14: Possible capacities of each arc & capacity designations

capacity designations, x_j	Possible capacities for each arc j					
	j=1	j=2	j=3	j=4	j=5	j=6
7		600				
6	500	500	500			
5	400	400	400			
4	300	300	300			
3	200	200	200			
2	100	100	100	100	100	100
1	0	0	0	0	0	0

So we can see from Table U21.14 that (read "➔" as "indicates that"):

$x_2=7$➔$C_2=600$.

$x_1=6$➔$C_1=500$; $x_2=6$➔$C_2=500$; $x_3=6$➔$C_3=500$;

$x_1=5$➔$C_1=400$; $x_2=5$➔$C_2=400$; $x_3=5$➔$C_3=400$;

$x_1=4$➔$C_1=300$; $x_2=4$➔$C_2=300$; $x_3=4$➔$C_3=300$;

$x_1=3$➔$C_1=200$; $x_2=3$➔$C_2=200$; $x_3=3$➔$C_3=200$;

$x_1=2$➔$C_1=100$; $x_2=2$➔$C_2=100$; $x_3=2$➔$C_3=100$; $x_4=2$➔$C_4=100$;
$x_5=2$➔$C_5=100$; $x_6=2$➔$C_6=100$;

$x_1=1$➔$C_1=0$; $x_2=1$➔$C_2=0$; $x_3=1$➔$C_3=0$; $x_4=1$➔$C_4=0$;
$x_5=1$➔$C_5=0$; $x_6=1$➔$C_6=0$;

We may also tabulate the cumulative probabilities, which are Pr[$X_j \le x_j$] for each arc j and each value it may take. For example,

$$\Pr[X_1 < 5] = \Pr[X_1 \le 4] = 0.2048 + 0.05120 + 0.0064 + 0.00032 = 0.2627$$

which come from Table U21.13 above.

These cumulative probabilities are given in Table U21.15 and will prove helpful (in our treatment of decomposition) in computing state probabilities.

Table U21.15: Cumulative probabilities for each capacity designation of each arc

capacity designations, x_j	Cumulative probabilities for each arc j					
	j=1	j=2	j=3	j=4	j=5	j=6
7		1.0				
6	1.0	.737860	1.0			
5	.67232	.344640	.67232			
4	.26272	.098880	.26272			
3	.05792	.016960	.05792			
2	.00672	.001600	.00672	1.0	1.0	1.0
1	.00032	.000064	.00032	.01	.01	.01

The capacity designations of Table U21.14 allow us to define the system state, $\underline{x}$ as

$$\underline{x} = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]$$

For example, the system state corresponding to maximum capacity of all elements would be

$$\underline{x} = [6 \quad 7 \quad 6 \quad 2 \quad 2 \quad 2]$$

which also happens to indicate the number of possible values for each arc, from which we can identify that there are 6×7×6×2×2×2=2016 system states.

How do we obtain the probability of a particular system state? Assuming that the capacity of the set i of elements is independent of the capacity of the set j of elements, the probability of a particular state is given by:

$$\Pr(\underline{x}) = \prod_{j=1}^{n} \Pr(x_j) \tag{U21.9}$$

For example, the probability of the state corresponding to maximum capacity $\underline{x}$=[6 7 6 2 2 2] is given by:

$$\Pr(\underline{x}) = \Pr(x_1) \times \Pr(x_2) \times \Pr(x_3) \times \Pr(x_4) \times \Pr(x_5) \times \Pr(x_6)$$
$$= 0.8^5 \times 0.8^6 \times 0.8^5 \times 0.99 \times 0.99 \times 0.99$$
$$= 0.3277 \times 0.2621 \times 0.3277 \times 0.99 \times 0.99 \times 0.99$$
$$= 0.02731149$$

This indicates it is not very likely that at any given moment, we will find all of the components up in this system! Systems with a large number of not-very reliable components are always like this (the 16 generators have availabilities of only 80%).

On the other hand, the probability of the state corresponding to minimum capacity $\underline{x}$=[1 1 1 1 1 1] is given by:

$$\Pr(\underline{x}) = \Pr(x_1) \times \Pr(x_2) \times \Pr(x_3) \times \Pr(x_4) \times \Pr(x_5) \times \Pr(x_6)$$
$$= 0.2^5 \times 0.2^6 \times 0.2^5 \times 0.01 \times 0.01 \times 0.01$$
$$= 0.00032 \times 0.000064 \times 0.00032 \times 0.01 \times 0.01 \times 0.01$$
$$= 6.5536 \times 10^{-18}$$

and so we see that it is extremely unlikely that at any given moment, we will find all of the components down in this system.

Notice that the above calculations are according to the binomial distribution, as given in Module U10, according to:

$$P_{X=r} = \Pr[X = r, n, p] = \frac{n!}{r!(n-r)!} p^r (1-p)^{(n-r)} \qquad \text{(U21.10)}$$

for r failures out of n components where each component has failure probability of p.

Just to illustrate, we use the binomial distribution to compute the probability of a general state, say, $\underline{x}$=[2 4 6 1 2 2], according to:

$$\begin{aligned} \Pr(\underline{x}) &= \Pr(x_1) \times \Pr(x_2) \times \Pr(x_3) \times \Pr(x_4) \times \Pr(x_5) \times \Pr(x_6) \\ &= 5(.2^4)(.8) \times 20(.2^3)(.8^3) \times 0.8^5 \times 0.01 \times 0.99 \times 0.99 \\ &= 0.0064 \times 0.0819 \times 0.3277 \times 0.01 \times 0.99 \times 0.99 \\ &= 1.6838 \times 10^{-6} \end{aligned}$$

Note that the convolution technique used to generate Table U21.13 is a more general way to get the individual probabilities Pr[x_j] used in the above calculations (the binomial distribution only works when all components have the same failure probability).

We define the states corresponding to the maximum and minimum capacities as $\underline{M}$ and $\underline{m}$, respectively, i.e.,

$\underline{M}$=[6 7 6 2 2 2] and $\underline{m}$=[1 1 1 1 1 1]

We may then enumerate all 2016 states from the minimum state to the maximum state as follows:

State 1: $\underline{m}$= [1 1 1 1 1 1]

State 2: [2 1 1 1 1 1]

State 3: [2 2 1 1 1 1]

…

State n: $\underline{M}$= [6 7 6 2 2 2]

It is clear that we can obtain the probability for any particular system state that we like. This fact motivates the following algorithm for computing loss of load probability.

```
LOLP=0
For i=1,n,
    Perform max flow for state i
    If failed state, LOLP=LOLP+Pr(state i)
End
```

However, it is obviously extremely computationally intensive, since we must perform a max-flow computation for every single state. We refer to this approach as "enumeration." Clearly, we need a better way!

There are 3 alternatives to enumeration, as follows:

1. Decomposition: This method treats groups of states rather than individual states by decomposing the states into sets.
2. Monte-Carlo Simulation: Here, states are sampled from the state-space and indices are computed by statistical inference.
3. Hybrid: Here, a combination of decomposition and simulation methods are employed, leading to the so-called decomposition-simulation approach.

