Unit Commitment, jdm@linux3.ece.iastate.edu

1.0 Introduction

The problem of unit commitment (UC) is to decide which units to interconnect over the next T hours, where T is commonly 24 or 48 hours, although it is reasonable to solve UC for a week at a time. The problem is complicated by the presence of inter-temporal constraints, i.e., what you do in one period constrains what you can do in the next period. The problem is also complicated because it involves integer decision variables, i.e., a unit is either committed (1) or not (0).

The UC problem forms the basis of today's day-ahead markets (DAMs). Most ISOs today are running so-called security-constrained unit commitment (SCUC) 24 hours ahead of the real-time (balancing) market.

If one has a very good solution method to solve the UC problem (or the SCUC problem), then the good solutions that come will save a lot of money relative to using a not-so-good solution method. Regardless of the solution method, however, the solutions may not save much money if the forecast of the demand that needs to be met contains significant error. Having a "perfect" solution for a particular demand forecast is not very valuable if the demand forecast is very wrong. Therefore demand forecasting is very important for solving the UC. Systems that are expecting high wind energy penetrations are concerned about this fact, since high wind penetration increases demand forecast uncertainty (the demand that the thermal units must meet is load-wind). This is why so much attention is being paid to improving wind power forecasting. It is also why so much attention is being paid to creating UC models and solvers that handle uncertainty.

We begin these notes by providing an explicit problem statement, in words, in Section 2.0, and Section 3.0 provides an analytic problem statement. Section 4.0 provides an overview of several good industry papers (which are posted on the website). Section 5.0 provides some illustrations.

2.0 Problem statement

The unit commitment problem is solved over a particular time period T; in the day-ahead market, the time period is usually 24 hours. It is articulated in [10], in words, as follows:

1. Min Objective=UnitEnergyCost+StartupCost+ShutdownCost
   +DemandBidCost

Subject to:
2. Area Constraints:
   a. Demand + Net Interchange
   b. Spinning and Operating Reserves
3. Zonal Constraints:
   a. Spinning and Operating Reserves
4. Security Constraints
5. Unit Constraints:
   a. Minimum and Maximum Generation limits
   b. Reserve limits
   c. Minimum Up/Down times
   d. Hours up/down at start of study
   e. Must run schedules
   f. Pre-scheduled generation schedules
   g. Ramp Rates
   h. Hot, Intermediate, & Cold startup costs
   i. Maximum starts per day and per week
   j. Maximum Energy per day and per study length

We describe the objective function and the various constraints in what follows.

2.1 Objective function

a. *UnitEnergyCost*: This is the total costs of supply over *T*, based on the supply offers made, in $/MWhr.

b. *StartupCost*: This is the total cost of starting units over *T*, based on the startup costs

c. *ShutdownCost*: This is the total cost of shutting down units over T, based on the shutdown costs.

d. *DemandBidCost*: This is the total "cost" of demand over *T*, based on the demand bids made, in $/MWhr. Revenue demand bids are added as negative costs so that by minimizing the objective the profit is maximized.

2.2 Area constraints

a. *Demand + Net Interchange*: The area demand plus the exports from the area (which could be negative, or imports).

b. *Spinning and Operating Reserves*: The spinning reserve is the amount of generation capacity $\Sigma(P_{gmax,k}-P_{gen,k})$ in MW that is on-line and available to produce energy within 10 minutes. Operating reserve is a broader term: the amounts of generating capacity scheduled to be available for specified periods of an Operating Day to ensure the security of the control area. Generally, operating reserve includes primary (which includes spinning) and secondary reserve, as shown in Fig. 1.
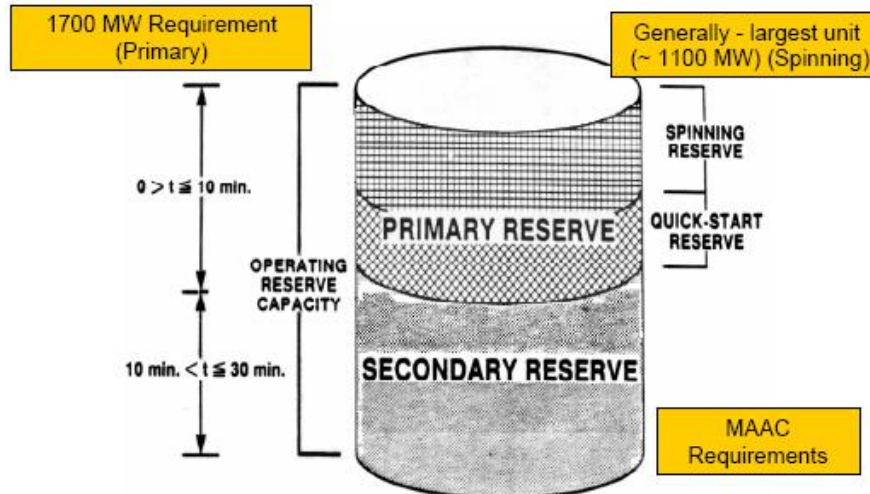
Fig. 1 [1]

2.3 Zonal constraints

Some regions within the control area, called zones, may also have spinning and operating reserve constraints, particularly if transmission interconnecting that region with the rest of the system is constrained.

2.4 Security constraints

These include constraints on branch flows under the no-contingency condition and also constraints on branch flows under a specified set of contingency conditions. The set is normally a subset of all N-1 contingencies.

2.5 Unit constraints
a. *Minimum and Maximum Generation limits*: Self explanatory.
b. *Reserve limits*: The spinning, primary, and/or secondary reserves must exceed some value, or some percentage of the load.
c. *Minimum Up/Down times*: Units that are committed must remain committed for a minimum amount of time. Likewise, units that are

de-committed must remain down for a minimum amount of time. These constraints are due to the fact that thermal units can undergo only gradual temperature changes.

d. *Hours up/down at start of study*: The problem must begin at some initial time period, and it will necessarily be the case that all of the units will have been either up or down for some number of hours at that initial time period. These hours need to be accounted for to ensure no unit is switched in violation of its minimum up/down times constraint.

e. *Must run schedules*: There are some units that are required to run at certain times of the day. Such requirements are most often driven by network security issues, e.g., a unit may be required in order to supply the reactive needs of the network to avoid voltage instability in case of a contingency, but other factors can be involved, e.g., steam supply requirements of co-generation plants.

f. *Pre-scheduled generation schedules*: There are some units that are required to generate certain amounts at certain times of the day. The simplest example of this is nuclear plants which are usually required to generate at full load all day. Import, export, and wheel transactions may also be modeled this way.

g. *Ramp Rates*: The rate at which a unit may increase or decrease generation is limited, therefore the generation level in one period is constrained to the generation level of the previous period plus the generation change achievable by the ramp rate over the amount of time in the period.

h. *Hot, Intermediate, & Cold startup costs*: A certain amount of energy must be used to bring a thermal plant on-line, and that amount of energy depends on the existing state of the unit. Possible states are: hot, intermediate, and cold. Although it costs less to start a hot unit, it is more expensive to maintain a unit in the hot state. Likewise, although it costs more to start a cold unit, it is less expensive to maintain a unit in the cold state. Whether a de-committed unit should be maintained in the hot, intermediate, or cold state, depends on the amount of time it will be off-line.

i. *Maximum starts per day and per week*: Starting a unit requires people. Depending on the number of people and the number of units at a plant, the number of times a particular unit may be started in a day, and/or in a week, is usually limited.

j. *Maximum Energy per day and per study length*: The amount of energy produced by a thermal plant over a day, or over a certain study time T, may be less than $P_{max} \times T$, due to limitations of other facilities in the plant besides the electric generator, e.g., the coal processing facilities. The amount of energy produced by a reservoir hydro plant over a time period may be similarly constrained due to the availability of water.

