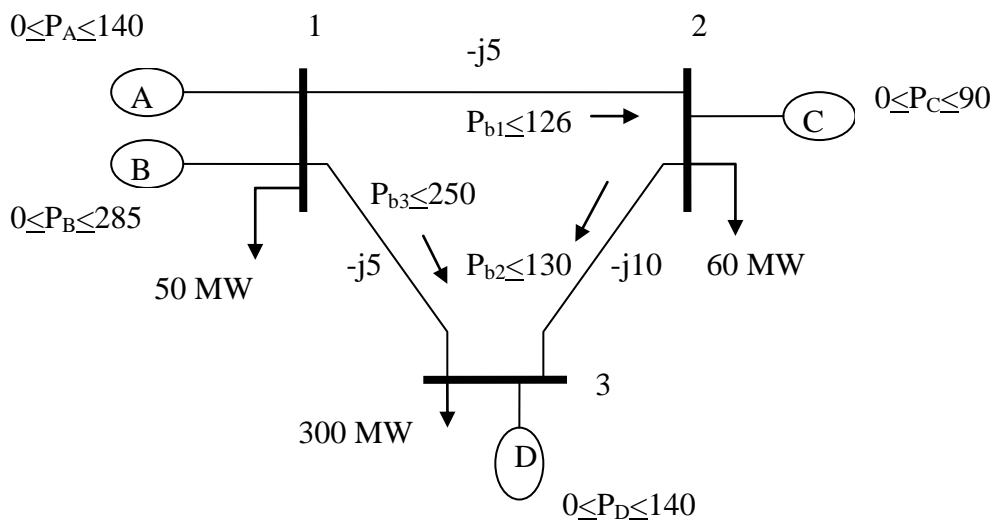


EE 458, Fall 2019: HW 7, Due Tuesday Nov 5

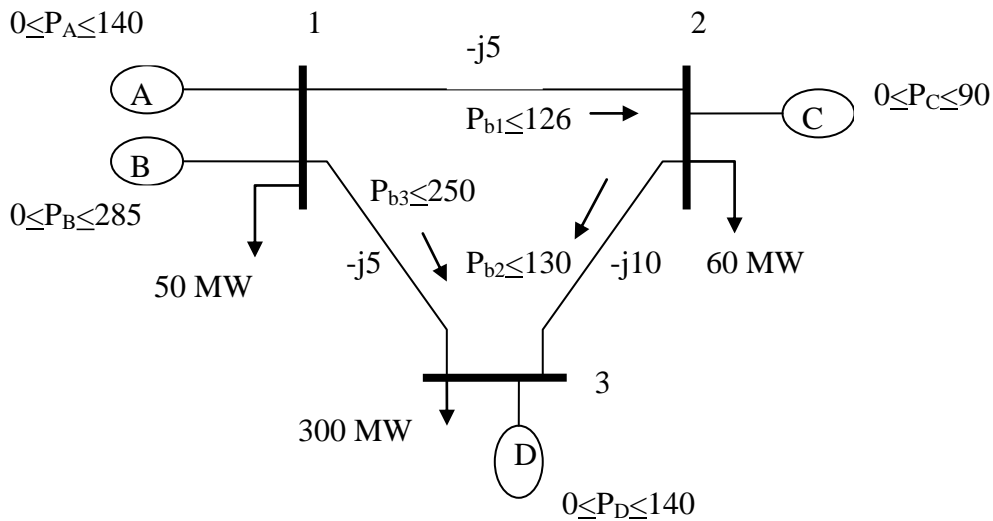
A. Set up a linear program in Matlab or CPLEX to solve the linearized optimal power flow problem for the system and data shown below. There are 4 generators: A, B, C, and D. Line admittances given on the diagram (-j5, -j5, -j10) are in per unit. Provide your Matlab or CPLEX code, the dispatch, flows, total cost, and all Lagrange multipliers.



The offers submitted from owners of units A, B, C, and D are 7.5, 6, 14, and 10, respectively, all in \$/MWhr.

