Unsymmetrical Fault Analysis 2

1.0 Introduction

Let's recall the basic steps in using symmetrical components for assessing faulted conditions (all quantities are assumed to be in pu).

A. For positive, negative, & zero sequence:
  1. Develop the sequence network for the system under analysis.
  2. Obtain the Thevenin equivalents looking into the network from the fault point.
B. Connect the networks to capture the influence of the particular fault type.
C. Compute the fault current from the circuit resulting from step B.
D. From step C, you will also determine the currents in all three of the networks (positive, negative, and zero sequence currents). This enables computation of the phase currents I_a, I_b, and I_c from $\underline{I}_{abc}=A\underline{I}_s$.

We now have the necessary background on developing the sequence representation of each component in the network (loads, lines, transformers, and generators). Step A-1 is completed by connecting this representation in a manner consistent with the network 1-line diagram.

We now investigate step A-2.

2.0 Obtaining Thevenin equivalents

The first thing to do is to draw all three sequence networks of the system to be analyzed, using information from the one-line diagram together with the component sequence models.

Then identify the fault location in each of the three sequence networks.

Then, for each of the three sequence networks, determine the Thevenin equivalent of the network as seen from the fault location.

Recall we
- Obtain the Thevenin impedance by idling all sources (short constant voltage sources and open constant current sources) and computing impedance see looking into the network from the desired location, and
- Obtain the Thevenin voltage by computing the voltage at the open-circuited terminals for the desired location.

Example:
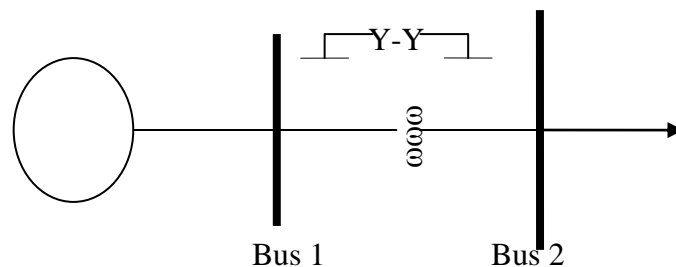Consider the system of Fig. 1.

Bus 1   Bus 2

Fig. 1

The transformer is Y-grounded to Y-grounded with rated line-to-line ratio of 13.8 kV/69 kV and power rating of 5 MVA. The transformer positive sequence reactance is 0.065 pu on its own base. The generator is also rated 5 MVA, 13.8 kV. The generator has a positive sequence subtransient reactance of 0.12 pu, and its negative and zero sequence reactances are 0.13 pu and 0.05 pu, respectively, all on the machine base. Determine the positive, negative, and zero-sequence Thevenin equivalents as seen from Bus 2, on a 1 MVA base.

Solution:
We must do a change of base. What is the formula for performing change of base? If you do not remember, then remember this:

$$Z_{B1} = \frac{(V_{B1})^2}{S_{B1}}; \qquad Z_{B2} = \frac{(V_{B2})^2}{S_{B2}}; \qquad Z_{\Omega} = Z_{pu1}Z_{B1} = Z_{pu2}Z_{B2}$$

$$\Rightarrow Z_{pu2} = \frac{Z_{B1}}{Z_{B2}}Z_{pu1} = \frac{(V_{B1})^2}{S_{B1}}\frac{S_{B2}}{(V_{B2})^2}Z_{pu1}$$

If there is no need for changing the voltage base, which is the case for the generator and transformer data, then

$$Z_{pu2} = \frac{S_{B2}}{S_{B1}} Z_{pu1}$$ (1)

So the generator data becomes:

$$X_g^+ = \frac{1}{5}(0.12) = 0.024$$

$$X_g^- = \frac{1}{5}(0.13) = 0.026$$

$$X_g^0 = \frac{1}{5}(0.05) = 0.01$$

The transformer reactance is 0.065 pu but it is also on a 5 MVA base. We need it on a 1 MVA base. Therefore:

$$X_{T1} = 0.065\frac{1}{5} = 0.013$$

The sequence networks are drawn on the left-hand side of Fig. 2.

