Stability 4

1.0 Introduction

In the previous notes (Stability 3), we developed the equal area criterion, which says that

> For stability, $A_1=A_2$, which means the decelerating energy (A_2) must equal the accelerating energy (A_1) in order for the system response to be stable.

Analytically, we have that

$$\underbrace{\int_{\delta_0}^{\delta_{clear}} P_M^0 - P_{fault} d\delta}_{A_1} = \underbrace{\int_{\delta_{clear}}^{\delta_{\max}} P_{fault} - P_M^0 d\delta}_{A_2} \tag{1}$$

Figure 1 below illustrates a stable case.

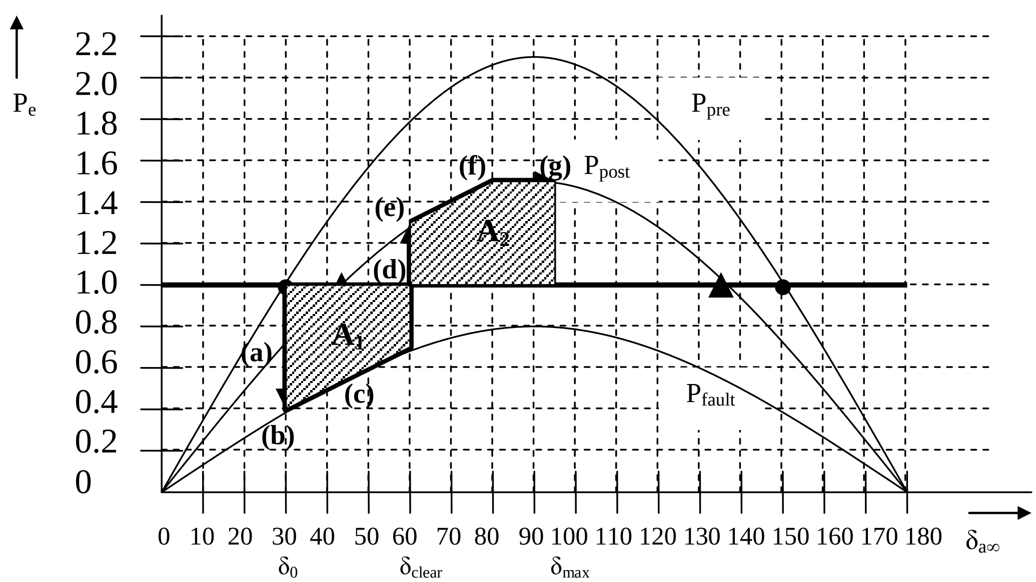

Fig. 1

Figure 2 below illustrates an unstable case.