Solution:

The system is shown below:



First let's develop the equality constraints for this problem. They correspond to the injection and flow equations, given by:

$$-\underline{P}_B + (\underline{D} \times \underline{A}) \times \underline{\theta} = \underline{0} \quad (1)$$

$$-\underline{P} + \underline{B}' \underline{\theta} = \underline{0} \quad (2)$$

So we need to obtain the matrices D, A, and B'. We get B' by first writing down the Y-bus matrix and then multiplying it by -1, which is, for this system:

$$Y = \begin{bmatrix} -10 & 5 & 5 \\ 5 & -15 & 10 \\ 5 & 10 & -15 \end{bmatrix} \rightarrow B' = \begin{bmatrix} 10 & -5 & -5 \\ -5 & 15 & -10 \\ -5 & -10 & 15 \end{bmatrix}$$

The matrix D is MxM (where M=3 is number of branches) containing the negative of the branch susceptances along the diagonal and zeros elsewhere. For this system, it is:

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

The matrix A is the MxN node-arc incidence matrix, where M is the number of branches and N the number of nodes, and row k has 1

and -1 in the columns corresponding to beginning and ending nodes, respectively, for branch k. The direction of the branches is shown in the system diagram above.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

So that the term DxA in eq. 1 is:

$$D \times A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 0 \\ 0 & 10 & -10 \\ 5 & 0 & -5 \end{bmatrix}$$

The solution vector is given by:

$$x = \begin{bmatrix} P_A \\ P_B \\ P_C \\ P_D \\ P_{b1} \\ P_{b2} \\ P_{b3} \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

The objective function is then given by $Z=c^T x$. However, we must recall that we are working in per-unit, and therefore the coefficients of the generation variables must be multiplied by 100 (to give \$/pu-hr instead of \$/MW-hr). So the objective function is:

$$Z = \begin{bmatrix} 750 & 600 & 1400 & 1000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 5 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 10 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 5 & 0 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 10 & -5 & -5 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -5 & 15 & -10 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -5 & -10 & 15 \end{bmatrix} \begin{bmatrix} P_A \\ P_B \\ P_C \\ P_D \\ P_{b1} \\ P_{b2} \\ P_{b3} \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

The equality constraints are given by: $A_{eq}x = b_{eq}$ which is comprised of eq. (1) at the top and eq.(2) at the bottom. In forming the eq. (2) part of A_{eq} , one must account for the fact that generators A and B are both located at bus 1 and will therefore both appear in the bus 1 injection equation. In addition, the loads will appear in b_{eq} . Finally, one must remember to work in per-unit because the DC power flow equations corresponding to eq. (2) were derived in per-unit and are only valid when working in per-unit.

Taking these issues into account, we get:

$$Z = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 & 0 & 5 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 10 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 5 & 0 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 10 & -5 & -5 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -5 & 15 & -10 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -5 & -10 & 15 \end{bmatrix} \begin{bmatrix} P_A \\ P_B \\ P_C \\ P_D \\ P_{b1} \\ P_{b2} \\ P_{b3} \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.5 \\ -0.6 \\ -3.0 \end{bmatrix}$$

Finally, the equality constraints are given by:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1.26 \\ -1.30 \\ -2.50 \\ -\pi \\ -\pi \\ -\pi \end{bmatrix} \leq \begin{bmatrix} P_A \\ P_B \\ P_C \\ P_D \\ P_{b1} \\ P_{b2} \\ P_{b3} \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \leq \begin{bmatrix} 1.40 \\ 2.85 \\ 0.90 \\ 0.85 \\ 1.26 \\ 1.30 \\ 2.50 \\ \pi \\ \pi \\ \pi \end{bmatrix}$$

The Matlab code to implement this optimization problem is:

```
%Build objective function vector.
```

```
c=[750 600 1400 1000 0 0 0 0 0 0];
```

```
%Build A matrix for inequality constraints Ax<b.
```

```
A=[];
```

```
%Build b, the right-hand-side of inequality constraints.
```

```
b=[];
```

```
%Build Aeq matrix for equality constraints.
```

```
Aeq=[0 0 0 0 -1 0 0 5 -5 0;
```

```
0 0 0 0 0 -1 0 0 10 -10;
```

```
0 0 0 0 0 0 -1 5 0 -5;
```

```
-1 -1 0 0 0 0 0 10 -5 -5;
```

```
0 0 -1 0 0 0 0 -5 15 -10;
```

```
0 0 0 -1 0 0 0 -5 -10 15];
```

```
%Build right-hand side of equality constraint. It will be vector of zeros
```

```
%except for element in first row, which is load-sum of minimum generation
```

```
beq=zeros(6,1);
```

```
beq(4)=-0.5;
```

```
beq(5)=-0.6;
```

```
beq(6)=-3.0;
```

```
%Build upper and lower bounds on decision variables.
```

```
LB=[0 0 0 0 -1.26 -1.30 -2.50 -pi -pi -pi]';
```

```
UB=[1.40 2.85 0.90 0.85 1.26 1.30 2.50 pi pi pi]';
```

```
%LB=[0 0 0 0 -500 -500 -500 -pi -pi -pi]';
```

```
%UB=[1.40 2.85 0.90 0.85 500 500 500 pi pi pi]';
```

```
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA]=LINPROG(c,A,b,Aeq,beq,LB,UB);
```

