Unsymmetrical Fault Analysis 1

1.0 Recap

We have seen in "SymmetricalComponents2," that positive, negative, and zero sequence networks are decoupled under the conditions that the phase impedance matrix for all components of the system have:

- Equal diagonal elements (phase impedances must be equal), i.e.,

$$Z_{aa} = Z_{bb} = Z_{cc} \qquad (1)$$

- Equal offdiagonal elements (offdiagonal phase impedances must be equal), i.e.,

$$Z_{ab} = Z_{ac} = Z_{bc} \qquad (2)$$

We also found that under the above conditions, the 0+- impedances are given by

$$Z_S^{0+} = Z_S^{+0} = Z_S^{0-} = Z_S^{-0} = Z_S^{+-} = Z_S^{-+} = 0 \qquad (3)$$

$$Z_S^0 = Z_{aa} + 2Z_{ab} \qquad (4)$$

$$Z_S^+ = Z_S^- = Z_{aa} - Z_{ab} \qquad (5)$$

The resulting sequence networks are shown in Fig. 1.

Zero sequence network

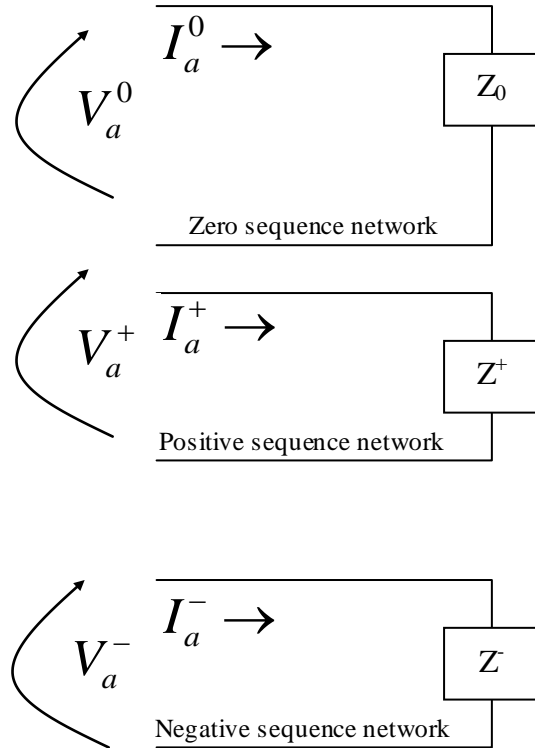Positive sequence network

Negative sequence network

Fig. 1

Distribution systems generally do not satisfy the requirements in (1) and (2) above since transposition is not used, that is, distribution systems are not symmetric.

Transmission systems generally do satisfy the requirements in (1) and (2).

2.0 Important concept

Section 12.2 of your text provides a very good conceptual description of using symmetrical components in fault analysis. I condense this discussion here for you. Please read it, and digest/absorb it!!!

- SC provides 3 decoupled systems for analysis of unbalanced sources applied to a symmetrical system.
- Faulted symmetrical systems (except for 3-phase faults) are not symmetrical systems, so it would appear that SC are not much good for SLG, 2LG, and LL faults.
- But we can replace the fault with an unbalanced source (substitution theorem), then the network becomes symmetric.
- Then get the sequence components of the (fictitious) unbalanced source at the fault point, then you can perform per-phase analysis on each sequence circuit.

3.0 Developing sequence networks

Basic steps in using symmetrical components for assessing faulted conditions are (all quantities are assumed to be in pu).

A. For positive, negative, & zero sequence:
  1. Develop the sequence network for the system under analysis.
  2. Obtain the Thevenin equivalents looking into the network from the fault point.
B. Connect the networks to capture the influence of the particular fault type.
C. Compute the fault current from the circuit resulting from step B.
D. From step C, you will also determine the currents in all three of the networks (positive, negative, and zero sequence currents). This enables computation of the phase currents I_a, I_b, and I_c from $\underline{I}_{abc}=A\underline{I}_s$.

We discuss each one of these steps in what follows.

We address loads, lines, transformers, & generators. For each of these, we may derive expressions for the 0+- sequence impedances via the following steps:

1. Express abc voltages as a function of abc currents and abc impedances.
2. Substitute symmetric components for abc voltages and currents (e.g., $\underline{V}_{abc}=\underline{A}\underline{V}_S$ and $\underline{I}_{abc}=\underline{A}\underline{I}_S$).
3. Manipulate to obtain the form $\underline{V}_S=\underline{Z}_S\underline{I}_S$.

If you refer back to "SymmetricalComponents2," you will see that this is the procedure we followed to obtain 0+- sequence impedances for a Y-connected load.

However, in what follows, we will not go through the analytical details but will rather just state the results.