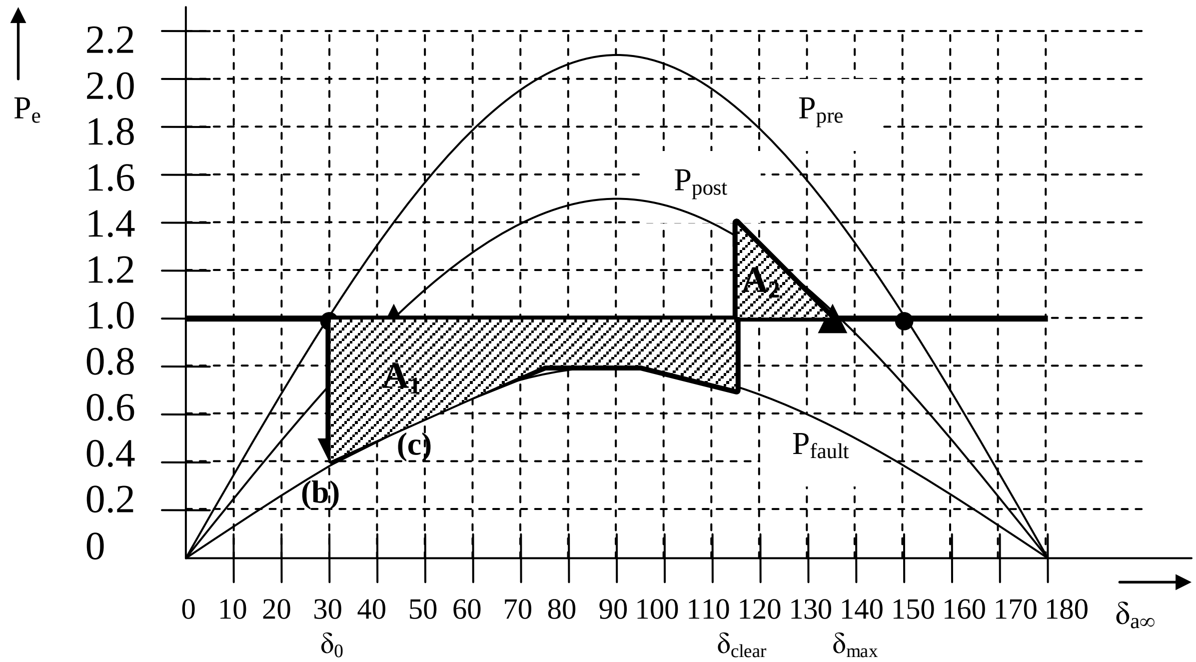

Fig. 2

Everything about Figs. 1 and 2 are the same with one exception, the clearing angle (δ_{clear}) in Fig. 2 is greater than the clearing angle in Fig. 1. In other words, Fig. 2 assumes that the speed of the protection system is slower than the speed of the protection system in Fig. 1.

In these notes, we want to develop expressions for computing critical clearing angle.

2.0 Critical clearing angle

The critical clearing angle will occur when the equal-area criterion is satisfied and the maximum angle is $\delta_{max}=180-\delta_0$. Such a case is illustrated in Fig. 3.

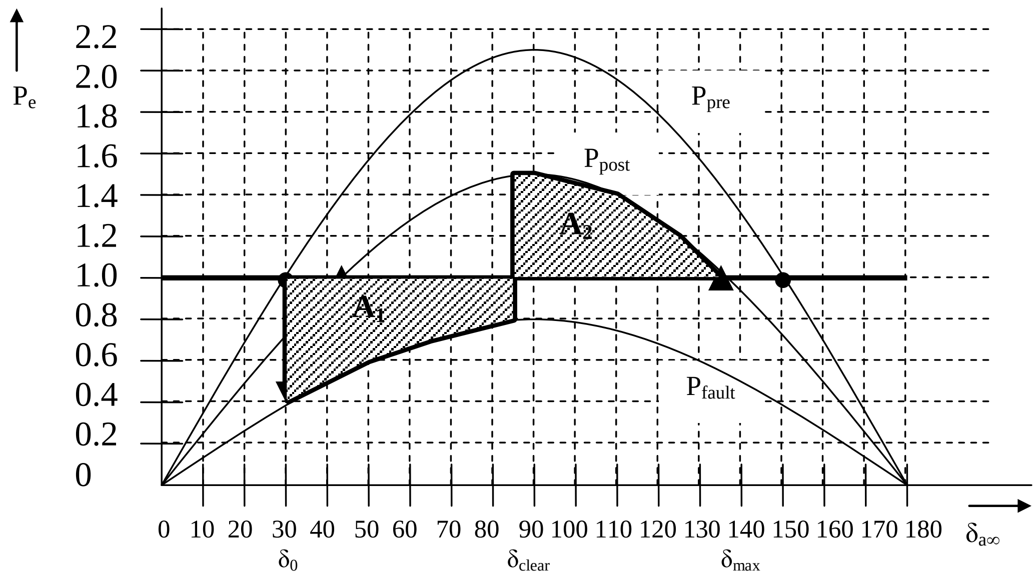

Fig. 3

We want to compute the critical clearing time. We will denote it as $\delta_{clear}=\delta_{cr}$.