The solution is (in per-unit):

$$x = \begin{bmatrix} P_A \\ P_B \\ P_C \\ P_D \\ P_{b1} \\ P_{b2} \\ P_{b3} \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0.50 \\ 2.85 \\ 0.0 \\ 0.75 \\ 1.26 \\ 0.66 \\ 1.59 \\ 0.1899 \\ -0.0621 \\ -0.1281 \end{bmatrix}$$

The total cost is \$2835/hr. (These values are in agreement with the solution given by the text by Kirschen & Strbac on page 163).

The Lagrange multipliers on equality constraints are given by Matlab (from LAMBDA.eqlin) as:

625.0
 0.0000
 0.0000
 750.0
 1125.0
 1000.0

But this is in units of \$/pu-hr. To get it in \$/MWhr, we divide by 100 to get

6.25
 0.0000
 0.0000
 7.50
 11.25
 10.0

This shows the spot prices (LMPs) for buses 1, 2, and 3 are 7.5, 11.25, and 10.0 \$/Mwhr, respectively.

The Lagrange multipliers on the lower bounds of the inequality constraints (from LAMBDA.lower) are:

0.0000
 0.0000
 275.0000
 0.0000
 0.0000
 0.0000
 0.0000
 0.0000
 0.0000
 0.0000

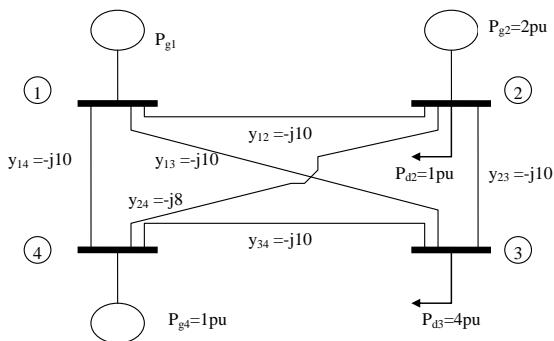
This indicates that only unit C is at its lower limit.

The Lagrange multipliers on the upper bounds of the inequality constraints (from LAMBDA.upper) are:

0.0000
 150.0000
 0.0000
 0.0000
 625.0000
 0.0000
 0.0000
 0.0000
 0.0000
 0.0000

This indicates that P_B and P_{b1} are at their upper limits.

B. You solved the following optimization problem in HW6.



B1. Using this same system, set up the optimal power flow as a linear program assuming the objective function is the same as used in the example in class when we investigated the case of demand bidding, i.e.,

$$Z = 1307P_{g1} + 1211P_{g2} + 1254P_{g4} - 1300P_{d2} - 1200P_{d3}.$$

Assume each generator has a lower limit of 100 MW and an upper limit of 300 MW, which will be (in per unit):

$$1 \leq P_{g1} \leq 3$$

$$1 \leq P_{g2} \leq 3$$

$$1 \leq P_{g3} \leq 3$$

and the loads are constrained as follows:

$$1 \leq P_{d2} \leq 2$$

$$2 \leq P_{d3} \leq 3$$

Assume the lines have infinite capacity.

Solution:

minimize

$$1307 \text{ pg1} + 1211 \text{ pg2} + 1254 \text{ pg4} - 1300 \text{ pd2} - 1200 \text{ pd3}$$

subject to

$$\text{theta1} = 0$$

$$-\text{pb1} + 10 \text{ theta1} - 10 \text{ theta4} = 0$$

$$-\text{pb2} + 10 \text{ theta1} - 10 \text{ theta2} = 0$$

$$-\text{pb3} + 10 \text{ theta2} - 10 \text{ theta3} = 0$$

$$-\text{pb4} - 10 \text{ theta3} + 10 \text{ theta4} = 0$$

$$-\text{pb5} + 10 \text{ theta1} - 10 \text{ theta3} = 0$$

$$-\text{pb6} + 8 \text{ theta2} - 8 \text{ theta4} = 0$$

$$\text{pg1} - 30 \text{ theta1} + 10 \text{ theta2} + 10 \text{ theta3} + 10 \text{ theta4} = 0$$

$$\text{pg2} - \text{pd2} + 10 \text{ theta1} - 28 \text{ theta2} + 10 \text{ theta3} + 8 \text{ theta4} = 0$$

$$-\text{pd3} + 10 \text{ theta1} + 10 \text{ theta2} - 30 \text{ theta3} + 10 \text{ theta4} = 0$$