3.1 Loads (see section 12.6 of text)

Recall from "SymmetricalComponents2" that for a Y-connected, balanced load grounded through a neutral impedance Z_n with impedance Z_Y per phase, the 0+-sequence circuits are as in Fig. 2.
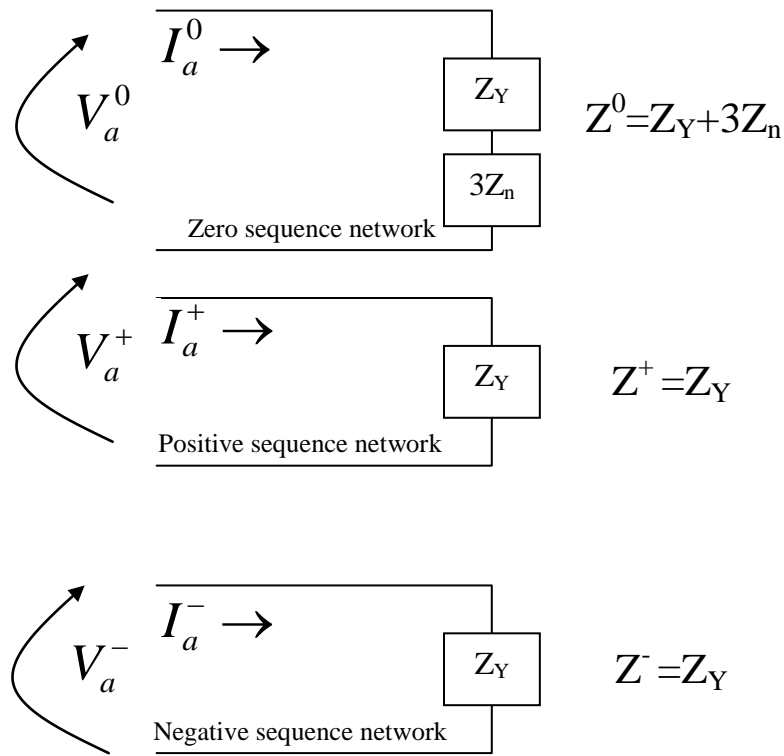

$I_a^0 \rightarrow$

V_a^0

Z_Y

$3Z_n$

Zero sequence network

$Z^0 = Z_Y + 3Z_n$

$I_a^+ \rightarrow$

V_a^+

Z_Y

Positive sequence network

$Z^+ = Z_Y$

$I_a^- \rightarrow$

V_a^-

Z_Y

Negative sequence network

$Z^- = Z_Y$

Fig. 2

where we see the sequence impedances are:

$$Z^0 = Z_Y + 3Z_n \qquad\qquad (6)$$
$$Z^+ = Z_Y \qquad\qquad (7)$$
$$Z^- = Z_Y \qquad\qquad (8)$$

If the neutral is solidly grounded, then $Z_n=0$ and eq. (6) above becomes $Z^0=Z_Y$.

If the neutral is ungrounded, then $Z_n=\infty$, and eq. (6) above becomes $Z^0=\infty$, i.e., the 0-sequence circuit is an open circuit, implying no 0-sequence current flows in an ungrounded Y connection.

For a delta-connected, balanced load, we simply convert to an equivalent Y using $Z_Y=Z_\Delta/3$ and then apply relations for an ungrounded Y connection, resulting in

$$Z^0 =\infty \qquad\qquad\qquad (9)$$
$$Z^1 = Z_\Delta/3 \qquad\qquad\qquad (10)$$
$$Z^- = Z_\Delta/3 \qquad\qquad\qquad (11)$$

Example: A delta connected balanced load with phase impedance Z_Δ is in parallel with a solidly grounded Y-connected load with phase impedance Z_Y. Draw the sequence networks for the entire paralleled load.

It is possible to develop the sequence networks using the 3-step approach given above (and I have notes that do that).

However, this is very painful. Intuition, which suggests that we should just obtain the sequence networks of the parallel combination as parallel combinations of the individual sequence networks, is right. Figure 3 shows the result.
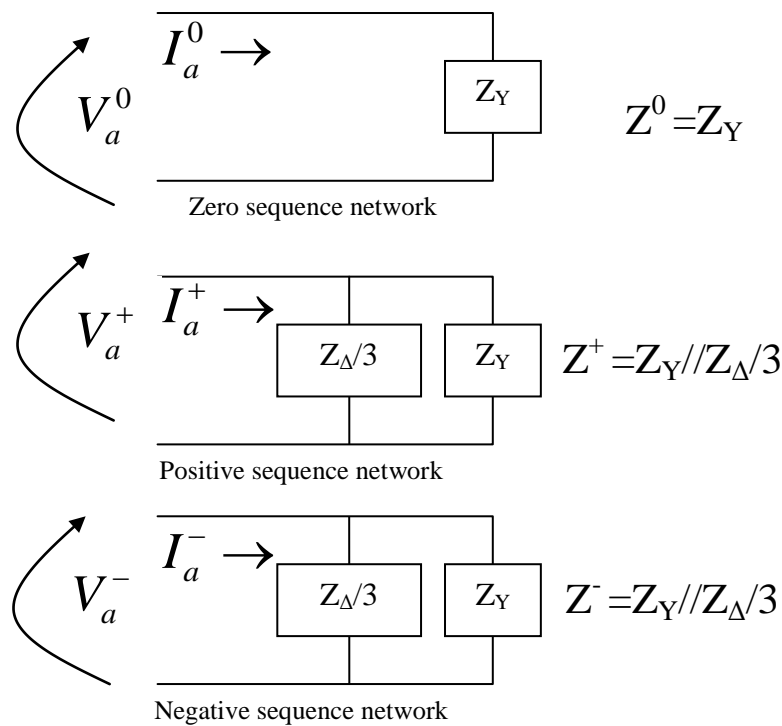