We will discuss only the decomposition approach.

U21.7.3 The decomposition approach

KEY IDEAS FOR THIS SECTION:

- **A-sets**: A set of acceptable states can be identified as all states ***between***
  - the state where all arc capacities are set to the max flow level of a success state, and
  - the maximum state.

  This set may be identified with one max flow calculation.

- Probability of an identified set of states can be efficiently computed using cumulative probabilities for each component.

- **L-sets**: A set of failure states may be identified as all states ***between***
  - the minimum state and
  - the state where all components are at maximum capacity except one, and the capacity of that one is just below the capacity that is required for success.

  This set may be identified with one max flow calculation.

- **U-sets**: Sets not identified as acceptable or failure sets are unidentified sets. We can further decompose unidentified sets in A-sets and L-sets.

The material in this subsection is adapted from [12], which was motivated by [17].

The decomposition approach proceeds by dividing all of the states into sets of three different types, described as follows:

- Sets of acceptable states, A_k: These sets consist of states that have the load satisfied in every area.
- Sets of system loss of load states, L_k: These sets consist of states that have at least one area experiencing loss of load (also called unacceptable states).
- Sets of unclassified states, U_k: The states in these sets have not been classified into acceptable or unacceptable states.

Initially, of course, all states are unclassified and therefore are contained in Set U. The approach is to decompose this initial set into A, L, and U subsets, and then repeat the procedure on the remaining U subset until a desired level of decomposition is achieved.

Consider an unclassified set S consisting of states $\{\underline{x}_1, \underline{x}_{2,\ldots}\}$ [note that $\underline{x}_1$ denotes "state 1" whereas x_1 (without underline) denotes "the capacity designation of arc 1"] defined by a maximum state $\underline{M}$ and a minimum state $\underline{m}$ such that

$$S=\{\underline{x}_i : \underline{m} \le \underline{x}_i \le \underline{M}\} \tag{U21.11}$$

The notion of what it means for one set to be less than or equal to another is similar to the notion of "between," but we will provide some more clarification. The above means that, for all j,

- the j[th] element of $\underline{m}$ must be less than or equal to the j[th] element of $\underline{x}_i$, and
- the j[th] element of $\underline{x}_i$ must be less than or equal to the j[th] element of $\underline{M}$.

Mathematically, we say that:

$$\underline{m} \le \underline{x}_i \le \underline{M} \Rightarrow m_j \le x_{ij} \le M_j \ \forall j \tag{U21.12}$$

So if $\underline{m}$=[1 1 1 1 1 1] and $\underline{M}$=[6 7 6 2 2 2], then the set defined by S={$\underline{x_i}$: $\underline{m} \leq \underline{x_i} \leq \underline{M}$}

- would include, for example,

[1 1 1 1 1 1],

[1 1 1 1 1 2],

…,

[6 7 6 2 2 1],

[6 7 6 2 2 2]

- but would not include, for example,

[1 1 1 1 1 0] and

[6 7 5 2 2 3]

Using a simpler example, let's assume the three transmission lines are assumed to be un-failable. Then their states are fixed to "2". So if $\underline{m}$=[1 1 1 2 2 2] and $\underline{M}$=[2 2 2 2 2 2], then the set defined by S={$\underline{x_i}$: $\underline{m} \leq \underline{x_i} \leq \underline{M}$}

- would include

[1 1 1 2 2 2],

[1 1 2 2 2 2],

[1 2 1 2 2 2],

[1 2 2 2 2 2],

[2 1 1 2 2 2],

[2 1 2 2 2 2],

[2 2 1 2 2 2],

[2 2 2 2 2 2],

- but would not include, for example,

[2 2 3 2 2 2]

We observe that in specifying $S=\{\underline{x}_i : \underline{m} \le \underline{x}_i \le \underline{M}\}$, the *ordering* of the set is unimportant; rather, what is important is the satisfaction of the inequality:

$$\underline{m} \le \underline{x}_i \le \underline{M} \Rightarrow m_i \le x_{ij} \le M_i \ \forall j \qquad \text{(U21.12)}$$

i.e., that each element of $\underline{m}$ be less than or equal to each corresponding element of $\underline{M}$.

With this definition, we can describe the first step of the decomposition approach where we identify the A-set.

Identification of the A-set

Suppose we set all the arc capacities of the network model equal to the capacities in the max state and make a max flow calculation such that the max flow is equal to the sum of the area loads and is therefore an acceptable state (if the max state is unacceptable, then LOLP=1.0, i.e., there are no success states). Then the max state is obviously a success state, i.e., no area suffers loss of load.

This information can be even more useful, however, if, in this max flow solution, we have arcs for which the flows are not at maximum capacity, because, for such arcs,

- another acceptable state can be identified immediately as the one with each arc capacity reduced to the flow level of the max state max flow
- and all states between this state and the max state are also acceptable.

Let's give some notation to this idea. Let $f_j(\underline{M})$ denote the flow through arc j for this max flow condition. Then a vector $\underline{u}$ can be so defined that its j^{th} element is given by:

u_j=capacity designation of arc j that has capacity equal to the flow through arc j, $f_j(\underline{M})$.

To illustrate, recall that Figure U21.17 gives the capacities and flows corresponding to the max state and max flow for our example system, repeated below for convenience:

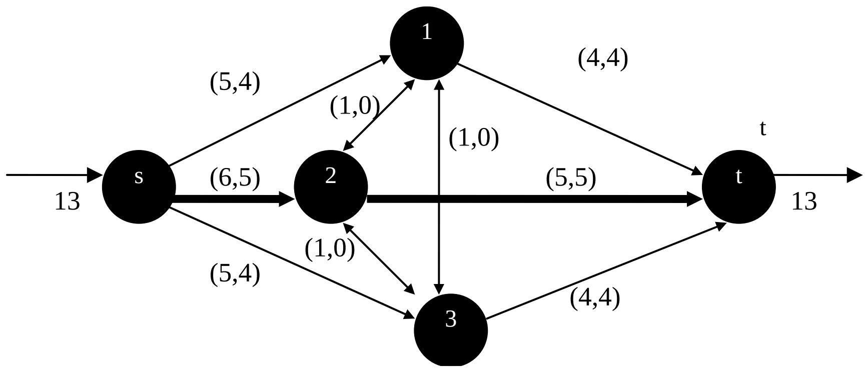


Fig. U21.19: Max flows for max state

Here, we observe that no generation or transmission arc flow is at maximum capacity, but rather $f_1[\underline{M}]=4\leq 5$, $f_2[\underline{M}]=5\leq 6$, $f_3[\underline{M}]=4\leq 5$, $f_4[\underline{M}]=0\leq 1$, $f_5[\underline{M}]=0\leq 1$, and $f_6[\underline{M}]=0\leq 1$. The capacity designations for these arcs corresponding to capacities equal to the flows are $u_1=5$, $u_2=6$, $u_3=5$, $u_4=1$, $u_5=1$, $u_6=1$, so that the vector $\underline{u}$ is:

$$\underline{u}=[5\ \ 6\ \ 5\ \ 1\ \ 1\ \ 1]$$

If a state is such that capacities of all the arcs are equal or higher than the corresponding arcs in $\underline{u}$, then that state will also be acceptable. Therefore all states between $\underline{u}$ and the max state constitute an A-set, that is,

$$A=\{\underline{x}_i : \underline{u} \leq \underline{x}_i \leq \underline{M}\} \qquad \text{(U21.13)}$$

More explicitly,

$$A=\{\underline{x}_i : [5\ \ 6\ \ 5\ \ 1\ \ 1\ \ 1] \leq \underline{x}_i \leq [6\ \ 7\ \ 6\ \ 2\ \ 2\ \ 2]\}$$

There are, for each arc, two possible capacities. Therefore, the number of acceptable states in this A-set is $2\times2\times2\times2\times2\times2=64$.

