Homework #9 Solution

Assignment: 11.8 - 11.12 and 11.16 – 11.18 Bergen & Vittal

Solutions:

11.8

\[ IC_1 = 8.0 + 0.003P_{G_1} \]

\[ IC_2 = 8.0 + 0.001P_{G_2} \]

\[ IC_3 = 7.5 + 0.002P_{G_3} \]

It helps to estimate the common IC graphically and then “fine tune” it by iteration.

Part A).
From the graph we estimate the common IC is approximately 8.1. Calculating the power produced with IC=8.1.

Rearranging the IC equation.

\[ P_{G_1} = \frac{IC_1 - 8}{0.003} = \frac{8.1 - 8}{0.003} = 33.333 \text{ MW} \]
\[ P_{G2} = \frac{IC_2 - 8}{.001} = \frac{8.1 - 8}{.001} = 100 \text{ MW} \]
\[ P_{G3} = \frac{IC_3 - 7.5}{.002} = \frac{8.1 - 7.5}{.002} = 300 \text{ MW} \]
\[ P_G = P_{G1} + P_{G2} + P_{G3} = 33.33 + 100 + 300 = 433.33 \text{ MW} \]

This is 66.66 too low! Try IC = 8.15
\[ P_{G1} = 50 \text{ MW} \]
\[ P_{G2} = 150 \text{ MW} \]
\[ P_{G3} = 325 \text{ MW} \]
\[ P_G = 525 \text{ MW} \] This is too high! After a few more iterations in a similar fashion we find IC = 8.136.
\[ P_{G1} = 46.33 \text{ MW} \]
\[ P_{G2} = 136 \text{ MW} \]
\[ P_{G3} = 318 \text{ MW} \]

Note: this is .66 MW off, but well within tolerance

Calculate costs:
\[ C_1(46.33) = 300 + 8.0(46.33) + .0015(46.33)^2 = 665.75 / \text{hr} \]
\[ C_2(136) = 450 + 8.0(136) + .0005(136)^2 = 1547.25 / \text{hr} \]
\[ C_3(318) = 700 + 7.5(318) + .001(318)^2 = 3186.12 / \text{hr} \]
\[ C_{total}(500) = 665.75 + 1547.25 + 3186.12 = 5399.12 / \text{hr} \]

**Part B.**
From the graph we estimate IC approximately 8.4. After a few iterations we find IC=8.409.

Using the same equations from Part A the corresponding production levels are:
\[ P_{G1} = 136.33 \text{ MW} \]
\[ P_{G2} = 409 \, MW \]
\[ P_{G3} = 454.5 \, MW \]

\[ C_1(136.33) = $1418.55/hr \]
\[ C_2(409) = 3805.64/hr \]
\[ C_3(454.5) = $4315.32/hr \]
\[ C_{total}(1000) = $9539.51/hr \]

**Part C).**

From the graph we estimate IC approximately 9.0. After a few iterations we find IC=8.955.

Using the same equations from Part A the corresponding production levels are:

\[ P_{G1} = 318.33 \, MW \]
\[ P_{G2} = 955 \, MW \]
\[ P_{G3} = 727.5 \, MW \]

\[ C_1(318.33) = $2998.67/hr \]
\[ C_2(955) = $8546.01/hr \]
\[ C_3(727.5) = $6685.51/hr \]
\[ C_{total}(2000) = $18230.19/hr \]
11.9
Part A).

\[ P_{G1} = P_{G2} = P_{G3} = \frac{500}{3} = 166.67 MW \]

\[ C_1(166.67) = 300 + 8.0(166.67) + .0015(166.67)^2 = \$1675/ hr \]

\[ C_2(166.67) = 450 + 8.0(166.67) + .0005(166.67)^2 = \$1797.22/ hr \]

\[ C_3(166.67) = 700 + 7.5(166.67) + .001(166.67)^2 = \$1977.78/ hr \]

\[ C_{total}(500) = 1675 + 1797.22 + 1977.78 = \$5450/ hr \]

