NOTE: Other "modules" referenced in these notes may be found at http://home.engineering.iastate.edu/~jdm/ee653/ee653schedule.htm. Also, in the below notes, text in blue will not be covered in class, though you should read them.

Updated: 2/14/2024

Module PE.PAS.U19.5

Generation adequacy evaluation

U19.1 Introduction

Probabilistic evaluation of generation adequacy is traditionally performed for one of two classes of decision problems. The first one is the generation capacity planning problem where one wants to determine the long-range generation needs of the system. The second one is the short-term operational problem where one wants to determine the unit commitment over the next few days or weeks.

We may think of the problem of generation adequacy evaluation in terms of Fig. U19.1.

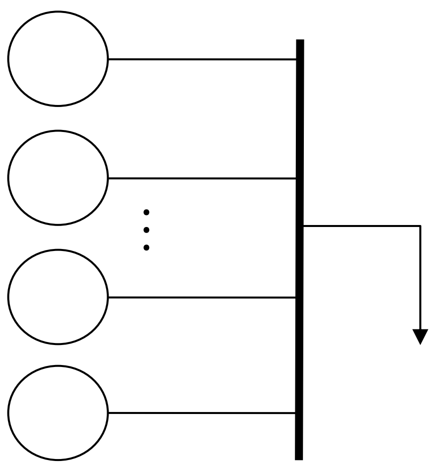

Fig. U19.1: Evaluation of Generation Adequacy

In Fig. U19.1, we see that there are a number of generation units, and there is a single lumped load. Significantly, we also observe that all generation units are modeled as if they were connected directly to the load, i.e., transmission is not modeled. The implication of this is that, in generation adequacy evaluation, transmission is assumed to be perfectly reliable.

We begin our treatment by first identifying the necessary modeling requirements in terms of, in Section U19.2, the generation side, and, in Section U19.3, the load side. Section U19.4 describes a common computational approach associated with the generation capacity planning problem, and Section U19.5 illustrates how this approach is typically used for capacity planning. Section U19.6 provides an alternative way of computing generation capacity planning indices. Section U19.7 briefly summarizes three important issues central to a more extended treatment of the topic.

U19.2 Generator model

In the basic capacity planning study, each individual generation unit is represented using the two-state Markov model illustrated in Fig. U19.2.

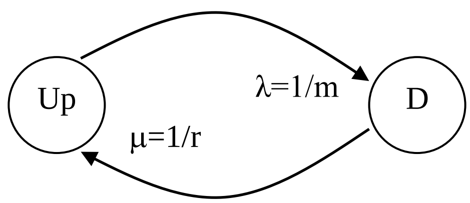

Fig. U19.2: Two-State Markov Model

This model was described in Sections U16.4-U16.5 and Section U18.2.3 of modules U16 & U18, respectively (suggest reviewing U16, U16.4-U16.5), using $\dot{\underline{p}}(t) = \underline{p}(t)\underline{A}$, where $\underline{p}(t)$ is a vector of state probabilities and, with λ_{jk} and λ_j as transition intensities, $\underline{A}$ is the transition intensity matrix given by (for the general case)

$$\underline{A} = \begin{bmatrix} -\lambda_1 & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & -\lambda_2 & \cdots & \lambda_{2n} \\ \vdots & \vdots\cdots & \vdots\cdots & \vdots \\ \lambda_{n1} & \cdots & \cdots & \lambda_{nn} \end{bmatrix}$$

Important relations for this model, in terms of long-run availability A and long-run unavailability U, are provided here again, for convenience, where m=MTTF, r=MTTR, μ and λ are transition rates (number of transitions per unit time) for repair (D to Up) and for failure (Up to D), respectively; T is the mean cycle time, and f is a “frequency” which gives expected number of direct transfers between states per-unit time. In U16.6.2, we show that f= λp_{up}

where p_{up} is the long-run probability of being up, i.e., it is $p_1(t\rightarrow\infty)$, $p(t)=[p_1(t), p_2(t)]$. We also show that

- whereas λ gives the expected number of transfers from Up to D per unit time, given the system is Up,
- f gives the expected number of transfers from Up to D per unit time, with no condition specified, and

$$A = p_{up} = \frac{\mu}{\lambda+\mu} = \frac{m}{m+r} = \frac{m}{T} = \frac{f}{\lambda} = \frac{SH}{FOH+SH} \qquad \text{(U19.1)}$$

$$U = p_D = FOR = \frac{\lambda}{\lambda+\mu} = \frac{r}{m+r} = \frac{r}{T} = \frac{f}{\mu} = \frac{FOH}{FOH+SH} \qquad \text{(U19.2)}$$

In (U19.2), the FOR is the *forced outage rate*. One should be careful to note that the FOR is not a rate at all but rather an estimator for a probability. The terms in the right-hand-expressions of (U19.1) and (U19.2) are defined as follows:

- Forced outage hours (FOH) is the number of hours a unit was in an unplanned outage state;
- Service hours (SH) is the number of hours a unit was in the in-service state. It does not include reserve shutdown hours (a reserve shutdown exists whenever a unit is available but is not synchronized[1]).

Module U18 also describes how one can approximate the effects of derating (the unit is operating but at reduced capacity due to, for example, the outages of auxiliary equipment such as pulverizers, water pumps, fans, or environmental constraints) and of reserve shutdown (very important for peaking units), by using the equivalent forced outage rate, EFOR, according to:

$$\text{EFOR} = \frac{\begin{matrix}\text{forced}\\\text{outage hours}\end{matrix} + \begin{matrix}\text{equivalent forced}\\\text{derated hours}\end{matrix}}{\begin{matrix}\text{forced}\\\text{outage hours}\end{matrix} + \begin{matrix}\text{service}\\\text{hours}\end{matrix} + \begin{matrix}\text{equivalent reserve}\\\text{shutdown forced}\\\text{derated hours}\end{matrix}}$$

[1] See Section 4.8.1, "Reserve Shutdowns," in "Powergads User Manual," Integ Enterprise Consulting, at https://www.pjm.com/~/media/etools/egads/egads-user-guide.ashx.

$$EFOR = \frac{FOH + EFDH}{FOH + SH + ERSFDH} \tag{U19.3}$$

The basis for (U19.3) is not simple, and so we will not address it here. But it is very well explained in Module U18 (see Fig. U18.3).

U19.2.1 Capacity outage table for identical units

A capacity table is simply a probabilistic description of the possible capacity states of the system being evaluated. The simplest case is that of the one-unit system, where there are two possible capacity states: 0 and C, where C is the maximum capacity of the unit. Table U19.1 shows capacities and corresponding probabilities.

Table U19.1: Capacity Table for 1 Unit System

Capacity	Probability
C	A
0	U

We may also describe this system in terms of *capacity outage states*. Such a description is generally given via a capacity outage table, as in Table U19.2.

Table U19.2: Capacity Outage Table for 1 Unit System

Capacity Outage	Probability
0	A
C	U

Figure U19.3 provides the probability mass function (pmf)[2] for this one-unit system.

[2] A PMF, for a discrete RV, provides for each value that the RV may assume, the probability of occurrence for the corresponding outcome of an experimental trial. Notationally, we write $f_X(x)$ to denote the PMF of the RV X. It may be interpreted as $P[X = x]$, or, in words, "the probability that X equals x", where x is any specific value that X may assume.

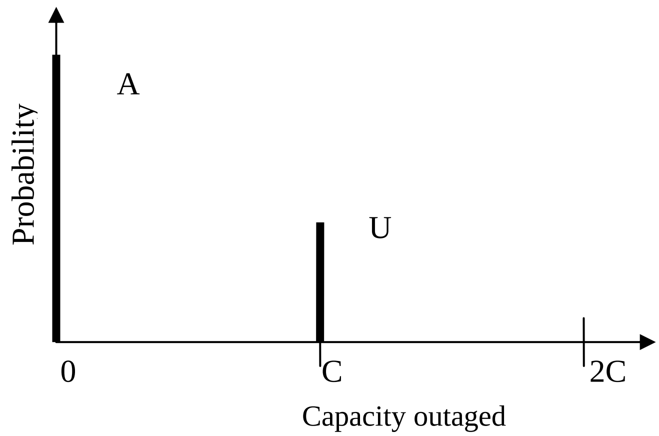

Fig. U19.3: pmf for Capacity Outage of One-Unit Example

Now consider a two-unit system, with both units of capacity C. We can obtain the capacity outage table by basic reasoning, resulting in Table U19.3.

Table U19.3: Capacity Outage Table for 2 Identical Units

Capacity Outage	Probability
0	A^2
C	AU
C	UA
2C	U^2

We may also more formally obtain Table U19.3 by considering the fact that it provides the pmf of the sum of two random variables. Define X_1 as the capacity outage random variable (RV) for unit 1 and X_2 as the capacity outage RV for unit 2, with pmfs $f_{X1}(x)$ and $f_{X2}(x)$, each of which appear as in Fig. U19.3. We desire $f_Y(y)$, the pmf of Y, where $Y=X_1+X_2$. Recall from Module U13, Section U13.3.2, that we obtain $f_Y(y)$ by convolving $f_{X1}(x)$ with $f_{X2}(x)$, i.e.,

$$f_Y(y)=\int_{-\infty}^{\infty} f_{X1}(t)\, f_{X2}(y-t)\,dt \qquad \text{(U19.4)}$$

But, inspection of $f_{X1}(x)$ and $f_{X2}(x)$, as given by Fig. U19.3, indicates that, since X_1 and X_2 are discrete random variables, their pmfs are comprised of impulses. Convolution of any function with an impulse function simply shifts and scales that function. The shift moves the origin of the original function to the location of the impulse, and the scale is by the value of the impulse. Fig. U19.4 illustrates this idea for the case at hand.

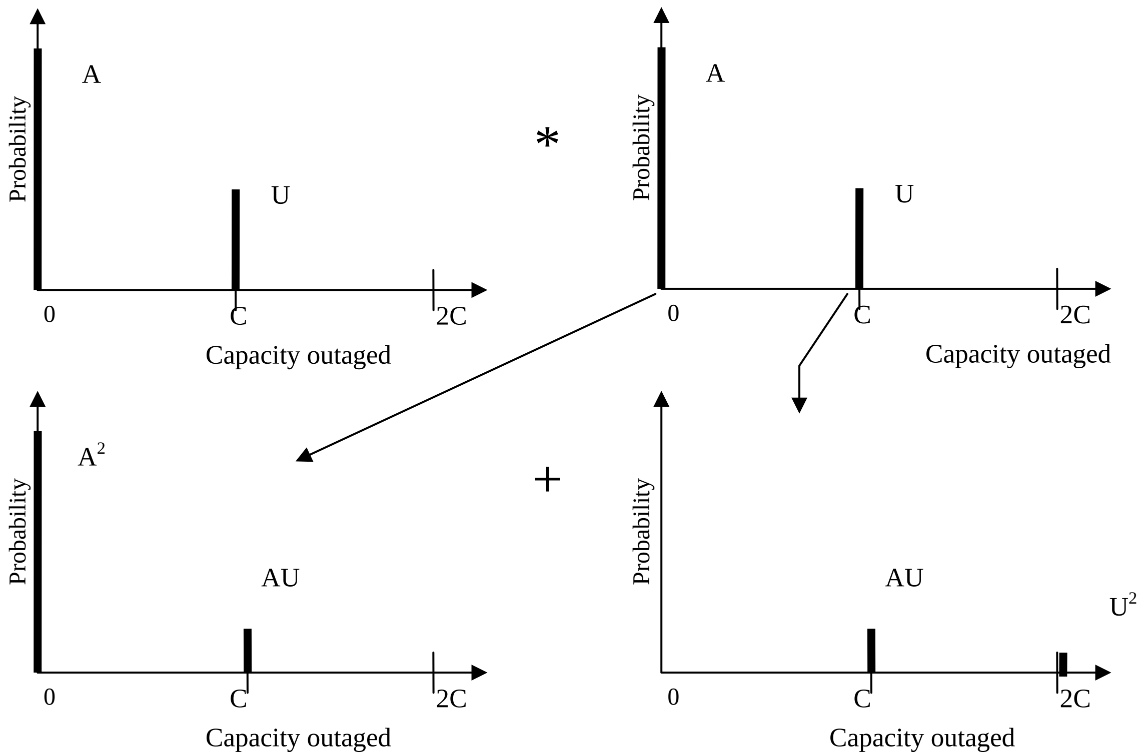

Fig. U19.4: Convolution of Generator Outage Capacity pmfs

Figure U19.5 shows the resultant pmf for the capacity outage for 2 identical units each of capacity C.