3.0 The UC problem (analytic statement)

The unit commitment problem is a mathematical program characterized by the following basic features.

• *Dynamic*: It obtains decisions for a sequence of time periods.

• *Inter-temporal constraints*: What happens in one time period affects what happens in another time period. So we may not solve each time period independent of solutions in other time periods.

• *Mixed Integer*: Decision variables are of two kinds:
  o Integer variables: For example, we must decide whether a unit will be up (1) or down (0). This is actually a special type of integer variable in that it is binary.
  o Continuous variables: For example, given a unit is up, we must decide what its generation level should be. This variable may be any number between the minimum and maximum generation levels for the unit.

There are many papers that have articulated an analytical statement of the unit commitment problem, more recent ones include [7, 8, 2, 3], but there are also more dated efforts that pose the problem well, although the solution method is not as effective as what we have today, an example is [4].

We provide a mathematical model of the security-constrained unit commitment problem in what follows. This model was adapted from the one given in [5, ch 1]. This model is a mixed integer linear program.

$$
\min \quad \underbrace{\sum_t \sum_i z_{it} F_{it}}_{\text{Fixed Costs}} + \underbrace{\sum_t \sum_i g_{it} C_{it}}_{\text{Production Costs}} + \underbrace{\sum_t \sum_i y_{it} S_{it}}_{\text{Startup Costs}} + \underbrace{\sum_t \sum_i x_{it} H_{it}}_{\text{Shutdown Costs}}
$$

(1)

subject to

power balance	$\sum_i g_{it} = D_t = \sum_i d_{it}$	$\forall\, t,$	(2)
reserve	$\sum_i r_{it} \geq SD_t$	$\forall\, t,$	(3)
min generation	$g_{it} \geq z_{it} MIN_i$	$\forall\, i,t,$	(4)
max generation	$g_{it} + r_{it} \leq z_{it} MAX_i$	$\forall\, i,t,$	(5)
max spinning reserve	$r_{it} \leq z_{it} MAXSP_i$	$\forall\, i,t,$	(6)
ramp rate pos limit	$g_{it} \leq g_{it-1} + MxInc_i$	$\forall\, i,t,$	(7)
ramp rate neg limit	$g_{it} \geq g_{it-1} - MxDec_i$	$\forall\, i,t,$	(8)
start if off-then-on	$z_{it} \leq z_{it-1} + y_{it}$	$\forall\, i,t,$	(9)
shut if on-then-off	$z_{it} \geq z_{it-1} - x_{it}$	$\forall\, i,t,$	(10)
normal line flow limit	$\sum_i a_{ki}(g_{it} - d_{it}) \leq MxFlow_k$	$\forall\, k,t,$	(11)
security line flow limits	$\sum_i a_{ki}^{(j)}(g_{it} - d_{it}) \leq MxFlow_k^{(j)}$	$\forall\, k,j,t,$	(12)

where the decision variables are:
- g_{it} is the MW produced by generator i in period t,
- r_{it} is the MW of spinning reserves from generator i in period t,
- z_{it} is 1 if generator i is dispatched during t, 0 otherwise,
- y_{it} is 1 if generator i starts at beginning of period t, 0 otherwise,
- x_{it} is 1 if generator i shuts at beginning of period t, 0 otherwise,

Other parameters are
- D_t is the total demand in period t,
- SD_t is the spinning reserve required in period t,
- F_{it} is fixed cost (\$/period) of operating generator i in period t,

- C_{it} is prod. cost (\$/MW/period) of operating gen i in period t;
- S_{it} is startup cost (\$) of starting gen i in period t.
- H_{it} is shutdown cost (\$) of shutting gen i in period t.
- $MxInc_i$ is max ramprate (MW/period) for increasing gen i output
- $MxDec_i$ is max ramprate (MW/period) for decreasing gen i output
- a_{ij} is linearized coefficient relating bus i injection to line k flow
- $MxFlow_k$ is the maximum MW flow on line k
- $a_{ki}^{(j)}$ is linearized coefficient relating bus i injection to line k flow under contingency j,
- $MxFlow_k^{(j)}$ is the maximum MW flow on line k under contingency j

The above problem statement is identical to the one given in [5] with the exception that here, we have added eqs. (11) and (12).
➔The addition of eq. (11) alone provides that this problem is a transmission-constrained unit commitment problem.
➔ The addition of eqs. (11) and (12) together provides that this problem is a security-constrained unit commitment problem.

One should note that our problem is entirely linear in the decision variables. Therefore this problem is a *linear* mixed integer program, and it can be compactly written as

$$\min \underline{c}^T \underline{x}$$

Subject to

$$\underline{A}\underline{x} \leq \underline{b}$$

There have four basic solution methods used in the past few years:
- Priority list methods
- Dynamic programming
- Lagrangian relaxation
- Branch and bound

The last method, branch and bound, is what the industry means when it says "MIP." It is useful to understand that the chosen method can have very large financial implications. This point is well-made in the chart [6] of Fig. 2.

	Current Approach	Planned Approach	Date of Planned Implementation of MIP	Estimated Annual Savings
Real-time market look ahead	LR used for 2 hour look ahead commitment and dispatch	MIP: 2 hour look ahead for dispatch. As long as 5 hours for commitment .	April 1, 2008	~$100,000-$1 million (0.1%-1%1 of 2006 RT Dispatch Costs and RT RMR Costs2: $97 million)
Residual unit commitment	Procedural based operator judgement advised by a MIP based UC with no network	Run a MIP, Full Network Model based on Residual Unit Commitment after Day-Ahead bid market.	April 1, 2008	~$100,000-$1 million (based on 0.1% - 1% of Total Minimum Load Costs for 2006: $106 million)
day-ahead market	Linear Programing: No unit commitment, No Energy Optimziation, Allocation of Transmission only using zonal model	Run a MIP based SCUC/SCED, Full Network Model program, Energy and A/S co-optimized	April 1, 2008	~$2.3-$23 million (Assumes an estimated 0.1%-1% reduction of $11.4 billion Energy and Ancillary Service)
Capacity market	None	Policy being considered	Policy being considered	No Estimate
Ancillary service market	Linear Programming sequential procured after Transmission Allocation	Run a MIP based SCUC/SCED, Full Network Model program co-optimized with energy	April 1, 2008	~$230,000-$2.3 million (0.1%-1%[1] of 2006 A/S costs[2] of $234 million)
planning	Powerflow studies	No immediate plans to incorporate MIP	No immediate plans to incorporate MIP	No Estimate

Fig. 2

4.0 UC and Day-ahead market

The main tool used to implement the day-ahead-markets (DAM) is the security-constrained unit commitment program, or SCUC. In this section, we review some basics about the DAM by looking at some descriptions given by a few industry authors. You are encouraged to review the papers from which these quotes were taken. Notice that any references made inside the quotations are given only in the bibliography of the subject paper and not in the bibliography of these notes. References made outside of the quotations are given in the bibliography of these notes.