To do this, let's define the following:

$$P_{pre} = P_{pre\max} \sin\delta \tag{2}$$

$$P_{fault} = P_{fault\max} \sin\delta = r_1 P_{pre\max} \sin\delta \tag{3}$$

$$P_{post} = P_{post\max} \sin\delta = r_2 P_{pre\max} \sin\delta \tag{4}$$

where

- P_{premax}, $P_{faultmax}$, and $P_{postmax}$ are the amplitudes of the power-angle curves for the pre-fault, fault-on, and post-fault networks, respectively;
- $0 \le r_1 \le 1$ where $r_1=0$ corresponds to a three-phase fault at the machine terminals, and $r_1=1$ corresponds to no-fault at all.
- $0 \le r_2 \le 1$ where $r_2=0$ corresponds to a three-phase fault at the machine terminals that is not cleared, and $r_2=1$ corresponds to a temporary fault (fault is removed without protective relay action to also remove a circuit and weaken the transmission)

Let's first compute A_1.

$$A_1 = \int_{\delta_0}^{\delta_{clear}} P_M^0 - P_{fault} d\delta$$

$$= \int_{\delta_0}^{\delta_{clear}} (P_M^0 - P_{fault\max} \sin\delta) d\delta$$

$$= P_M^0 \delta + P_{fault\max} \cos\delta \Big|_{\delta_0}^{\delta_{clear}} \tag{5}$$

$$= P_M^0 (\delta_{clear} - \delta_0) + P_{fault\max} (\cos\delta_{clear} - \cos\delta_0)$$

Now let's compute A_2.

$$A_2 = \int_{\delta_{clear}}^{\delta_{\max}} P_{post} - P_M^0 d\delta$$

$$= \int_{\delta_{clear}}^{\delta_{\max}} P_{post\max} \sin\delta - P_M^0 d\delta$$

$$= -P_{post\max} \cos\delta - P_M^0 \delta \Big|_{\delta_{clear}}^{\delta_{\max}} \tag{6}$$

$$= P_{post\max} (\cos\delta_{clear} - \cos\delta_{\max}) + P_M^0 (\delta_{clear} - \delta_{\max})$$

If the system is stable, then A_1=A_2. So let's equate the expressions in eq. (5) and (6), below.

$$A_1 = P_M^0(\delta_{clear} - \delta_0) + P_{fault\max}(\cos\delta_{clear} - \cos\delta_0)$$
$$= P_{post\max}(\cos\delta_{clear} - \cos\delta_{\max}) + P_M^0(\delta_{clear} - \delta_{\max}) = A_2 \quad (7)$$

Expand:

$$P_M^0\delta_{clear} - P_M^0\delta_0 + P_{fault\max}\cos\delta_{clear} - P_{fault\max}\cos\delta_0$$
$$= P_{post\max}\cos\delta_{clear} - P_{post\max}\cos\delta_{\max} + P_M^0\delta_{clear} - P_M^0\delta_{\max} \quad (8)$$

Notice there is a $P_M^0\delta_{cr}$ on both sides, and so:

$$-P_M^0\delta_0 + P_{fault\max}\cos\delta_{clear} - P_{fault\max}\cos\delta_0$$
$$= P_{post\max}\cos\delta_{clear} - P_{post\max}\cos\delta_{\max} - P_M^0\delta_{\max} \quad (9)$$

Let's put all terms with $\cos\delta_{\mathrm{clear}}$ on the left side and everything else on the right:

$$P_{fault\max}\cos\delta_{clear} - P_{post\max}\cos\delta_{clear}$$
$$= P_{fault\max}\cos\delta_0 - P_{post\max}\cos\delta_{\max} - P_M^0\delta_{\max} + P_M^0\delta_0 \quad (10)$$

Factor out the $\cos\delta_{\mathrm{clear}}$ term on the left and the P_M^0 term on the right:

$$\cos\delta_{clear}(P_{fault\max} - P_{post\max})$$
$$= P_{fault\max}\cos\delta_0 - P_{post\max}\cos\delta_{\max} + P_M^0(\delta_0 - \delta_{\max}) \quad (11)$$

Divide by the term in parentheses on the left:

$$\cos\delta_{clear}$$
$$= \frac{P_{fault\max}\cos\delta_0 - P_{post\max}\cos\delta_{\max} + P_M^0(\delta_0 - \delta_{\max})}{(P_{fault\max} - P_{post\max})} \quad (12)$$

Now eq. (12) is true as long as the system response is stable (if it is not stable, then the equal-area criterion is not satisfied and therefore eq. (7) is invalid).