Thus, we can see that by making one max-flow calculation, we have been able to classify 64 states as acceptable. In the straightforward enumeration scheme, this would have required 64 max-flow calculations. So we made a considerable computational savings. This is good progress, addressing 64/2016=3.2% of states, but we still have 2016-64=1952 states to evaluate.

What about the total probability corresponding to this A-set. Clearly the brute-force approach is to simply compute the probability of each and every state in the set and then sum these state probabilities.

However, a simpler approach results from observing that we want to obtain the joint probability of all six events designated by $u_j \leq x_{ij} \leq M_j$ for j=1,…,6. If we assume these are independent events (which is reasonable for components that fail independently), then we have:

$$\Pr[A]=\Pr[(u_1\leq x_{i1}\leq M_1)\cap(u_2\leq x_{i2}\leq M_2)\cap(u_3\leq x_{i3}\leq M_3)$$
$$\cap(u_4\leq x_{i4}\leq M_4)\cap (u_5\leq x_{i5}\leq M_5)\cap(u_6\leq x_{i6}\leq M_6)]$$
$$=\Pr[u_1\leq x_{i1}\leq M_1]\times\Pr[u_2\leq x_{i2}\leq M_2]\times\Pr[u_3\leq x_{i3}\leq M_3]\times\Pr[u_4\leq x_{i4}\leq M_4]$$
$$\times\Pr[u_5\leq x_{i5}\leq M_5]\times\Pr[u_6\leq x_{i6}\leq M_6]=\prod_{j=1}^{6}\Pr[u_j \leq x_{ij} \leq M_j] \qquad \text{(U21.14)}$$

Now what is $\Pr[u_j\leq x_{ij}\leq M_j]$? This is nothing more than

$$\Pr[u_j\leq x_{ij}\leq M_j]=\Pr[(x_{ij}=u_j)\cup\ldots\cup(x_{ij}=M_j)]=\Pr[x_{ij}=u_j]+\ldots+\Pr[x_{ij}=M_j] \qquad \text{(U21.15)}$$

Substitution of (U21.15) into (U21.14) yields:

$$\Pr[A]=\prod_{j=1}^{6}\Pr[u_j \leq x_{ij} \leq M_j]$$
$$=\prod_{j=1}^{6}\left\{\Pr[x_{ij}=u_j]+\ldots+\Pr[x_{ij}=M_j]\right\} \qquad \text{(U21.16)}$$

Thus, we see that to compute the probability of the A-set (or any specified range of sets), we first calculate the sum of probabilities of all states between the max and min state for a given arc. Then we multiply these cumulated arc probabilities to find the set probability. In practice, the sum can be found more readily by taking the difference in the cumulative probabilities, i.e.,

$$\Pr[A] = \prod_{j=1}^{6} \left\{ \Pr[x_{ij} \le M_j] - \Pr[x_{ij} \le (u_j - 1)] \right\} \qquad \text{(U21.17)}$$

For our example problem, we have 6 arcs, so we must compute 6 cumulated probabilities. Recalling that the A-set is specified by

$$A=\{\underline{x}_j : [5\ 6\ 5\ 1\ 1\ 1] \le \underline{x}_j \le [6\ 7\ 6\ 2\ 2\ 2]\}$$

we see that:

- The arc 1 cumulated probability is given by:

  $$\Pr[x_{i1}=5]+\Pr[x_{i1}=6]$$

  which are the probabilities of having 1 or 0 Area 1 gen-units out of service, respectively. Recall Table U21.15, repeated here for convenience.

capacity designations, x_j	Cumulative probabilities for each arc j					
	j=1	j=2	j=3	j=4	j=5	j=6
7		1.0				
6	1.0	.737860	1.0			
5	.67232	.344640	.67232			
4	.26272	.098880	.26272			
3	.05792	.016960	.05792			
2	.00672	.001600	.00672	1.0	1.0	1.0
1	.00032	.000064	.00032	.01	.01	.01

Using cumulative probabilities results in (from Table U21.15)

$$\Pr[x_{i1} \le 6] - \Pr[x_{i1} \le 4] = 1.0 - .26272 = .73728$$

(Notice $\Pr[x_{i1} \le 6] - \Pr[x_{i1} \le 4] = \Pr[x_{i1}=6,5,4,3,2,1] - \Pr[x_{i1}=4,3,2,1]$ $= \Pr[x_{i1}=6,5]$).

Once again, recalling that the A-set is specified by

$$A = \{\underline{x}_j : [5\ \ 6\ \ 5\ \ 1\ \ 1\ \ 1] \le \underline{x}_j \le [6\ \ 7\ \ 6\ \ 2\ \ 2\ \ 2]\}$$

then…

- The arc 2 probabilities are:

  $$\Pr[x_{i2}=6] + \Pr[x_{i2}=7] = \Pr[x_{i2} \le 7] - \Pr[x_{i2} \le 5] = 1.0 - .34464 = .65536$$

- The arc 3 probabilities are:

  $$\Pr[x_{i3}=5] + \Pr[x_{i3}=6] = \Pr[x_{i3} \le 6] - \Pr[x_{i3} \le 4] = 1.0 - .26272 = .73728$$

- The arc 4 probabilities are:

  $$\Pr[x_{i4}=1] + \Pr[x_{i4}=2] = \Pr[x_{i4} \le 2] - \Pr[x_{i4} \le 0] = 1.0 - 0 = 1.0$$

- The arc 5 probabilities are:

  $$\Pr[x_{i5}=1] + \Pr[x_{i5}=2] = \Pr[x_{i5} \le 2] - \Pr[x_{i5} \le 0] = 1.0 - 0 = 1.0$$

- The arc 6 probabilities are:

  $$\Pr[x_{i6}=1] + \Pr[x_{i6}=2] = \Pr[x_{i6} \le 2] - \Pr[x_{i6} \le 0] = 1.0 - 0 = 1.0$$

The probability of the A-set then becomes:

$.73728 \times .65536 \times .73728 \times 1.0 \times 1.0 \times 1.0 = .356242$

With 1 max-flow calculation, although we only addressed 3.2% of states, we obtained knowledge of 35.6% of the probability space!

Identification of the L-set

1. Essential idea

Recall that an L-set is a set of failure states.