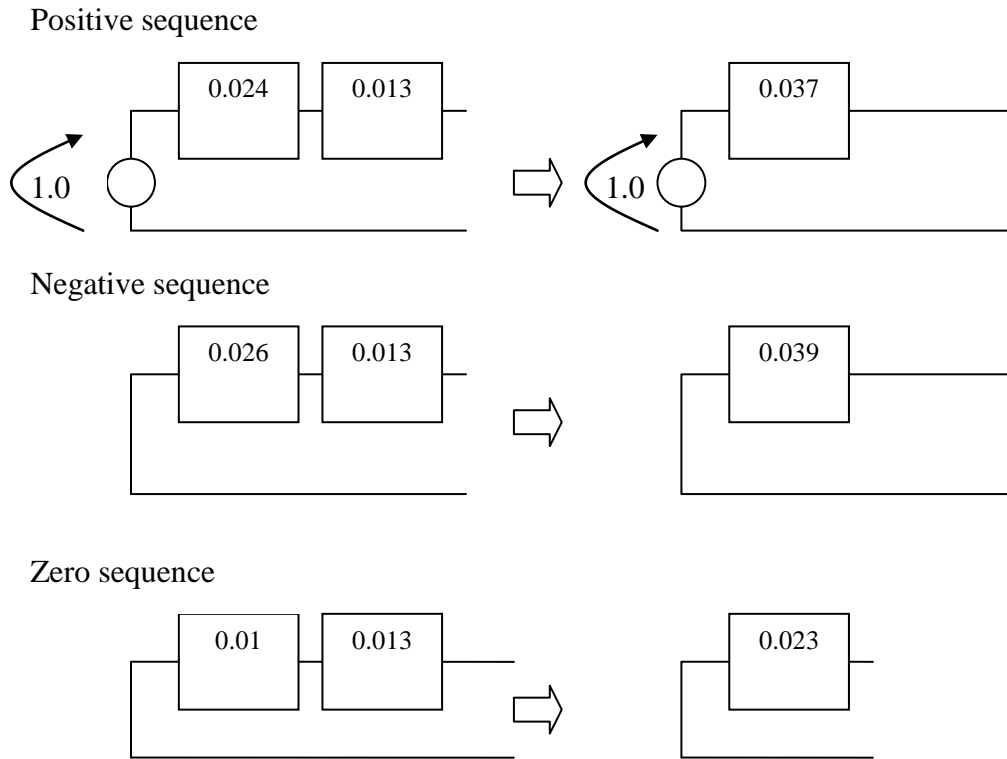The Thevenin equivalents, pictured on the right of Fig. 2, are very easy to obtain.

Positive sequence

Negative sequence

Zero sequence

Fig. 2

3.0 Connecting the networks

We will develop connections between the sequence networks for each fault type and apply them to our example.

3.1 Three-phase fault

A three phase fault has all three phases connected to each other and to ground. It is a symmetrical one; even with the fault, the network is still symmetric.

To analyze it, we denote fault currents in phases a, b, and c as $I_{af}\angle\_0°$, $I_{af}\angle\_-120°$, $I_{af}\angle\_120°$. Then the 0+- currents are computed as

$$\underline{I}_S = \begin{bmatrix} I_s^0 \\ I_s^+ \\ I_s^- \end{bmatrix} = \underline{A}^{-1}\underline{I}_{abc} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_{af}\angle 0° \\ I_{af}\angle -120° \\ I_{af}\angle 120° \end{bmatrix} = \begin{bmatrix} 0 \\ I_{af} \\ 0 \end{bmatrix}$$

This result shows what we already knew. Only positive sequence currents flow for a 3-phase (symmetric) fault!

So we use only positive-sequence network to analyze this fault. Fig. 3 shows, for our example, the network corresponding to a 3-phase fault.
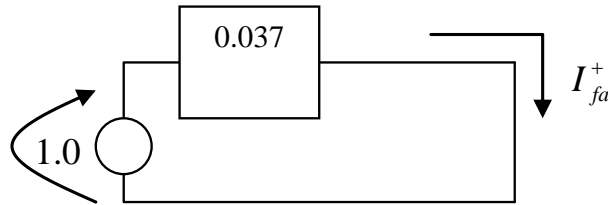

Fig. 3

From Figure 18, we compute:

$$I^1_{fa} = \frac{1.0}{0.037} = 27.03 \, pu$$

And of course,

$$I^0_{fa} = I^-_{fa} = 0$$

Now obtain the abc currents.

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \underline{A} \begin{bmatrix} I^0_{fa} \\ I^+_{fa} \\ I^-_{fa} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 27.03 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 27.03 \\ 27.03\angle -120 \\ 27.03\angle 120 \end{bmatrix}$$

So the a-phase fault current is 27.03 pu.

To get this in amperes, we obtain the current base somewhere – let's choose the low side of the transformer:

$$I_b = \frac{S_{b3\phi}}{\sqrt{3}V_{LLb}} = \frac{1E6}{\sqrt{3}(13.8E3)} = 41.84$$

Therefore the a-phase current on the low voltage side of the transformer, for a three-phase fault at bus 2, is 27.03(41.84)=1131A.

Conclusion: Analysis of a 3-phase fault involves shorting the associated positive sequence Thevenin network and computing associated positive sequence current. That is consistent with our work on symmetrical fault analysis!