This is \$50.88/hr more than 7.2a

Part B).

\[ P_{G1} = P_{G2} = P_{G3} = \frac{1000}{3} = 333.33 MW \]

\[ C_{total}(1000) = \$9616.67/ hr \]

This is \$77.16/hr more than 7.2b

Part C).

\[ P_{G1} = P_{G2} = P_{G3} = \frac{2000}{3} = 666.67 MW \]

\[ C_{total}(2000) = \$18450/ hr \]

This is \$219.18/hr more than 7.2c
11.10
Adding generator limits to the graph in 11.8

Part A).
In 11.8 when Pd was equal to 500 MW, Pg1 was lower than 50 MW.
Therefore in this problem we know that Pg1 will be pegged at its lower limit 50 MW.

\[ P_{G1} = 50 \text{ MW} \]

Then:
\[ P_{G2} + P_{G3} = 500 - 50 = 450 \text{ MW} \]

We can solve directly as follows:

\[ 8.0 + 0.001 P_{G2} = 7.5 + 0.002(450 - P_{G2}) \Rightarrow P_{G2} = 133.33 \]
\[ P_{G3} = 450 - 133.33 = 316.67 \text{ MW} \]

\[ C_{total}(500) = $5408.59/hr \]

Part B).
We estimate all three generators are away from their limits. Therefore we perform the problem in the exact same manner as 11.8. From the graph we estimate IC approximately 8.4. After a few iterations we find IC=8.409.

Using the same equations from Part A the corresponding production levels are:

\[
P_{G1} = 136.33 \text{ MW} \\
P_{G2} = 409 \text{ MW} \\
P_{G3} = 454.5 \text{ MW}
\]

\[
C_1 (136.33) = $1418.55 / hr \\
C_2 (409) = 3805.64 / hr \\
C_3 (454.5) = $4315.32 / hr \\
C_{total} (1000) = $9539.51 / hr
\]

**Part C.**

In 11.8 we found the Pg2 was over its upper limit. Therefore we peg Pg2 at its upper limit and solve for Pg1 and Pg3 as we did in Part A.

\[
P_{G2} = 800 \text{ MW}
\]

Then:

\[
P_{G1} + P_{G2} = 2000 - 800 = 1200 \text{ MW}
\]

We can solve directly as follows:

\[
8.0 + .003P_{G1} = 7.5 + .002(1200 - P_{G1}) \implies P_{G1} = 380 \\
P_{G3} = 1200 - 380 = 820 \text{ MW}
\]

\[
C_{total} (2000) = $18249 / hr
\]

In cases a and c (with limits imposed) it costs more than problem 11.8.
11.11
For this problem we use the graph from 11.10 to determine the “cut in” and “cut out” points of the three generators

<table>
<thead>
<tr>
<th>IC</th>
<th>Pd</th>
<th>Regulating gens (gens not at a limit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.6</td>
<td>150</td>
<td>G3</td>
</tr>
<tr>
<td>8.05</td>
<td>375</td>
<td>G2,G3</td>
</tr>
<tr>
<td>8.15</td>
<td>525</td>
<td>G1,G2,G3</td>
</tr>
<tr>
<td>8.8</td>
<td>1716.67</td>
<td>G1,G3</td>
</tr>
<tr>
<td>9.2</td>
<td>2050</td>
<td>G3</td>
</tr>
<tr>
<td>9.5</td>
<td>2200</td>
<td>G3</td>
</tr>
</tbody>
</table>
11.12
We first assume that there are no generators at their limits. Performing the same steps as problem 11.10b. We iterate to find \( IC = 8.85 \).