The essential idea for identifying L-sets is as follows. For any particular component j, it may be possible to identify a capacity v_j for which any lower capacity $x_{ij} < v_j$ necessarily results in loss of

Attempt to tell you how we will proceed from here…we will approach the identification of the L-set in the following way:

1. Essential idea
2. What to do with it once you have it: probability calculation
3. What to do with it once you have it: overlapping sets.
4. How to find an L-set? Concept
5. How to find an L-set? Algorithm
6. How to find an L-set? Picture.
7. How to find an L-set? Underlying rationale.
8. Example.

2. What to do with it once you have it: probability calculation.

load, independent of the capacities of other arcs. If this is the case, then all states with $x_{ij}<v_j$ are members of the L-set.

Therefore, if $\underline{m}$ and $\underline{M}$ are the minimum and maximum states, and if we can find v_1 for component 1, then

$$L_1 = \begin{cases} v_1 - 1 & M_2 & M_3 & \cdots & M_n \\ m_1 & m_2 & m_3 & \cdots & m_n \end{cases} = \begin{cases} \overline{V}_1 \\ \underline{V}_1 \end{cases} \qquad \text{(U21.18)}$$

where the L_1 set is comprised of all states between the lower state, denoted $\underline{V}_1$, and the upper state, denoted $\overline{V}_1$, i.e.,

$$L_1=\{\underline{x}_i : \underline{V}_1 \le \underline{x}_i \le \overline{V}_1 \} \qquad \text{(U21.19)}$$

The significance of this L_1 set is, if arc 1 is between m_1 and v_1-1 (inclusive), we have loss of load irrespective of other arc values.

The probability of this L-set, $\Pr[L_1]$, is computed in the same way that we computed the probability of the A-set from (U21.17), i.e.,

$$\Pr[L_1] = \prod_{j=1}^{n} \left\{ \Pr[x_{ij} \le \overline{V}_{1j}] - \Pr[x_{ij} \le (\underline{V}_{1j} - 1)] \right\} \qquad \text{(U21.20)}$$

Expanded, (U21.20) becomes,

$$\begin{aligned} \Pr[L_1] = &\left\{ \Pr[x_{i1} \le (v_1 - 1)] - \Pr[x_{i1} \le (m_1 - 1)] \right\} \\ &\times \left\{ \Pr[x_{i2} \le M_2] - \Pr[x_{i2} \le (m_2 - 1)] \right\} \\ &\times \left\{ \Pr[x_{i3} \le M_3] - \Pr[x_{i3} \le (m_3 - 1)] \right\} \\ &\vdots \\ &\times \left\{ \Pr[x_{in} \le M_n] - \Pr[x_{in} \le (m_n - 1)] \right\} \end{aligned} \qquad \text{(U21.21)}$$

The 2^{nd} probability in each term is 0 since no state may be $<m_j$.

A similar idea holds for the other arcs as well. For example, we need to find v_2 such that there is no failure but if the system goes to the next lower state v_2-1, there will be failure (loss of load). This also identifies an L-set.

$$L'_2 = \begin{cases} M_1 & v_2 - 1 & M_3 & \cdots & M_n \\ m_1 & m_2 & m_3 & \cdots & m_n \end{cases} \quad \text{(U21.22)}$$

3. What to do with it once you have it: overlapping sets.

Now this is a legitimate L-set, i.e., all states are loss of load states. However, there is overlap between L_1 and L'_2 that include the following states:

$$L_2 \cap L'_2 = \begin{cases} v_1 - 1 & v_2 - 1 & M_3 & \cdots & M_n \\ m_1 & m_2 & m_3 & \cdots & m_n \end{cases} \quad \text{(U21.23)}$$

It is easier to compute probabilities that we can use in the final LOLP calculation if we maintain disjoint (nonintersecting or nonoverlapping) L-sets. Therefore, we define the second L-set as:

$$L_2 = \begin{cases} M_1 & v_2 - 1 & M_3 & \cdots & M_n \\ v_1 & m_2 & m_3 & \cdots & m_n \end{cases} = \begin{cases} \overline{V}_2 \\ \underline{V}_2 \end{cases} \quad \text{(U21.24)}$$

where it is clear that component j=1 is constrained to take on only values that are outside of the L_1 set. Therefore, this set is comprised of failed states that are not included in L_1.

The probability of this set is given similar to (U21.20):

$$\Pr[L_2] = \prod_{j=1}^{n} \left\{ \Pr[x_j \le \overline{V}_{2j}] - \Pr[x_j \le (\underline{V}_{2j} - 1)] \right\} \quad \text{(U21.25)}$$

Similarly, the third L-set is given by:

$$L_3 = \begin{cases} M_1 & M_2 & v_3 - 1 & \cdots & M_n \\ v_1 & v_2 & m_3 & \cdots & m_n \end{cases} = \begin{cases} \overline{V}_3 \\ \underline{V}_3 \end{cases} \quad \text{(U21.26)}$$

and its probability given by

$$\Pr[L_3] = \prod_{j=1}^{n} \left\{ \Pr[x_j \le \overline{V}_{3j}] - \Pr[x_j \le (\underline{V}_{3j} - 1)] \right\} \quad \text{(U21.27)}$$

In general, if there are n arcs, there will be n L-sets generated, and the kth set is given by:

$$L_k = \begin{cases} M_1 & M_2 & M_3 & \cdots & M_{k-1} & v_k - 1 & M_{k+1} & \cdots & M_n \\ v_1 & v_2 & v_3 & \cdots & v_{k-1} & m_k & m_{k+1} & \cdots & m_n \end{cases} = \begin{cases} \overline{V}_k \\ \underline{V}_k \end{cases} \quad \text{(U21.28)}$$

An important question is, at this point:

4. How to find an L-set? ➔Concept.

> How to find v_k, the capacity of the kth component such that there is no failure (loss of load) but if the system goes to the next lower state v_k-1, there will be failure (loss of load)?

The obvious approach is, beginning with the maximum state $\underline{M}$, decrease the kth component capacity by 1, and run the max flow to see if the tested state is a failure state. If we repeat this over and over, we are guaranteed to identify v_k, or, alternatively, to identify that changes to the kth component's capacity cannot cause system failure. However, the computational cost of doing so is significant, since it requires that we run a max flow for every tested capacity of the kth component's arc.

Another method that would decrease this computational cost would be the so-called bisection approach where our first capacity tested is halfway between M_k and m_k. If it is a failed state, then we test the one halfway between it and M_k. If it is not a failed state, then we test the one halfway between it and m_k, continuing in this manner until we identify v_k or until we identify that changes to the kth component's capacity cannot cause system failure.

Yet, there is a better method that only requires a single max flow. It is based on the following premise.

Let arc k be connected from node i to node j.

- If, in the max flow calculation of the maximum state, arc k carries flow f_k to its terminating node j, and
- if, without arc k, the network has e_k residual capacity to carry flow from the source to that terminating node j,

- then the state with greatest capacity of arc k (and all other arcs at maximum capacity) that is a failure state is when arc k capacity is decreased by more than e_k.

