Stability 2

1.0 Introduction

We ended our last set of notes, concluding that the following equation characterizes the electromechanical dynamics of a synchronous machine.

$$M\ddot{\delta}(t) = P_M^0 - \frac{|E_a||V_\infty|}{X_d}\sin\delta \tag{1}$$

Now I want to do an example of the most simple power system that we can consider – the so-called one-machine against an infinite bus.

2.0 Example

Consider the power system in Fig. 1. It is referred to as a one-machine against an infinite bus. There are no modern day power systems like this, although there are portions of actual systems which behave in a similar way, and this system serves well to illustrate these basic kinds of behavior.

So many engineers use it to provide conceptual basis for understanding fundamental machine behavior. It would not be used, however, to provide precise machine response as the computer serves well for this purpose.

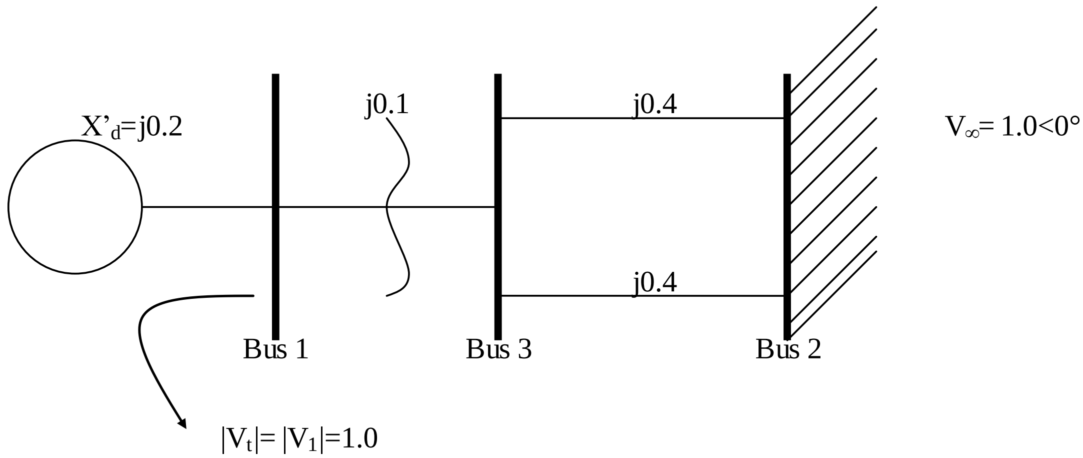


Fig. 1

Bus 2, the infinite bus, is so-called because it has a voltage and angle that is constant under all conditions, and it can absorb infinite power. Although there is no real infinite bus in power systems, a single small machine connected to a very large power system behaves as if it is connected to an infinite bus.

Given that the machine is delivering 1.0 per unit power under steady-state conditions, we have the following objectives in this problem.

1. **Determine the voltage phasor E_a.**
2. **Draw the power-angle (P-δ) curve.**
3. **Determine the steady-state operating point corresponding to the 1.0 pu power condition on the *pre-fault* power angle curve.**
4. **For a three-phase fault in the middle of one of the lines between buses 3 and 2, determine the *fault-on* power angle curve.**
5. **Determine the *post-fault* power-angle curve after protection has operated to clear the fault.**
6. **Determine the steady-state operating point corresponding to the 1.0 pu power condition on the *post-fault* power angle curve.**
7. **Use the three curves to describe what happens to the angle δ during the three periods: pre-fault, fault-on, and post-fault.**

1. Determine the voltage phasor E_a.

The pre-fault circuit diagram is given in Fig. 2.

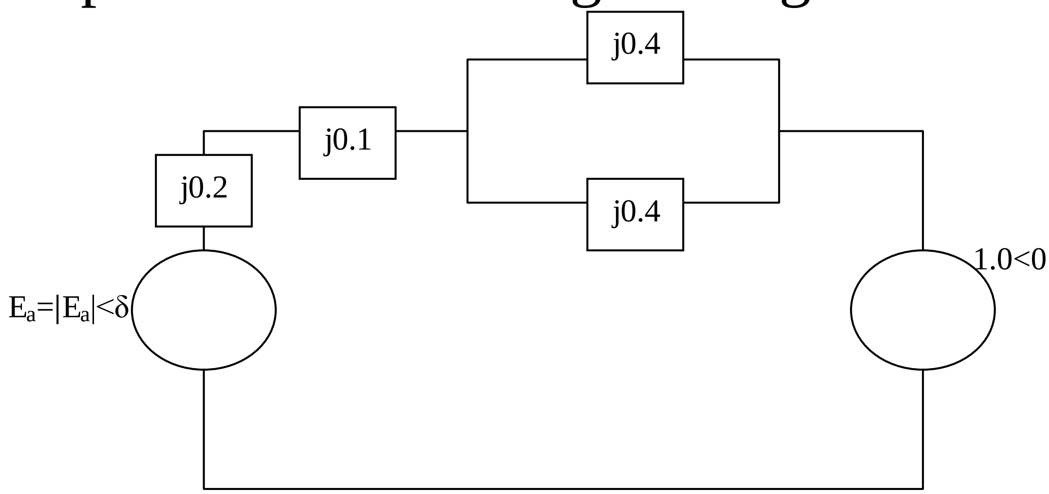


Fig. 2

Compute the impedance between the generator terminals and the infinite bus:

$$X_{1\infty} = 0.1 + 0.4 // 0.4 = 0.1 + 0.2 = 0.3$$

Note that we have two bus voltage magnitudes, the reactance between them, and the power flowing out of one of them, so our familiar relation for power flowing out of generator terminals will allow us to solve for $\delta_{1\infty}$, i.e.,

$$P_{pre} = \frac{|V_1||V_\infty|}{X_{1\infty}} \sin\delta_{1\infty} = \frac{(1)(1)}{0.3} \sin\delta_{1\infty} = 1.0$$

where $\delta_{1\infty}$ is the angle between E_a and bus 2.

$$\Rightarrow \quad \delta_{1\infty} = 17.458°$$

$$\Rightarrow \quad V_1 = 1\angle 17.458°$$

From this we can compute the current flowing from the machine terminals (bus 1) to the infinite bus, according to

$$I = \frac{1.0\angle 17.458° - 1.0\angle 0°}{j0.3} = 1.012\angle 8.729°$$

And from this, we may compute the internal voltage phasor E_a, according to

$$E_a = V_1 + I(j0.2)$$

$$= 1.0\angle 17.458° + 1.012\angle 8.729°(j0.2)$$

$$= 1.05\angle 28.44°$$

The above procedure is typical of what is done in full-scale commercial power flow programs where the program will begin from a power flow solution, from which it computes the current flow from every gen bus, and then it computes each generator's internal voltage as we have done in the above.

2. Draw the power-angle (P-δ) curve.

We can draw the power angle curve for different angles. Some of the choices are given below:

$$P_{pre} = \frac{|V_1||V_\infty|}{X_{1\infty}} \sin\delta_{1\infty} \tag{2}$$

$$P_{pre} = \frac{|E_a||V_\infty|}{X_{a\infty}} \sin\delta_{a\infty} \tag{3}$$

$$P_{pre} = \frac{|V_2||V_\infty|}{X_{2\infty}} \sin\delta_{2\infty} \tag{4}$$

Note that the electrical power (left-hand-side) is the same in all three cases since there is no resistance in this circuit. We should choose the most restrictive power angle curve, i.e., the one that gives the largest angle for the same power. Since the voltages are all reasonably close, the most restrictive curve is determined by the one with the largest reactance – this would be eq (3).