4.1 Paper by Chow & De Mello:
Reference [7] offers an overall view of the sequence of functions used by an ISO, as given in Fig. 3. Observe that the "day-ahead

scheduling" and the "real time commitment and dispatch" both utilize the SCUC.
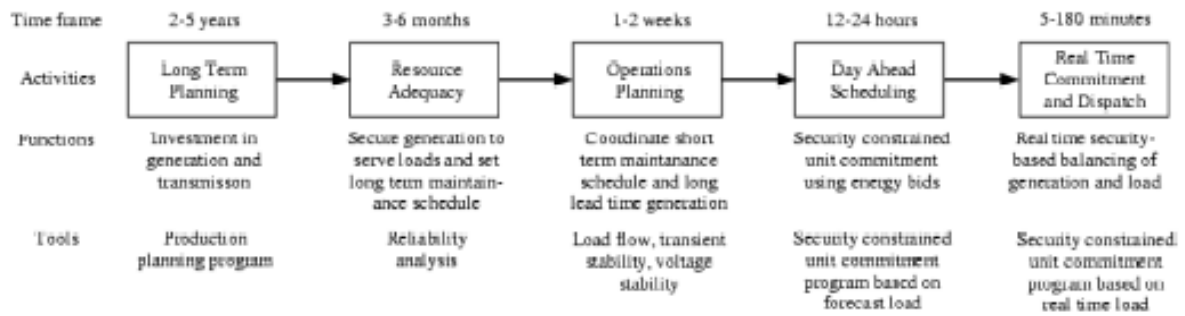

Fig. 3

They state:

"Electricity is a commodity that cannot be effectively stored and the energy-supplying generators have limits on how quickly they can be started and ramped up or down. As a result, both the supply and demand become more inelastic and the electricity market becomes more volatile and vulnerable as it gets closer to real time [34]. To achieve a stable margin as well as to maintain the system reliability, a forward market is needed to provide buyers and sellers the opportunity to lock in energy prices and quantities and the ISO to secure adequate resources to meet predicted energy demand well in advance of real time. Thus architecturally, many ISOs (e.g. PJM, ISO New England, New York ISO) take a multisettlement approach for market design…."

"The two main energy markets, each producing a financial settlement, in a multisettlement system, are the following.
1) DAM: schedules resources and determines the LMPs for the 24 h of the following day based on offers to sell and bids to purchase energy from the market participants.
2) Real-time market: optimizes the clearing of bids for energy so that the real-time system load matching and reliability requirements are satisfied based on actual system operations. LMPs are computed for settlement at shorter intervals, such as 5–10 min…."

"Fig. 6 shows the timeline of the multiple-settlement systems used in NYISO, PJM, and ISO-NE, which are typical of those used in practice. Supply and demand bids are submitted for the DAM, typically 12–24 h ahead of the real-time operation. Then the day-ahead energy prices are computed and posted, 6–12 h ahead of real-time operation…."

"The DAM typically consists of supply and demand bids on an hourly basis, usually from midnight to the following midnight. The supply bids include generation supply offers with start-up and no-load costs, incremental and decremental bids[1], and external transactions schedules. The demand bids are submitted by loads individually or collectively through load-serving entities. In scheduling the supply to meet the demand, all the operating constraints such as transmission network constraints, reserve requirements, and external transmission limits must not be violated. This process is commonly referred to as an SCUC problem, which is to determine hourly commitment schedules with the objective of minimizing the total cost of energy, start-up, and spinning at no-load while observing transmission constraints and physical resources' minimum runtime, minimum downtime, equipment ramp rates, and energy limits of energy-constrained resources. Based on the commitment schedules for physical resources, SCUC is used to clear energy supply offers, demand bids, and transaction schedules, and to determine LMPs and their components at all defined price nodes including the hubs, zones, and aggregated price nodes for the DAM settlement. The SCUC problem is usually optimized using a Lagrangian relaxation (LR) or a mixed-integer programming (MIP) solver…."

---

[1] Decremental bids are similar to price-sensitive demand bids. They allow a marketer or other similar entity without physical demand to place a bid to purchase a certain quantity of energy at a certain location if the day-ahead price is at or below a certain price. Incremental offers are the flip side of decremental bids. Usually, a decremental bid is a fee paid by suppliers to the ISO when it no longer requires the full amount of energy previously contracted for, due to congestion. The ISO must purchase electricity elsewhere to make up the shortfall, and the generator reimburses the ISO. A bilateral generator with a decremental bid is saying: "Schedule me as a bilateral, must-run plant unless the spot price falls to (or below) my bid. In that event, don't schedule me as must run; I will supply my bilateral load from the spot market."

"A critical part of the DAM is the bid-in loads, which is a day-ahead forecast of the real-time load. The load estimate depends on the season, day type (weekday, weekend, holiday), and hour of the day. Most ISOs have sophisticated load forecasting programs, some with neural network components [36], [37], to predict the day-ahead load to within 3%−5% accuracy and the load forecasts are posted. LSEs with fully hedged loads through long-term bilateral contracts tend to bid in the amount corresponding to the ISO predicted loads. Some other LSEs may bid in loads that are different from those posted by the ISO. In such cases, if the LSE bid load exceeds the ISO load, the LSE bid load is taken as the load to be dispatched. Otherwise, the ISO load will supersede the LSE bid load and the SCUC will commit generators to supply the ISO forecasted load in a reliability stage. Then the generation levels of the committed generators will be allocated to supply LSE bid loads. Committing extra generators outside the DAM will be treated as uplifts and be paid by the LSEs…."

4.2 Paper by Papalexopoulos:
Reference [8] states:
"The Must Offer Waiver (MOW) process is basically a process of determining which Must Offer units should be committed in order to have enough additional capacity to meet the system energy net short which is the difference between the forecast system load and the Day-Ahead Market energy schedules. This commitment process ensures that the resulting unit schedule is feasible with respect to network and system resource constraints. Mathematically, this can be stated as a type of a SCUC problem [3]. The objective is to minimize the total start up and minimum load costs of the committed units while satisfying the power balance constraint, the transmission interface constraints, and the system resource constraints, including unit inter-temporal constraints…."

"The most popular algorithms for the solutions of the unit commitment problems are Priority-List schemes [4], Dynamic Programming [5], and Mixed Integer Linear Programming [6]. Among these approaches the MILP technique has achieved significant progress in the recent years [7]. The MILP methodology has been applied to the SCUC formulation to solve this MOW problem. Recent developments in the implementation of MILP-based algorithms and careful attention to the specific problem formulation have made it possible to meet accuracy and performance requirements for solving such large scale problems in a practical competitive energy market environment. In this section the MILP-based SCUC formulation is presented in detail…."

4.3 Paper by Ott:

Reference [9] states:
"In addition to the LMP concept, the fundamental design objectives of the PJM day-ahead energy market are: 1) to provide a mechanism in which all participants have the opportunity to lock in day-ahead financial schedules for energy and transmission; 2) to coordinate the day-ahead financial schedules with system reliability requirements; 3) to provide incentive for resources and demand to submit day-ahead schedules; and 4) to provide incentive for resources to follow real-time dispatch instructions…."

4.4 Paper by AREVA and PJM:

Reference [10] states:
"As the operator of the world's largest wholesale market for electricity, PJM must ensure that market-priced electricity flows reliably, securely and cost-effectively from more than 1100 Generating resources to serve a peak load in excess of 100,000 MW. In doing so, PJM must balance the market's needs with thousands of reliability-based constraints and conditions before it can schedule

and commit units to generate power the next day. The PJM market design is based on the Two Settlement concept [4]. The Two-Settlement System provides a Day-ahead forward market and a real-time balancing market for use by PJM market participants to schedule energy purchases, energy sales and bilateral contracts. Unit commitment software is used to perform optimal resource scheduling in both the Day-ahead market and in the subsequent Reliability Analysis…."