But if the system response is marginally stable, then the clearing angle will be the maximum possible angle for which we can clear and still retain stability, i.e., it is the critical clearing angle, and so in this case, $\delta_{clear}=\delta_{cr}$. In addition, the maximum angle must be the unstable equilibrium, which is $\delta_{max}=180-\delta_0$. Making these substitutions into eq. (12) results in

$$\cos\delta_{cr} = \frac{P_{fault\max}\cos\delta_0 - P_{post\max}\cos(\pi-\delta_0) + P_M^0(\delta_0-\pi+\delta_0)}{(P_{fault\max}-P_{post\max})} \tag{13}$$

Recalling that $\cos(\pi-x)=-\cos(x)$, and noting on the right-hand-side that we can combine the two δ_0 terms inside the brackets, we get:

$$\cos\delta_{cr} = \frac{P_{fault\max}\cos\delta_0 + P_{post\max}\cos\delta_0 + P_M^0(2\delta_0 - \pi)}{(P_{fault\max} - P_{post\max})} \quad (14)$$

Now recall eqs. (3) and (4), which imply that:

$$P_{fault\max} = r_1 P_{pre\max} \quad (15)$$

$$P_{post\max} = r_2 P_{pre\max} \quad (16)$$

Substituting eqs. (15) and (16) into (14), we get:

$$\cos\delta_{cr} = \frac{r_1 P_{pre\max}\cos\delta_0 + r_2 P_{pre\max}\cos\delta_0 + P_M^0(2\delta_0 - \pi)}{(r_1 P_{pre\max} - r_2 P_{pre\max})} \quad (17)$$

Factoring out the P_{premax} from the bottom and dividing it through all terms in the top, and rearranging, results in

$$\cos\delta_{cr} = \frac{r_1\cos\delta_0 + r_2\cos\delta_0 + \dfrac{P_M^0}{P_{pre\max}}(2\delta_0 - \pi)}{(r_1 - r_2)} \quad (18)$$

Let’s consider a few cases:

Case 1, Temporary fault at machine terminals:

The fact that it is a temporary fault means that the post-disturbance network is the same as the pre-disturbance network, therefore $r_2=1$.

The fact that it is a three-phase fault at the machine terminals means that the ability to transmit power to the infinite bus, during the fault-on period, is zero. Therefore $r_1=0$.

Applying these values to eq. (18) results in:

$$\cos\delta_{cr} = \frac{\cos\delta_0 + \dfrac{P_M^0}{P_{pre\max}}(2\delta_0 - \pi)}{-1}$$

$$= \frac{P_M^0}{P_{pre\max}}(\pi - 2\delta_0) - \cos\delta_0 \qquad (19)$$

But recall that

$$P_M^0 = P_{pre\max}\sin\delta_0 \qquad (20)$$

Substitution of (20) into (19) results in

$$\cos\delta_{cr} = \frac{P_{pre\max}\sin\delta_0}{P_{pre\max}}(\pi - 2\delta_0) \qquad (21)$$

Or,

$$\cos\delta_{cr} = \sin\delta_0(\pi - 2\delta_0) - \cos\delta_0 \qquad (22)$$

The above is a closed form solution for the critical clearing angle for the condition of a temporary three phase fault at the machine terminals.