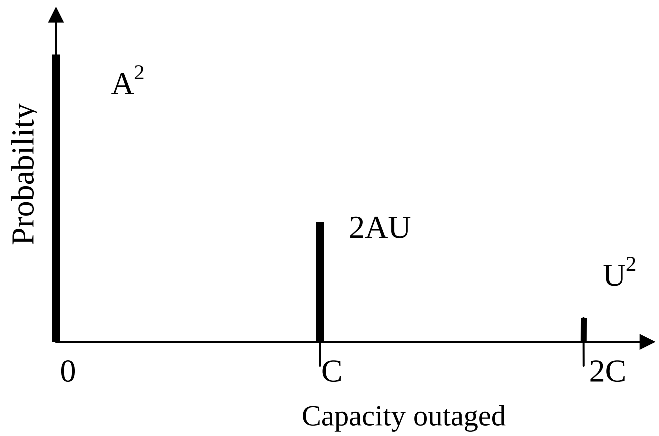

Fig. U19.5: pmf for Capacity Outage of 2 Unit Example

We note that Fig. U19.5 indicates there are only 3 states, but in Table U19.3, there are 4. One may reason from Table U19.3 that there are two possible ways of seeing a capacity outage of C, either unit 1 goes down or unit 2 goes down. Since these two states are the same, we may combine their probabilities, resulting in Table U19.4, which conforms to Fig. U19.5.

Table U19.4: Capacity Outage Table for 2 Identical Units

Capacity Outage	Probability
0	A^2
C	2AU
2C	U^2

In fact, we saw this same kind of problem in Section U10.2 of module U10, where we showed that the probabilities can be handled using a binomial distribution, since each unit may be considered as a "trial" with only two possible outcomes (up or down). We may then write the probability of having r units out of service as:

$$P_{X=r} = \Pr[X = r, n, U] = \frac{n!}{r!(n-r)!} U^r (A)^{(n-r)} \quad \text{(U19.5)}$$

where n is the number of units.

It is interesting to note we may also think about this problem via a state-space model, as shown in U19.6 where we have indicated the state of each unit together with the capacity outage level associated with each system state. Note that we are not representing the possibility of common mode or dependent failures.

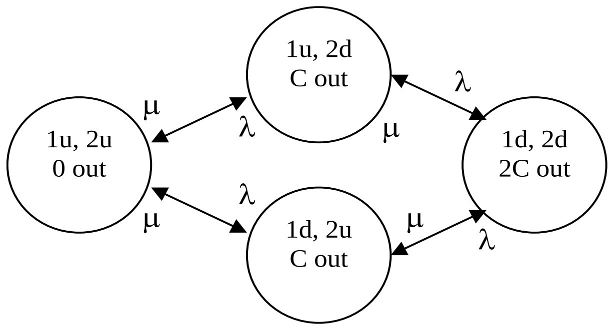

Fig. U19.6: State Space Model for 2-Unit System

From Section U16.8 of module 16, since the two middle states of Fig. U19.3 satisfy the merging condition (a group of (internal) states can be merged if the transition intensities to any external states are the same from each internal state) and they satisfy rule 3 (two states should be combined only if they are of the same state classification – in this case, the same capacity), we may combine them using rule 1 (when two (internal) states have transition rates that are identical to common external states, those two states can be merged into one; entry rates are added, exit rates remain the same.) Therefore, Fig. U19.6 becomes Fig. U19.7.

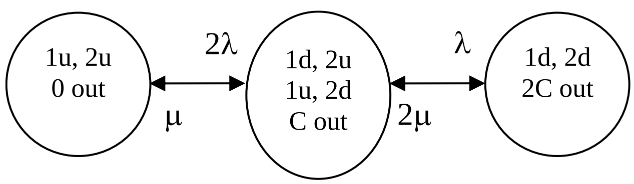

Fig. U19.7: Reduced State Space Model for 2 Unit System

The 2λ transition in Fig. U19.7 reflects the fact that the “0 out” state may transition to the “C out” state because of unit 1 or because of unit 2, but it *does not* reflect a common mode outage since the middle state is a state in which only 1 unit is failed.

Similarly, the 2μ transition in Fig. U19.6 reflects the fact the “2C out” state may transition to the “C out” state because of repair to unit 1 or repair to unit 2, but it *does not* reflect a common mode repair since the middle state is a state in which only 1 unit is repaired.

One may also compute frequency and duration for each state in Fig. U19.7 according to (U16.32) and (U16.33) from module U16, repeated here for convenience:

$$f_j = p_{j,\infty} \sum_{k \neq j} \lambda_{jk} \quad \text{(U19.6)}$$

$$T_j = \frac{1}{\sum_{k \neq j} \lambda_{jk}} \quad \text{(U19.7)}$$

Table U19.5 tabulates all of the information.

Table U19.5: Capacity Outage Table for 2 Identical Units with Frequencies and Durations

Capacity Outage	Probability	Frequency	Duration
0	A^2	$2\lambda A^2$	$1/2\lambda$
C	2AU	$2AU\mu\lambda$	$1/\mu\lambda$
2C	U^2	$2\mu U^2$	$1/2\mu$

U19.2.2 Capacity outage table for units having different capacities

Reference [1] provides a simple example for the more realistic case of having multiple units with different capacities, which we adapt and present here. Consider a system with two 3 MW units (units 1 and 2) and one 5 MW unit (unit 3), all of which have forced outage rates (FOR) of 0.02. (The fact that all units have the same FOR means that we could handle this using the binomial distribution, which would not be applicable if any unit had a different FOR).

The pmfs of the two identical 3 MW units can be convolved as in Section U19.2.1 to give the pmf of Fig. U19.8 and the corresponding capacity outage table of Table U19.6.

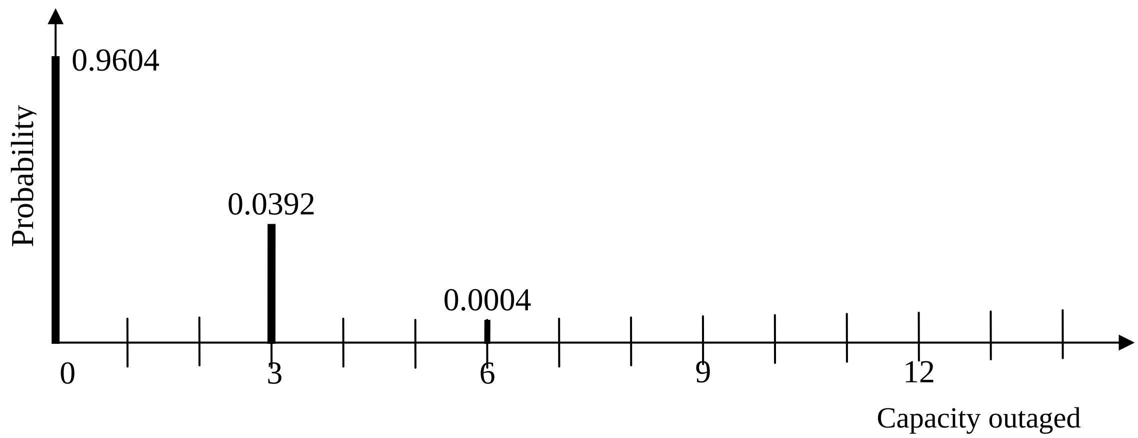

Fig. U19.8: pmf for Capacity Outage of Convolved 3 MW Units

Table U19.6: Capacity Outage Table for Convolved 3 MW Units

Capacity Outage	Probability
0	$0.98^2=0.9604$
3	2(0.98)(0.02)=0.0392
6	$0.02^2=0.0004$

Now we want to convolve in the 5 MW unit. The pmf for this unit is given by Fig. U19.9.

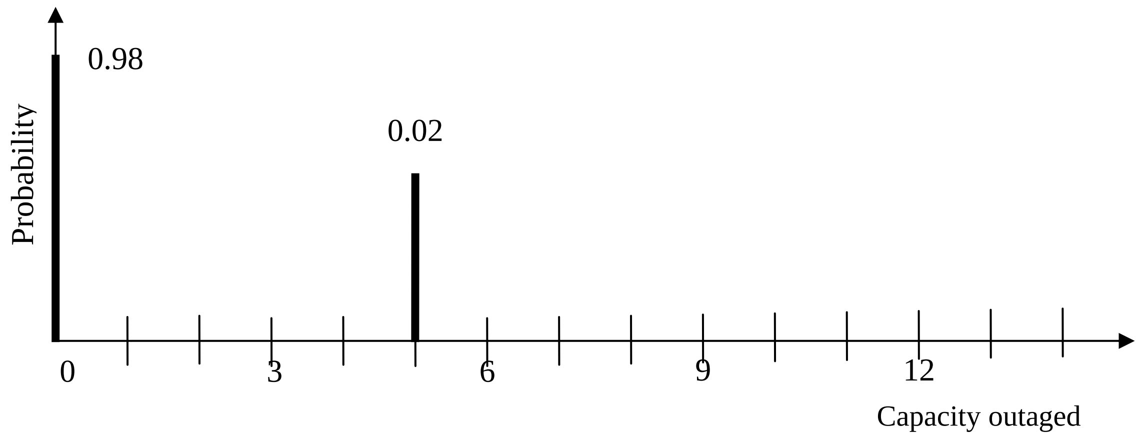

Fig. U19.9: pmf for 5 MW Capacity Outage

Convolving the pmf of Fig. U19.8 with the pmf of Fig. U19.9 results in the pmf illustrated in Fig. U19.10, with the corresponding capacity outage table given in Table U19.7.

Table U19.7: Capacity Outage Table for Convolved 3 MW Units and 5 MW Unit

Capacity Outage	Description	Probability
0	All units up.	0.98×0.9604=0.941192
3	U3 up, U1 or U2 down.	0.98×0.0392=0.038416
5	U3 down, U1 and U2 up.	0.02×0.9604=0.019208
6	U3 up, U1 and U2 down.	0.98×0.0004=0.000392
8	U3 down, U1 or U2 down.	0.02×0.0392=0.000784
11	All units down	0.02×0.0004=0.000008

