EE/Econ 458: HW 7, Due Wednesday Oct 26

A. You solved the following optimization problem in HW6. Using the same system you analyzed in problem 1, shown below, set up the optimal power flow as a linear program. Assume the objective function is the same as used in the example in class when we investigated the case of demand bidding, i.e., $Z = 1307P_{g1} + 1211P_{g2} + 1254P_{g4} - 1300P_{d2} - 1200P_{d3}$. Also, assume (as in HW6) each generator has a lower limit of 100 MW and an upper limit of 300 MW, which will be (in per unit):

$1 \le P_{g1} \le 3$

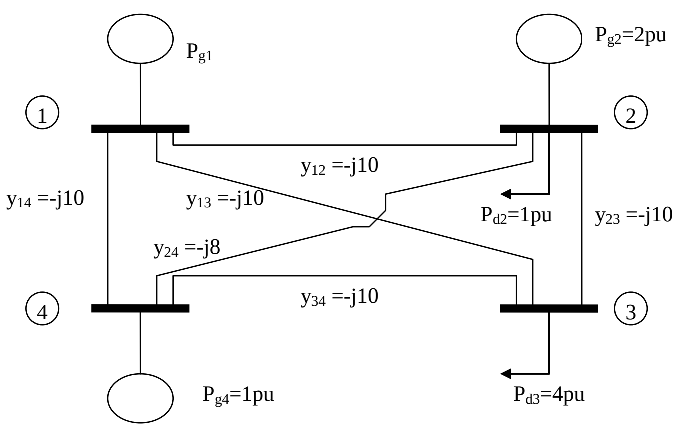
$1 \le P_{g2} \le 3$

$1 \le P_{g3} \le 3$

and the loads are constrained as follows:

$1 \le P_{d2} \le 2$

$2 \le P_{d3} \le 3$



<u>Problem A1</u>: Provide the objective function and all primal decision and auxiliary variables and dual variables at the optimal solution. Also identify the settlement, i.e., identify how much each load pays and how much each generator pays.

<u>**Solution**</u>:

<u>Objective: </u>-17.0$

<u>Primal decision and auxiliary variables</u>:

pg1	1.000000
pg2	2.000000

pg4	1.000000
pd2	2.000000
pd3	2.000000
pb1	-0.013889
theta4	0.001389
pb2	0.263889
theta2	-0.026389
pb3	0.486111
theta3	-0.075000
pb4	0.763889
pb5	0.750000
pb6	-0.222222

Dual variables:

c8	1211.000000
c9	1211.000000
c10	1211.000000
c11	1211.000000
c12	-96.000000
c16	-43.000000
c19	-89.000000
c20	-11.000000

All other dual prices in the range 1-33 are 0.

Settlement:

Problem A2: Now constrain the flow on branch 3 to $P_{b3}=P_{23}=0.45$ pu and resolve. Provide the objective function and all solution variables and dual variables at the optimal solution. Also identify the settlement, i.e., identify how much each load pays and how much each generator pays. Finally, compute the congestion charges as the difference between total load payments made and total generator payments received, and using the dual variable of the constrained branch.

Objective: -11.41$

Primal decision and auxiliary variables:

pg1	1.000000

pg2	1.870000
pg4	1.130000
pd2	2.000000
pd3	2.000000
pb1	-0.050000
theta4	0.005000
pb2	0.300000
theta2	-0.030000
pb3	0.450000
theta3	-0.075000
pb4	0.800000
pb5	0.750000
pb6	-0.280000

All other variables in the range 1-15 are 0.

Dual variables:

c1	0.000000
c4	-154.800000
c8	1251.850000
c9	1211.000000
c10	1290.550000
c11	1254.000000
c12	-55.150000
c19	-89.000000
c20	-90.550000
c27	-154.800000

All other dual prices in the range 1-33 are 0.

Settlement:

B. Use branch-and-bound to solve problems B1 and problem B2.
For both problems:

a. Solve them using successive LP-relaxations, where each LP-relaxation is solved using the CPLEX (or Matlab) LP-solver.
b. Solve them using the CPLEX MIP-solver.

Problem B1:

Max $z = 5x1 + 2x2$
s.t. $3x1 + x2 \leq 12$
$x1 + x2 \leq 5$
$x1, x2, \geq 0$; $x1, x2$ integer

<u>Solution to B1</u>:

a. Using successive LP-relaxations:

First solve as a relaxed LP:
P1: Max $z=5x1+2x2$
s.t. $3x1+x2\leq 12$
$x1 + x2 \leq 5$
➔ $z^*=20.5$, $x1^*=3.5$, $x2^*=1.5$

P2: Max $z=5x1+2x2$
s.t. $3x1+x2\leq 12$
$x1 + x2 \leq 5$
$x1>=4$
$x2 \geq 0$
$z^*=20$, $x1^*=4$, $x2^*=0$
This solution is integer, so $z^*=20$ becomes our best feasible solution.