Using the numerical data for eq. (3), we have:

$$P_{pre} = \frac{|E_a||V_\infty|}{X_{a\infty}} \sin\delta_{a\infty} = \frac{(1.05)(1)}{0.5} \sin\delta_{a\infty} = 2.1\sin\delta_{a\infty} \tag{5}$$

Fig. 3 illustrates this curve.

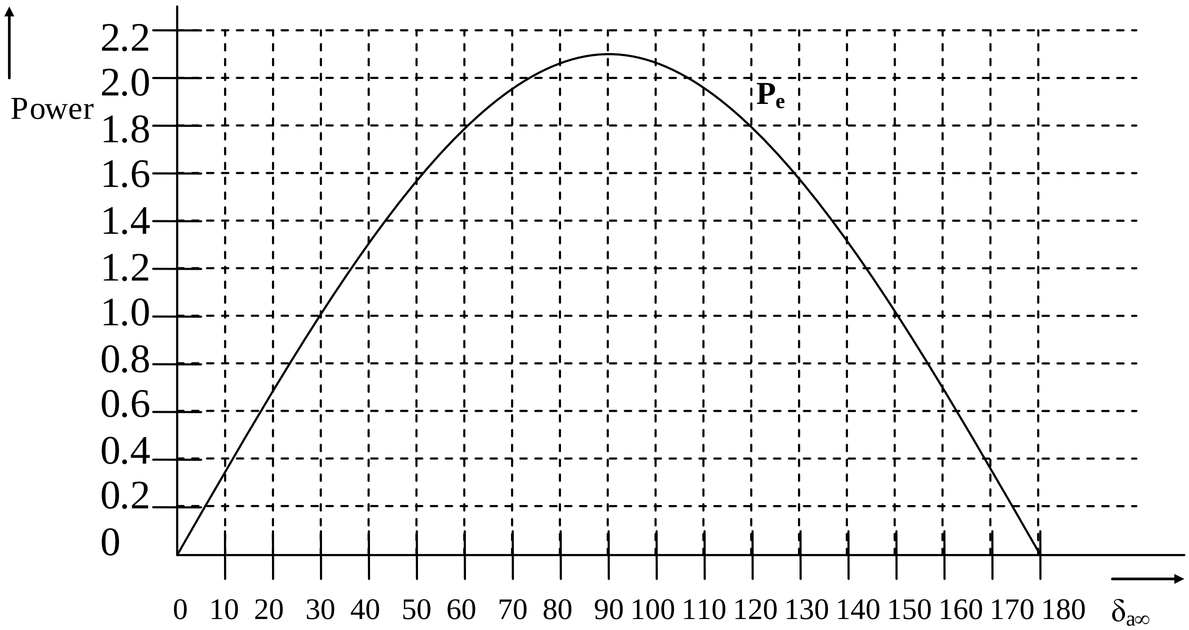


Fig. 3

3. Determine the steady-state operating point corresponding to the 1.0 pu power condition on the *pre-fault* power angle curve.

This is where P_e=1.0, i.e.,

$$P_{pre} = 2.1\sin\delta_{a\infty} = 1.0 \tag{6}$$

Solving for $\delta_{a\infty}$, we get $\delta_{a\infty}$=28.44°. We can show this point on the pre-fault power-angle curve, using dots as in Fig. 4.

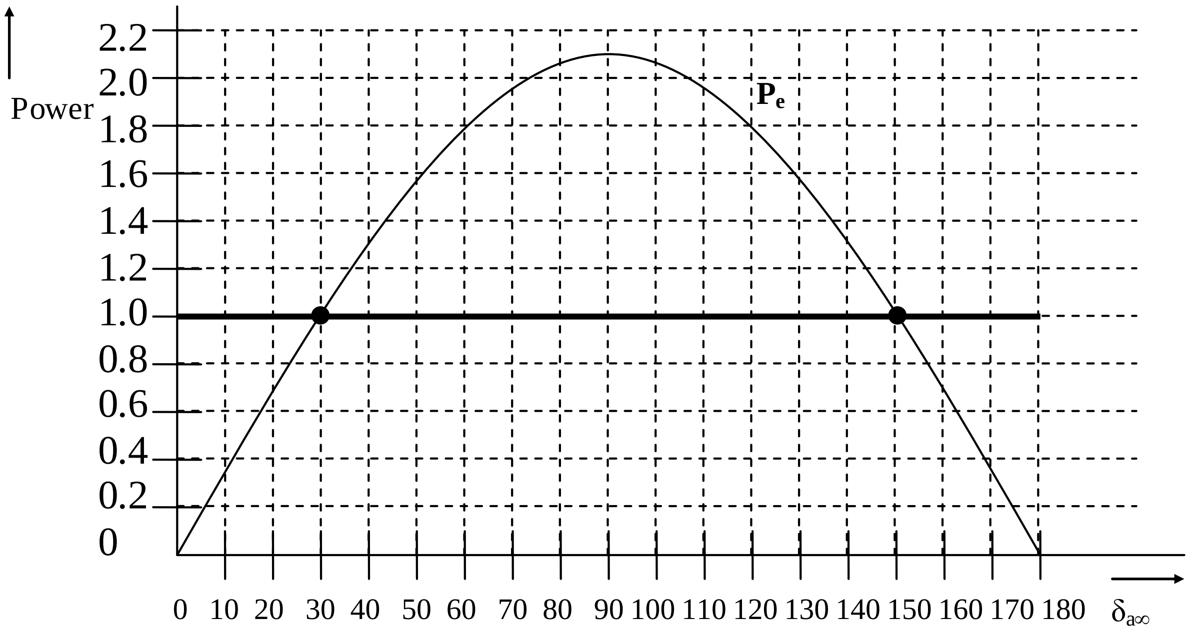


Fig. 4

Figure 4 shows, however, that there are really two solutions, one at 28.44° and the other at 180-28.44=151.56°. Both of these points constitute equilibria, i.e., a location in terms of the problem variables where all equations are satisfied, and, if unperturbed, the system would be able to lie in rest. We shall show later, however, that the point at 28.44° is a stable equilibrium, and the point at 151.56° is an unstable equilibrium.

4. For a three-phase fault in the middle of one of the lines between buses 3 and 2, determine the *fault-on* power angle curve.

The faulted system is shown in Fig. 5.

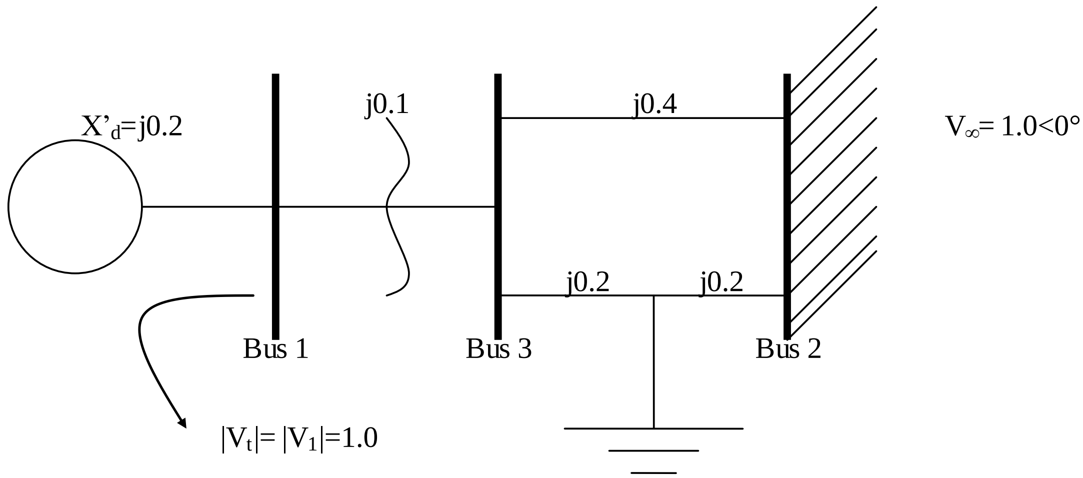


Fig. 5

The circuit diagram corresponding to the faulted system is shown in Fig. 6.