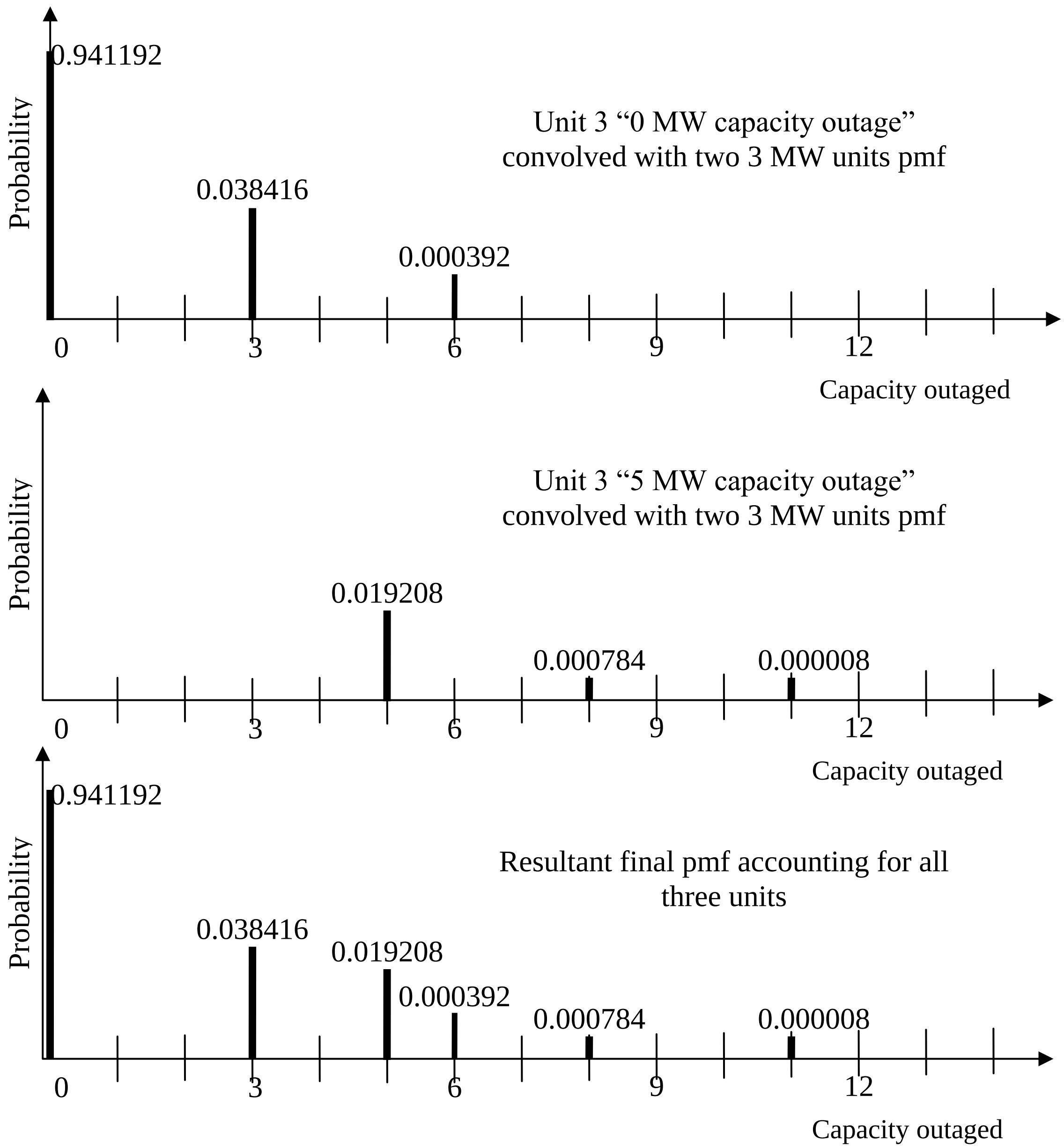

Fig. U19.10: Procedure for convolving Two 3 MW units with 5 MW Unit (top two plots) and final 3 unit pmf

U19.2.3 Convolution algorithm

The procedure illustrated above can be expressed algorithmically, which is advantageous in order to code it.

Two-state model:

The algorithm is simplest if we assume that all units are represented using two-state models.

Let k denote the k^{th} unit to be convolved in, A_k and U_k its availability and FOR, respectively, and C_k its capacity.

The composite capacity outage pmf before a convolution is denoted by $f_{Yold}(y)$, and after by $f_{Ynew}(y)$, so that for unit k, the capacity outage random variables are related by $Y_{new}=Y_{old}+X_k$. We assume that there are N units to be convolved.

The algorithm follows.

1. Let k=1.
2. Convolve in the next unit according to:

$$f_{Ynew}(y) = A_k\, f_{Yold}(y) + U_k\, f_{Yold}(y - C_k) \qquad \text{(U19.8)}$$

   for all values of y for which $f_{Yold}(y) \neq 0$ and/or $f_{Yold}(y-C_k) \neq 0$.
3. If k=N, stop, else k=k+1 and go to 2.

Note that in (U19.8) the influence of the argument in the last term $f_{Yold}(y-C_k)$ is to shift the function $f_{Yold}(y)$ to the right by an amount equal to C_k. This corresponds to the shift influence of the k^{th} unit pmf impulse at $X_k=C_k$.

U19.2.4 Deconvolution

An interesting situation frequently occurs, particularly in operations, but also in production costing programs, when the composite pmf has been computed for a large number of units, and capacity outage probabilities are fully available. Then one of the units is decommitted, and the existing composite pmf no longer applies. How to obtain a new one?

One obvious approach is to simply start over and perform the convolution for each and every unit. But this is time-consuming, and besides, there is a much better way! We have a better approach based on the following fact:

The computation of $f_{Ynew}(y)$ is independent of the order in which the units are convolved.

Consider, in (U19.8), the term $f_{Yold}(y)$. This is the composite pmf just before the "last" unit was convolved in.

Given we have $f_{Ynew}(y)$, we assume that the "last" unit convolved in was the unit that we would like to decommit.

It may not have been the last unit, in actuality, but because the computation of $f_{Ynew}(y)$ is independent of order, we can make this assumption without loss of generality.

In that case, we may "convolve out" the decommited unit.

How to do that? Consider solving (U19.8) for $f_{Yold}(y)$, resulting in:

$$f_{Yold}(y) = \frac{f_{Ynew}(y) - U_k f_{Yold}(y - C_k)}{A_k} \qquad \text{(U19.9)}$$

The problem with the above is that the function we want to compute on the left-hand-side, $f_{Yold}(y)$, is also on the right-hand-side, as $f_{Yold}(y-C_k)$.

There is a way out of this, however. It stems from two facts.

Fact 1: The probability of having capacity outage less than 0 is zero, i.e., the "best" that we can do is that we have no capacity outage, in which case the capacity outage is zero. Therefore any valid capacity outage pmf must be zero to the left of the origin.

Fact 2: $f_{Yold}(\bullet)$ is a valid capacity outage pmf.

Implication: For values of y such that $0 \leq y < C_k$, $f_{Yold}(y-C_k)$ evaluates to the left of the origin and therefore, since f_{Yold} is a valid capacity outage pmf, it MUST BE ZERO in this range. As a result,

$$f_{Yold}(y) = \frac{f_{Ynew}(y)}{A_k}, \qquad 0 \leq y < C_k \qquad \text{(U19.10)}$$

But what about the case of $C_k \leq y \leq IC$, where IC is the total installed capacity? Here, we must use (U19.9). But let's assume that we have already computed $f_{Yold}(y)$ for $0 \leq y < C_k$. Then the first time we use (U19.9) is when $y=C_k$. Then we have:

$$f_{Yold}(C_k) = \frac{f_{Ynew}(C_k) - U_k f_{Yold}(0)}{A_k}$$

But we already have computed $f_{Yold}(0)$ from (U19.10)!

And we will be able to use the values of $f_{Yold}(y)$, $0 \leq y < C_k$, in computing all values of $f_{Yold}(y)$, $C_k \leq y < 2C_k$. In fact, we will be able to compute all of the remaining values of $f_{Yold}(y)$ in this way!

As an example, try deconvolving one of the 3 MW units from the capacity outage table of Table U19.7 (which is also illustrated at the bottom of Fig. U19.10). In this case, $C_3=3$, $A_3=0.98$, $U_3=0.02$. The computations are given in Table U19.8.

Note that, since $f_{Yold}(y-C_k)=0$ for $y<C_k$, (U19.9) includes the case of (U19.10), and we can express the algorithm using (U19.9) only. The deconvolution algorithm is given below. There is just one step. We assume that we are deconvolving unit k.

1. Compute:

   $$f_{Yold}(y) = \frac{f_{Ynew}(y) - U_k f_{Yold}(y - C_k)}{A_k}$$

   <u>consecutively</u> for y=0, ….,IC such that

   $$f_{Ynew}(y) \neq 0 \text{ and/or } f_{Yold}(y-C_k) \neq 0,$$

   where IC is the installed capacity of the system before deconvolution.

2. Stop.

Table U19.8: Computations for Deconvolution Example

Capacity Outage y	$f_{Ynew}(y)$	$f_{Yold}(y)$
0	0.94119200	$f_{Yold}(0)=\frac{f_{Ynew}(0)}{A_3}=\frac{.941192}{.98}=.9604$
3	0.0384160	$f_{Yold}(3)=\frac{f_{Ynew}(3)-U_3\times f_{Yold}(3-3)}{A_3}$ $=\frac{.0384160-0.02\times .9604}{.98}=.0196$
5	0.019208	$f_{Yold}(5)=\frac{f_{Ynew}(5)-U_3\times f_{Yold}(5-3)}{A_3}$ $=\frac{.019208-.02\times 0}{.98}=.0196$
6	0.00039200	$f_{Yold}(6)=\frac{f_{Ynew}(6)-U_3\times f_{Yold}(6-3)}{A_3}$ $=\frac{.000392-.02\times .0196}{.98}=0$
8	0.00078400	$f_{Yold}(8)=\frac{f_{Ynew}(8)-U_3\times f_{Yold}(8-3)}{A_3}$ $=\frac{.000784-.02\times .0196}{.98}=.0004$
11	0.00000800	$f_{Yold}(11)=\frac{f_{Ynew}(11)-U_3\times f_{Yold}(11-3)}{A_3}$ $=\frac{.000008-.02\times .0004}{.98}=0$

U19.2.5 Multi-state models

We have so far addressed only the case where all units are represented by two-state models. It may be, however, that we would like to account for derated units, in which case we need to address the multi-state model as well. This situation presents no additional conceptual difficulty relative to the two-state model, as the pmf for each unit will still consist of only impulses, except now, each unit will have a pmf consisting of as many impulses as it has states, instead of only two.

We do, however, need to generalize the algorithms for convolution and deconvolution.

Convolution algorithm for multi-state case:

With N the total number of units:

1. Let k=1.
2. Convolve in the next unit according to:

$$f_{Ynew}(y) = \sum_{j=1}^{n_k} p_{kj}\, f_{Yold}(y - C_{kj}) \qquad \text{(U19.11)}$$

   for all values of y for which $f_{Yold}(y)$ or $f_{Yold}(y-C_{kj})$ are non-zero. Here, n_k is the number of states for unit k; p_{kj} is the j^{th} state probability for unit k; C_{kj} is the j^{th} capacity outage for unit k.
3. If k=N, stop, else k=k+1 and go to 2.

Note (U19.11) is the same as (U19.8) if n_k=2, with $A_k=p_{k1}$, $U_k=p_{k2}$.

$$f_{Ynew}(y) = A_k\, f_{Yold}(y) + U_k\, f_{Yold}(y - C_k) \qquad \text{(U19.8)}$$

Deconvolution algorithm for multi-state case:

We again assume that we are deconvolving unit k. To determine the deconvolution equation for the multi-state case, rewrite (U19.11) by extracting from the summation the first term, according to:

$$f_{Ynew}(y) = p_{k1} f_{Yold}(y) + \sum_{j=2}^{n_k} p_{kj} f_{Yold}(y - C_{kj})$$

where we have assumed that the first capacity outage state for unit k is zero, i.e., $C_{k1}=0$. Solving for $f_{Yold}(y)$, we have:

$$f_{Yold}(y) = \frac{f_{Ynew}(y) - \sum_{j=2}^{n_k} p_{kj} f_{Yold}(y - C_{kj})}{p_{k1}}$$

We assume that we are deconvolving unit k. The algorithm is:

1. Compute:

$$f_{Yold}(y) = \frac{f_{Ynew}(y) - \sum_{j=2}^{n_k} p_{kj} f_{Yold}(y - C_{kj})}{p_{k1}} \qquad \text{(U19.12)}$$

   consecutively for y=0, ….,IC, and y such that $f_{Ynew}(y)\neq 0$, $f_{Yold}(y-C_{kj})\neq 0$, where IC is the installed capacity of the system before deconvolution.

2. Stop.

U19.3 Load model

Consider Fig. U19.11, instantaneous demand as function of time.

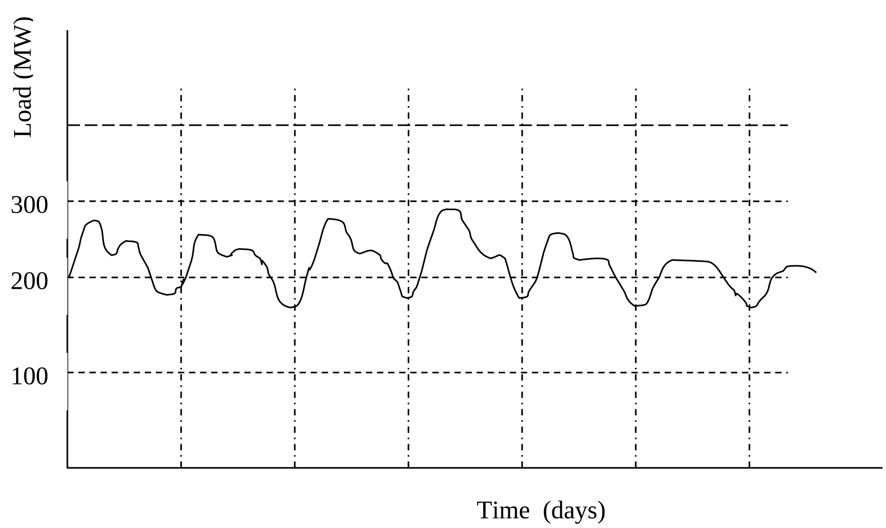

Fig. U19.11: Instantaneous demand vs. time

Although this curve is only illustrated for seven days, one could easily imagine extending the curve to cover a full year.