$$\text{pg4} + 10 \text{ theta1} + 8 \text{ theta2} + 10 \text{ theta3} - 28 \text{ theta4} = 0$$

$$-\text{pg1} \leq -1$$

$$\text{pg1} \leq 3$$

$$-\text{pg2} \leq -1$$

$$\text{pg2} \leq 3$$

$$-\text{pg4} \leq -1$$

$$\text{pg4} \leq 3$$


```

-pd2 <= -1
pd2 <= 2
-pd3 <= -2
pd3 <= 3
-pb1 <= 500
pb1 <= 500
-pb2 <= 500
pb2 <= 500
-pb3 <= 500
pb3 <= 500
-pb4 <= 500
pb4 <= 500
-pb5 <= 500
pb5 <= 500
-pb6 <= 500
pb6 <= 500

```

Bounds

```

-500 <= pb1 <= 500
-500 <= pb2 <= 500
-500 <= pb3 <= 500
-500 <= pb4 <= 500
-500 <= pb5 <= 500
-500 <= pb6 <= 500
-3.14159 <= theta1 <= 3.14159
-3.14159 <= theta2 <= 3.14159
-3.14159 <= theta3 <= 3.14159
-3.14159 <= theta4 <= 3.14159

```

end

B2: Provide the value of the objective function and values for all primal decision and auxiliary variables and dual variables at the optimal solution. Also identify the settlement, i.e., identify how much each load pays and how much each generator pays.

Solution:

Objective: -17.0\$

Primal decision and auxiliary variables:

pg1	1.000000
pg2	2.000000
pg4	1.000000
pd2	2.000000
pd3	2.000000
pb1	-0.013889
theta4	0.001389
pb2	0.263889
theta2	-0.026389
pb3	0.486111
theta3	-0.075000
pb4	0.763889
pb5	0.750000
pb6	-0.222222

Dual variables:

c8	1211.000000
c9	1211.000000
c10	1211.000000
c11	1211.000000
c12	-96.000000
c16	-43.000000
c19	-89.000000
c20	-11.000000

All other dual prices in the range 1-33 are 0.

Settlement:

Sum the products of LMP and Pg for supply side.

Sum the products of LMP and Pd for demand side.

Should be equal.

B3: Now constrain the flow on branch 3 to $P_{b3}=P_{23}=0.45$ pu and resolve. Provide the value of the objective function and values for all solution variables and dual variables at the optimal solution. Also identify the settlement, i.e., identify how much each load pays and how much each generator pays. Finally, compute the congestion

charges as the difference between total load payments made and total generator payments received, and also using the dual variable of the constrained branch.

Objective: -11.41\$

Primal decision and auxiliary variables:

pg1	1.000000
pg2	1.870000
pg4	1.130000
pd2	2.000000
pd3	2.000000
pb1	-0.050000
theta4	0.005000
pb2	0.300000
theta2	-0.030000
pb3	0.450000
theta3	-0.075000
pb4	0.800000
pb5	0.750000
pb6	-0.280000

All other variables in the range 1-15 are 0.

Dual variables:

c1	0.000000
c4	-154.800000
c8	1251.850000
c9	1211.000000
c10	1290.550000
c11	1254.000000
c12	-55.150000
c19	-89.000000
c20	-90.550000
c27	-154.800000

All other dual prices in the range 1-33 are 0.

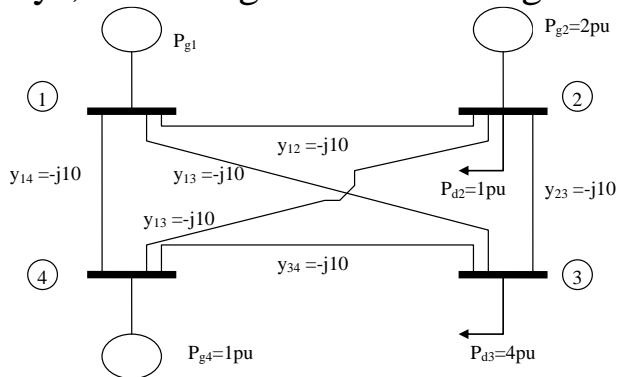
Settlement:

Sum the products of LMP and Pg for supply side.

Sum the products of LMP and Pd for demand side.

Should be different; the excess of demand-side over supply side is the congestion charges.

C. Using the system below, compute the \underline{T} matrix of generation shift factors for every branch. Make the computation in two different ways, according to the following instructions.



a. Assume bus 1 compensates for all changes, i.e., bus 1 is the slack bus.