3.2 Single-phase fault

A single phase fault, or more commonly known as a single-line-to-ground (SLG) fault, has one phase connected to ground.

To analyze a SLG fault, denote fault currents in phases a, b, and c as I_{af}, 0, and 0 (observe we are not changing the impedance matrix but rather adding a current source, consistent with the substitution theorem).

Then the 0+- currents are computed as

$$\underline{I}_S = \begin{bmatrix} I_s^0 \\ I_s^+ \\ I_s^- \end{bmatrix} = \underline{A}^{-1} \underline{I}_{abc} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_{af} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} I_{af} \\ I_{af} \\ I_{af} \end{bmatrix}$$

We observe that the same current is flowing in all three sequence networks! The circuit connection for which this is true is a series connection of the positive, negative, and zero-sequence circuits.

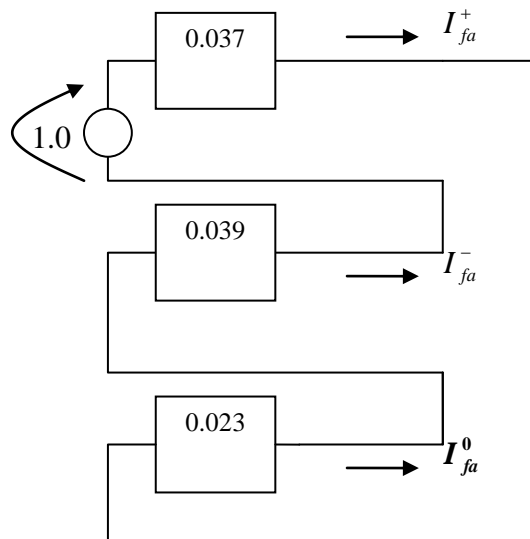We show the connected sequence circuits for our example in Fig. 4.

Fig. 4

From Figure 4, we compute:

$$I_{fa}^{+} = I_{fa}^{-} = I_{fa}^{0} = \frac{1.0}{0.037 + 0.039 + 0.023} = 10.10\,pu$$

Now obtain the abc currents.

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \underline{A} \begin{bmatrix} I_{fa}^{0} \\ I_{fa}^{1} \\ I_{fa}^{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2} \end{bmatrix} \begin{bmatrix} 10.10 \\ 10.10 \\ 10.10 \end{bmatrix}$$

$$= \begin{bmatrix} 30.30 \\ 0 \\ 0 \end{bmatrix}$$

So the a-phase fault current is 30.30 pu.

Recalling that the base current on the low voltage side is 41.84A, the a-phase current for a SLG fault at bus 2 is 30.30(41.84)=1268A. We could obtain it on the high voltage side, where:

$$I_{b} = \frac{S_{b3\phi}}{\sqrt{3}V_{LLb}} = \frac{1E6}{\sqrt{3}(69E3)} = 8.37$$

Then, 27.03(8.37)=226.17A.

<u>Conclusion</u>: Analysis of a single line to ground fault involves connecting the positive, negative, and zero-sequence circuits in series.

3.3 Line-to-line fault

A line-to-line (LL) fault has one phase connected to another. Assuming phase b is connected to phase c, we have $I_{af}=0$, $I_{cf}=-I_{bf}$.

To analyze a LL fault, denote fault currents in phases a, b, and c as 0, I_{bf}, and $-I_{bf}$. Then the 0+- currents are computed as

$$\underline{I}_S = \begin{bmatrix} I_s^0 \\ I_s^+ \\ I_s^- \end{bmatrix} = \underline{A}^{-1}\underline{I}_{abc} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_{bf} \\ -I_{bf} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ \sqrt{3}I_{bf}\angle 90° \\ \sqrt{3}I_{bf}\angle -90° \end{bmatrix}$$

We observe that the zero sequence circuit is **<u>dead</u>**, but positive and negative sequence

currents are equal in magnitude but opposite in direction! The circuit connection for which this is true is one where the positive sequence terminals are directly connected to the negative sequence terminals.

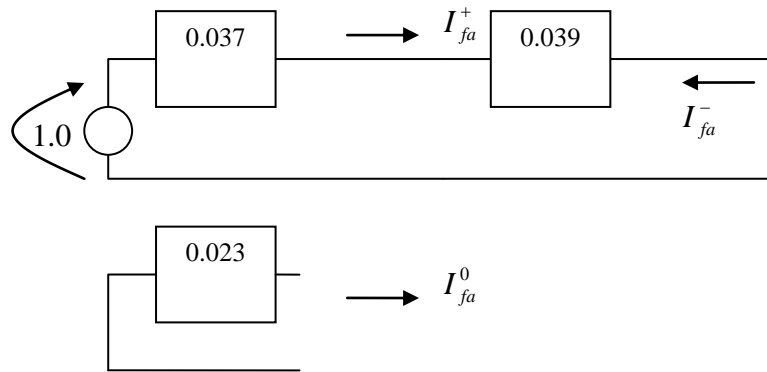We show the connected sequence circuits for our example in Fig. 5.