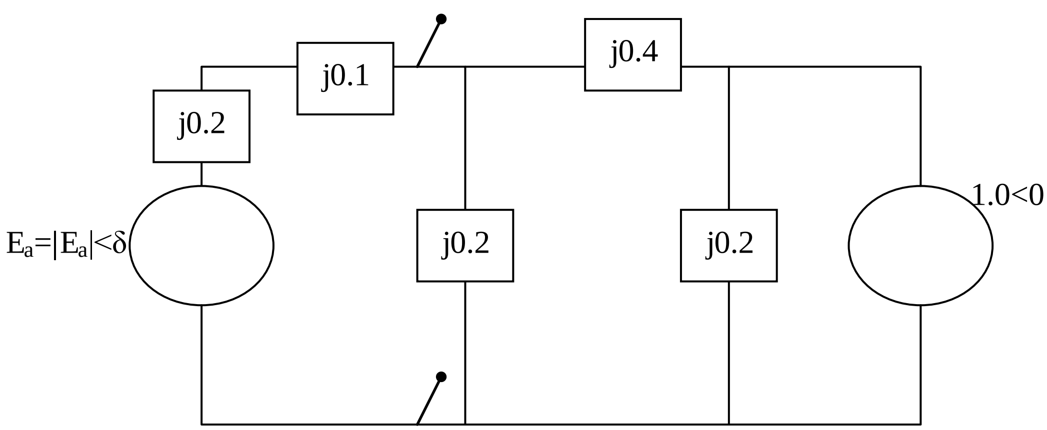


Fig. 6

So we want to be able to write another equation like eq. (5), except this time, the electrical power out will not be P_{pre} but rather P_{fault}.

To write such an equation, however, we will need the series reactance between the two voltage sources. This series reactance is not obvious from the circuit diagram of Fig. 6. We can get it, however, if we replace the circuit to the right of the two marked nodes in Fig. 6 with its Thevenin equivalent. The relevant part of the circuit is shown in Fig. 7.

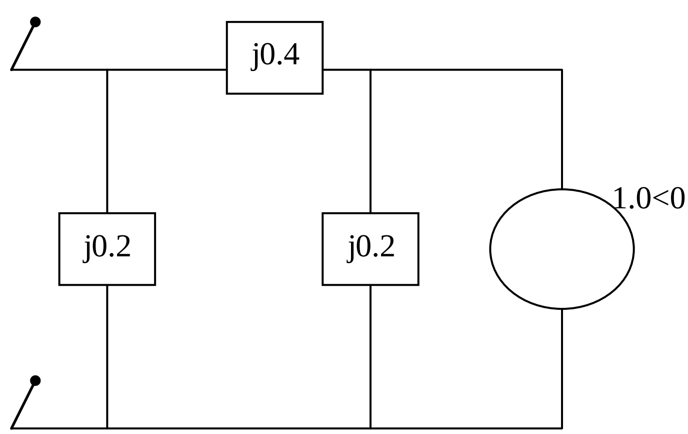


Fig. 7

We obtain the Thevenin voltage from the circuit of Fig. 7 as the voltage seen at the left-hand terminals. We can use voltage division to get it.

$$V_{thev} = 1.0\angle 0° \frac{0.2}{0.2+0.4} = 0.3333\angle 0° \quad (7)$$

We get the Thevenin impedance by idling the source and computing the composite impedance, as shown in Fig. 8.

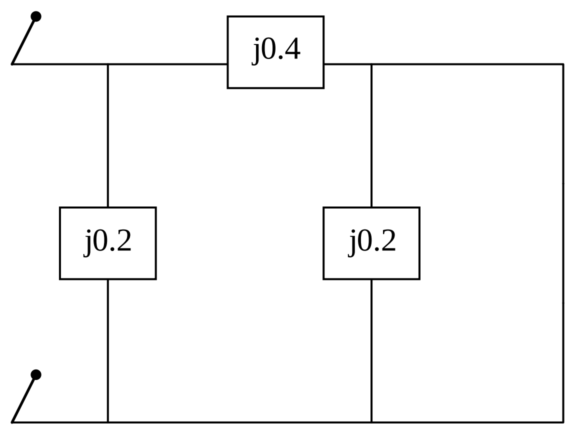


Fig. 8

In Fig. 8, we recognize that the j0.2 impedance on the right is shorted, therefore the impedance seen looking in from the terminals on the left is just the parallel combination of the j0.2 impedance on the left and the j0.4 impedance at the top. This is j(0.2)(0.4)/0.6=j0.1333. The faulted circuit with the Thevenin equivalent is given in Fig. 9.

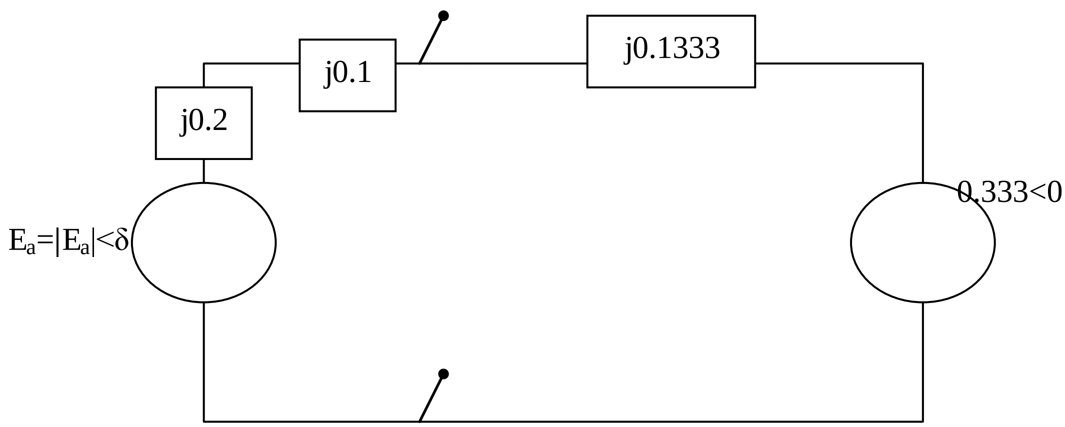


Fig. 9

From Fig. 9, we can immediately see that the impedance between the sources is

$$X_{aThev} = 0.2 + 0.1 + 0.1333 = 0.4333$$

write down the power-angle equation as:

$$P_{fault} = \frac{|E_a||V_{Thev}|}{X_{aThev}} \sin\delta_{aThev} = \frac{(1.05)(0.333)}{0.4333} \sin\delta_{aThev}$$
$$= 0.8077 \sin\delta_{aThev} \tag{8}$$

This curve is plotted in Fig. 10.