The implication is that if the flow in arc k is reduced by e_k, this much flow can be sent through the unused capacity of the remaining network without having system loss of load. Thus, v_k (which indicates the capacity just higher than the capacity necessary for failure) corresponds to the state with capacity equal to ~~or just greater than~~ f_k-e_k.

This algorithm begins from the U set being decomposed. At the beginning, this "U-set" is the full set, and the "maximum capacities of the U set being decomposed" is M.

5. How to find an L-set? Algorithm.

We summarize the steps for identifying v_k as follows. Assume that arc k connects node i to node j.

1. Set all states of the network to the maximum capacities of the U set being decomposed (initially, this would be M).
2. Find the max flow using the max flow algorithm.
3. If the max flow found is less than the total demand, then there is loss of load in at least one area; thus the entire set U is an L set.
4. Identify v_k as follows:

As we have seen, the max flow solution is not unique. This means there is not a unique decomposition process, i.e., each max-flow solution can generate its own series of A, L, and U sets. I believe the final result will always be a unique set of A, L, and U sets. However, I have not proved this, nor have I been able to find a proof for this in the literature, though I have not tried/looked too hard.

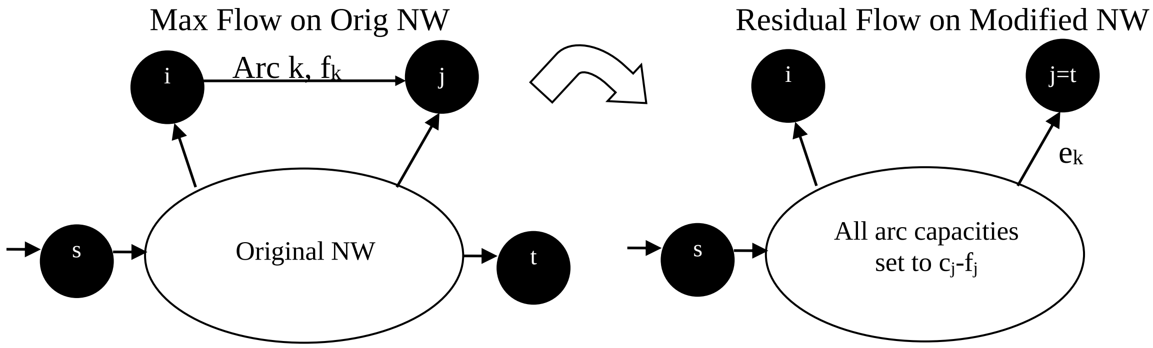
   a. Remove the k^{th} arc. Retain(preserve) flows found in step 2 on all other arcs by letting new capacities of all remaining arcs equal to their original capacities less their flows. In other words, let new arc capacities be the capacity remaining of each arc in the step 2 max flow.

   b. Find the maximum additional flow from node s to node j, or, equivalently, find the maximum flow from node s to node j with all arcs at their new capacities. This is done by simply identifying node j as the sink node and running the max-flow algorithm. Denote the maximum additional flow from node s to node j as e_k.

   c. Identify the desired component k capacity designation such that the corresponding capacity is equal to ~~or just greater than~~ f_k-e_k. This is v_k. Failure occurs for any component k capacity designation less than v_k.

6. How to find an L-set? Picture.

The picture below illustrates (arc k is really internal to the bubble).



7. How to find an L-set? Underlying rationale.

The reason this works is as follows:

- If, with arc k in the network, the max flow from s to t is F, with f_k on arc k, and this is a success (all load arcs to t at capacity),
- and if the max additional flow the network is capable of providing from s to j is e_k (established by our max-flow s to j calculation with arc k out and other arc capacities adjusted),
- then reducing arc k capacity below f_k-e_k must result in a max flow less than F (which is therefore a failed state), because the arc k capacity is below f_k-e_k, yet there is only e_k capacity in the rest of the network.

Since a max flow is not unique, is it possible that reducing arc k capacity below f_k-e_k might still result in a max flow of F because the max flow algorithm discovers a different flow pattern? The answer is "no" but need to give explanation/proof.

8. Example.

To illustrate using an example, recall that Figure U21.20 gives the capacities and flows corresponding to the max state for our example system, repeated below for convenience:

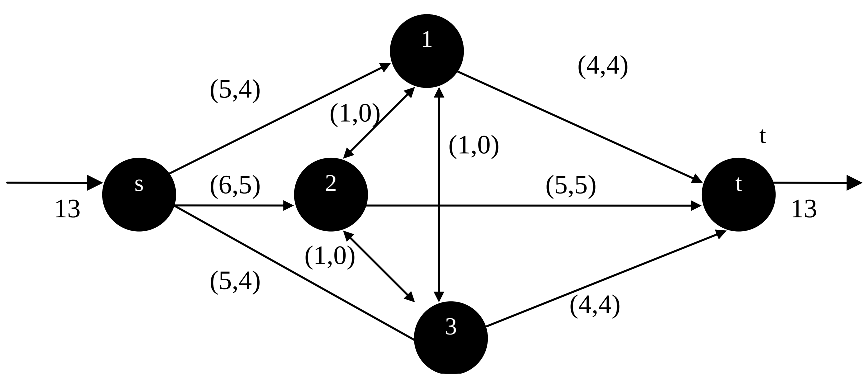


Fig. U21.20: Max flows for max state

Also, recall that $\underline{m}$=[1 1 1 1 1 1] and $\underline{M}$=[6 7 6 2 2 2]. Thus, by (U21.18), L_1 is given by:

$$L_1 = \begin{cases} v_1 - 1 & M_2 & M_3 & \cdots & M_n \\ m_1 & m_2 & m_3 & \cdots & m_n \end{cases} = \begin{cases} \overline{V}_1 \\ \underline{V}_1 \end{cases}$$
$$= \begin{cases} v_1 - 1 & 7 & 6 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{cases} \quad \text{(U21.29)}$$

Applying step 4 to find v_1, we remove arc 1 (generation for Area 1). Fig. U21.21 shows the network with arc 1 removed and other flows as in the solution to the max-state max-flow problem given in Fig. U21.20.

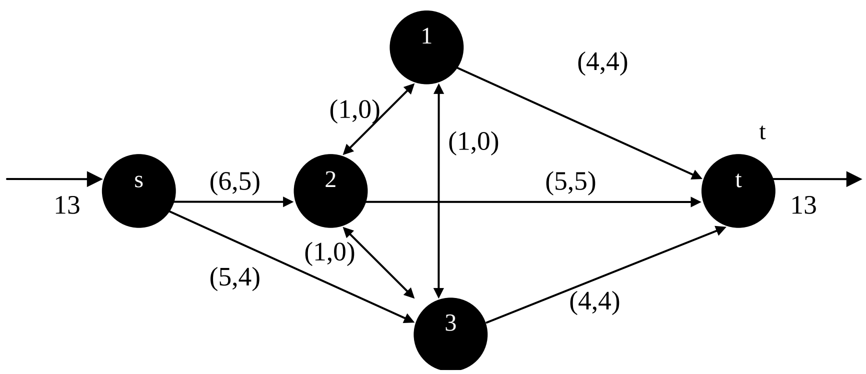


Fig. U21.21: Max flows for max state with arc 1 removed

Fig. U21.22 shows the arc capacity values adjusted to the new values corresponding to the difference between the flows in the max-flow solution and the old capacities, i.e., the residual capacities. Note that all load arcs have zero capacity, and node 1 (the terminating node for arc 1) is now modeled as the sink node.