"As the market was projected to more than double its original size, PJM identified the need to develop a more robust approach for solving the unit commitment problem. The LR algorithm was adequate for the original market size, but as the market size increased, PJM desired an approach that had more flexibility in modeling transmission constraints. In addition, PJM has seen an increasing need to model Combined-cycle plant operation more accurately. While these enhancements present a challenge to the LR formulation, the use of a MIP formulation provides much more flexibility. For these reasons, PJM began discussion with its software vendors, in late 2002, concerning the need to develop a production grade MIP-based approach for large-scale unit commitment problems…."

"The Day-ahead market clearing problem includes next-day generation offers, demand bids, virtual bids and offers, and bilateral transactions schedules. The objective of the problem is to minimize costs subject to system constraints. The Day-ahead market is a financial market that provides participants an operating plan with known compensation: If their generation (or load) is the same in the real-time market, their revenue (or cost) is the same. Compensation for any real-time deviations is based on real-time prices, providing participants with opportunities to improve profit (or reduce cost) if they have flexibility to adjust their schedules…."

"In both problems, unit commitment accepts data that define bids (e.g., generator constraints, generator costs, and costs for other resources) and the physical system (e.g., load forecast, reserve requirements, security constraints). In real time, the limited responsiveness of units and additional physical data (e.g., state estimator solution, net-interchange forecast) further constrains the unit commitment problem."

"The Unit Commitment problem is a large-scale non-linear mixed integer programming problem. Integer variables are required for modeling: 1) Generator hourly On/Off-line status, 2) generator Startups/Shutdowns, 3) conditional startup costs (hot, intermediate & cold). Due to the large number of integer variables in this problem, it has long been viewed as an intractable optimization problem. Most existing solution methods make use of simplifying assumptions to reduce the dimensionality of the problem and the number of combinations that need to be evaluated. Examples include priority-based methods, decomposition schemes (LR) and stochastic (genetic) methods. While many of these schemes have worked well in the past, there is an increasing need to solve larger (RTO-size) problems with more complex (e.g. security) constraints, to a greater degree of accuracy. Over the last several years, the number of units being scheduled by RTOs has increased dramatically. PJM started with about 500 units a few years ago, and is now clearing over 1100 each day. MISO cases will be larger still…."

"The classical MIP implementation utilizes a Branch and Bound scheme. This method attempts to perform an implicit enumeration of all combinations of integer variables to locate the optimal solution. In theory, the MIP is the only method that can make this claim. It can, in fact, solve non-convex problems with multiple local minima. Since the MIP methods utilize multiple Linear Programming (LP) executions, they have benefited from recent advances in both computer hardware and software [6]…"

"This section presents results from using the CPLEX 7.1 and CPLEX 9.0 MIP solvers on a large-scale RTO Day Ahead Unit Commitment problem. This problem has 593 units and a 48 hour time horizon…."

The below reference provides a brief description of the Midwest ISO's current implementation:

> M. Tackett, "Experience with implementing simultaneous co-optimization in the midwest ISO energy and operating reserve markets," Power Systems Conference and Exposition, 2009. PSCE '09. IEEE/PES.

"The Midwest ISO will operate a Day-Ahead Energy and Operating Market, a Reliability Assessment Commitment process and a Real-Time Energy and Operating Reserve Market.

- The Day-Ahead Energy and Operating Reserve Market is a financially binding market that clears energy, regulating reserve, spinning reserve and supplemental reserve on an hourly basis.

- The Reliability Assessment Commitment (RAC) process is a process to commit resources, schedule regulating reserve on committed resources and/or release emergency operating ranges on resources when appropriate on an hourly basis for use in the Real-Time Energy and Operating Reserve Market. The RAC process can be executed on a multi-day, day-ahead and/or intra-day basis.

- The Real-Time Energy and Operating Reserve Market is a financially and physically binding market that clears energy, regulating reserve, spinning reserve and supplemental reserve on a five-minute basis.

Three functions:
-DAM/ORM
-RAC
-RTM/ORM

The Midwest ISO will utilize a simultaneously co-optimized Security Constrained Unit Commitment (SCUC) algorithm and a simultaneously co-optimized Security Constrained Economic Dispatch (SCED) algorithm to operate the Day-Ahead Energy and Operating Reserve Market. The simultaneously co-optimized SCUC algorithm is used in the Day-Ahead Energy and Operating Reserve Market to commit resources, schedule regulating reserves on committed resources and/or release emergency operating ranges on resources in the Day-Ahead Energy and Operating Reserve Market. The simultaneously co-optimized SCED algorithm is used in the Day-Ahead Energy and Operating Reserve Market to clear and price energy, regulating reserve, spinning reserve

DAM/ORM require SCUC

DAM/ORM require SCED for hourly.

and supplemental reserve on an hourly basis. Demand curves are utilized to price Energy and Operating Reserve during times of scarcity.

The Midwest ISO will utilize a simultaneously co-optimized Security Constrained Unit Commitment (SCUC) algorithm to implement the RAC process and a simultaneously co-optimized Security Constrained Economic Dispatch (SCED) algorithm to operate the Real-Time Energy and Operating Reserve Market. The simultaneously cooptimized SCUC algorithm is used in the RAC process to commit resources, schedule regulating reserves on committed resources and/or release emergency operating ranges on resources for the Real-Time Energy and Operating Reserve Market. The simultaneously co-optimized SCED algorithm is used in the Real-Time Energy and Operating Reserve Market to dispatch and price energy, regulating reserve, spinning reserve and supplemental reserve on a five-minute basis. Demand curves are utilized to price Energy and Operating Reserve during times of scarcity.

The SCUC algorithms used in the Day-Ahead Energy and Operating Reserve Market and the RAC process incorporate Mixed Integer Programming (MIP) solvers to commit resources, schedule regulating reserve on resources and release emergency operating ranges on resources (minimum or maximum) when inadequate capacity exists to meeting energy demand plus operating reserve requirements. The SCED algorithms used in the Day-Ahead Energy and Operating Reserve Market and the Real-Time Energy and Operating Reserve Market use Linear Programming (LP) solvers to clear and price energy, regulating reserve, spinning reserve and supplemental reserve in a manner that minimizes production costs.

In both the Day-Ahead and Real-Time Energy and Operating Reserve Markets, reserve requirement constraints are modeled against cumulative reserve requirements to ensure operating reserve pricing is consistent with operating reserve priority for each of the three operating reserve products. Reserve Zones are also utilized to ensure dispersion of operating reserve throughout the market in a manner that allows for deliverability and good utility practice. Reserve zones are established quarterly and reserve zone requirements are updated daily based on the results of off-line studies."

RAC requires SCUC.

RTM/ORM requires SCED.

Some good description of the Midwest ISO's reliability assessment commitment (RAC) is found in the below paper:

Xingwang Ma, Yonghong Chen, Jie Wan, "Midwest ISO Co-Optimization Based Real-Time Dispatch And Pricing of Energy and Ancillary Services," 2009.