Zero sequence network

$Z^0 = Z_Y$

Positive sequence network

$Z^+ = Z_Y // Z_\Delta/3$

Negative sequence network

$Z^- = Z_Y // Z_\Delta/3$

Fig. 3

3.2 Lines (see section 12.9 of text)

In "SymmetricalComponents2," the work we did to answer the question of "What if the load (or line, or load and line) is not symmetric?" led to another question, which was: "So what are the conditions for the offdiagonal elements of $\underline{Z}_S$ to be 0?"

We have already reviewed the answer to this question in Section 1.0 above, which was:

▪ Equal diagonal elements (phase impedances must be equal), i.e.,

$$Z_{aa} = Z_{bb} = Z_{cc} \qquad (1)$$

▪ Equal offdiagonal elements (offdiagonal phase impedances must be equal), i.e.,

$$Z_{ab} = Z_{ac} = Z_{bc} \qquad (2)$$

In this case, the 0+- sequence impedances are [1, p. 28-30]:

$$Z_S^{0+} = Z_S^{+0} = Z_S^{0-} = Z_S^{-0} = Z_S^{+-} = Z_S^{-+} = 0 \quad (3)$$

$$Z_S^0 = Z_{aa} + 2Z_{ab} \qquad (4)$$

$$Z_S^+ = Z_S^- = Z_{aa} - Z_{ab} \qquad (5)$$

Equation (3) simply says that all off-diagonal elements on the 0+- sequence impedance matrix are zero. Equations (4) and (5) provide the actual expressions that we need for the 0, positive, and negative sequence impedances. These expressions apply to transmission lines because transposition makes conditions (1, 2) true.

The sequence networks are given in Fig. 4.

Zero sequence network

$$Z^0 = Z_{aa} + 2Z_{ab}$$

Positive sequence network

$$Z^+ = Z_{aa} - Z_{ab}$$

Negative sequence network
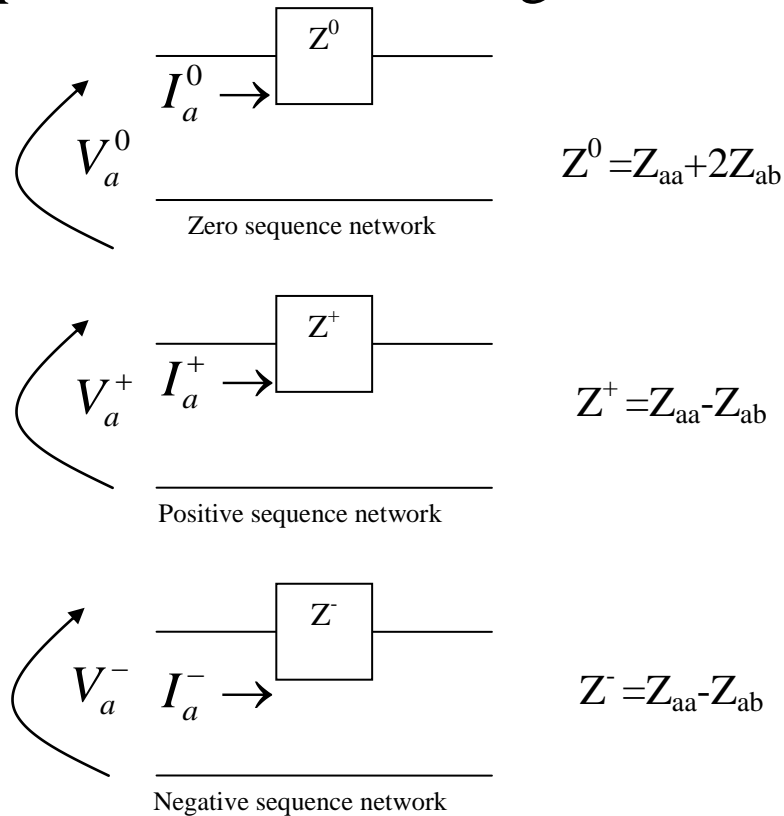
$$Z^- = Z_{aa} - Z_{ab}$$

Fig. 4

It is interesting to compare eqs. (4) and (5) for a symmetric line, and note that they indicate that the zero-sequence quantity is larger than the positive and negative sequence quantities by $3Z_{ab}$.