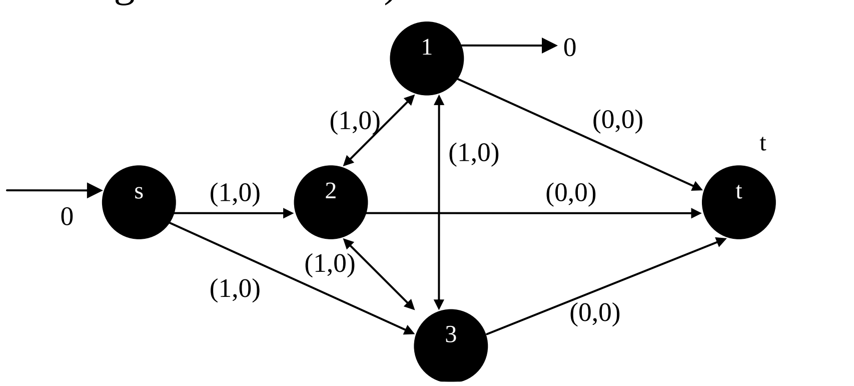


Fig. U21.22: Network with arc 1 removed using residual capacities

Applying the max-flow algorithm to the network of Fig. U21.22, we obtain the flows indicated in Fig. U21.23.

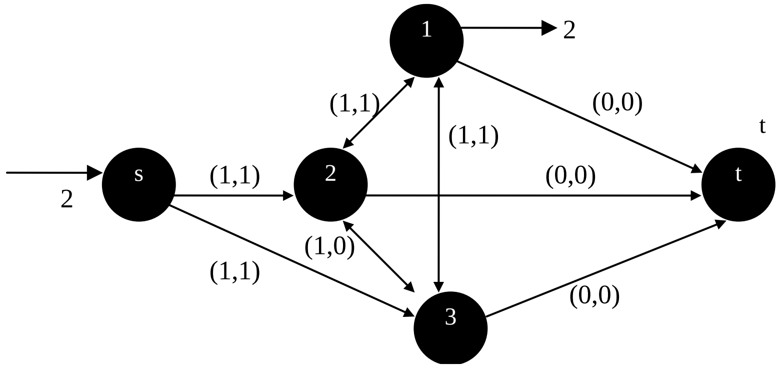


Fig. U21.23: Network with arc 1 removed using residual capacities

Thus, we see that the residual capacity in the network is e_1=2. Since from Fig. U21.20, the arc 1 flow in the max-state max flow condition was f_1=4, we can conclude that with arc 1 capacity at f_1-e_1=4-2=2, the state will be *just* acceptable (any additional capacity decrease in arc 1 will result in a failure state). This means that v_1 corresponds to the arc 1 capacity of 2 (or 200 MW). Reference to Table U21.14, repeated below for convenience,

Table U21.14: Possible capacities of each arc & capacity designations

capacity designations, x_j	Possible capacities for each arc j					
	j=1	j=2	j=3	j=4	j=5	j=6
7		600				
6	500	500	500			
5	400	400	400			
4	300	300	300			
3	200	200	200			
2	100	100	100	100	100	100
1	0	0	0	0	0	0

indicates that the 200 MW capacity designation for arc 1 is 3, thus, v_1=3. Therefore, (U21.18), repeated here for convenience,

$$L_1 = \begin{cases} v_1 - 1 & M_2 & M_3 & \cdots & M_n \\ m_1 & m_2 & m_3 & \cdots & m_n \end{cases} = \begin{cases} \overline{V}_1 \\ \underline{V}_1 \end{cases} \quad \text{(U21.18)}$$

is, in this case,

$$L_1 = \begin{cases} 2 & 7 & 6 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{V}_1 \\ \underline{V}_1 \end{cases} \quad \text{(U21.30)}$$

This is 2×7×6×2×2×2=672 states, which is 672/2016=33% of the states.

Similarly, we may repeat the step 4 procedure for arcs 2 and 3, obtaining e_2=2 and e_3=2. It is unnecessary to repeat the procedure for arcs 4, 5, and 6 (the transmission arcs) since their flows in the max-state max flow are zero, implying that it is not possible to change their capacity in a way that will cause a failure state. Effectively, this means that the L-sets for these arcs, L_4, L_5, and L_6, are empty.

Since from Fig. U21.20, the arcs 2 and 3 flows in the max-state max flow condition were f_2=5 and f_3=4, we can conclude that with arcs 2 and 3 capacities at f_2-e_2=5-2=3 and f_3-e_3=4-2=2 in their respective states, these states will be *just* acceptable (any additional capacity decrease in arc 2 in its state or in arc 3 in its state will result in a failure state). This means that v_2 corresponds to the arc 2 capacity of 3 (or 300 MW) and v_3 corresponds to the arc 3 capacity of 2 (or 200 MW).

Reference to Table U21.14 indicates that the 300 MW capacity designation for arc 2 is 4, thus, v_2=4. Therefore, (U21.21) is

$$L_2 = \begin{cases} 6 & 3 & 6 & 2 & 2 & 2 \\ 3 & 1 & 1 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{V}_2 \\ \underline{V}_2 \end{cases} \quad \text{(U21.31)}$$

This is 4×3×6×2×2×2=576 states or 576/2016=28.6% of the states.

Note that the element corresponding to the component 1 in the lower state was set to v_1 so as to maintain disjoint sets.

Reference to Table U21.14 indicates that the 200 MW capacity designation for arc 3 is 3, thus v_3=3. Therefore, (U21.26) is

$$L_3 = \begin{cases} 6 & 7 & 2 & 2 & 2 & 2 \\ 3 & 4 & 1 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{V}_3 \\ \underline{V}_3 \end{cases} \qquad \text{(U21.32)}$$

This is 4×4×2×2×2×2=256 states or 256/2016=12.7% of the states.