Fig. 5

From Figure 5, we compute:

$$I_{fa}^{+} = -I_{fa}^{-} = \frac{1.0}{0.037 + 0.09} = 13.16\,pu$$

Now obtain the abc currents.

$$
\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \underline{A} \begin{bmatrix} I_{fa}^0 \\ I_{fa}^+ \\ I_{fa}^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 13.16 \\ -13.16 \end{bmatrix}
$$

$$
= \begin{bmatrix} 0 \\ \sqrt{3}(13.16)\angle 90° \\ \sqrt{3}(13.16)\angle -90° \end{bmatrix} = \begin{bmatrix} 0 \\ 22.79\angle 90° \\ 22.79\angle -90° \end{bmatrix}
$$

So the a-phase fault current is 22.79 pu.
Recalling that the base current on the secondary side is 41.84A, the a-phase current on the low voltage side, for a SLG fault at bus 2, is 22.79(41.84)=954A, or, on the high voltage side, is 22.79(8.37)=191A.

3.4 Two-line to ground fault

Similar analysis results as before results in the conclusion that a 2-line-to-ground fault (LLG) fault can be analyzed using a parallel combination of all three sequence networks, as shown in Fig. 6.
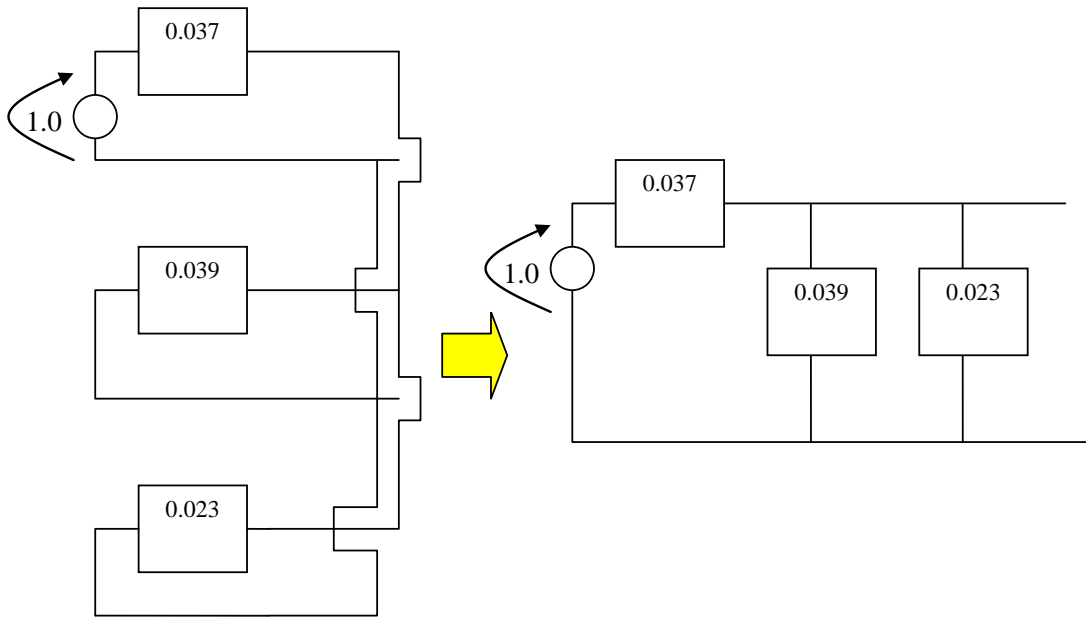
Fig. 6

From Figure 6, we compute:

$$I_{fa}^+ = \frac{1.0}{0.037 + 0.039 // .023} = \frac{1.0}{0.037 + 0.0145} = 19.42\,pu$$

The negative and zero sequence currents are then computed using current division as

$$I_{fa}^- = -19.42 \frac{.023}{0.039 + .023} = -7.2\,pu$$

$$I_{fa}^0 = -19.42 \frac{.039}{0.039 + .023} = -12.22\,pu$$

Now obtain the abc currents.

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \underline{A} \begin{bmatrix} I_{fa}^0 \\ I_{fa}^+ \\ I_{fa}^- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} -12.22 \\ 19.42 \\ -7.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 29.45\angle 129° \\ 29.45\angle -129° \end{bmatrix}$$