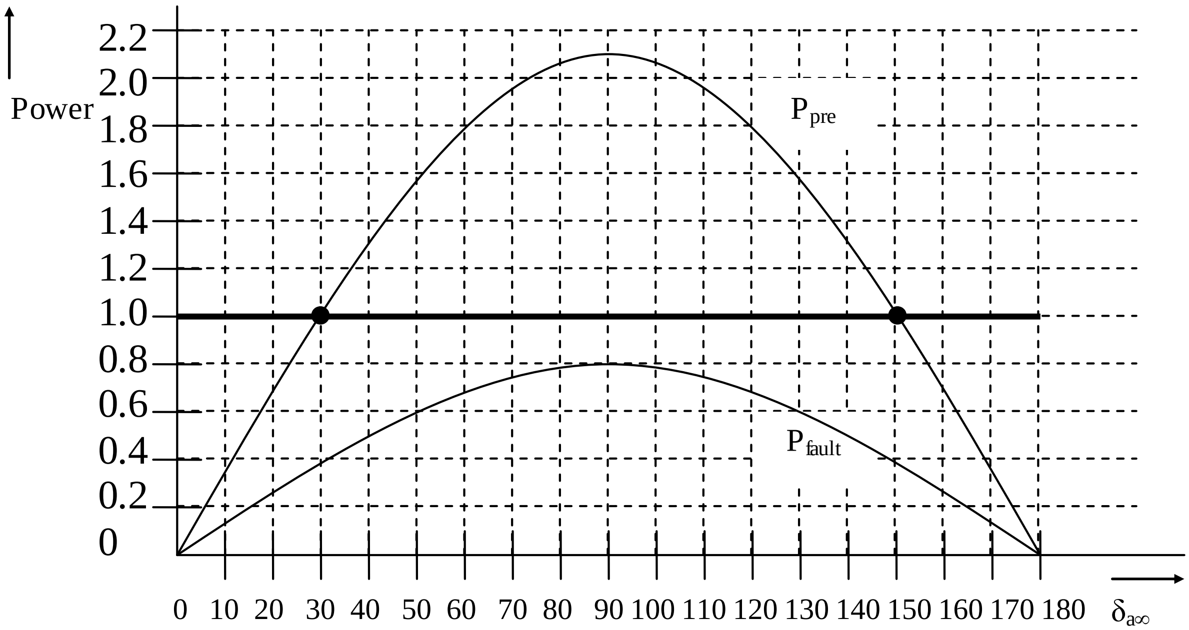


Fig. 10

5. **Determine the *post-fault* power-angle curve after protection has operated to clear the fault**.

The post-fault system is obtained from understanding of basic protective relaying which results in removing the faulted circuit. The resulting one-line diagram is shown in Fig. 11, and the corresponding circuit diagram is shown in Fig. 12.

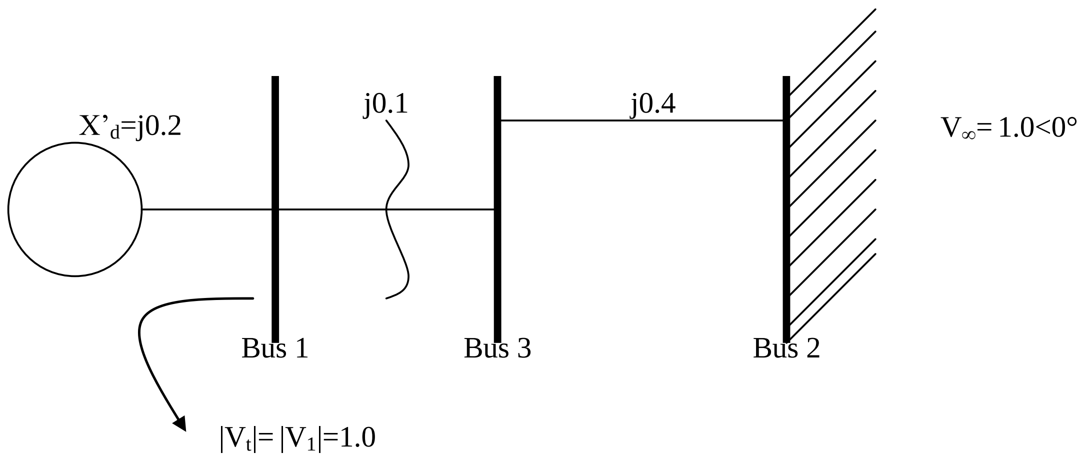


Fig. 11

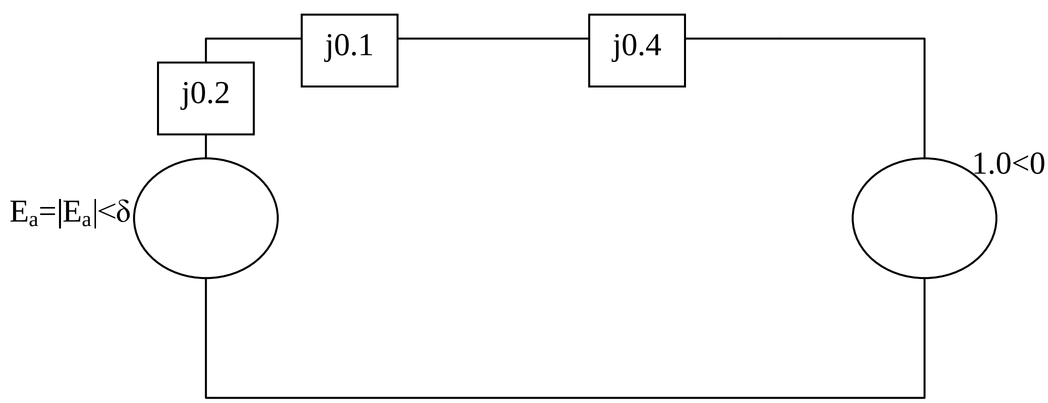


Fig. 12

Again, we need the series reactance between the two voltage sources, but this time, it is very easy to see that this series reactance is

$$X_{a\infty} = 0.2 + 0.1 + 0.4 = 0.7$$

Therefore, the post-fault power-angle curve is given by

$$P_{post} = \frac{|E_a||V_\infty|}{X_{a\infty}} \sin\delta_{a\infty} = \frac{(1.05)(1.0)}{0.7} \sin\delta_{a\infty}$$
$$= 1.5 \sin\delta_{a\infty} \tag{9}$$

This curve is plotted in Fig. 13.