"Real-time grid reliability is at the center of Midwest ISO's cooptimized energy and AS design. While resource schedules are cleared as financially, not physically binding, the day-ahead market is critically linked to real-time operation through the reliability assessment commitment (RAC) and the two-settlement mechanism that guarantees resource adequacy for reliability and enables participants to arbitrage price differences between dayahead and real-time markets respectively. The DA market cleared financially binding resource schedules form the basis for the DA RAC by which sufficient resources are committed using the security-constrained unit commitment (SCUC) to meet Midwest ISO's demand forecasts and AS requirements subject to transmission limits. The DA RAC resource commitment schedules make the operating plan for the next day. During the operating day, more accurate information about demand forecasts, net scheduled interchanges (NSI) and transmission limitations is available; the RAC algorithm may be executed several times during the operating day, called intra-day (ID) RAC process, to further update the operating plan. The intra-day operating plan updates allow Midwest ISO operations to prepare sufficient resources at the right locations to manage load-generation-NSI balances and transmission congestions under normal and emergency conditions. With this integrated market-driven scheduling process, Midwest ISO uses the security-constrained economic dispatch (SCED) to achieve real-time reliable grid operation at the lowest costs. Energy deliveries and AS dispatches are priced based on actual system conditions after the fact."

RAC links DAM to real-time operations.

RAC is used to update the DA-schedules as new info becomes available.

5.0 Illustrations by MIP

Here, we provide some data to use in solving our UC problem.
We illustrate using an example that utilizes the same system we have been using in our previous notes, where we had 3 generator buses in a 4 bus network supplying load at 2 different buses, but this time we will model each generator with the ability to submit 3 offers.
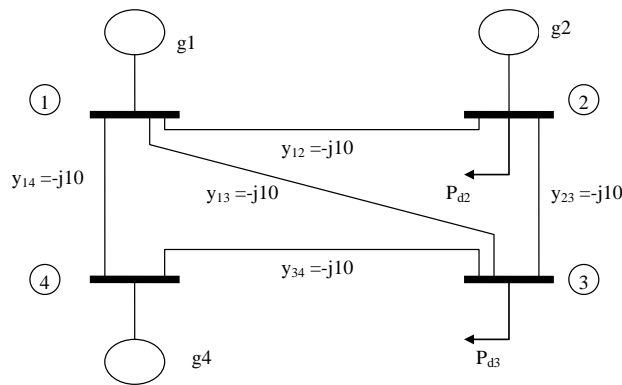

Fig. 4: One line diagram for example system

The offers, in terms of fixed costs, production costs, and corresponding min and max generation limits are as follows

Production costs (in $/pu-hr):

Unit, k	Fixed costs ($/hr)	Startup Costs ($)	Shutdown Costs ($)	Production Costs ($/pu-hr) g_{k1t}	g_{k2t}	g_{k4t}
1	50	100	20	1246	1307	1358
2	50	100	20	1129	1211	1282
4	50	100	20	1183	1254	1320

Notice that for each unit, the offers increase with generation, i.e., $g_{k1t} < g_{k2t} < g_{k3t}$. This prevents use of a higher generation level before a lower generation level. It also says that our offer function is convex.

The constraints on the offers are given below.

$$
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} g_{11t} \\ g_{12t} \\ g_{13t} \\ g_{21t} \\ g_{22t} \\ g_{23t} \\ g_{41t} \\ g_{42t} \\ g_{43t} \end{bmatrix} \leq \begin{bmatrix} 0.50 \\ 0.60 \\ 0.40 \\ 0.35 \\ 0.60 \\ 0.20 \\ 0.45 \\ 0.50 \\ 0.40 \end{bmatrix}, \forall t
$$

The UC problem is for a 24 hour period, with loading data given as below. Figure 5, the load curve, illustrates variation of load with time over the 24 hour period.

Hour, t	Load, D_t (pu)
1	1.50
2	1.40
3	1.30
4	1.40
5	1.70
6	2.00
7	2.40
8	2.80
9	3.20
10	3.30
11	3.30
12	3.20
13	3.20
14	3.30
15	3.35
16	3.40
17	3.30
18	3.30
19	3.20
20	2.80
21	2.30
22	2.00
23	1.70
24	1.60

Notice between the hours of t=20 and t=21 that the load drops 0.5 pu. We must have the reserves available to handle such a drop!
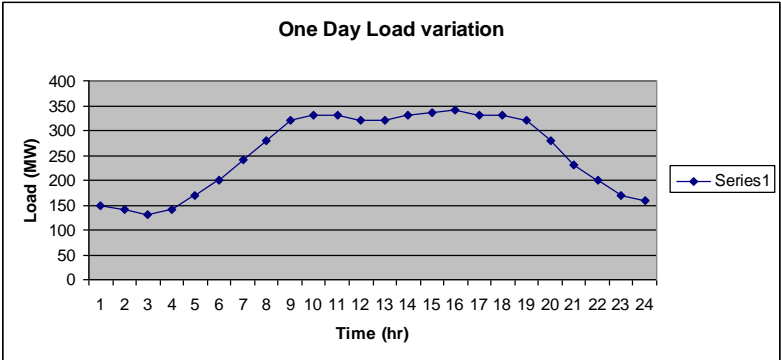
Fig. 5: Load curve

5.1 Example – 4 hours

For this solution, we will only include startup and shutdown constraints. In order to illustrate all data entered, we will analyze only the first four hours. The CPLEX code to do this is given below.

```
minimize
  50 z11 +50 z12 + 50 z13 + 50 z14
 +50 z21 +50 z22 + 50 z23 +50 z24
 +50 z41 +50 z42 + 50 z43 +50 z44
 +1246 g111 + 1307 g121 + 1358 g131
  +1129 g211 + 1211 g221 + 1282 g231
  +1183 g411 + 1254 g421 + 1320 g431
 +1246 g112 + 1307 g122 + 1358 g132
  +1129 g212 + 1211 g222 + 1282 g232
  +1183 g412 + 1254 g422 + 1320 g432
 +1246 g113 + 1307 g123 + 1358 g133
  +1129 g213 + 1211 g223 + 1282 g233
  +1183 g413 + 1254 g423 + 1320 g433
 +1246 g114 + 1307 g124 + 1358 g134
  +1129 g214 + 1211 g224 + 1282 g234
  +1183 g414 + 1254 g424 + 1320 g434
 +100 y12 + 100 y13 +100 y14
 +100 y22 + 100 y23 +100 y24
 +100 y42 + 100 y43 +100 y44
 +20 x12 + 20 x13 +20 x14
 +20 x22 + 20 x23 + 20 x24
 +20 x42 + 20 x43 +20 x44
subject to
 loadhr1: g111+g121+g131+g211+g221+g231+g411+g421+g431=1.5
 loadhr2: g112+g122+g132+g212+g222+g232+g412+g422+g432=1.4
 loadhr3: g113+g123+g133+g213+g223+g233+g413+g423+g433=1.3
 loadhr4: g114+g124+g134+g214+g224+g234+g414+g424+g434=1.4
 initialu1: z11=0
 initialu2: z21=1
 initialu4: z41=1
 starthr21u1: z12-z11-y12<=0
 starthr32u1: z13-z12-y13<=0
 starthr43u1: z14-z13-y14<=0
 starthr21u2: z22-z21-y22<=0
 starthr32u2: z23-z22-y23<=0
 starthr43u2: z24-z23-y24<=0
 starthr21u4: z42-z41-y42<=0
 starthr32u4: z43-z42-y43<=0
 starthr43u4: z44-z43-y44<=0
 shuthr21u1: z12-z11+x12>=0
 shuthr32u1: z13-z12+x13>=0
 shuthr43u1: z14-z13+x14>=0
 shuthr21u2: z22-z21+x22>=0
 shuthr32u2: z23-z22+x23>=0
 shuthr43u2: z24-z23+x24>=0
 shuthr21u4: z42-z41+x42>=0
 shuthr32u4: z43-z42+x43>=0
 shuthr43u4: z44-z43+x44>=0
```

Fixed costs.