A typical overhead line has $Z_{ab} \approx (2/5)Z_{aa}$. In this case, it is easy to show that $Z^0 = 3Z^+$, suggesting that finding zero sequence impedance 3 times as large as positive sequence impedance, is quite typical (see bottom of page 474 in text).

3.3 Transformers (see Sec. 12.8 of text)

There are five different types of transformer connections to assess. These are:
1. Grounded Y to grounded Y.
2. Grounded Y to Y or Y to grounded Y.
3. Δ-Δ
4. Grounded Y to Δ or Δ to grounded Y
5. Y-Δ or Δ-Y.

As before, we can perform our 3-step procedure given at the end of Section 3.0 (pg. 5), where we express abc quantities, substitute in symmetrical components, and then obtain the decoupled equations. Here, however, we must repeat this for both sides of the transformer and then relate the two sets of equations.

We will not perform this tedious work but will instead simply observe general guides for drawing appropriate sequence circuits. In forming these guides, we assume:

- Exciting current is negligible so shunt path is infinite impedance and we only have the series Z (winding resistance and leakage reactance) in our transformer abc model.
- Transformers in Δ-Y or Y-Δ configuration are always connected so that positive sequence voltages on the high side lead positive sequence voltages on the low side by 30° (per industry convention). See pp. 139-140 for more on xfmr 30° phase shift.

General guidelines for transformer 0+-sequence circuits:

1. Positive and negative sequence impedances are equal, i.e.,

$$Z^+ = Z^- = Z_{series}$$

where Z_{series} is the transformer winding resistance and leakage reactance.

2. For connection types 4 and 5 in the above list (pg. 11), the phase shift is included from low side to high side as
   - +30° for positive sequence
   - -30° for negative sequence

   Let's take a brief "aside" to look at this.
   a. Why does a Δ-Y or Y-Δ xfmr have a 30° phase shift for positive sequence quantities? Consider a Y-Δ. Across the winding, it is V_{an}/V_{AB}, but on the Y-side, the line-line voltage is $V_{ab} = \sqrt{3}V_{an}/\_30°$, so line-line voltage ratio is $V_{ab}/V_{AB} = \sqrt{3}V_{an}/\_30°/V_{AB}$.
   b. Why does negative sequence use -30°? Consider the Y-side in our Y-Δ. From KVL, $V_{ab} = V_{an} - V_{bn}$. See Fig. 5.

i. Positive sequence: $V_{bn} = V_{an} / \underline{} -120$

ii. Negative sequence: $V_{bn} = V_{an} / \underline{} +120$

$V_{ab} = V_{an} - V_{bn}$

V_{bn}

V_{bn}

$120°$

$30°$

V_{an}

V_{an}

$30°$

$120°$

V_{bn}

Positive sequence relation
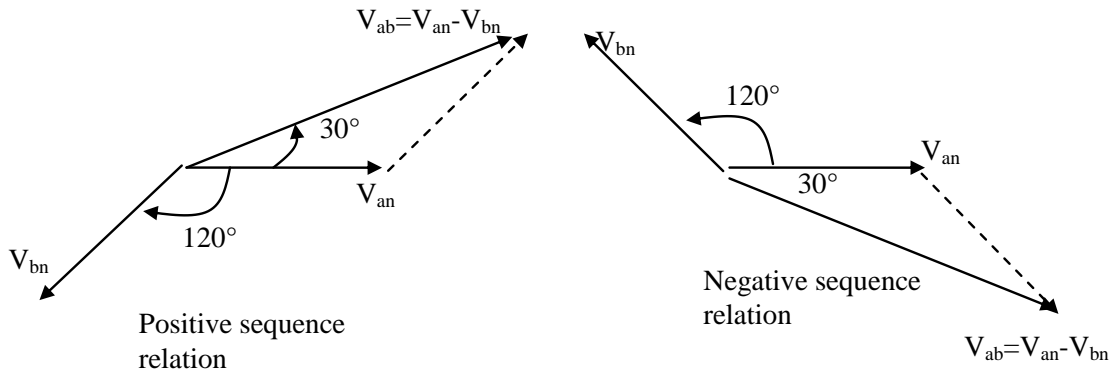
Negative sequence relation

$V_{ab} = V_{an} - V_{bn}$

Fig. 5

3. For zero-sequence network,

a. We get a complete open circuit ($I^0 = 0$) if there is an ungrounded Y on one or both sides.

b. We get isolation of primary from secondary if there is a Δ on one or both sides. This means that Δ connection prevents pass-through of zero-sequence currents. However, we may still get zero-sequence current flowing if the other side is grounded Y or Δ.

c. We get no isolation if both sides are grounded-Y.