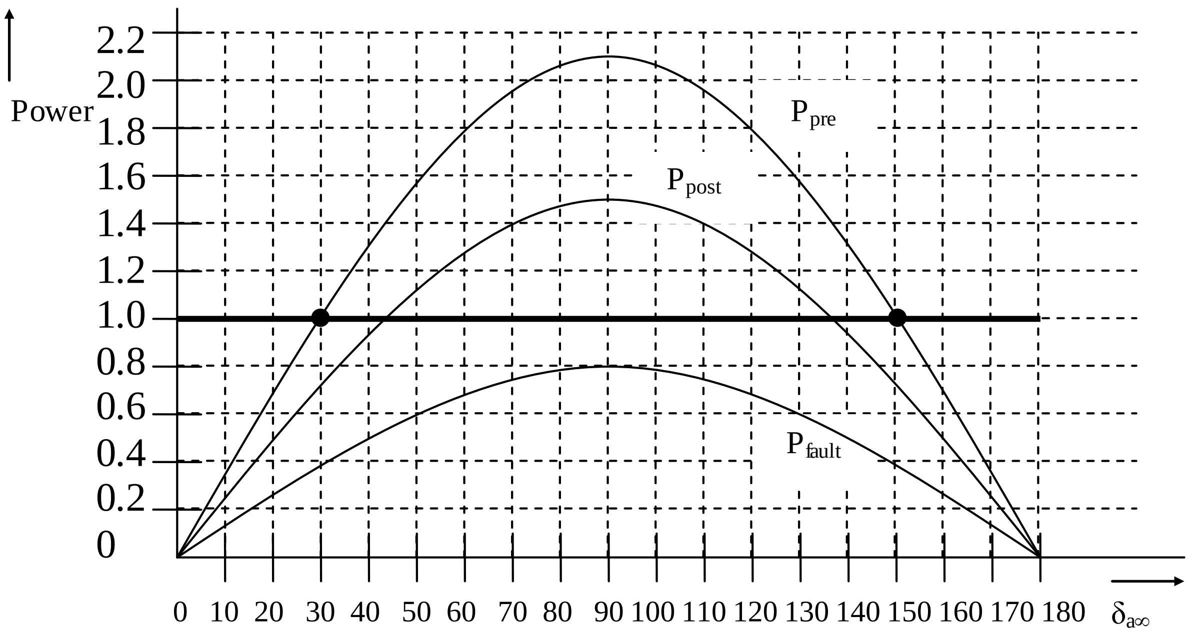


Fig. 13

Note from Fig. 13 that the pre-fault curve is highest, the fault-on curve is lowest, the post-fault curve is in between. This reflects the relative "strength" of the systems to transfer power from source to the infinite bus, where "strength" is determined by the impedance magnitude between source and infinite bus and voltage magnitudes at these two buses. More transmission makes systems stronger.

6. Determine the steady-state operating point corresponding to the 1.0 pu power condition on the *post-fault* power angle curve.

This is where P_e=1.0, i.e.,

$$P_{post} = 1.5 \sin \delta_{a\infty} = 1.0 \qquad (6)$$

Solving for $\delta_{a\infty}$, we get $\delta_{a\infty}$=41.81°. We can show this point on the pre-fault power-angle curve using triangles, as in Fig. 14.

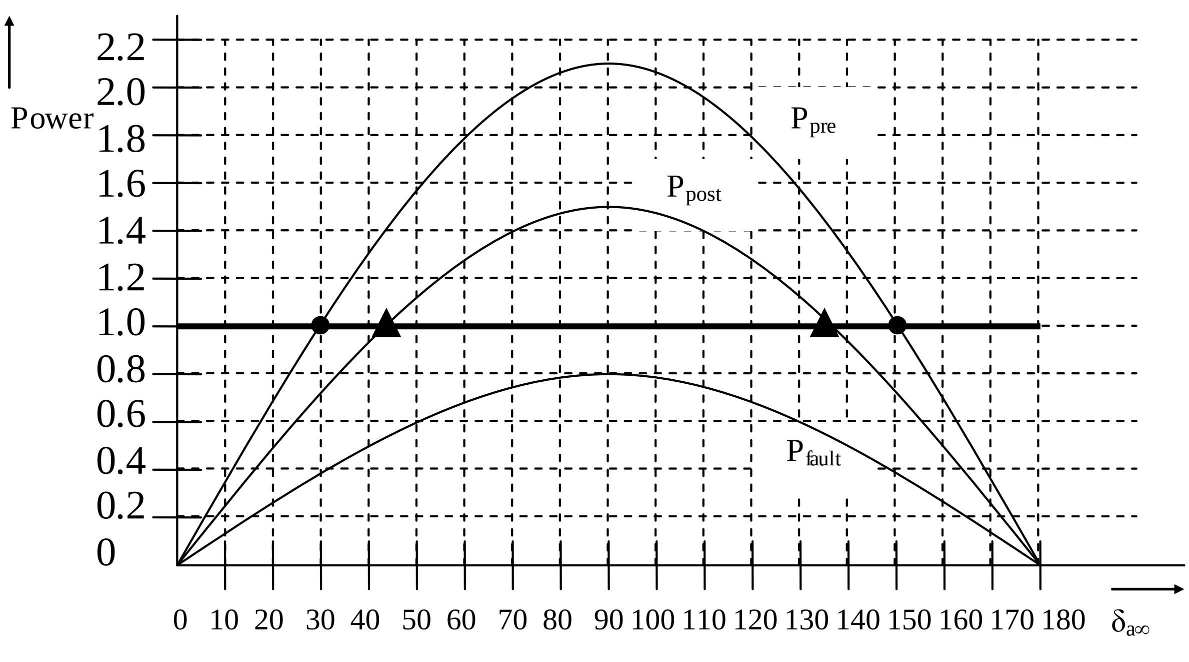


Fig. 14

Again, we see that there are two equlibria.

7. Use the three curves to describe what happens to the angle δ during the three periods: pre-fault, fault-on, and post-fault.

We have the pre-fault equilibrium (at 28.44°) identifying where this systems "starts" (just before and just after being faulted) and the post-fault equilibrium (at 41.81°) identifying where this system "ends" (after fault is cleared and after all transients die out).

Question is: What happens in between these two points in time?

Let's review the sequence, to be clear.

a. Prefault condition.
b. t=0: fault occurs
c. t=4 cycles (typical clearing): fault is cleared
d. t=many seconds: transients die out and system returns to rest.

So we want to know what happens between steps b and d. Figure 15 tells this story.

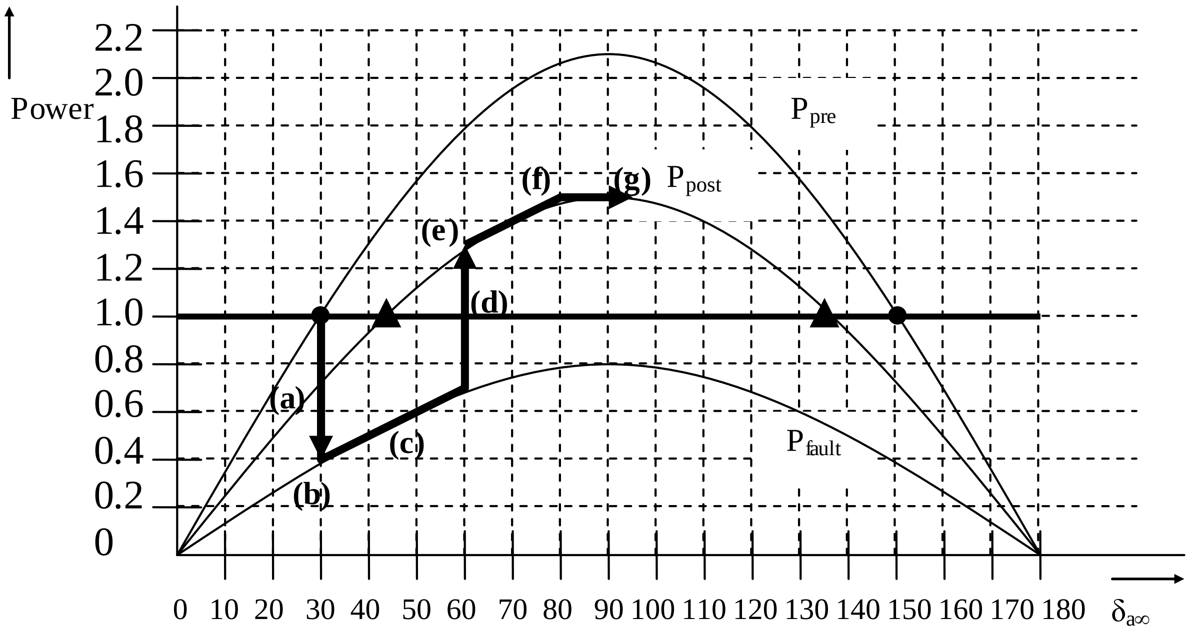


Fig. 15

Each stage of Fig. 15 (a, b, c, d, e, f, g) is described in what follows:

(a) On occurrence of the fault, the electrical power out of the machine immediately drops due to the change in power-angle curves caused by the change in the network (from pre-fault network, Fig. 2, to the fault-on network, Fig. 9). However, because the power angle δ, characterizes the mechanical angle of the rotor, it cannot change instantaneously, and therefore it remains at 28.44° during this transition.