Production costs.

Startup costs.

Shutdown costs.

Power balance constraint for each hour.

Initial conditions

Constraints associated with starting. For example, $Z_{12} \leq Z_{11} + Y_{12}$, or more generally, $Z_{kt} \leq Z_{k,t-1} + Y_{kt}$, which says

Status of unit k in time t ≤status of unit k in time t-1+start flag in time t

Constraints associated with shutting. For example, $Z_{12} \geq Z_{11} - X_{12}$, or more generally, $Z_{kt} \geq Z_{k,t-1} - X_{kt}$, which says

Status of unit k in time t ≥status of unit k in time t-1-shut flag in time t

g111 - 0.5 z11<= 0
g112 - 0.5 z12<= 0
g113 - 0.5 z13<= 0
g114 - 0.5 z14<= 0
g121 - 0.6 z11<= 0
g122 - 0.6 z12<= 0
g123 - 0.6 z13<= 0
g124 - 0.6 z14<= 0
g131 - 0.4 z11<= 0
g132 - 0.4 z12<= 0
g133 - 0.4 z13<= 0
g134 - 0.4 z14<= 0

Constraints relating generation values for unit 1 in time period t to on-off status of unit 1 in time period t.
- If $z_{1t}=0$, then generation value g_{1jt} must also be zero.
- If $z_{1t}=1$, then generation value g_{1jt} must be $\leq$ maximum value for the offer.

g211 - 0.35 z21<= 0
g212 - 0.35 z22<= 0
g213 - 0.35 z23<= 0
g214 - 0.35 z24<= 0
g221 - 0.6 z21<= 0
g222 - 0.6 z22<= 0
g223 - 0.6 z23<= 0
g224 - 0.6 z24<= 0
g231 - 0.2 z21<= 0
g232 - 0.2 z22<= 0
g233 - 0.2 z23<= 0
g234 - 0.2 z24<= 0

Constraints relating generation values for unit 2 in time period t to on-off status of unit 2 in time period t.
- If $z_{1t}=0$, then generation value g_{2jt} must also be zero.
- If $z_{1t}=1$, then generation value g_{2jt} must be $\leq$ maximum value for the offer.

g411 - 0.45 z41<= 0
g412 - 0.45 z42<= 0
g413 - 0.45 z43<= 0
g414 - 0.45 z44<= 0
g421 - 0.5 z41<= 0
g422 - 0.5 z42<= 0
g423 - 0.5 z43<= 0
g424 - 0.5 z44<= 0
g431 - 0.4 z41<= 0
g432 - 0.4 z42<= 0
g433 - 0.4 z43<= 0
g434 - 0.4 z44<= 0

Constraints relating generation values for unit 3 in time period t to on-off status of unit 4 in time period t.
- If $z_{1t}=0$, then generation value g_{4jt} must also be zero.
- If $z_{1t}=1$, then generation value g_{4jt} must be $\leq$ maximum value for the offer.

Bounds
 0<= g111
 0<= g112
 0<= g113
 0<= g114
 0<= g121
 0<= g122
 0<= g123
 0<= g124
 0<= g131
 0<= g132
 0<= g133
 0<= g134

 0<= g211
 0<= g212
 0<= g213
 0<= g214
 0<= g221
 0<= g222
 0<= g223
 0<= g224
 0<= g231
 0<= g232
 0<= g233
 0<= g234

 0<= g411
 0<= g412
 0<= g413
 0<= g414
 0<= g421
 0<= g422
 0<= g423
 0<= g424
 0<= g431
 0<= g432
 0<= g433
 0<= g434
Integer
 z11 z12 z13 z14
 z21 z22 z23 z24
 z41 z42 z43 z44
 y12 y13 y14
 y22 y23 y24
 y42 y43 y44
 x12 x13 x14
 x22 x23 x24
 x42 x43 x44
end

Result: CPLEX gives an objective function value of 7020.7 \$.

CPLEX> display solution variables -
Variable Name   Solution Value
z21   1.000000
z22   1.000000
z23   1.000000
z24   1.000000
z41   1.000000
z42   1.000000
z43   1.000000
z44   1.000000
g211   0.350000
g221   0.600000
g411   0.450000
g421   0.100000 | Offer 4,2 is the marginal unit in time period 1. |
g212   0.350000
g222   0.600000 | Offer 2,2 is the marginal unit in time period 2. |
g412   0.450000
g213   0.350000
g223   0.500000 | Offer 2,2 is the marginal unit in time period 3. |
g413   0.450000
g214   0.350000
g224   0.600000 | Offer 2,2 is the marginal unit in time period 4. |
g414   0.450000
All other variables in the range 1-66 are 0.

Note that all y- and x-variables are zero, therefore there is no starting up or shutting down.
One should check that the generation in each hour equals the demand in that hour:
g211+g221+g411+g421=0.35+0.6+0.45+0.1=1.5
g212+g222+g412=0.35+0.6+0.45=1.4
g213+g223+g413=0.35+0.5+0.45=1.3
g214+g224+g414=0.35+0.6+0.45=1.4

This very simple solution was obtained as a result of the fact that the initial solution of

initialu1: z11=0
initialu2: z21=1
initialu4: z41=1

was in fact the best one for the initial loading condition, and since the loading condition hardly changed during the first four hours, there was no reason to change any of the units.

Let's try a different initial condition:

initialu1: z11=1
initialu2: z21=0
initialu4: z41=1

Result: CPLEX gives an objective function value of 7208.9 $.

CPLEX> display solution variables -

Variable Name	Solution Value	
z11	1.000000	
z22	1.000000	
z23	1.000000	
z24	1.000000	
z41	1.000000	
z42	1.000000	
z43	1.000000	
z44	1.000000	
g111	0.500000	
g121	0.050000	
g411	0.450000	
g421	0.500000	
g212	0.350000	What happened here relative to hour 1?
g222	0.600000	
g412	0.450000	
g213	0.350000	Now unit 2 backs off a bit. What is the marginal unit?
g223	0.500000	
g413	0.450000	
g214	0.350000	
g224	0.600000	
g414	0.450000	
y22	1.000000	
x12	1.000000	

All other variables in the range 1-66 are 0.
Why was this solution more expensive?

➔ Because we initialized the solution with more expensive units, to get back to the less expensive solution, notice that the program forces unit 2 to start up ($y_{22}=1$) and unit 1 to shut down ($x_{12}=1$) at the beginning of period 2. Apparently, the additional cost of starting unit 2 ($100) and shutting down unit 1 ($20) was less than the

savings associated with running the more efficient unit (unit 2) over the remaining three hours of the simulation, and so the program ordered starting of unit 2 and shutting down unit 1.

Let's test our theory by increasing the startup costs of unit 2 from $100 to $10,000. The objective function value in this case is $7281.25 (higher than the last solution). The decision variables are:

Variable Name	Solution Value
$z11$	1.000000
$z12$	1.000000
$z13$	1.000000
$z14$	1.000000
$z41$	1.000000
$z42$	1.000000
$z43$	1.000000
$z44$	1.000000
$g111$	0.500000
$g121$	0.050000
$g411$	0.450000
$g421$	0.500000
$g112$	0.500000
$g412$	0.450000
$g422$	0.450000
$g113$	0.500000
$g413$	0.450000
$g423$	0.350000
$g114$	0.500000
$g414$	0.450000
$g424$	0.450000

All other variables in the range 1-66 are 0.