From such a yearly curve, we may identify the percent of time for which the demand exceeds a given value. If we assume that the curve is a forecasted curve for the next year, then this percentage is equivalent to the probability that the demand will exceed the given value in that year.
The procedure for obtaining the percent of time for which the demand exceeds a given value is as follows.

1. Discretize the curve into N equal time segments, so that the value of the discretized curve in each segment takes on the maximum value of the continuous curve in that segment.
2. The percentage of time the demand ***exceeds a value*** d is obtained by counting the number of segments having a value greater than d and dividing by N.
3. Plot the demand d against the percent of time the demand exceeds a value d. A typical such plot is illustrated in Fig. U19.12.

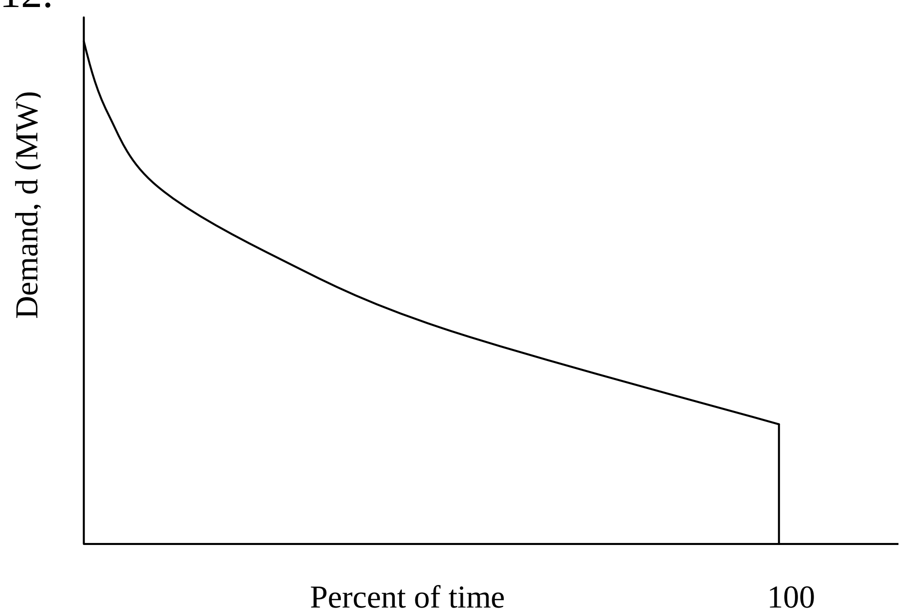

Fig. U19.12: Load duration curve

Fig. U19.12 is often generically referred to as a *load duration curve (LDC)*. However, one should be aware that there is a significant difference between LDCs based on hourly segments and LDCs based on daily segments.

- Hourly: the load duration curve indicates the percentage of hours through the year that the hourly peak exceeds a value d.
- Daily: the load duration curve indicates the percentage of days through the year that the daily peak exceeds a value d.

Thus, one must realize that the load duration curve gives the percentage of time through the year that the load exceeds a value d, only under the assumption that

- Hourly: the load is constant throughout the hour at the hourly peak.
- Daily: the load is constant throughout the day at the daily peak.

Clearly, the smaller the segment, the better approximation is given by the LDC to the actual percentage of time through the year that the load exceeds a value d. Nonetheless, both daily and hourly-based LDCs are used in practice.

The LDC may also be drawn in another way that is convenient for computation. Consider first normalizing the abscissa (x-coordinate) by dividing all values by 100, so that we obtain all abscissa values in the range of 0 to 1.
The abscissa then represents the probability that the demand exceeds the corresponding value d. We denote this probability using the notation for a cumulative distribution function (cdf), $F_D(d)$. However, one should realize that it is actually the complement of a true cdf, i.e.,

$$F_D(d) = P(D > d) = 1 - P(D \leq d)$$

Here, D is a random variable and d are the values it may take.

Finally, we can switch the axes of the LDC so that we plot $F_D(d)$ as a function of d. Figure U19.12 illustrates the curve, which we refer to as the *load shape curve* or the *load model* for the given time period.

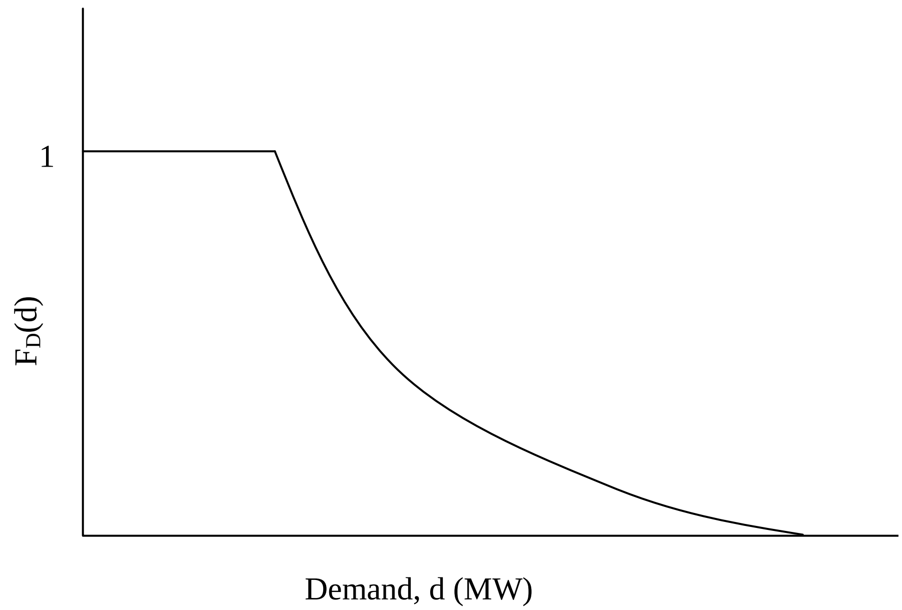

Fig. U19.12: Load shape

Note that Chanan Singh in his notes on "Load Modeling" gives an algorithm for getting the load model from one scan of the hourly load data [12].

U19.4 Calculation by Capacity Outage Tables

Module U17 identifies the loss of load probability (LOLP) and the loss of load expectation (LOLE) as two indices for characterizing generation adequacy risk. The LOLP is the probability of losing load throughout the time interval (year). LOLE is the expected number of time units (hours or days) per time interval (year) for which the load will exceed the demand.

Fig. U19.13 illustrates a typical load-capacity relationship [1] where the load model is shown as a continuous curve for a period of 365 days. The capacity outage state, C_k, is shown so that one observes that load interruption only occurs under the condition that the load exceeds the installed capacity less the capacity outage, i.e., $d>IC-C_k$. The minimum demand for which this is the case is $d_k=IC-C_k$. Thus, the probability of having an outage of capacity C_k and of having the demand exceed d_k is given by the capacity outage pmf and $F_D(d_k)$, i.e., $f_Y(C_k)F_D(d_k)= f_Y(C_k)F_D(IC-C_k)$. (This assumes independence between outage events and demand).

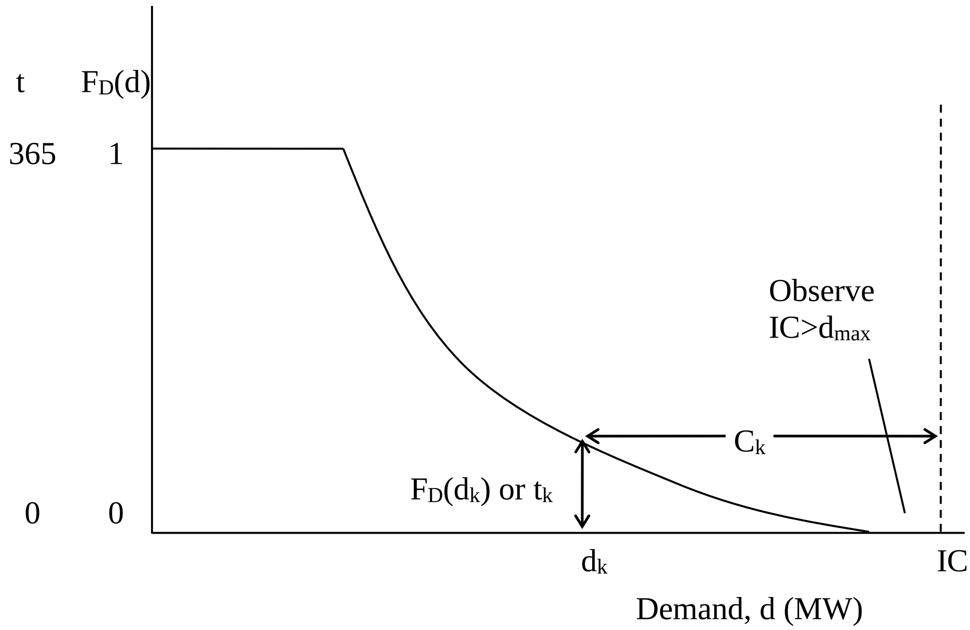

Fig. U19.13: Relationship between capacity outage, load model [1]

The LOLP is computed as the sum over all capacity outage states:

$$LOLP = \sum_{k=1}^{N} f_Y(C_k)F_D(IC - C_k) \qquad \text{(U19.13)}$$

and the LOLE as:

$$LOLE = \sum_{k=1}^{N} f_Y(C_k)F_D(IC - C_k) * 365 = \sum_{k=1}^{N} f_Y(C_k)t_k \qquad \text{(U19.14)}$$

where N is the total number of capacity outage states.

<u>Example</u>: Compute the LOLP and the LOLE for the capacity outage table of Table U19.7, for the (daily) load shape curve given by Fig. U19.14. Table U19.7 is repeated below for convenience.

Table U19.7: Capacity Outage Table for Convolved 3 MW Units and 5 MW Unit

Capacity Outage	Description	Probability
0	All units up.	0.98×0.9604=0.941192
3	U3 up, U1 or U2 down.	0.98×0.0392=0.038416
5	U3 down, U1 and U2 up.	0.02×0.9604=0.019208
6	U3 up, U1 and U2 down.	0.98×0.0004=0.000392
8	U3 down, U1 or U2 down.	0.02×0.0392=0.000784
11	All units down	0.02×0.0004=0.000008

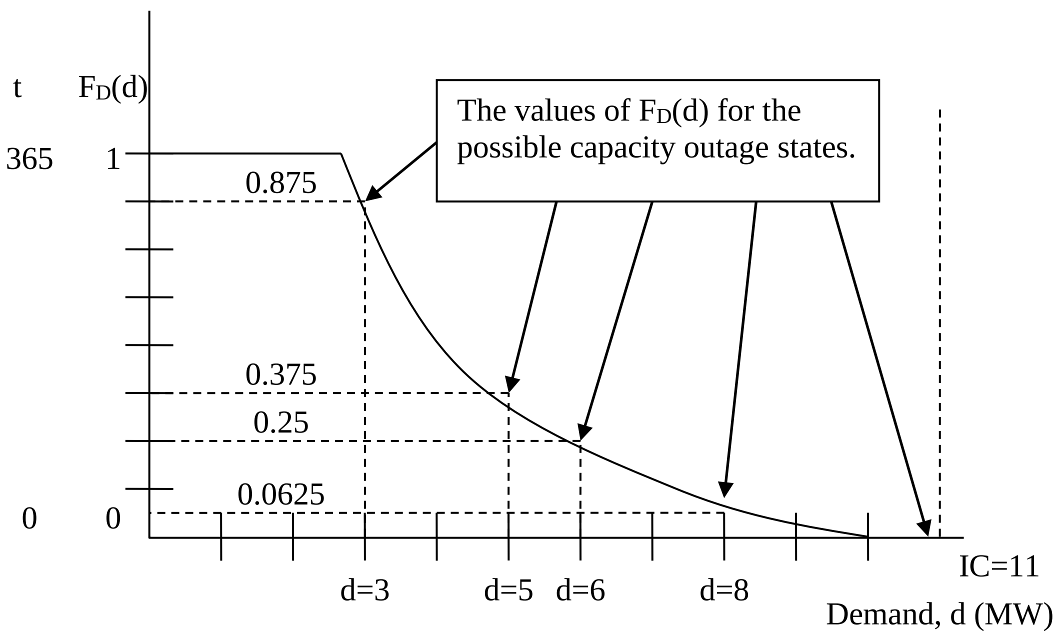

Fig. U19.14: Load shape curve for example

From (U19.13), we then have:

$$\begin{aligned}
LOLP &= \sum_{k=1}^{N} f_Y(C_k) F_D(IC - C_k) \\
&= f_Y(0)F_D(11) + f_Y(3)F_D(8) + f_Y(5)F_D(6) \\
&+ f_Y(6)F_D(5) + f_Y(8)F_D(3) + f_Y(11)F_D(0) = \\
&= .941192*0 + .038416*.0625 + .019208*.25 \\
&+ .000392*.375 + .000784*.875 + .000008*1 \\
&= 0.008044 / year
\end{aligned}$$

We could compute LOLE using (U19.14),

$$LOLE = \sum_{k=1}^{N} f_Y(C_k) F_D(IC - C_k) * 365 = \sum_{k=1}^{N} f_Y(C_k) t_k \quad \text{(U19.14)}$$

but it is easier to just recognize that

LOLE=LOLP*365=0.008044*365 =2.93606 days/year.