(b) Although the electrical power out of the machine has decreased from 1.0 to about 0.4 pu, the mechanical power into the machine is still 1.0 pu. Therefore the accelerating power $P_a=P_M-P_e$ is no longer zero, rather it is positive (about 0.6), and so the machine begins to accelerate. This means that its rotational velocity begins to change. Whereas before the fault, it had a velocity equal to that of the synchronously rotating reference frame (and so a relative velocity $\dot{\delta}$ of 0), after the fault, due to acceleration, that velocity begins to increase (relative velocity $\dot{\delta}$ increases from 0).

(c) Because the relative velocity $\dot{\delta}$ is positive, the angle δ, which is relative to the reference angle (the infinite bus angle), increases.

(d) At some point in time, let's say 4 cycles after the fault, the protective system causes the breakers at both ends of the faulted circuit to operate and clear the fault. This results in a new network (post-fault, Fig. 12), and correspondingly a new power-angle curve. But because the angle cannot change instantaneously, it remains at the angle it was at the moment just before the fault was cleared. Figure 15 indicates this angle is about 60°.

(e) Now the electrical power out of the machine has increased to about 1.3 pu. But the mechanical power into the machine is still 1.0 pu. Therefore $P_a=P_M-P_e$ is negative (about -0.3), and so the machine begins to decelerate.

(f) Because of the acceleration associated with stage (c), the velocity is still positive and therefore the angle is increasing. But because of deceleration, the velocity is decreasing.

(g) At some point (assuming the behavior is stable), the velocity becomes zero, and because $P_a=P_M-P_e$ is still negative (so machine is still decelerating), the velocity will go negative. When the velocity goes negative, the angle δ begins to decrease.

The next stages (h, i, j, k) are shown in Fig. 16.

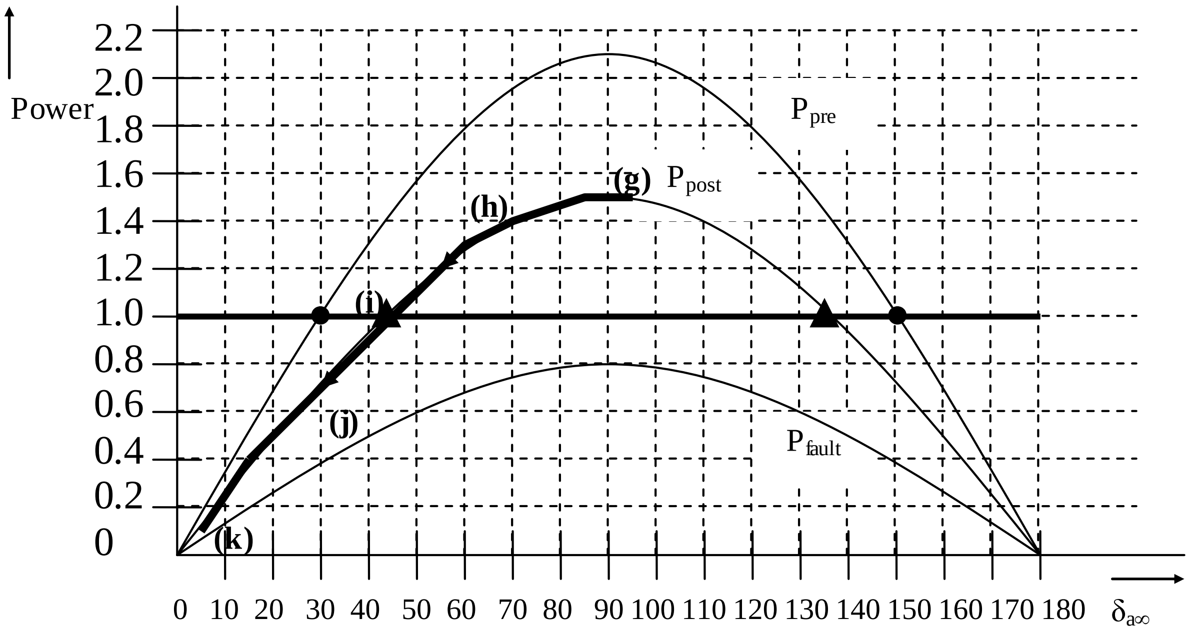


Fig. 16

(h) Acceleration P_a=P_M-P_e is still negative in this stage and therefore velocity continues to decrease (i.e., increase in negative direction), and because velocity is negative, the angle δ continues to decrease.

(i) Here, the accelerating power is 0, since P_M=P_e, but because velocity is still negative, the machine moves through this point.

(j) Now P_a=P_M-P_e is again positive, therefore the machine begins to accelerate again resulting in an increase in velocity. But the velocity is negative, and so acceleration causes this negative velocity to move towards 0. But because the velocity is negative during this stage, the angle δ continues to decrease.

(k) The velocity has reached 0 again, and because $P_a=P_M-P_e$ is still positive, the machine continues to accelerate, and so the velocity becomes positive. As the velocity becomes positive, the angle begins to increase again.

The next stages, represented by (l, m, n), are shown in Fig. 17.

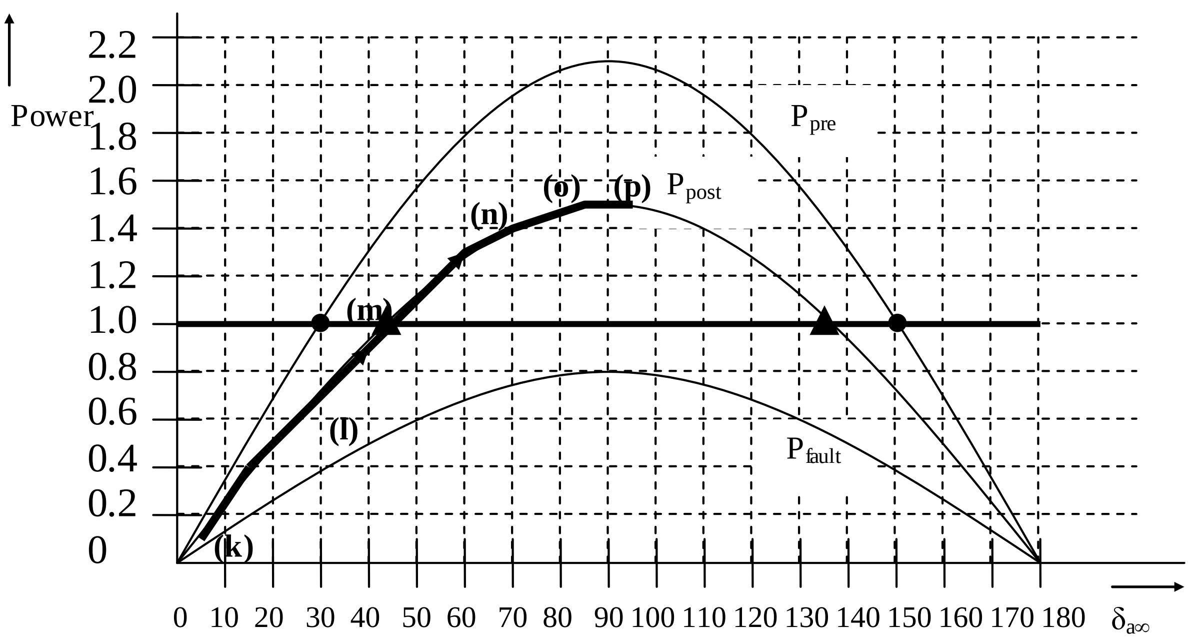


Fig. 17

(l) $P_a=P_M-P_e$ is positive and so the machine continues to accelerate. Velocity is positive, and so angle increases.