We observe that unit 1 was on-line the entire four hours, i.e, there was no switching, something we expected since the start-up cost of unit 2 was so very high.

5.2 Example – 24 hours

We refrain from providing the data in this case because it is extensive, having 426 variables:

     72 z-variables

     69 y-variables

     69 x-variables

     216 g-variables

Rather, we have posted the dataset on the web page under "UC24 Data."

The solution was initialized at

initialu1: z11=0
initialu2: z21=1
initialu4: z41=1

which is the most economic solution for this loading level.

The output is most easily analyzed by using

"display solution variables -"

and then searching the output variables for y-variables and/or x-variables that are listed (and therefore 1). These variables indicate changes in the unit commitment. In studying the load curve, what kind of changes do you expect?

The result, objective value=$77667.3, shows that the only x and y variables that are non-zero are y1,8 and x1,21. This means that the changes in the unit commitment occur only for unit 1 and only at hours 8 and 21. A pictorial representation of the unit commitment through the 24 hour period is shown below.


One Day Load variation

Unit 1

Unit 2

Unit 3
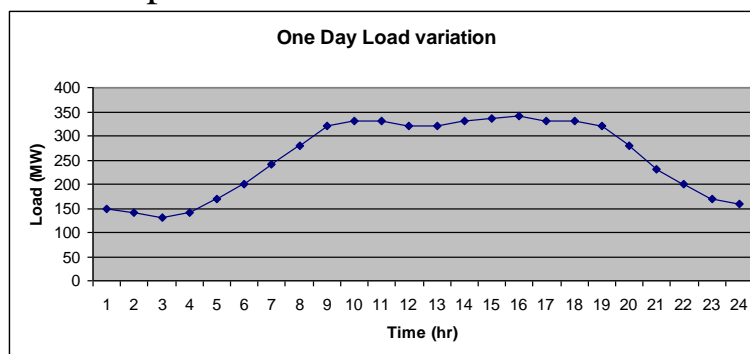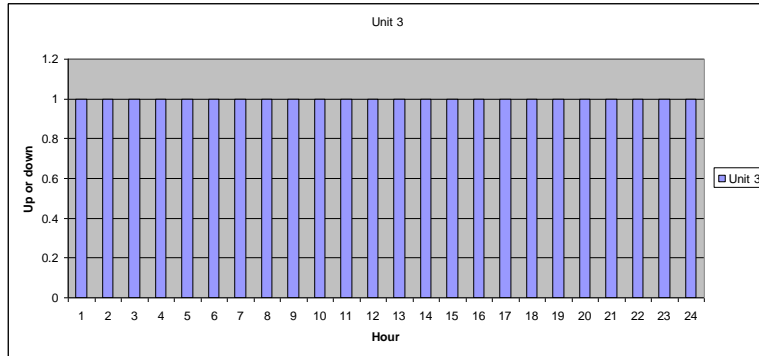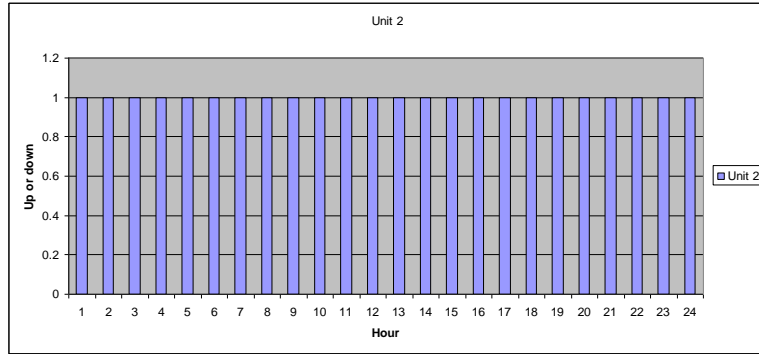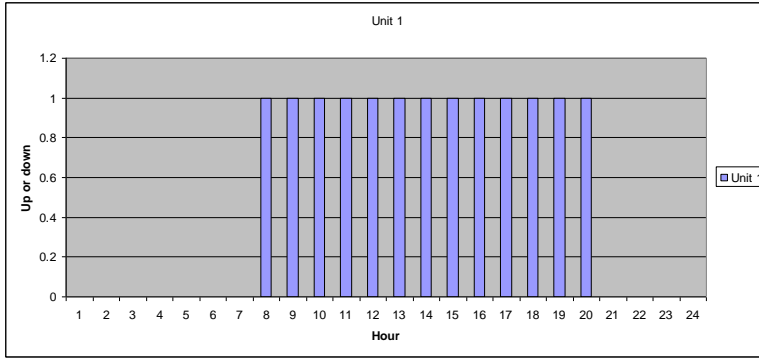
To perform additional investigation, the load curve was modified as shown below (UC24a.lp). All other data remained as before. The result, with objective function value of $?, shows that the only x and y variables that are non-zero are y1,8, x1,20, and x4,24. A pictorial representation of the UC through the 24 hour period is shown below.

One day load variation

Unit 1

Unit 2

Unit 3
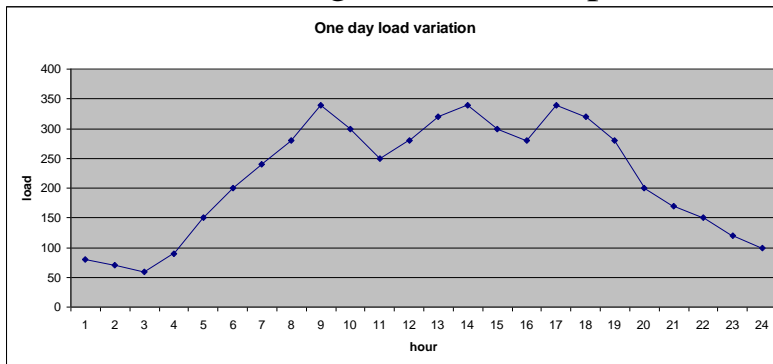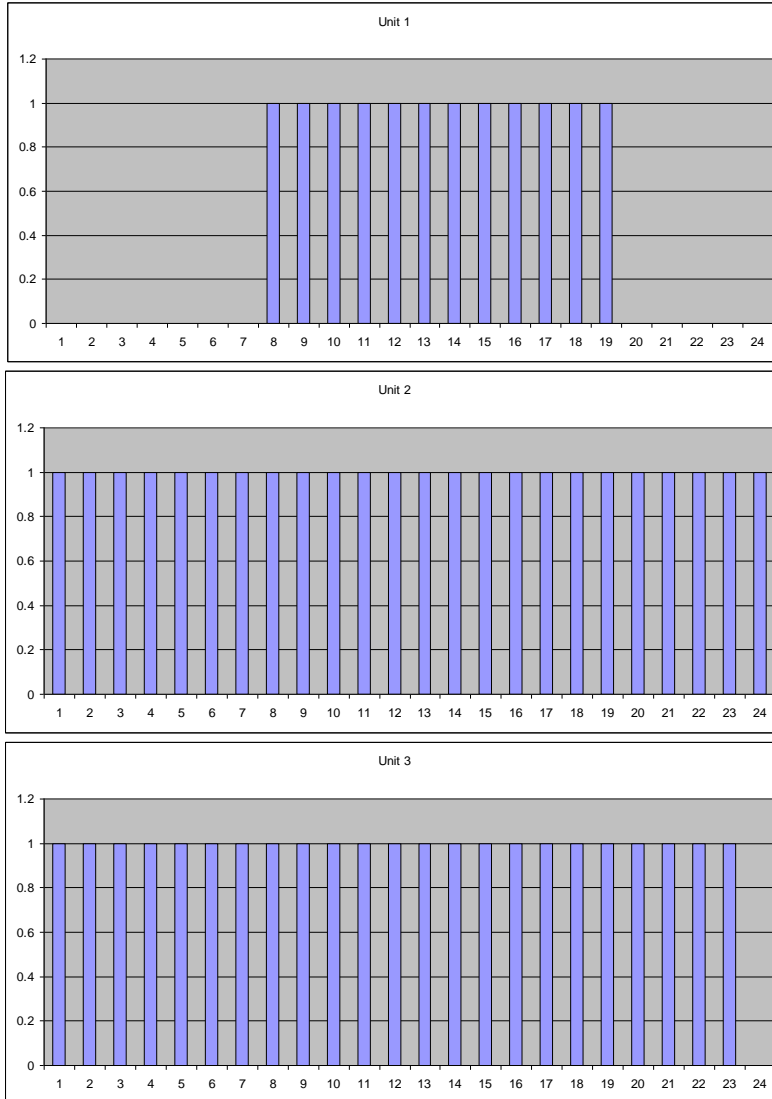
In a last investigation, the load curve remained modified, and startup costs were reduced to $10, shutdown costs reduced to $2. All other data remained as before (UC24b.lp). The result, with objective function value of $66,867.95, shows that the only x and y variables that are non-0 are y1,8, y1,12, y4,5, x1,11, x1,20, x4,2, x4,24. A pictorial representation of the UC through the 24 hour period is shown below.