This means that we can expect to see 2.93606 complete days of load interruption each year, assuming that the peak load per day lasts all day. Another index often cited is the years/day, in this case, 1/2.93606=0.3406 years/day. This is the number of years that must pass before we see a full day of load interruption. A well-known metric is “1 day in 10” indicating LOLE=0.1 day/year, or years/day should be 10.

Two important qualifiers should be emphasized:

- This LOLE is the load outage time expected as a result of generation unavailability and does not include the effects of transmission or distribution system components unavailability.
- This amount of outage time would correspond to the long-run average of this system only if
  - all 3 units are always committed, i.e., no reserve shutdown, and there is no maintenance;
  - the demand remains at its daily peak throughout each day.

These qualifiers are obviously pointing towards inaccuracies in the model and as a result, indicate that the indices computed should not be perceived as accurate in an absolute sense. However, the indices do serve well for comparative purposes.

U19.5 A capacity planning example

Reference [1] provides an illustrative example showing how the generation adequacy calculation procedure in the previous section can be applied to the capacity planning problem. We adapt that example here.

Consider a system containing five 40 MW units each with a FOR=0.01, so that the installed capacity is 200 MW. We can go through a convolution process for this, or else we can observe that there are six distinct capacity outage levels: outage of 0, 40, 80, 120, 160, and 200 MW, and that for each of these capacity outage levels, there are a number of states equal to the combination of 5 things taken r at a time, where r is 0, 1, 2, 3, 4, and 5, respectively. It is a combination, where order does not matter, instead of a permutation, where order does, because, for example, the status vector {U1, U2, U3, U4, U5} characterizes the same state (and corresponding probability) as the status vector {U2, U1, U3, U4, U5}. Therefore we may identify the number of each specific combination of states with 0, 1, 2, …,5 outages as $C_r^n = n! / (n-r)!r!$. For example, the number of states with a capacity outage level of 80 MW (2 units out) will be

$$C_2^5 = 5! / (5-2)!(2!) = 5 \times 4 / 2 = 10$$

The probability of any one such state will be (0.01)(0.01)(0.99)(0.99)(0.99)=0.00009703, and so the probability of this state will be 10*(0.00009703)=0.0009703.

Proceeding in this way (or alternatively, performing the convolution), the capacity outage table for the system is developed and is shown in Table U19.9, where capacity outage states having probabilities less than 10^{-6} have been neglected.

Table U19.9: Capacity outage table for example [1]

Capacity Outage	Description	Probability
0	All units up	0.950991
40	1 unit down	0.048029
80	2 units down	0.000970
120	3 units down	0.000009

The next year's system load model is represented by the load shape curve of Fig. U19.15a, which is a linear approximation of an actual load shape curve. Note that the forecasted annual system peak load is 120 MW.

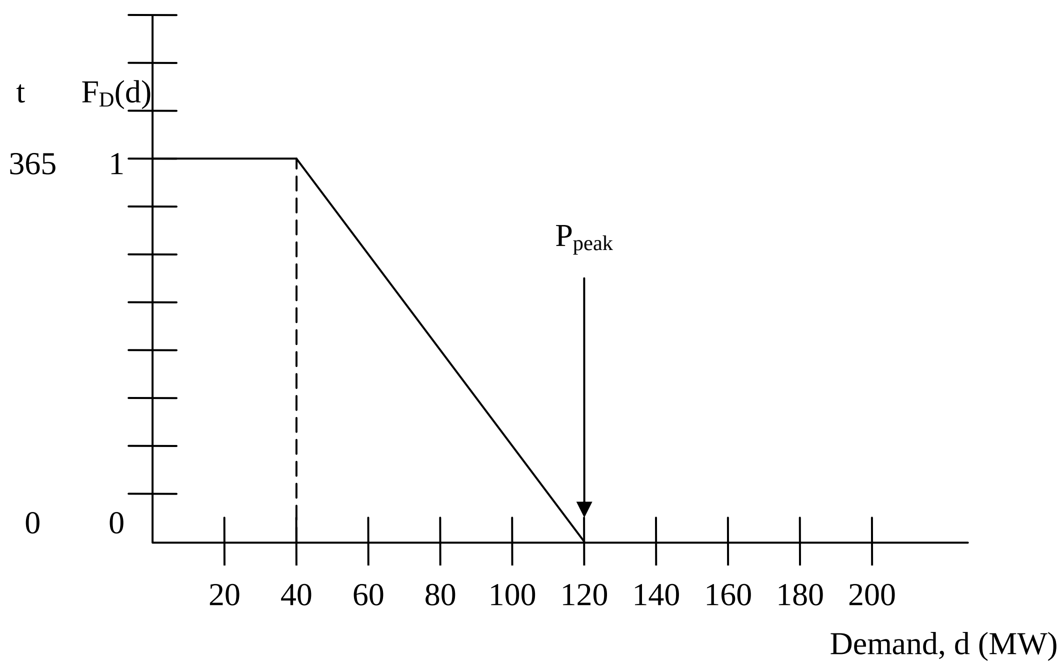

Fig U19.15a: Load shape curve for example [1]

The procedure of the previous section was applied, i.e., the following expression was evaluated,

$$LOLE = \sum_{k=1}^{N} f_Y(C_k)F_D(IC - C_k)*365 \qquad \text{(U19.14)}$$

and the LOLE and years/day were computed as 0.002005 days/year and 498 years/day, respectively. Certainly this is a very reliable system! The reason for the high reliability is, of course, that the installed capacity is so much greater than the system annual peak.

However, the load will grow in the future, so it is of interest to see how these indices vary as peak load increases. Table U19.10 summarizes LOLE and years/day for the system peak beginning at 120 MW and increasing to 200 MW in units of 10 (this is to just illustrate the effect on the indices; the 10 MW increment should not be interpreted as an annual load growth). This is imposed on the load model by shifting it to the right by 10 MW for each 10 MW increment, as illustrated by the dark line in Fig. U19.15b for a 160 MW system peak. A similar thing is done for 130, 140, 150, 170, 180, 190, and 200 MW system peaks, and indices are recomputed for each.

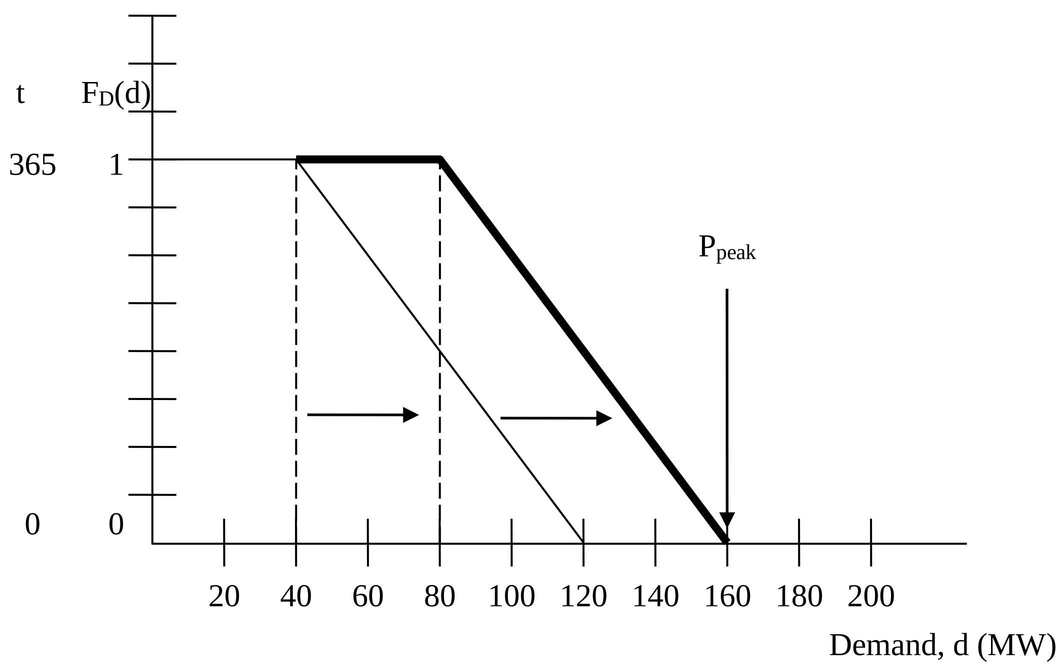

Fig U19.15b: Load shape curve for example [1]

Table U19.10: Variation in LOLE with System Annual Peak [1]

System annual peak (MW)	Indices	
	LOLE, days/year	Years/day
120	0.002005	498
130	0.04772	20.96
140	0.08687	11.51
150	0.1208	8.28
160	0.1506	6.64
170	1.895	0.53
180	3.447	0.29
190	4.837	0.21
200	6.083	0.16

The LOLE (days/year) is plotted on semi-log scale in Fig. U19.16.

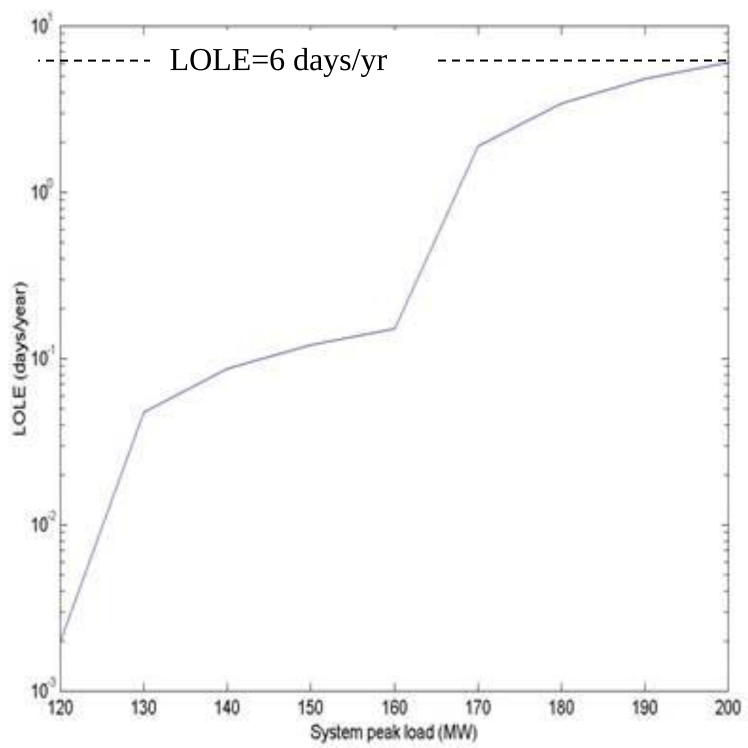

Fig. U19.16: LOLE as a function of system annual peak load [1]

Obviously, we must add some capacity before we reach an annual peak demand of 200 MW, as the LOLE is about 6 days/year at that level (very high!). But at what peak demand level should capacity be added?