No flow.

Flow, but no pass-through..

Flow and pass-through

d. $Z^0 = Z_{series} + 3Z_{np} + 3Z_{ns}$ where:

- Z_{np}: neutral impedance on primary
- Z_{ns}: neutral impedance on secondary

Two concepts important in understanding the points under (3) above are:

- There must be a connection to ground on the primary (secondary) side for zero-sequence current to flow between the primary (secondary)-side _system_ and the primary (secondary) side of the transformer.

- **If zero-sequence currents cannot flow on primary** (secondary) side of the transformer, **then** because currents on the secondary (primary) side of the transformer can only arise through induction of currents on the primary (secondary) side of the transformer, **zero-sequence currents also cannot flow on the secondary** (primary) side of the transformer.

So let's draw 0+- sequence circuits for various transformer connections....
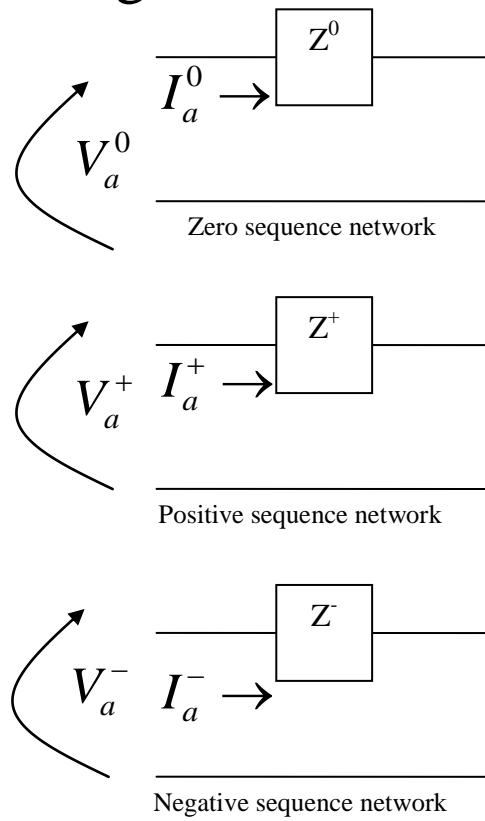
1.Grounded Y to grounded Y.

$$I_a^0 \rightarrow \boxed{Z^0}$$

$$V_a^0$$

Zero sequence network

$$V_a^+ \quad I_a^+ \rightarrow \boxed{Z^+}$$

Positive sequence network

$$V_a^- \quad I_a^- \rightarrow \boxed{Z^-}$$

Negative sequence network

Fig. 5

2. Grounded Y to Y or Y to grounded Y.

Z_0

$I_a^0 \rightarrow$

V_a^0

Zero sequence network

Z^+

V_a^+ $I_a^+ \rightarrow$

Positive sequence network

Z^-

V_a^- $I_a^- \rightarrow$
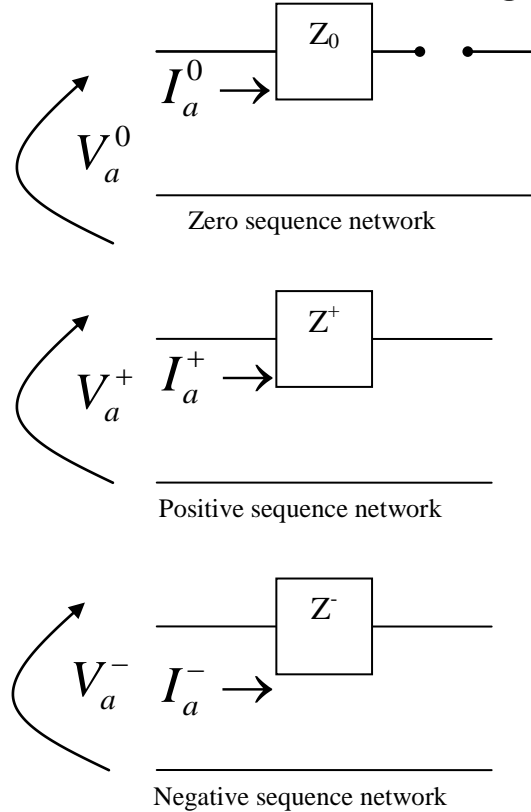
Negative sequence network

Fig. 6

Here, there is no place for zero-sequence currents to flow on the Y side (since there is no neutral and sum of phase currents, which equals $3I_a^0$, must be 0). Therefore, there can be no zero-sequence currents flowing on the other side either. So $I_a^0 = 0$ for this connection.

Y to grounded Y is the same.