Recall the example introduced in the notes called "Stability 2" for the below system:

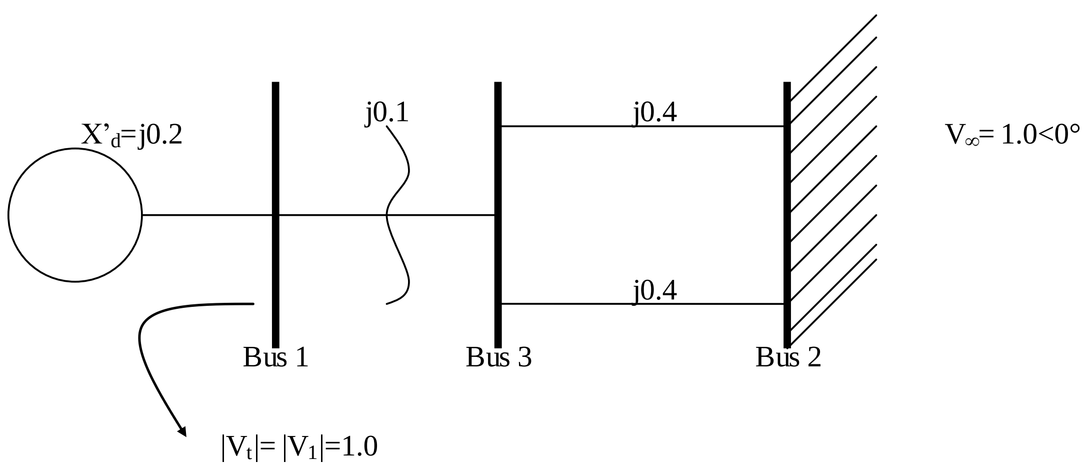

Fig. 4

In those notes, we determined that the angle between the generator internal voltage and the infinite bus is $\delta_{a\infty}$=28.44°. This is δ_0, and it is for the same system that is characterized in these notes by Fig. 3. Using δ_0=28.44°=0.4964 rad in eq. (22) results in

$$\cos\delta_{cr} = \sin\delta_0(\pi - 2\delta_0) - \cos\delta_0$$
$$= \sin(0.4964)(\pi - 2(0.4964)) - \cos(0.4964)$$
$$= 0.4763(\pi - 0.998) - 0.8793$$
$$= 1.021 - 0.8793 = 0.1417$$

Therefore we have that

$$\cos\delta_{cr} = 0.1417 \rightarrow \delta_{cr} = \cos^{-1} 0.1417 = 1.4286 rad$$

In degrees, this is 81.85°. Reference to Fig. 5, which is a "hand-approximation" for this case, suggests this angle is quite reasonable.

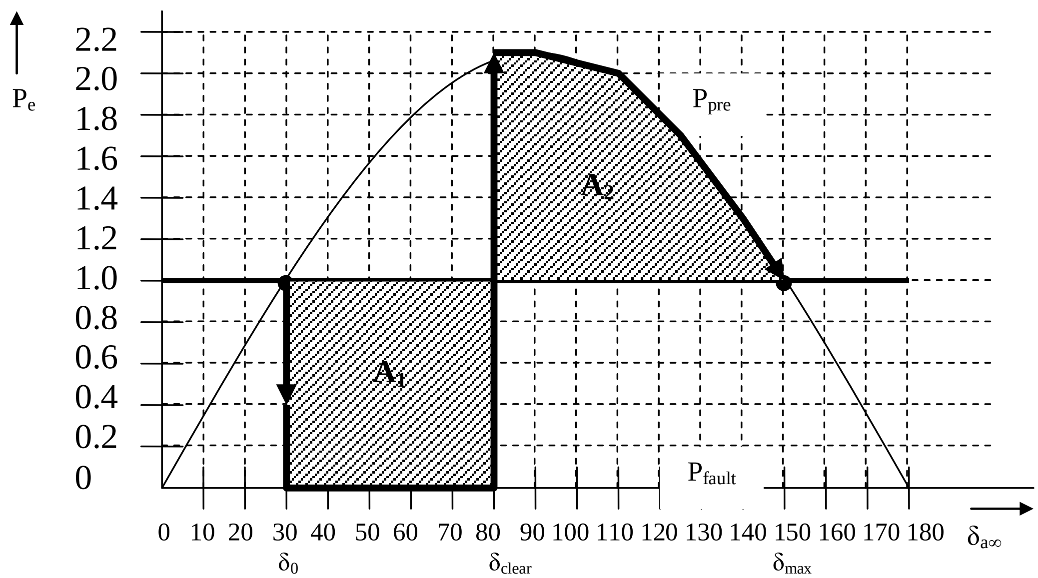

Fig. 5

3.0 Critical clearing time

Let's consider our case of a temporary three-phase fault at the machine terminals. To obtain information on clearing time, we need to look at the differential equation characterizing this system, which is:

$$\frac{2H}{\omega_{e0}}\ddot{\delta}(t) = P_{a,pu} = P_M^0 - P_e \tag{23}$$

The right hand-side is of course 0 before the fault (no acceleration), but just after the fault, in this case, P_e goes instantly to 0. We therefore have that

$$\frac{2H}{\omega_{e0}}\ddot{\delta}(t) = P_M^0 \tag{24}$$

And so we see that just after the fault, there is non-zero acceleration, but that acceleration is constant since the right-hand-side is constant!

Equation (22) may be rewritten as

$$\ddot{\delta}(t)=\frac{\omega_{e0}}{2H}P_M^0 \qquad (25)$$

Rewrite the left-hand-side of eq. (25) as

$$\ddot{\delta}(t)=\frac{d^2\delta}{dt^2}=\frac{d\omega}{dt}=\frac{\omega_{e0}}{2H}P_M^0 \qquad (26)$$

Multiply both sides by dt:

$$d\omega=\frac{\omega_{e0}}{2H}P_M^0 dt \qquad (27)$$

Now integrate on the left from ω(0)=0 (initial state is zero velocity) to ω(t) and on the right from $t=0^+$ to t:

$$\int_{\omega(0)=0}^{\omega(t)} d\omega(t)=\frac{\omega_{e0}}{2H}P_M^0\int_{0^+}^{t} dt \qquad (28)$$

$$\omega(t)=\frac{\omega_{e0}}{2H}P_M^0 t \qquad (29)$$

Now express the left-hand-side as the derivative of δ(t)

$$\frac{d\delta}{dt} = \frac{\omega_{e0}}{2H} P_M^0 t \tag{30}$$

Multiply both sides by dt:

$$d\delta = \frac{\omega_{e0}}{2H} P_M^0 t dt \tag{31}$$

Now integrate the left-hand-side from δ_0 to $\delta(t)$, and the right-hand-side from $t=0^+$ to t:

$$\int_{\delta_0}^{\delta(t)} d\delta = \int_{0^+}^{t} \frac{\omega_{e0}}{2H} P_M^0 t dt \tag{32}$$

$$\delta(t) - \delta_0 = \frac{\omega_{e0}}{4H} P_M^0 t^2 \tag{33}$$

Now recall we have the critical clearing angle $\delta(t)=\delta_{cr}$, and we are attempting to find the time for which we reach this angle. So solve eq. (33) for time to obtain:

$$t = \sqrt{\left(\delta(t) - \delta_0\right) \frac{4H}{P_M^0 \omega_{e0}}} \tag{34}$$

When the angle is the critical clearing angle, we obtain:

$$t_{cr} = \sqrt{\left(\delta_{cr}(t) - \delta_0\right)\frac{4H}{P_M^0 \omega_{e0}}} \qquad (35)$$

So, let's compute the critical clearing time for our machine. The only other thing we need to know is the inertia constant H. We can assume that it is H=3.0 sec on the machine base. With $P_M^0 = 1.0$, ω_{e0}=377, δ_0=28.44°=0.4964 rad, and $\delta_{cr} = 1.4286 rad$, we have

$$t_{cr} = \sqrt{\left(1.4286 - 0.4964\right)\frac{4*3}{1*377}}$$

$$= 0.1723 \sec$$

What if the inertia constant was 5? In this case, we would obtain:

$$t_{cr} = \sqrt{\left(1.4286 - 0.4964\right)\frac{4*5}{1*377}}$$

$$= 0.2224 \sec$$

➔The larger the machine, the longer it takes to accelerate it.