One day load variation

Unit 1

Unit 2

Unit 3
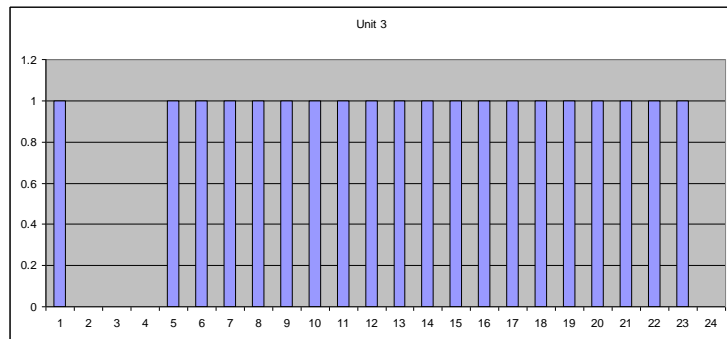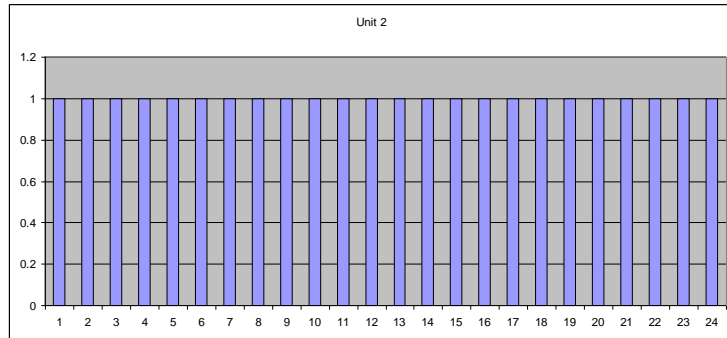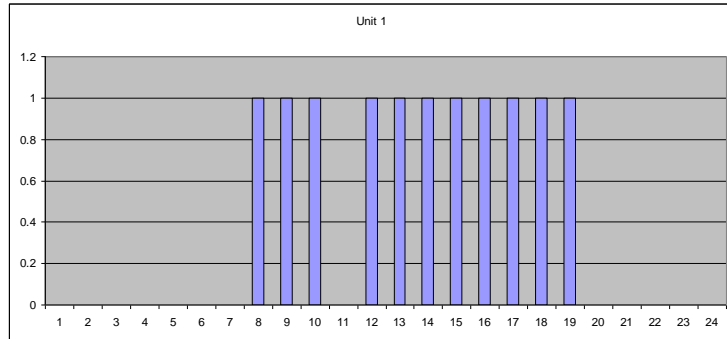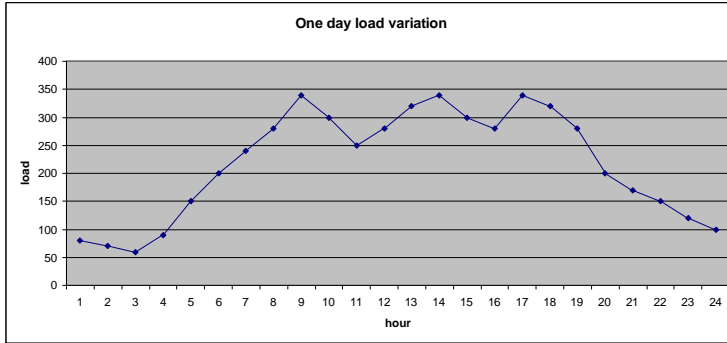
[1] "PJM Emergency Procedures,"
www.pjm.com/etools/downloads/edart/edart-training-pres/edart-training-instantaneous-revserve-check.pdf.

[2] H. Pinto, F. Magnago, S. Brignone, O. Alsaç, B. Stott, "Security Constrained Unit Commitment: Network Modeling and Solution Issues," Proc. of the 2006 IEEE PES Power Systems Conference and Exposition, Oct. 29 2006-Nov. 1 2006, pp. 1759 – 1766.

[3] R. Chhetri, B. Venkatesh, E. Hill, "Security Constraints Unit Commitment for a Multi-Regional Electricity Market," Proc. of the 2006 Large Engineering Systems Conference on Power Engineering, July 2006, pp. 47 – 52.

[4] J. Guy, "Security Constrained Unit Commitment," IEEE Transactions on Power Apparatus and Systems Vol. PAS-90, Issue 3, May 1971, pp. 1385-1390.

[5] B. Hobbs, M. Rothkopf, R. O'Neill, and H. Chao, editors, "The Next Generation of Electric Power Unit Commitment Models," Kluwer, 2001.

[6] M. Rothleder, presentation to the Harvard Energy Policy Group, Dec 7, 2007.

[7] J. Chow, R. De Mello, K. Cheung, "Electricity Market Design: An Integrated Approach to Reliability Assurance," Proceedings of the IEEE, Vol. 93, No. 11, November 2005.

[8] Q. Zhou, D. Lamb, R. Frowd, E. Ledesma, A. Papalexopoulos, "Minimizing Market Operation Costs Using A Security-Constrained Unit Commitment Approach," 2005 IEEE/PES Transmission and Distribution Conference & Exhibition: Asia and Pacific Dalian, China.

[9] A. Ott, "Experience with PJM Market Operation, System Design, and Implementation," IEEE Transactions on Power Systems, Vol. 18, No. 2, May 2003, pp. 528-534.

[10] D. Streiffert, R. Philbrick, and A. Ott, "A Mixed Integer Programming Solution for Market Clearing and Reliability Analysis," Power Engineering Society General Meeting, 2005. IEEE 12-16 June 2005 , pp. 2724 - 2731 Vol. 3.