The corresponding power levels are:

\[
\begin{align*}
P_{G_1} &= 283.33 \text{ MW} \\
P_{G_2} &= 850 \text{ MW} \quad \text{This is over P} \text{g}2 \text{ 800 MW max!} \\
P_{G_3} &= 675 \text{ MW}
\end{align*}
\]

Now we set \( P_{G_2} = 800 \) and perform the problem with the same steps as 11.10c.

\[
P_{G_2} = 800 \text{ MW}
\]

Then:

\[
P_{G_1} + P_{G_3} = 1800 - 800 = 1000 \text{ MW}
\]

We can solve directly as follows:

\[
\begin{align*}
8.0 + 0.003P_{G_1} &= 7.5 + 0.002(1000 - P_{G_1}) \Rightarrow P_{G_1} = 300 \\
P_{G_3} &= 1000 - 300 = 700 \text{ MW}
\end{align*}
\]

Check G1 and G3 limits: OK!

Use any equation to solve for IC

\[
IC = 8.0 + 0.003P_{G_1} = 8 + 0.003 \times 300 = \$8.9/\text{hr additional cost}
\]
There are numerous ways to verify the approximation here’s one:

\[ P_L = |I|^2 \frac{R}{Z} = \frac{(V_1 - V_2)(V_1 - V_2)^*}{|Z|^2} R = \frac{|V_1|^2 + |V_2|^2 - V_1V_2^* - V_2V_1^*}{|Z|^2} R \]

Introducing \(|V_1| = |V_2|\)

\[ P_L = 2|V_1|^2 (1 - \cos(\theta_{12})) \frac{R}{|Z|^2} \approx |V_1|^2 \theta_{12} \frac{R}{|Z|^2} \]

\[ S_{12} = \frac{|V_1|^2}{|Z|} e^{j\theta_{12}} (1 - e^{j\theta_{12}}) = \frac{|V_1|^2}{|Z|^2} Z(1 - e^{j\theta_{12}}), \text{ then} \]

\[ P_{12} = \frac{|V_1|^2}{|Z|^2} [R (1 - \cos \theta_{12}) + X \sin \theta_{12}] \approx \frac{|V_1|^2}{|Z|^2} X \theta_{12} \]

Eliminating \(\theta_{12}\), we find:

\[ P_L = \frac{R}{|V_1|^2} P_{12}^2 \]
\[ \frac{\partial P_L}{\partial P_{G2}} = 0.016(P_{G2} - 100) = 0.016P_{G2} - 16 \]

Using (11.52), we find the penalty factors

\[ L_1 = 1.0, \quad L_2 = \frac{1}{1.16 - 0.0016P_{G2}} \]

Applying the appropriate optimal dispatch rule

\[ L_1 \frac{\partial C_1}{\partial P_{G1}} = 0.002P_{G1} + 7.0 = \lambda \Rightarrow P_{G1} = \frac{\lambda - 7.0}{0.002} \]

\[ L_2 \frac{\partial C_2}{\partial P_{G2}} = \frac{0.002P_{G2} + 7.0}{1.16 - 0.0016P_{G2}} = \lambda \Rightarrow P_{G2} = \frac{1.16\lambda - 7.0}{0.002 + 0.0016\lambda} \]

Iterating lambda we get
\[ \lambda = 7.551 \]

\[ P_{G1} = 275.5 \text{ MW} \]

\[ P_{G2} = 124.93 \text{ MW} \]

\[ P_L = 0.497 \]

\[ P_{G1} + P_{G2} - P_L = 349.9 \text{ OK!!!} \]

\[ \Delta C_t = \lambda P_D = $7.55/\text{hr} \]
11.18
We have:
\[2P_{G2} - .0008(P_{G2} - 100)^2 = 400\]

Solving the quadratic we get:
\[P_{G2} = 204.356\]
\[P_{G2} = P_{G1}\]
\[P_L = 8.712\]

\[IC_i = .002P_{Gi} + 7.0 \Rightarrow C_i = \alpha + 7.0P_{Gi} + .001P_{Gi}^2\]

With \(P_{g1}=P_{g2}=204.356\) MW:
\[C_T = 2\alpha + 2944.51\]

With \(P_{g1}=275.5; P_{g2}=124.93\) MW:
\[C_T = 2\alpha + 2894.52\]

The savings are $49.99/hour.