The answer to this question can be identified if we select a threshold risk level beyond which we will not allow. This is basically a management decision, but of course, all management decisions can be facilitated by quantitative analysis. We will forego such analysis here and instead arbitrarily select 0.15 days/year as the threshold risk level.

Assume:

- we have forecasted a 10% per year load growth (very high) and so the peak loads will be, for years 0, 1, 2, 3, 4, 5, 6, 7, 8, given by 120, 132, 145, 160, 176. 193, 213, 234, and 257 MW, respectively;
- we have decided to add one 50 MW unit at a time, each with FOR=0.01, as the load grows, in order to ensure the system satisfies the identified threshold risk level.

The question is: when do we add the units?

To answer this question, we will repeat the analysis of Table U19.10, except for four different installed capacities: 200 MW, 250 MW, 300 MW, and 350 MW, corresponding to additional units of 0, 1, 2, and 3, respectively.

Table U19.11 summarizes the calculations. Fig. U19.17 illustrates the variation in LOLE with peak load for each case, together with vertical lines indicating the peak load value for each year.

Table U19.11: LOLE Calculations for Example [1]

System annual peak (MW)	**LOLE (days/year)**			
	200 MW	**250 MW**	**300 MW**	**350 MW**
100	0.001210	-	-	-
120	0.002005	-	-	-
140	0.08687	0.001301	-	-
160	0.1506	0.002625	-	-
180	3.447	0.06858	-	-
200	6.083	0.1505	0.002996	-
220	-	2.058	0.03615	-
240	-	4.853	0.1361	0.002980
250	-	6.083	0.1800	0.004034
260	-	-	0.6610	0.01175
280	-	-	3.566	0.1075
300	-	-	6.082	0.2904
320	-	-	-	2.248
340	-	-	-	4.880
350	-	-	-	6.083

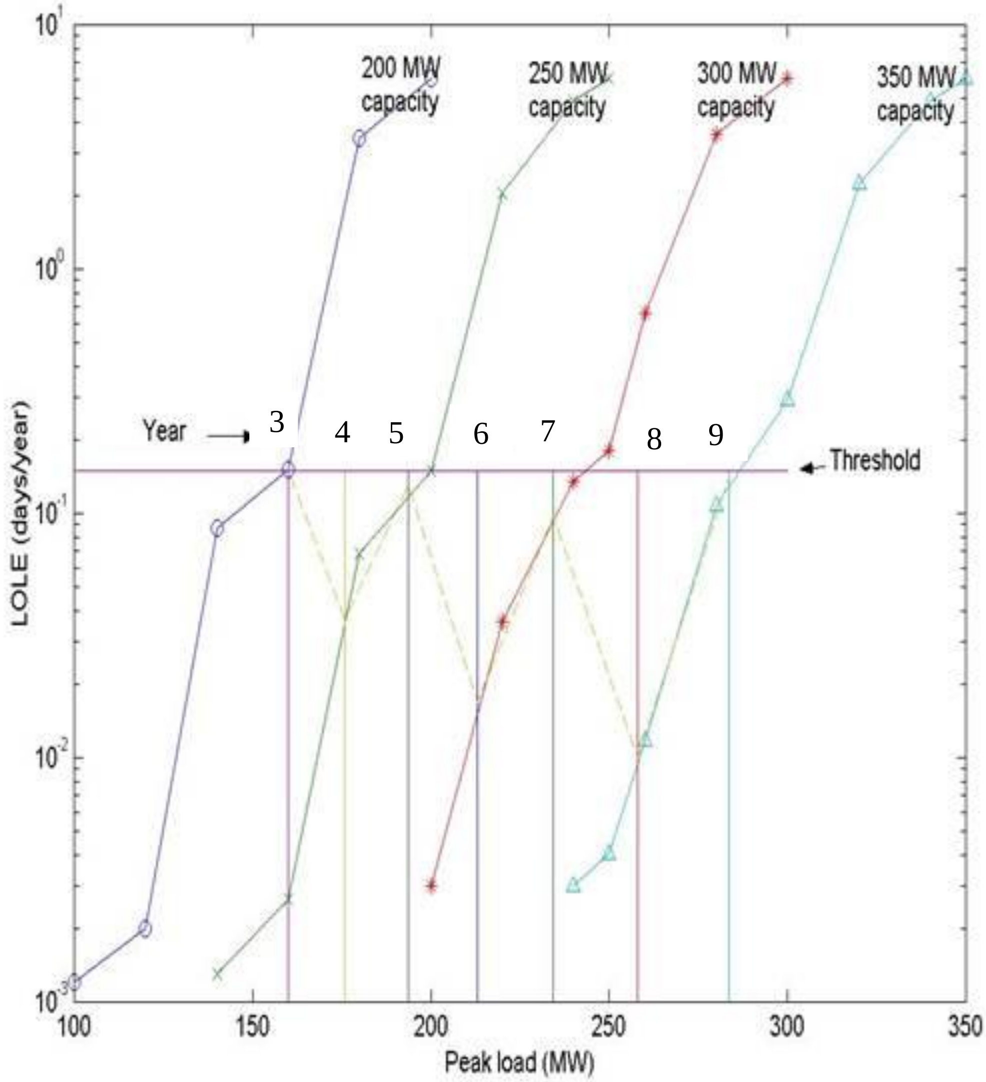

Fig. U19.17: Capacity planning example [1]

The unit additions would need to be made in years 3, 5, and 7. The dotted line tracks the year-by-year risk variation.

This approach ensures that the stated reliability criteria are met; however, the other influence to the decision-making process is, as always, economic. Recall that we assumed that we would solve our capacity problem by adding capacity at increments of 50 MW at a time. It would be quite atypical if this were the only solution approach considered. For example, one might consider larger or smaller increments, or more or less reliable units (different FOR).

Different decisions would have different influence on the system risk; they would also have different present worth values. The influence on risk and present worth would need be weighed one against another in order to arrive a “good” decision.

Question:

Why would you want to perform this kind of calculation for a system in which generators are built by electricity market participants rather than a centralized vertically integrated utility company?

U19.6 The effective load approach

Most of what we have seen in sections U19.1-U19.5 characterize the view taken by [1]. We now provide another view, based on [2].

U19.6.1 Preliminary Definitions

Let’s characterize the load shape curve with t=g(d), as illustrated in Fig. U19.18. It is important to note that the load shape curve characterizes the (forecasted) future time period and is therefore a probabilistic characterization of the demand.

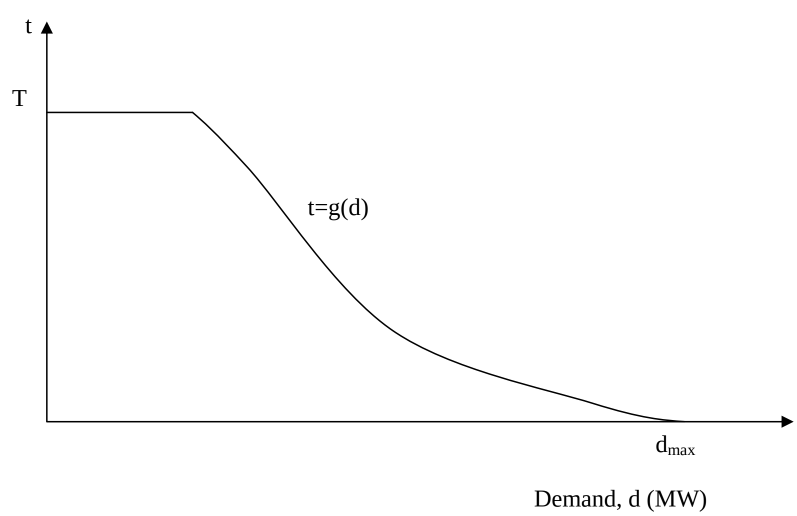

Fig. U19.18: Load shape t=g(d)

Here:

- d is the system load
- t is the number of time units in the interval T for which the load is greater than d and is most typically given in hours or days
- t=g(d) expresses the functional dependence of t on d
- T represents, most typically, a day, week, month, or year

The cumulative distribution function (cdf) introduced in Section U19.3 is given by

$$F_D(d) = P(D > d) = \frac{t}{T} = \frac{g(d)}{T} \qquad \text{(U19.15)}$$

One may also compute the *total energy* E_T consumed in the period T as the area under the curve, i.e.,

$$E_T = \int_0^{d_{\max}} g(\lambda)d\lambda \qquad \text{(U19.16)}$$

The *average demand* (a power quantity) in the period T is obtained from

$$d_{avg} = \frac{1}{T}E_T = \frac{1}{T}\int_0^{d_{\max}} g(\lambda)d\lambda = \int_0^{d_{\max}} F_D(\lambda)d\lambda \qquad \text{(U19.17)}$$

Now let's assume that the planned system generation capacity, i.e., the installed capacity, is C_T, and that $C_T<d_{max}$. This is an undesirable situation, since we will not be able to serve some demands, even when there is no capacity outage! Nonetheless, it serves well to understand the relation of the load duration curve to several useful indices. The situation is illustrated in Fig. U19.19.

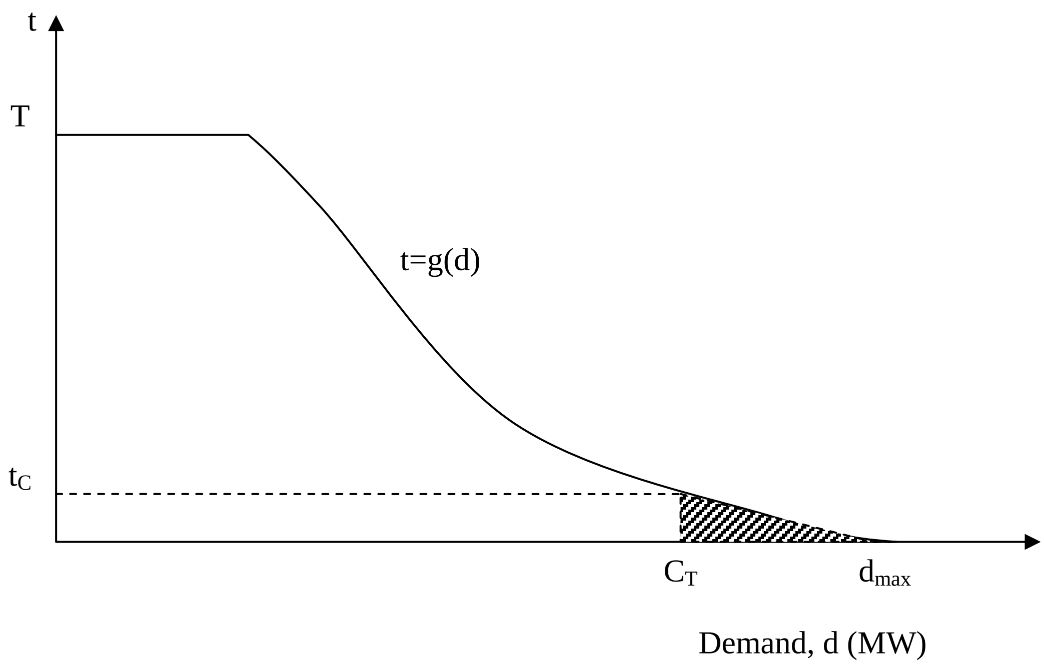

Fig. U19.19: Illustration of Unserved Demand

Then, under the assumption that the given capacity C_T is perfectly reliable, we may express three useful reliability indices:

- Loss of load expectation, LOLE: the number of time units that the load will exceed the capacity,

$$LOLE = t_{C_T} = g(C_T) \qquad \text{(U19.18)}$$

- Loss of load probability, LOLP: the probability that the load will be interrupted during the time period T

$$LOLP = P(D > C_T) = F_D(C_T) \qquad \text{(U19.19)}$$

  One may think that, given $C_T < d_{max}$, then LOLP=1, i.e., the event "load interruption during T" is certain. The reason why it is not certain is because the load model is probabilistic. So LOLP is simply reflecting the uncertainty associated with demand, i.e., the demand may or may not exceed C_T, according to $F_D(C_T)$.