3.Δ-Δ

Zero sequence network
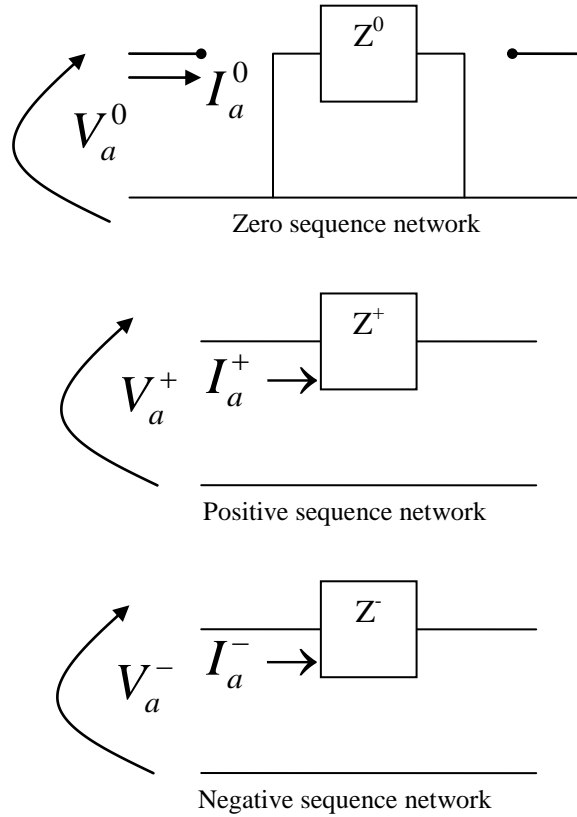
Positive sequence network

Negative sequence network

Fig. 7

Here, zero sequence currents cannot enter or leave either Δ winding, so for all practical purposes, the zero-sequence circuit is an open on both sides.