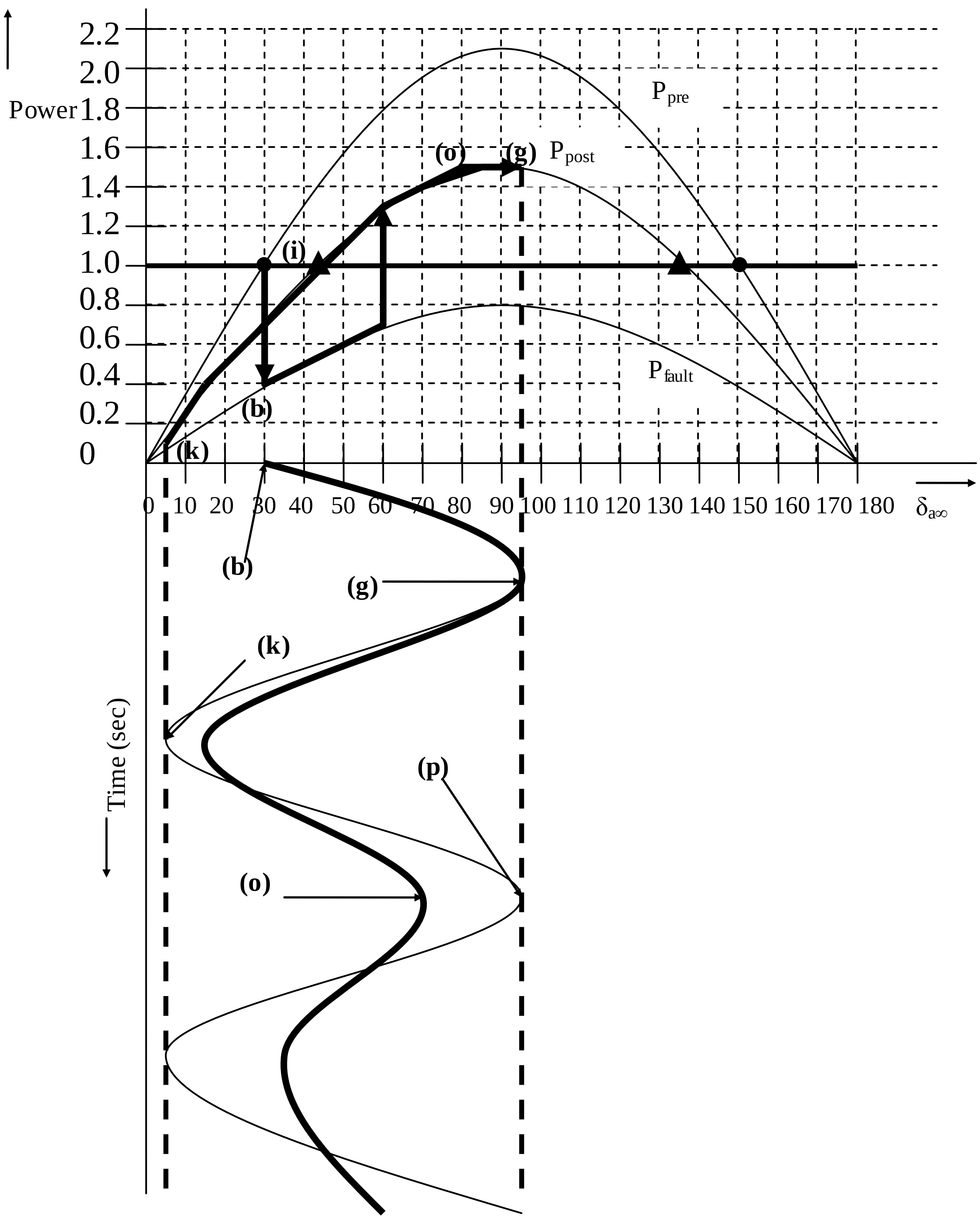
(m) Here, $P_a=P_M-P_e$ is zero ($P_M=P_e$) but because velocity is positive, machine moves through this point.

(n) $P_a=P_M-P_e$ is negative, so machine decelerates. But velocity is positive, so angle continues to increase. At some point, velocity reaches 0, and angle begins to decrease.

(o) If damping is present, the point at which the angle begins to decrease (the "turn-around" point) will occur "before" the turn-around point seen in the previous oscillation.

(p) If damping is not present, the "turn around point" will be the same as the one in the last oscillation (labeled point (p) here but previously labeled (g)).

One last thing with respect to this example. We have plotted power vs. angle, but you should be aware that the angle is actually a function of time. This relationship is conveniently illustrated in Fig. 18, where we also indicate a few stages previously discussed (b, g, i, k).



Power
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
P pre
(o)
(g)
P post
(i)
P fault
(b)
(k)
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180
δ a∞
(b)
(g)
(k)
(p)
(o)
Time (sec)

Fig. 18

In Fig. 18, the curves extending downwards are using the horizontal axis of the power-angle curve (the δ-axis) as the vertical axis of the angle-time curve. There are two angle-time curves shown, and both are associated with stable system behavior. The thin-lined curve shows the angular oscillation following the disturbance if the system has no damping. Such a system oscillates forever. The solid-lined curve shows the angular oscillation following the disturbance if the system has damping. One observes that the amplitude of the oscillations of this curve diminish with time (this is the realistic case).

3.0 Equilibria

We mentioned on page 8 of these notes that there are two equilibria for our pre-fault system, as shown in Fig. 4, which is repeated in Fig. 19, except we have designated the two points x and y. We also indicated that the one at 28.44° (x) is a stable equilibrium, and the one at 180-28.44=151.56° (y) is an unstable equilibrium.

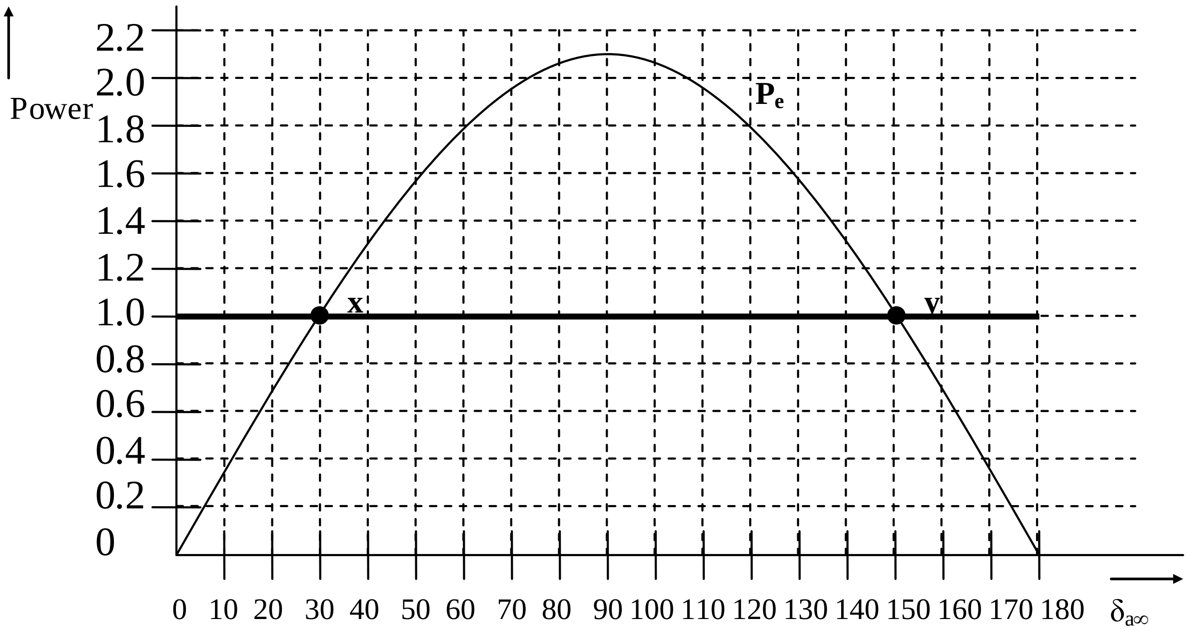


Fig. 19

We can give an analogy here to a ball in a bowl. The stable equilibrium corresponds to when the ball is resting at the bottom of the bowl. The unstable equilibrium corresponds to when the ball is resting on the edge of the ball. This is illustrated in Fig. 20a.