Now compute the probabilities. The probability of the L_1 set is given by (U21.20) or (U21.21). Using the appropriate cumulative probabilities from Table U21.15, we find:

$$\Pr[L_1] = \prod_{j=1}^{6} \left\{ \Pr[x_j \le \overline{V}_{1j}] - \Pr[x_j \le (\underline{V}_{1j} - 1)] \right\}$$

$$= \left\{ \Pr[x_1 \le 2] - \Pr[x_1 \le 0] \right\}$$
$$\times \left\{ \Pr[x_2 \le 7] - \Pr[x_2 \le 0] \right\}$$
$$\times \left\{ \Pr[x_3 \le 6] - \Pr[x_3 \le 0] \right\}$$
$$\times \left\{ \Pr[x_4 \le 2] - \Pr[x_4 \le 0] \right\}$$
$$\times \left\{ \Pr[x_5 \le 2] - \Pr[x_5 \le 0] \right\}$$
$$\times \left\{ \Pr[x_6 \le 2] - \Pr[x_6 \le 0] \right\}$$

$$= \{.00672 - 0\} \times \{1 - 0\} \times \{1 - 0\} \times \{1 - 0\} \times \{1 - 0\} \times \{1 - 0\}$$

$$= .00672$$

The probability of the L_2 set is given by (U21.22). Using the appropriate cumulative probabilities from Table U21.15, we find:

$$\Pr[L_2] = \prod_{j=1}^{n} \left\{ \Pr[x_j \le \overline{V}_{2j}] - \Pr[x_j \le (\underline{V}_{2j} - 1)] \right\}$$

$$
\begin{aligned}
&= \left\{\Pr[x_1 \le 6] - \Pr[x_1 \le 2]\right\} \\
&\times \left\{\Pr[x_2 \le 3] - \Pr[x_2 \le 0]\right\} \\
&\times \left\{\Pr[x_3 \le 6] - \Pr[x_3 \le 0]\right\} \\
&\times \left\{\Pr[x_4 \le 2] - \Pr[x_4 \le 0]\right\} \\
&\times \left\{\Pr[x_5 \le 2] - \Pr[x_5 \le 0]\right\} \\
&\times \left\{\Pr[x_6 \le 2] - \Pr[x_6 \le 0]\right\}
\end{aligned}
$$

$$= \{1 - .00672\} \times \{.01696 - 0\} \times \{1 - 0\} \times \{1 - 0\} \times \{1 - 0\} \times \{1 - 0\}$$

$$= .016846$$

The probability of the L_3 set is given by (U21.27). Using the appropriate cumulative probabilities from Table U21.15, we find:

$$\Pr[L_3] = \prod_{j=1}^{n} \left\{\Pr[x_j \le \overline{V}_{3j}] - \Pr[x_j \le (\underline{V}_{3j} - 1)]\right\}$$

$$
\begin{aligned}
&= \left\{\Pr[x_1 \le 6] - \Pr[x_1 \le 2]\right\} \\
&\times \left\{\Pr[x_2 \le 7] - \Pr[x_2 \le 3]\right\} \\
&\times \left\{\Pr[x_3 \le 2] - \Pr[x_3 \le 0]\right\} \\
&\times \left\{\Pr[x_4 \le 2] - \Pr[x_4 \le 0]\right\} \\
&\times \left\{\Pr[x_5 \le 2] - \Pr[x_5 \le 0]\right\} \\
&\times \left\{\Pr[x_6 \le 2] - \Pr[x_6 \le 0]\right\}
\end{aligned}
$$

$$= \{1 - .00672\} \times \{1 - .01696\} \times \{.00672 - 0\} \times \{1 - 0\} \times \{1 - 0\} \times \{1 - 0\}$$

$$= .00656164$$

The total probability of the L-sets is given by

$\Pr[L]=\Pr[L_1]+\Pr[L_2]+\Pr[L_3]=.00672+.016846+.00656164$

$=.03012764$

If we had no unclassified states, this would be the LOLP. However, the number of states identified so far for A, L1, L2, L3 is 64, 672, 576, 256, respectively, for total of 1568 (1568/2016= 77.8%), so there are 2016-1568=448 remaining; we need to check unclassified states. This is important because it provides us with an accuracy indication of using the total probability of the L-sets that have been identified so far as the LOLP.

Recall the probabilities of the various sets A, L1, L2, and L3 are 0.356242, 0.00672, 0.016846, and 0.00656164, respectively, for a total probability of 0.38636964, and so, although there are only 448 states unclassified, those unclassified states comprise 1-0.38636964=0.61363036 of the probability.

Identification of the U-set

We need to determine which states are unclassified. Recall that we have identified:

- Acceptable (A) states as those between $\underline{u}$ and $\underline{M}$. The basic criterion here, for an acceptable state, is to say that ***all*** *arcs must be equal to or above their u-state capacity* (*the u-state is the one with each arc capacity reduced to the flow level of the max state max flow).*
- Loss of load (L) states as those between $\underline{V}_j$ and $\overline{V}_j$ for j=1,…,n. The basic criterion here, for a loss of load state, is to say that ***at least one*** *arc must be below its v-state capacity (the v-state is the one with one arc k capacity equal to f_j-e_j, which is the amount just higher than the capacity necessary for failure).*

Thus, we can say that a state remains unclassified if:

- ***At least one*** *arc is below its u-state capacity* and
- ***All*** *arcs are equal to or above their v-state capacity*.

With these criteria, we may immediately write down a set of unclassified states as:

$$U_1 = \begin{cases} u_1 - 1 & M_2 & M_3 & \cdots & M_n \\ v_1 & v_2 & v_3 & \cdots & v_n \end{cases} = \begin{cases} \overline{U}_1 \\ \underline{U}_1 \end{cases} \qquad \text{(U21.33)}$$

Similarly, we may write down another one as

$$U'_2 = \begin{cases} M_1 & u_2 - 1 & M_3 & \cdots & M_n \\ v_1 & v_2 & v_3 & \cdots & v_n \end{cases} \qquad \text{(U21.34)}$$

However, U'_2 would not be disjoint with U_1, i.e., the following states would be included in both sets:

$$U_1 \cap U'_2 = \begin{cases} u_1 - 1 & u_2 - 1 & M_3 & \cdots & M_n \\ v_1 & v_2 & v_3 & \cdots & v_n \end{cases} \qquad \text{(U21.35)}$$

As indicated in identifying L-states, this would create difficulties in computing the total probability of the unclassified states. Therefore, the second U-state should be:

$$U_2 = \begin{cases} M_1 & u_2 - 1 & M_3 & \cdots & M_n \\ u_1 & v_2 & v_3 & \cdots & v_n \end{cases} = \begin{cases} \overline{U}_2 \\ \underline{U}_2 \end{cases} \qquad \text{(U21.36)}$$

The third U-state is:

$$U_3 = \begin{cases} M_1 & M_2 & u_3 - 1 & \cdots & M_n \\ u_1 & u_2 & v_3 & \cdots & v_n \end{cases} = \begin{cases} \overline{U}_3 \\ \underline{U}_3 \end{cases} \qquad \text{(U21.37)}$$

In general, there will be n U-sets generated, with the k^{th} U-set given by:

$$U_k = \begin{cases} M_1 & M_2 & M_3 & \cdots & M_{k-1} & u_k - 1 & M_{k+1} & \cdots & M_n \\ u_1 & u_2 & u_3 & \cdots & u_{k-1} & v_k & v_{k+1} & \cdots & v_n \end{cases} = \begin{cases} \overline{U}_k \\ \underline{U}_k \end{cases} \qquad \text{(U21.38)}$$

The probabilities are computed as usual.