4.Grounded Y to Δ or Δ to grounded Y

Z^0

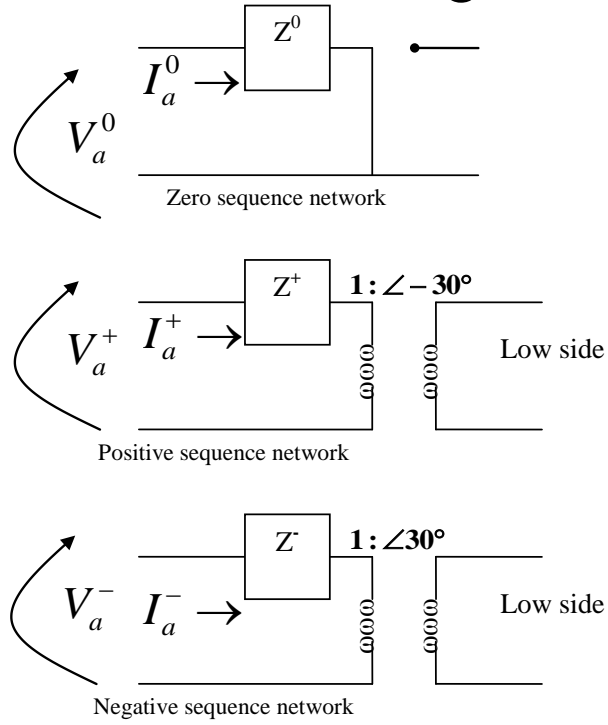$I_a^0 \rightarrow$

V_a^0

Zero sequence network

Z^+ **1 : ∠ − 30°**

V_a^+ $I_a^+ \rightarrow$

Low side

Positive sequence network

Z^- **1 : ∠30°**

V_a^- $I_a^- \rightarrow$

Low side

Negative sequence network

Fig. 8: Grounded Y to Δ

Z^0

$I_a^0 \rightarrow$

V_a^0

$I_A^0 \rightarrow$

Zero sequence network

Z^+ **1 : ∠ − 30°**

V_a^+ $I_a^+ \rightarrow$

Low side

Positive sequence network

Z^- **1 : ∠30°**

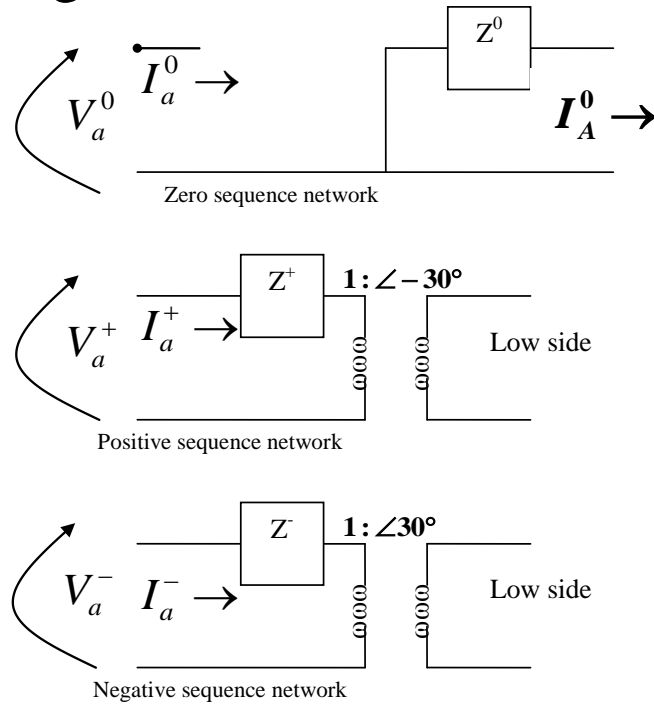V_a^- $I_a^- \rightarrow$

Low side

Negative sequence network

Fig. 9: Δ to Grounded Y

In Figs. 8 and 9, we observe that zero sequence currents can flow out of the grounded Y side, which means they also must be able to flow within the Δ (but not out of the Δ).

We also observe that, in both Figs. 8 and 9:

- -30º phase shift occurs of low side quantities relative to high side quantities for positive sequence (which implies high side leads low side by 30º for positive sequence quantities, in conformance with industry convention)
- 30º phase shift occurs of low side quantities relative to high side quantities for negative sequence (which implies high side quantities lag low side by 30º for negative sequence, in conformance with industry convention).
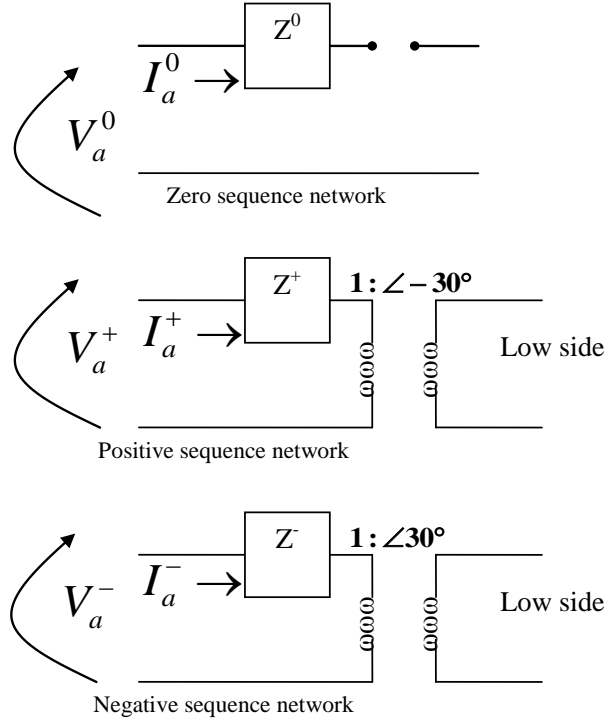
5. Y-Δ or Δ-Y.

Zero sequence network

Positive sequence network

Negative sequence network

Fig. 10: Y to Δ

Zero sequence network
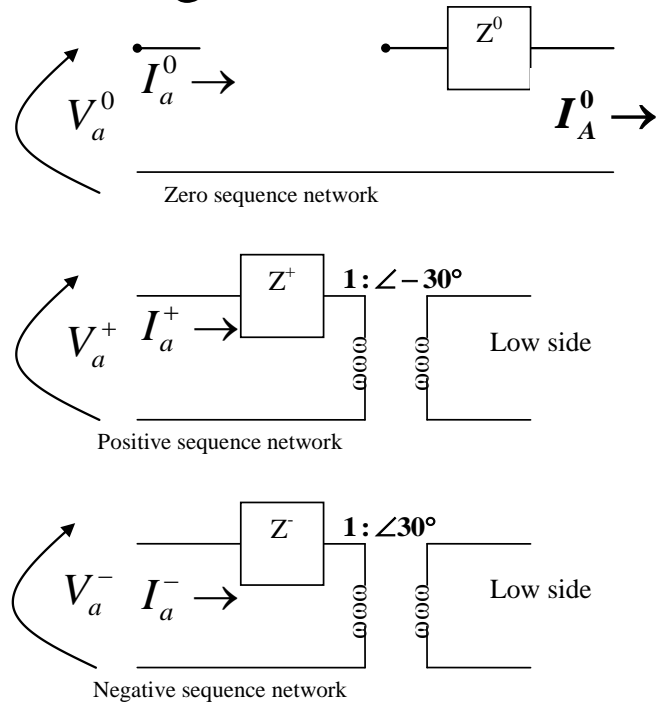
Positive sequence network

Negative sequence network

Fig. 11: Δ to Y

Observe that Figs. 10 and 11 are exactly like Figs. 8 and 9 in the positive and negative sequence circuits. The only difference is the zero-sequence circuit, where we see that, in Figs. 10 and 11, not only can zero-sequence currents not pass through (which is the case in Figs. 8 and 9) but they cannot flow at all.