- Expected demand not served, EDNS: If the average (or expected) demand is given by (U19.17), then it follows that the expected demand not served would be:

$$EDNS = \int_{C_T}^{d_{max}} F_D(\lambda) d\lambda \qquad \text{(U19.20)}$$

which would be the same area as in Fig. U19.19 when the ordinate is normalized to provide $F_D(d)$ instead of t. Reference [2] provides a rigorous derivation for (U19.20).

- Expected energy not served, EENS: This is the total amount of time multiplied by the expected demand not served, i.e.,

$$EENS = T \int_{C_T}^{d_{max}} F_D(\lambda) d\lambda = \int_{C_T}^{d_{max}} g(\lambda) d\lambda \qquad \text{(U19.21)}$$

which is the area shown in Fig. U19.19.

U19.6.2 Effective load

The notion of effective load is used to account for the unreliability of the generation, and it is essential for understanding the view taken by [2].

The basic idea is that the total system capacity is always C_T, and the effect of capacity outages are accounted for by changing the load model in an appropriate fashion, and then the different indices are computed as given in (U19.18), (U19.19), and (U19.20).

A capacity outage of C_i is therefore modeled as an increase in the demand, not as a decrease in capacity!

We have already defined D as the random variable characterizing the demand. Now we define two more random variables:

- D_j is the random increase in load for outage of unit i.
- D_e is the random load accounting for outage of all units and represents the *effective* load.

Thus, the random variables D, D_e, and D_j are related according to:

$$D_e = D + \sum_{j=1}^{N} D_j \qquad \text{(U19.21)}$$

It is important to realize that, whereas C_j represents the capacity of unit j and is a deterministic value, D_j represents the increase in load corresponding to outage of unit j and is a random variable. The probability mass function (pmf) for D_j is assumed to be as given in Fig. U19.20, i.e., a two-state model. We denote the pmf for D_j as $f_{Dj}(d_j)$

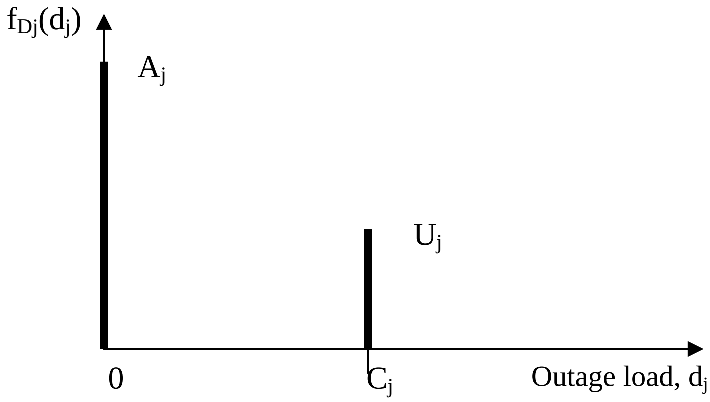

Fig. U19.20: Two state generator outage model

Recall from module U13 that the pdf of the sum of two random variables is the convolution of their individual pdfs. In addition, it is true that the cdf of two random variables can be found by convolving the cdf of one of them with the pdf (or pmf) of the other, that is, for random variables X and Y, with Z=X+Y, that

$$\boldsymbol{F_Z(z) = \int_{\lambda=-\infty}^{\infty} F_X(z-\lambda) f_Y(\lambda) d\lambda} \qquad \text{(U19.22)}$$

Let's consider the case for only one unit, i.e., from (U19.21),

$$D_e = D + D_j \qquad \text{(U19.23)}$$

Then, by (U19.22), we have that:

$$F_{D_e}^{(1)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(0)}(d_e - \lambda) f_{D_j}(\lambda) d\lambda \qquad \text{(U19.24)}$$

where the notation $F_D^{(j)}(\cdot)$ indicates the cdf after the j^th unit is convolved in. Under this notation, then, (U19.23) becomes

$$D_e^{(j)} = D_e^{(j-1)} + D_j \qquad \text{(U19.25)}$$

and the general case for (U19.24) is:

$$F_{D_e}^{(j)}(d_e) = \int_{\lambda=-\infty}^{\infty} F_{D_e}^{(j-1)}(d_e - \lambda) f_{D_j}(\lambda) d\lambda \qquad \text{(U19.26)}$$

which expresses the equivalent load after the j^th unit is convolved in.

Since $f_{Dj}(d_j)$ is discrete (i.e., a pmf), we may rewrite (U19.26) as

$$F_{D_e}^{(j)}(d_e) = \sum_{d_j \neq 0} F_{D_e}^{(j-1)}(d_e - d_j) f_{D_j}(d_j) \qquad \text{(U19.27)}$$

From an intuitive perspective, (U19.27) is providing the convolution of the load shape $F_D^{(j-1)}(\cdot)$ with the set of impulse functions comprising $f_{Dj}(d_j)$. When using a 2-state model for each generator, $f_{Dj}(d_j)$ is comprised of only 2 impulse functions, one at 0 and one at C_j. Recalling that the convolution of a function with an impulse function simply shifts and scales that function, (U19.27) can be expressed for the 2-state generator model as:

$$F_{D_e}^{(j)}(d_e) = A_j F_{D_e}^{(j-1)}(d_e) + U_j F_{D_e}^{(j-1)}(d_e - C_j) \qquad \text{(U19.28)}$$

So the cdf for the effective load following convolution with capacity outage pmf of the j^th unit, is the sum of

- the original cdf, scaled by A_j and
- the original cdf, scaled by U_j and right-shifted by C_j.

<u>Example</u>: Fig. U19.21 illustrates the convolution process for a single unit C_1=4 MW supplying a system having peak demand d_{max}=4 MW, with demand cdf given as in plot (a) based on a total time interval of T=1 year.

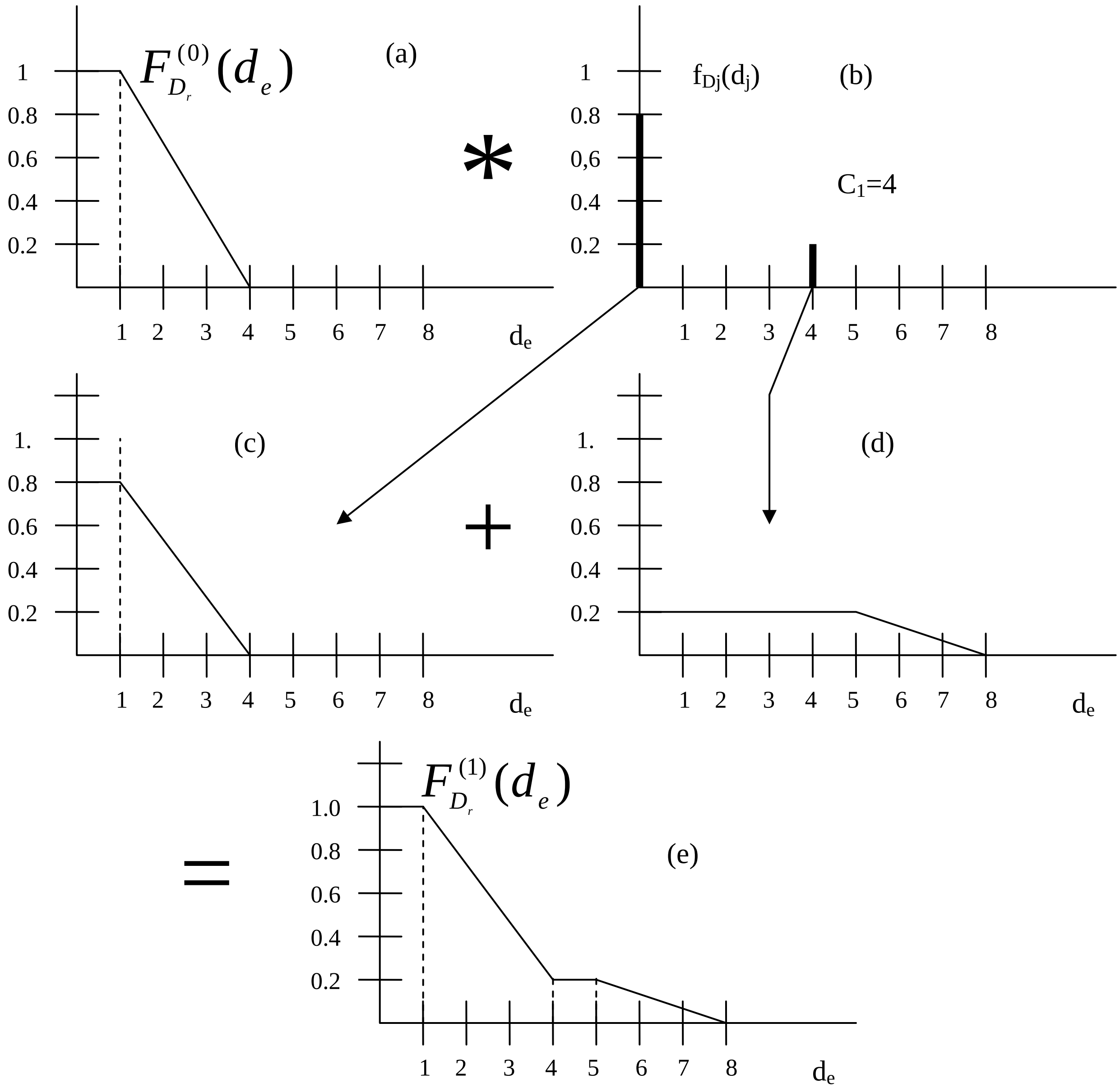

Fig. U19.21: Convolving in the first unit

Plots (c) and (d) represent the intermediate steps of the convolution where the original cdf $F^{(0)}_{D_e}(d_e)$ was scaled by A_1=0.8 and U_1=0.2, respectively, and right-shifted by 0 and C_1=4, respectively. Note the effect of convolution is to spread the original cdf.

Plot (d) may raise some question since it appears that the constant part of the original cdf has been extended too far to the left. The reason for this apparent discrepancy is that all of the original cdf, in plot (a), was not shown. The complete cdf is illustrated in Fig.

U19.22 below, which shows clearly that $F_{D_e}^{(0)}(d_e)=1$ for $d_e<0$, reflecting the fact that $P(D_e>d_e)=1$ for $d_e<0$.

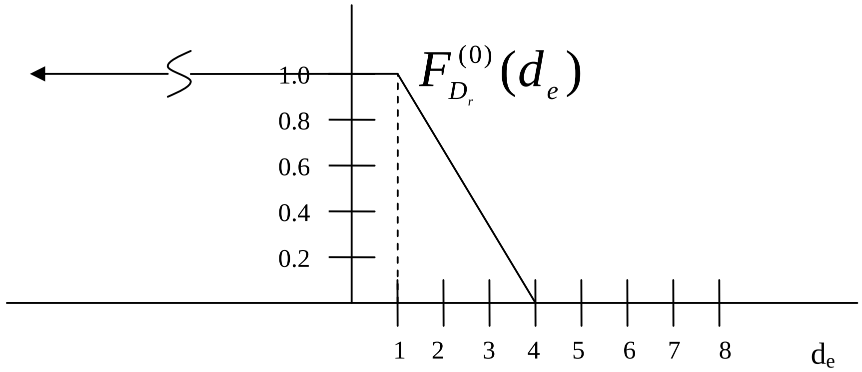

Fig. U19.22: Complete cdf including values for $d_e<0$

Let's consider that the "first" unit we just convolved in is actually the only unit. If that unit were perfectly reliable, then, because $C_1=4$ and $d_{max}=4$, our system would never have loss of load. This would be the situation if we applied the ideas of Fig. U19.19 to Fig. U19.21, plot (a).

However, Fig. U19.21, plot (e) tells a different story. Fig. U19.23 applies the ideas of Fig. U19.19 to Fig. U19.21, plot (e) to show how the cdf on the *equivalent load* indicates that, for a total capacity of $C_T=4$, we do in fact have some chance of losing load.