For our example, we have:

$$U_1 = \begin{cases} 4 & 7 & 6 & 2 & 2 & 2 \\ 3 & 4 & 3 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{U}_1 \\ \underline{U}_1 \end{cases}$$

$$U_2 = \begin{cases} 6 & 5 & 6 & 2 & 2 & 2 \\ 5 & 4 & 3 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{U}_2 \\ \underline{U}_2 \end{cases}$$

$$U_3 = \begin{cases} 6 & 7 & 4 & 2 & 2 & 2 \\ 5 & 6 & 3 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{U}_3 \\ \underline{U}_3 \end{cases}$$

Total unclassified sets is

U1: 2×4×4×2×2×2=256 states. U2: 2×2×4×2×2×2=128 states.

U3: 2×2×2×2×2×2=64 states. Total is 448 states, which is the same the number of remaining states we obtained from adding up all the A-set states and the L-set states.

ASIDE

Here is a good question I once received:

> "I attempted a problem, however, I don't feel that I understand conceptually what I am being asked to compute. Could you talk a little more about the analysis method that you went through on Monday today in class? I think my question boils down to:
>
> if L1 = [2 7 6 2 2 2; 1 1 1 1 1 1] then
>
> why is U1 = [4 7 6 2 2 2; 3 4 3 1 1 1]?
>
> Shouldn't anything above [2 7 6 2 2 2] be considered **acceptable**
>
> such as [3 7 6 2 2 2] and [4 7 6 2 2 2]?"

SHORT ANSWER: L1 guarantees everything "between" [1 1 1 1 1 1] and [2 7 6 2 2 2] is failed, but it says nothing about something "above" [2 7 6 2 2 2] such as [3 7 6 2 2 2] and [4 7 6 2 2 2].

The only guarantee on acceptable states comes from the A-set, which is

$A=\{\underline{x}_j : \underline{u} \le \underline{x}_j \le \underline{M}\}=\{\underline{x}_j : [5\ \ 6\ \ 5\ \ 1\ \ 1\ \ 1] \le \underline{x}_j \le [6\ \ 7\ \ 6\ \ 2\ \ 2\ \ 2]\}$

States "between" L-set upper bounds and A-set lower-bounds are unclassified.

LONG ANSWER:

How did we find u-state? By setting capacities to the flows in the max flow solution!

Remember that the criteria for an unclassified state includes:

- ***At least one** arc is below its u-state capacity*

A single vector gives the u-state capacities, from the A-set specification:

$A=\{\underline{x}_j : \underline{u} \le \underline{x}_j \le \underline{M}\}=\{\underline{x}_j : \mathbf{[5\ \ 6\ \ 5\ \ 1\ \ 1\ \ 1]} \le \underline{x}_j \le [6\ \ 7\ \ 6\ \ 2\ \ 2\ \ 2]\}$

If just one element is below its u-state capacity, we cannot be certain the state is acceptable. The u-state capacity that the first unclassified set needs to be below is u_1=5, i.e., u_1-1=4. The remaining elements can be anything, because if element 1 is 4 or below, then it cannot be guaranteed to be within the acceptable set. Thus:

$$U_1 = \begin{cases} u_1-1 & M_2 & M_3 & \cdots & M_n \\ v_1 & v_2 & v_3 & \cdots & v_n \end{cases} = \begin{cases} \overline{U}_1 \\ \underline{U}_1 \end{cases} = \begin{cases} 4 & 7 & 6 & 2 & 2 & 2 \\ v_1 & v_2 & v_3 & \cdots & v_n \end{cases}$$

How did we find each component's v-state capacity? By lowering each component's capacity (with other components in max-state capacity) until we failed the system, then the v-state capacity is the next capacity up.

The criteria for an unclassified state also includes:

- ***All** arcs are equal to or above their v-state capacity*.

The v-state capacity for element j, v_j, is found from L_j, which will have v_j-1 in column j. Thus, we inspect L_1, L_2, and L_3 below (notice underlined numbers), and observe that

$$L_1 = \begin{cases} \underline{2} & 7 & 6 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{V}_1 \\ \underline{V}_1 \end{cases} \quad v_1-1=2 \Rightarrow v_1=3;$$

$$L_2 = \begin{cases} 6 & \underline{3} & 6 & 2 & 2 & 2 \\ 3 & 1 & 1 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{V}_2 \\ \underline{V}_2 \end{cases} \quad v_2-1=3 \Rightarrow v_2=4;$$

$$L_3 = \begin{cases} 6 & 7 & \underline{2} & 2 & 2 & 2 \\ 3 & 4 & 1 & 1 & 1 & 1 \end{cases} = \begin{cases} \overline{V}_3 \\ \underline{V}_3 \end{cases} \quad v_3-1=2 \Rightarrow v_3=3;$$

If all elements are above their v-state capacity, we cannot be certain that the state is failed. Since it is not possible to limit the transmission lines enough to definitely fail the system (independent of the other element states), we consider their v-states to be their minimum states 1. Therefore:

$$U_1 = \begin{cases} u_1-1 & M_2 & M_3 & \cdots & M_n \\ v_1 & v_2 & v_3 & \cdots & v_n \end{cases} = \begin{cases} \overline{U}_1 \\ \underline{U}_1 \end{cases} = \begin{cases} 4 & 7 & 6 & 2 & 2 & 2 \\ 3 & 4 & 3 & 1 & 1 & 1 \end{cases}$$

END ASIDE

Note that U_4, U_5, and U_6 would have all three arcs 1, 2, and 3 above or equal to their u-capacities of u_1, u_2, and u_3, respectively, according to (U21.34). Any states for which this is true have already been classified as acceptable, since the max-state max flow indicated 0 flow on arcs 4, 5, and 6. Therefore, U_4, U_5, and U_6 are empty sets and we can obtain Pr[U] as the sum of probabilities of U_1, U_2, and U_3. The appropriate calculations are given below:

$Pr(U_1)=(.26272-.00672)\times(1-.01696)\times(1-.00672)\times1\times1\times1=.24997$

$Pr(U_2)=(1-.26272)\times(.34464-.01696)\times(1-.00672)\times1\times1\times1=.23997$

$Pr(U_3)=(1-.26272)\times(1-.34464)\times(.26272-.00672)\times1\times1\times1=.1237$

$Pr(U)=Pr(U_1)+Pr(U_2)+Pr(U_3)=.61337$

This completes the first stage of decomposition. At this stage, we know that LOLP≥P(L)=.03012764.

It is of interest to note the sum of probabilities for the three identified sets, i.e.,

$Pr(A)+Pr(L)+Pr(U)=.356242+.03012764+.61337=.99974$

This probability should be 1.0, as all states have been classified; the small difference can be attributed to round-off error.

However, we see that Pr(U) is quite large. This indicates that we need to do some more work by decomposing the identified U-sets into their resulting A-sets, L-sets, and U-sets. This should continue until either no U set remains or the total probability of all U-sets is below a certain threshold.

It is also possible to obtain area indices. This is done by decomposing the L-sets into sets having identical area load loss characteristics. Reference [12] addresses this issue together with several other issues, including:

- Use of Monte-Carlo sampling for estimating contribution to reliability indices from remaining nondecomposed U-sets.

- Analysis including load uncertainty.
- Composite system analysis

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