3.4 Rotating machines (see sec 12.7, text)

Development of sequence impedances for the synchronous machine requires significant effort together with background in two-reactance theory, including the Park's transformation. We do not have that background. Your text offers some of that background in chapter 7, and in Appendix 5. Additional references include [1, chap. 6], [2, chap. 1]. Here we simply provide some comments.

<u>Positive sequence reactance</u>: As in the symmetrical fault analysis, we will just use X_d, X'_d, or X''_d, depending on what time frame of interest we have. This is quite reasonable for smooth rotor machines, but approximate for salient pole-machines.

<u>Negative sequence reactance</u>: The negative sequence currents set up flux in the air gap that rotates opposite to the rotor and therefore sweeps rapidly over the face of the rotor, inducing currents in the iron which counteract the original flux. This condition is similar to the rapidly changing flux immediately upon the occurrence of a short circuit at the machine terminals. As a result, the negative sequence reactance is generally assumed equal to X''_d.

Zero-sequence: Two comments:

1. The zero sequence reactance is typically quite small. The reason for this is that the zero sequence currents in the a, b, and c windings are in-phase. Their individual fluxes in the air gap sum to zero and therefore induce no voltage, so the only effect to be modeled is due to leakage reactance. We call this Z_g^0 .

2. As with loads, if the neutral is grounded through an impedance Z_n, because 3 times the zero sequence current flows through Z_n, we model $3Z_n$ in the zero sequence network.

Therefore we have, for generators, that

$$Z^0 = Z_g^0 + 3Z_n \qquad (12)$$

Voltage source: Finally, because generators produce balanced positive sequence voltages, generators produce no negative or zero sequence voltages. Therefore, we model a voltage source only in the positive sequence circuit.

When working in per-unit (as we have been assuming throughout this discussion), the positive sequence source voltage is typically assumed to be $E_{an}=1.0$. Although it actually may be something a little different than 1.0, the influence on final short circuit currents calculated is negligible.

$$Z^0 \qquad I_a^0 \rightarrow \qquad V_a^0$$

Zero sequence network

$$Z^+ \qquad I_a^+ \rightarrow \qquad V_a^+$$
$$\overset{+}{E_{an}} \\ -$$

Positive sequence network

$$Z^- \qquad I_a^- \rightarrow \qquad V_a^-$$
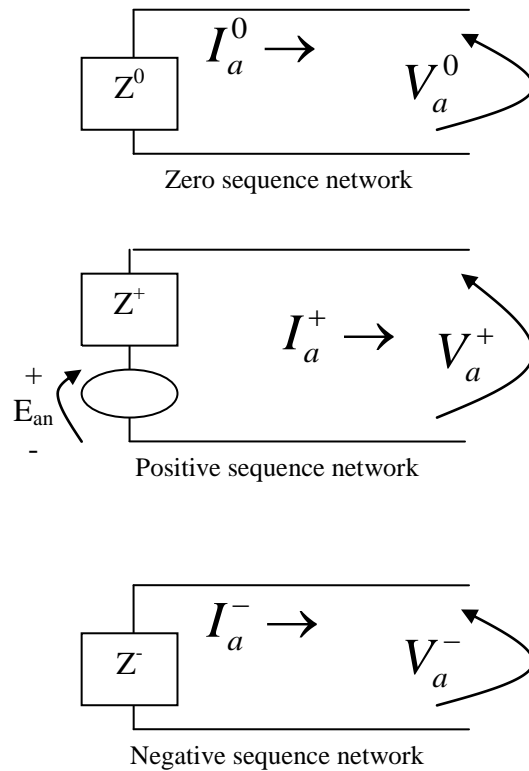
Negative sequence network

Fig. 12

Your text provides a table of typical synchronous machine reactances, see Table 12.1 of pp. 468. You can compare this table with a similar one below that I obtained from another reference [3]. All values are in per-unit on the MVA base of the machine.

		Smooth Rotor	Salient Pole	Synchronous Condensers	Motor
X^+	X_d	1.1	1.15	1.8	1.2
	X'_d	0.23	0.37	0.4	0.35
	X''_d	0.12	0.24	0.25	0.30
X^-		0.13	0.29	0.27	0.35
X^0		0.05	0.11	0.09	0.16

HW4: Due Tuesday, Feb 10, 2015.
In your text:
12.2, 12.3, 12.9, 12.10, 12.11, 12.12, 12.17

[1] P. Anderson, "Analysis of Faulted Power Systems," Iowa State University Press, 1973.
[2] E. Kimbark, "Power system stability, Vol. II, Synchronous Machines," 1995 by IEEE (Originally published in 1955).
[3] J. Glover and M. Sarma, "Power system analysis and design," PWS Publishers, 1987