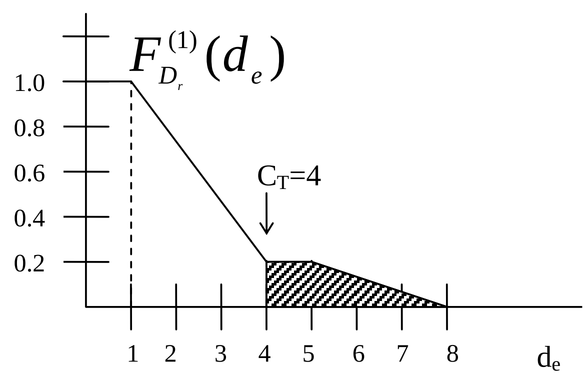

Fig. U19.23: Illustration of loss of load region

The desired indices are obtained from (U19.18), (U19.19), and (U19.20) as:

$$LOLE = t_{C_T} = g_e(C_T) = T \times F_{D_r}(C_T = 4) = 1 \times 0.2 = 0.2 years$$

A LOLE of 0.2 years is 73 days, a very poor reliability level that reflects the fact we have only a single unit with a high FOR=0.2.

The LOLP is given by:

$$LOLP = P(D_e > C_T) = F_{De}(C_T) = 0.2$$

and the EDNS is given by:

$$EDNS = \int_{C_T}^{d_{e,\max}} F_{De}(\lambda) d\lambda$$

which is just the shaded area in Fig. U19.23, most easily computed using the basic geometry of the figure, according to:

$$0.2(1) + \frac{1}{2}(3)(0.2) = 0.5MW$$

The EENS is given by

$$EENS = T \int_{C_T}^{d_{e,\max}} F_{De}(\lambda) d\lambda = \int_{C_T}^{d_{e,\max}} g_e(\lambda) d\lambda$$

or T×EDNS=1(0.5)=0.5MW-years, or 8760(0.5)=4380MWhrs.

U19.7 Four additional issues

A more extended treatment of generation adequacy evaluation would treat a number of additional issues. Here, we just point to these issues with a brief overview of each so that the interested reader may follow up on them as desired. The main issues are model uncertainty (U16.7.1), maintenance (U16.7.2), convolution techniques (U16.7.3), and frequency and duration approach (U16.7.4).

U19.7.1 Model uncertainty

We have modeled uncertainty in our analysis of generation adequacy. However, we have assumed that our uncertainty models are precise, i.e., the unit FORs and the load forecast used to obtain the load duration curves are both perfectly accurate. The fact of the matter is that the unit FORs and the load forecast are estimates of the “true” parameters, and they will always be estimates no matter how much data is collected! Therefore, it is of interest to model uncertainty in the model parameters and then identify the influence of these uncertainties on the resulting adequacy indices.

One method of modeling parameter uncertainty is to represent each parameter with a numerical distribution. Then repeatedly draw values from each distribution, and calculate the reliability indices using those values. If the parameter values are drawn as a function of their probabilities, as indicated by the distribution, then the computed reliability indices will also form a distribution, from which we may compute their statistics, e.g., mean, variance, etc.

For example, if the peak load is normally distributed, then the distribution may be discretized, and each interval of the distribution can be assigned to an interval on (0,1) in proportion to its area under the normal curve. Then a random draw on (0,1), which is then converted to the peak load value through the assignment, will reflect the desired normal distribution. Figure U19.24 illustrates the process.

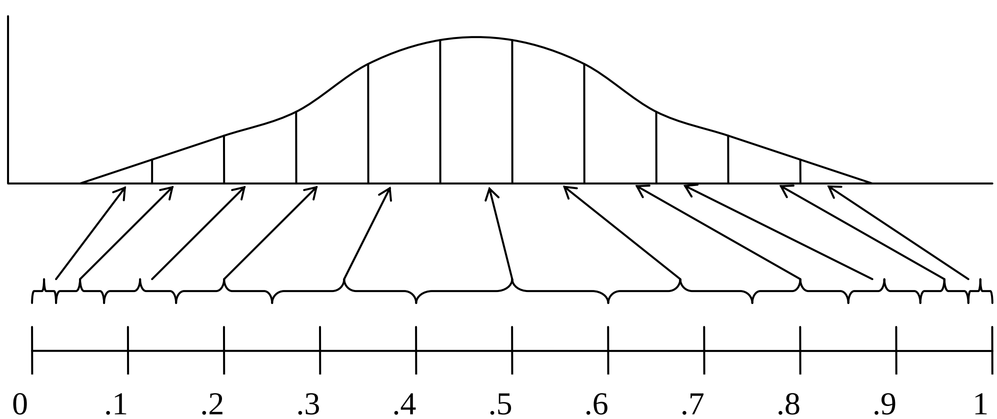

Fig. U19.24: Monte Carlo Simulation

This process is called Monte Carlo simulation (MCS) and is almost always an available option for computing reliability indices under parameter uncertainty. The advantage to MCS is that it is conceptually simple to implement. The disadvantage is that it is computationally intensive.

<u>Load forecast uncertainty</u>:

There are two basic methods. The first, well articulated in [1], is the most computational but the easiest to understand. The approach is to model the peak load using a discretized normal distribution, as shown in Fig. U19.24, where the mean of the distribution corresponds to the forecasted load.

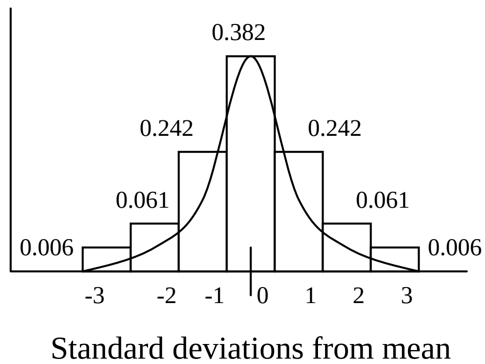

Fig. U19.25: Modeling of load uncertainty [1]

The load shape curve is adjusted for each of the load values corresponding to the seven standard deviations from the mean (-3, -2, -1, 0, 1, 2, 3), where 1 standard deviation is estimated based on the load forecasting program used and the amount of time over which the forecast is being done. A reasonable value could be 2%, for example.

Then the indices are computed for each different load shape and composite indices are computed as a weighted function of the individual indices, where the weights are the probabilities given in Fig. U19.25.

A second method is given in [1] but perhaps more thoroughly described in [2]. The basic idea is that a single cdf is constructed that reflects the uncertainty of the peak load forecast, using

$$F_D^{(0)}(d) = \sum_{\theta} F_D^{(0)}(d \mid \theta) f_{\Theta}(\theta) \qquad \text{(U19.29)}$$

Once this cdf is obtained, the indices are computed using one of our standard approaches.

It is important to realize that modeling of uncertainty in load forecast always results in indices reflecting poorer reliability because the rate of increase of the indices is nonlinear with peak load, in that it is higher at higher load levels than at lower load levels.

<u>FOR uncertainty</u>:

References [1, 4] address inclusion of FOR uncertainty using a covariance matrix corresponding to the capacity outage table. The method is based on [5]. One important conclusion from this work is that although FOR uncertainty certainly affects the distribution of the reliability indices, it does not affect their expected values. This is in contrast to load forecast uncertainty.

U19.7.2 Maintenance

The conceptually simplest method for including unit maintenance is through the capacity outage approach according to the following:

1. Compute a “full” capacity outage table.
2. Divide the year into N_y intervals and obtain a unique load shape cdf $F_{Dp}(d)$ for each period p.
3. For each interval p=1, N_y
   a. Identify the units out on maintenance in this interval
   b. Deconvolve each outaged unit from the capacity outage table to get a capacity outage table for period p, using the

algorithm of Section U19.2.4. Denote the resulting capacity outage pmf as $f_{Yp}(y)$.

c. Compute the LOLE for period p as (similar to (U19.14)):

$$LOLE_p = \sum_{k=1}^{N_p} f_{Yp}(C_k) F_{Dp}(IC - C_k) * N_{days} = \sum_{k=1}^{N} f_{Yp}(C_k) t_{kp}$$

(U19.30)

where N_p is the total number of capacity outage states for period p and N_{days} are the number of days in period p.

4. The annual LOLE is then given as the sum of the $LOLE_p$, i.e.,

$$LOLE = \sum_{p=1}^{N_Y} LOLE_p \qquad \text{(U19.31)}$$

U19.7.3 Convolution techniques

We have seen that convolution plays a major role in both the capacity table approach and the effective load approach. The convolution method illustrated for both approaches is called the recursive method. One drawback of this method is that it is quite computationally intensive and can require significant computer resources when it is used for systems having a large number of units and/or units with a large number of derated states.

As a result, there has been a great deal of research effort into developing faster convolution methods. This work has resulted in, in addition to the recursive method, the following methods [3]:

- Fourier transform [6]
- Method of cumulants [7]
- Segmentation method [8, 9, 10]
- Energy function method [3]

Of these, the method of cumulants is very fast, and the recursive method very is accurate. The segmentation method is said to achieve a good tradeoff between speed and accuracy. Chanan Singh summarizes the method of cumulants in his notes [12].

U19.7.4 Frequency and duration approach

The methods presented in this module so far provide the ability to compute LOLP, LOLE, EDNS, and EENS, but they do not provide the ability to compute

- Frequency of occurrence of an insufficient capacity condition
- The duration for which an insufficient capacity condition is likely to exist.

A competing method which provides these latter quantities goes, quite naturally, under the name of the *frequency and duration* (F&D) approach. The F&D approach is based on state space diagrams and Markov models. We touched on this at the end of Section U19.2.1 above by showing that we may represent a 2 generator system via a Markov model and then compute state probabilities, frequencies, and durations for each of the states.

The underlying steps for the F&D approach, outlined in chapter 10 of [11], are:

1. Develop the Markov model and corresponding state transition matrix, $\underline{A}$ for the system.
2. Use the state transition matrix to solve for the long-run probabilities from $\underline{0}=\underline{pA}$ and $\sum p_j=1$ (note that we have dropped the subscript ∞ for brevity, but it should be understood that all probabilities in this section are long-run probabilities).
3. Evaluate the frequency of encountering the individual states from (U16.31), repeated here for convenience:

$$f_j = \sum_{k\neq j} \lambda_{jk} p_{j,\infty} = p_{j,\infty} \sum_{k\neq j} \lambda_{jk} \qquad \text{(U19.32)}$$

which can be expressed as:

$$f_j = p_{j,\infty}[\text{total rate of departure from state j}]$$

4. Evaluate the mean duration of each state, i.e., the mean time of residing in each state, from (U16.33), repeated here for convenience:

$$T_j = \frac{1}{\sum_{k \neq j} \lambda_{jk}} = \frac{p_{j,\infty}}{f_j} \tag{U19.33}$$

(Note that [11] uses m_j to denote the duration for state j and uses T_j to denote the cycle time for state j, which is the reciprocal of the state j frequency f_j. One should carefully distinguish between the cycle time and the mean duration.

- The cycle time is the mean time between entering a given state to next entering that same state.
- The duration is the mean time of remaining in a given state.)

5. Identify the states corresponding to failure, lumped into a cumulative state denoted as J.
6. Compute the cumulative probability of the failure states p_J as the sum of the individual state probabilities:

$$p_J = \sum_{j \in J} p_j \tag{U19.34}$$

7. Compute the cumulative frequency f_J of the failure states (see section U16.8.2) as the total of the frequencies leaving a failure state j for an non-failure state k:

$$f_J = \sum_{k \notin J} \sum_{j \in J} f_{jk} \tag{U19.35}$$

Because (see (U16.29)) $f_{jk}=\lambda_{jk}\, p_{j,\infty}$, (U19.35) can be expressed as

$$f_J = \sum_{k \notin J} \sum_{j \in J} \lambda_{jk} p_{j,\infty} = \sum_{j \in J} \sum_{k \notin J} \lambda_{jk} p_{j,\infty}$$
$$= \sum_{j \in J} p_{j,\infty} \sum_{k \notin J} \lambda_{jk} \qquad \text{(U19.36)}$$

8. Compute the cumulative duration for the failure states, as:

$$T_J = \frac{p_J}{f_J} \qquad \text{(U19.37)}$$

The above approach is quite convenient for a system of just a very few states, and it is important for our purposes because it lays out the underlying principles on which the F&D is based.

However, for a large system, the above approach is not very useful because of step 1 where we must develop the Markov model. This difficulty is circumvented by building the capacity outage table using recursive relations for the capacity outage (e.g. state) probabilities together with additional recursive relations for state transitions and state frequencies [1, 2, 4, 11].

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