Solution:

$$\underline{T} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 30 & -10 & -10 \\ -10 & 30 & -10 \\ -10 & -10 & 30 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -0.25 & -0.25 & -0.5 \\ 0 & -0.5 & -0.25 & -0.25 \\ 0 & 0.25 & -0.25 & 0 \\ 0 & 0 & -0.25 & 0.25 \\ 0 & -0.25 & -0.5 & -0.25 \\ 0 & 0.25 & 0 & -0.25 \end{bmatrix}$$

b. Assume a distributed slack, with all buses compensating equally.

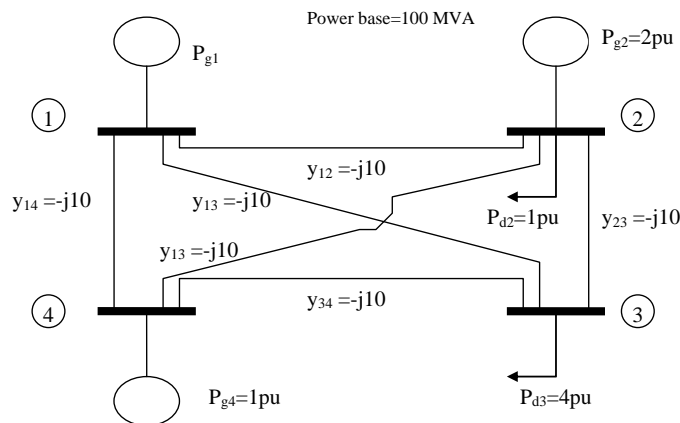
Solution:

$$\underline{T} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 30 & -10 & -10 \\ -10 & 30 & -10 \\ -10 & -10 & 30 \end{bmatrix}^{-1} \begin{bmatrix} -0.333 & 1 & -0.333 & -0.333 \\ -0.333 & -0.333 & 1 & -0.333 \\ -0.333 & -0.333 & -0.333 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3333 & 0 & 0 & -0.3333 \\ 0.3333 & -0.3333 & 0 & 0 \\ 0 & 0.3333 & -0.3333 & 0 \\ 0 & 0 & -0.3333 & 0.3333 \\ 0.3333 & 0 & -0.3333 & 0 \\ 0 & 0.3333 & 0 & -0.3333 \end{bmatrix}$$

- c. Use the elements of the appropriate T-matrix above to compute the flows for a change in generation at bus 2 from 2 to 4 per unit, compensated by reduction at bus 1. Perform the calculations for each branch to get the new branch flow using the original branch flow plus the change in branch flow, where the change in branch flow is computed from the appropriate elements of the appropriate \underline{T} matrix. The original branch flows are given by

$$\begin{bmatrix} P_{B1} \\ P_{B2} \\ P_{B3} \\ P_{B4} \\ P_{B5} \\ P_{B6} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 1.25 \\ 1.25 \\ 1.5 \\ 0 \end{bmatrix}$$



Solution:

The change made is compensated by the swing bus therefore we should use the distribution factors computed based on a bus 1 swing bus, which are:

$$\underline{T} = \begin{bmatrix} 0 & -0.25 & -0.25 & -0.5 \\ 0 & -0.5 & -0.25 & -0.25 \\ 0 & 0.25 & -0.25 & 0 \\ 0 & 0 & -0.25 & 0.25 \\ 0 & -0.25 & -0.5 & -0.25 \\ 0 & 0.25 & 0 & -0.25 \end{bmatrix}$$

The new branch flow is given by the old branch flow (which comes from the problem statement, and that comes from HW6-Part 1) plus the change in branch flows (using the above distribution factors) resulting from changing the bus 2 generation from 2 to 4 (a change of +2 at bus 2):

$$\begin{bmatrix} P_{B1} \\ P_{B2} \\ P_{B3} \\ P_{B4} \\ P_{B5} \\ P_{B6} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 1.25 \\ 1.25 \\ 1.5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -0.25 & -0.25 & -0.5 \\ 0 & -0.5 & -0.25 & -0.25 \\ 0 & 0.25 & -0.25 & 0 \\ 0 & 0 & -0.25 & 0.25 \\ 0 & -0.25 & -0.5 & -0.25 \\ 0 & 0.25 & 0 & -0.25 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 1.25 \\ 1.25 \\ 1.5 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.5 \\ -1 \\ 0.5 \\ 0 \\ -0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -0.25 \\ -0.75 \\ 1.75 \\ 1.25 \\ 1.0 \\ 0.5 \end{bmatrix}$$