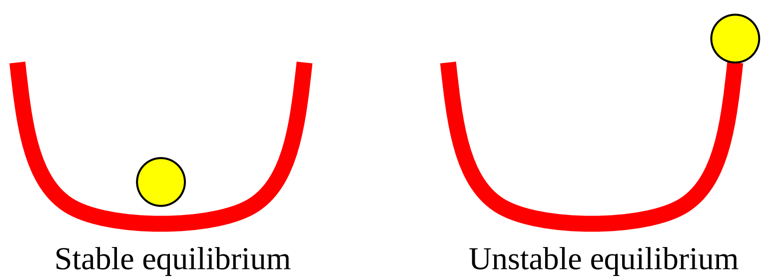


Fig. 20a

It is possible for the ball to be at rest (in equilibrium) in both positions. However, the stable equilibrium is resilient to disturbances. If you shake the table, or if you give the ball a push, the ball in the stable equilibrium will move but then return to the stable equilibrium.

On the other hand, the unstable equilibrium is not resilient to disturbances. If you shake the table, or if you give the ball a push, the ball in the unstable equilibrium will move down the bowl and come to rest at the stable equilibrium, or it will move out of the bowl altogether. If we consider the ball-bowl as a system, the latter movement is analogous to instability.
A metal rod pendulum provides a similar analogy, shown in Fig. 20b [[1]].

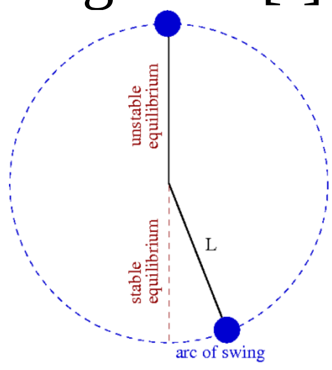


Fig. 20b

We are now in a position to understand why one is a stable equilibrium and one is an unstable equilibrium. Consider perturbing the machine at the stable equilibrium, point x. The perturbation could be a fault, but let's maintain simplicity as much as possible and assume the perturbation is just a small increase in mechanical power P_M. The key point is that the moment just after the perturbation, the mechanical power is greater than 1, say 1.1, whereas the electrical power is still 1, and the machine begins to accelerate. The situation is illustrated in Fig. 21.

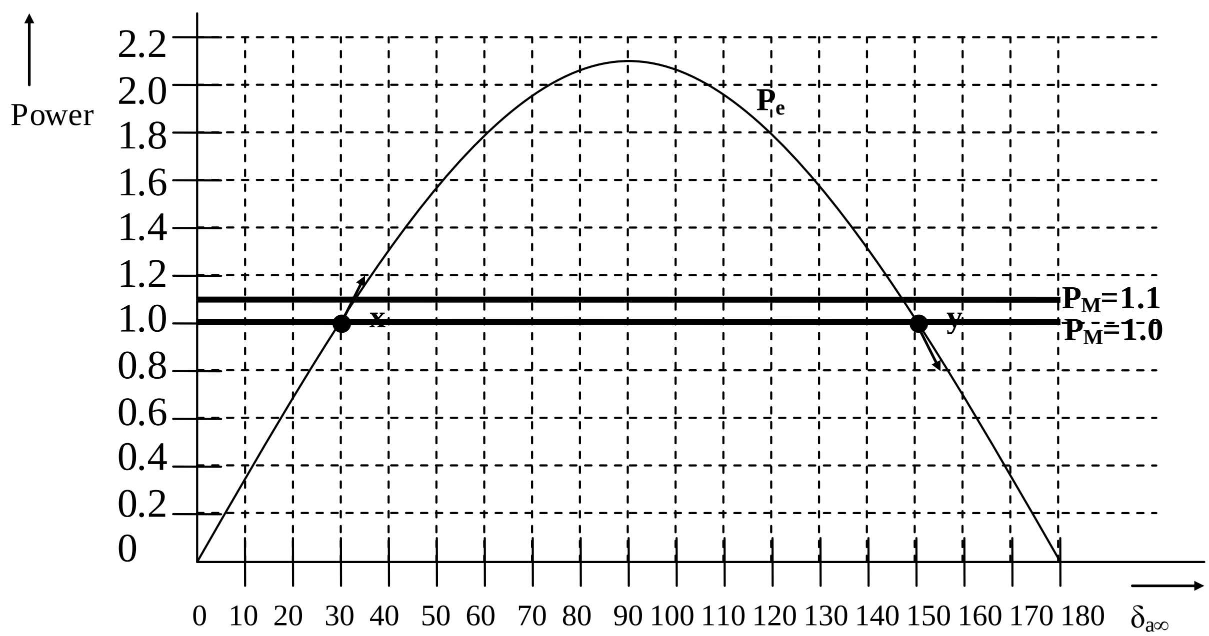


Fig. 21

This means the velocity becomes positive and the angle begins to increase.

Question: As the angle increases (the point begins to move up and to the right, as shown), what happens to the accelerating power?

Answer: $P_a=P_M-P_e$ decreases. This means that the rate of change in velocity is decreasing. Once the point moves above the solid line corresponding to $P_M=1.1$, the machine will begin to decelerate, i.e., rate of change in velocity will go negative and the velocity will begin to decrease.

The fact that the perturbation causes acceleration resulting in motion that inherently decreases that acceleration is the reason this is a stable equilibrium.

One can go through similar logic in relation to the situation when mechanical power is decreased to, say, 0.9. Then the machine decelerates, the angle decreases, and as angle decreases, the decelerating power decreases.

Now let's consider the unstable equilibrium, point y, and let's again assume a small increase in mechanical power P_M to, say, 1.1 pu. Does the angle increase or decrease?
Because $P_a=P_M-P_e$ is positive, the velocity goes positive, resulting in an increase in angle.

Question: As the angle increases (the point begins to move down and to the right, as shown), what happens to the accelerating power?

Answer: $P_a=P_M-P_e$ increases! This means that the rate of change in velocity is increasing. The point does not move above the solid line corresponding to $P_M=1.1$, and so the machine never decelerates. In fact, the accelerating power just continues to increase, equivalent to the ball falling of the edge.

The fact that the perturbation causes acceleration resulting in motion that inherently increases that acceleration is the reason this is an unstable equilibrium.

One can go through similar logic in relation to the situation when mechanical power is decreased to, say, 0.9. Then the machine decelerates, the angle decreases, and as angle decreases, the decelerating power increases. This will continue until point y moves all the way back to point x, the stable equilibrium. This is equivalent to the ball rolling down into the bowl.

[[1]] http://galileospendulum.org/2011/05/31/physics-quanta-pendulums-revisited