P3: Max $z=5x1+2x2$
s.t. $3x1+x2\leq 12$
$x1 + x2 \leq 5$
$x1<=3$
$x2 \geq 0$
$z^*=19$, $x1^*=3$, $x2^*=2$
The objective function value is not as good as that of our best feasible solution and therefore we may stop pursuing this branch. Let's go back and force x2 to be integer.

P4: Max $z = 5x1 + 2x2$
s.t. $3x1 + x2 \leq 12$
$x1 + x2 \leq 5$
$x2 \geq 2$
$x1 \geq 0$
$x1, x2, \geq 0$
$z^*=19, x1^*=3, x2^*=2$
The objective function value is not as good as that of our best feasible solution and therefore we may stop pursuing this branch.

P5: Max $z = 5x1 + 2x2$
s.t. $3x1 + x2 \leq 12$
$x1 + x2 \leq 5$
$x2 \leq 1$
$x1 \geq 0$
$x1, x2, \geq 0$
$z^*=20.3, x1^*=3.667, x2^*=1$
The objective function value is better than that of our best feasible solution, but this solution is not integer. Therefore we need to continue pursuing this branch. Let's force x1=3.

P6: Max $z = 5x1 + 2x2$
s.t. $3x1 + x2 \leq 12$
$x1 + x2 \leq 5$
$x2 \leq 1$
$x1 \leq 3$
$x1, x2, \geq 0;$
$z^*=17, x1^*=3, x2^*=1$
The objective function value is not as good as that of our best feasible solution and therefore we may stop pursuing this branch.
Let's force x1=3.:

P7: Max $z = 5x1 + 2x2$
s.t. $3x1 + x2 \leq 12$
$x1 + x2 \leq 5$
$x2 \leq 1$
$x1 \geq 4$
$x1, x2, \geq 0$;
$z^*=20$, $x1^*=4$, $x2^*=0$
Which is the same solution as that obtained in P2. All possible branches have been considered, therefore the solution to P2 becomes the solution to the problem: $z^*=20$, $x1^*=4$, $x2^*=0$.

b. Using CPLEX MIP solver:

$z^*=20$, $x1^*=4$, $x2^*=0$.

Problem B2:
Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1, x2, \geq 0$; x2 integer

Solution to Problem B2:
Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1, x2, \geq 0$; x2 integer

a. Using successive LP-relaxations:

First solve as a relaxed LP:
P1: Max $z = 3x1 + x2$

s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1, x2, \geq 0$;
$z^*=5.667$, $x1^*=1.333$, $x2^*=1.667$

Force x1=1.

P2: Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1 \leq 1$
$x1, x2, \geq 0$;
$z^*=5.5$, $x1^*=1$, $x2^*=2.5$

Force x2=2.

P3: Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1 \leq 1$
$x2 \leq 2$
$x1, x2, \geq 0$;
$z^*=5.0$, $x1^*=1$, $x2^*=2$

This is a feasible solution (all integer) and therefore $z^*=5$ represents our best bound found so far. Now let's branch the other way from P2 by forcing x2=3.

P4: Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$

$x1 \leq 1$
$x2 \geq 3$
$x1, x2, \geq 0$;
$z^*=5.4$, $x1^*=0.8$, $x2^*=3$

P4 is not feasible but it has a better bound than our best bound so far (of 5.0). So we need to branch further on P4. Force x1 to 1.

P5: Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1 \geq 1$
$x2 \geq 3$
$x1, x2, \geq 0$;
This one is infeasible. Go the other way by forcing x1 to 0.

P6: Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1 \leq 0$
$x2 \geq 3$
$x1, x2, \geq 0$;
$z^*=5.0$, $x1^*=0$, $x2^*=5$
This is a feasible solution and it has objective function value equal to our best bound so far (P3).

So now go back to P1 and branch the other way by forcing x1=2.

P7: Max $z = 3x1 + x2$
s.t. $5x1 + 2x2 \leq 10$
$4x1 + x2 \leq 7$
$x1 \geq 2$

$x1, x2, \geq 0$;
This problem is infeasible, and so we are done. There are two answers:
From P3: $z^*=5.0$, $x1^*=1$, $x2^*=2$
From P6: $z^*=5.0$, $x1^*=0$, $x2^*=5$

b. Using MIP solver:

$z^*=5$, $x1^*=0$, $x2^*=5$.