Homework #7 Solution

Assignment: 11.1 through 11.6 Bergen & Vittal

Solutions:

11.1

\[ \Delta P_M = -\frac{1}{R} \Delta \omega \]

Modified Equation 11.6 because gen. speed not fed back

\[ \Delta \omega = -R \times \Delta P_M = -(0.01 \text{ rad/MW sec})(100 \text{ MW}) = -1 \text{ rad/sec} \]

\[ \Delta \text{speed} = (-1) \left( \frac{1}{2\pi} \right)(60) = -9.55 \text{ r.p.m.} \]

The new speed is therefore: \(3600 + (-9.55) = 3590.45 \text{ r.p.m.}\)

11.2 (Copied directly from class notes)

For the isolated generating station with local load shown in Fig. 1 below, it is observed that \(\Delta P_L=0.1\) brings about \(\Delta \omega=-0.2\) in the steady-state.

(a) Find 1/R.

Solution:

We need the transfer function between \(\Delta \omega\) and \(\Delta P_L\). To get this, write down \(\Delta \omega\) as a function of what is coming into it:

\[ \begin{align*}
\Delta \omega &= \frac{1}{s+1} + \frac{10}{1+10s} \\
&= \frac{1}{s+1} + \frac{10}{1+10s}
\end{align*} \]
\[ \Delta \hat{\omega} = \frac{10}{1 + 10s} \left[ \left( \frac{1}{s + 1} \right) \left( \Delta \hat{P}_C - \frac{1}{R} \Delta \hat{\omega} \right) + - \Delta \hat{P}_L \right] \]

Now solve for \( \Delta \omega \). Expanding:

\[ \Delta \hat{\omega} = \frac{10}{1 + 10s} \left[ \left( \frac{1}{s + 1} \right) \left( \Delta \hat{P}_C - \frac{1}{R} \Delta \hat{\omega} \right) - \Delta \hat{P}_L \right] \]

\[ = \Delta \hat{P}_C \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) - \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) \left( \frac{1}{R} \Delta \hat{\omega} \right) - \frac{10 \Delta \hat{P}_L}{1 + 10s} \]

Bringing terms in \( \Delta \omega \) to the left-hand-side:

\[ \Delta \hat{\omega} + \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) \frac{1}{R} \Delta \hat{\omega} = \Delta \hat{P}_C \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) - \frac{10 \Delta \hat{P}_L}{1 + 10s} \]

Factoring \( \Delta \omega \):

\[ \Delta \hat{\omega} \left[ 1 + \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) \frac{1}{R} \right] = \Delta \hat{P}_C \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) - \frac{10 \Delta \hat{P}_L}{1 + 10s} \]

Dividing:

\[ \Delta \hat{\omega} = \frac{\Delta \hat{P}_C \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) - \frac{10 \Delta \hat{P}_L}{1 + 10s}}{1 + \frac{10}{1 + 10s} \left( \frac{1}{s + 1} \right) \frac{1}{R}} \]

Multiply through by \((1+10s)(s+1)\):

\[ \Delta \hat{\omega} = \frac{10 \Delta \hat{P}_C - 10 \Delta \hat{P}_L (s + 1)}{(1 + 10s)(s + 1) + \frac{10}{R}} \]
Rearrange the top and expand the bottom:

\[
\Delta \hat{\omega} = \frac{10\Delta \hat{P}_C - 10(s+1)\Delta \hat{P}_L}{10s^2 + 11s + (1+10/R)}
\]

(\*)

Now we consider \( \Delta P_C=0, \Delta P_L=0.1 \), and assume it is a step change. Therefore:

\[
\Delta \hat{P}_L = \frac{\Delta P_L}{s}
\]

Substituting into (\*), we get:

\[
\Delta \hat{\omega} = \frac{-10(s+1)\Delta P_L}{10s^2 + 11s + (1+10/R) s}
\]

The above expression is a LaPlace function (i.e., in \( s \)). The problem gives data for the steady-state (in time). We may apply the final-value theorem to the above expression to obtain:

\[
\Delta \omega = \lim_{t \to \infty} \Delta \omega(t)
\]

\[
= \lim_{s \to 0} s\Delta \hat{\omega} = \lim_{s \to 0} s \frac{-10(s+1)\Delta P_L}{10s^2 + 11s + (1+10/R) s}
\]

\[
= \lim_{s \to 0} \frac{-10(s+1)\Delta P_L}{10s^2 + 11s + (1+10/R)} = \frac{-10\Delta P_L}{1+10/R}
\]

that is,

\[
\Delta \omega = \frac{-10\Delta P_L}{1+10/R}
\]

Solving for \( R \), we obtain:
\[ \frac{10}{R} = \frac{-10 \Delta P_L}{\Delta \omega} - 1 = \frac{-10 \Delta P_L - \Delta \omega}{\Delta \omega} \]

\[ R = \frac{-10 \Delta \omega}{10 \Delta P_L + \Delta \omega} \]

Substituting \( \Delta P_L = 0.1 \) and \( \Delta \omega = -0.2 \), we obtain:

\[ R = \frac{-10(-0.2)}{10(0.1) + (-0.2)} = 2.5 \]

The problem was specified with power in per-unit and \( \Delta \omega \) in rad/sec. Reference to the block diagram indicates that the left-hand-side summing junction outputs \( \Delta P_C - \Delta \omega/R \). To make this sum have commensurate units, it must be the case that \( R \) has units of (rad/sec)/pu power.

The problem asks for \( 1/R \), which would be \( 1/2.5 = 0.4 \) pu power/(rad/sec).

One might also express \( R \) and \( 1/R \) in units of pu frequency/pu power. This would be:

\( R_{pu} = 2.5/60 = 0.0417 \)
\( 1/R_{pu} = 24 \)

Recalling the NERC specification that all units should have \( R = 0.05 \), then this \( R \) should be adjusted upwards.

**Question:** What does an \( R = 0.0417 \) mean relative to an \( R = 0.05 \)?

**Answer:** Recalling that \( R_{pu} = -\Delta \omega_{pu} / \Delta P_{m,pu} \), we can say that \( R_{pu} \) expresses the steady-state frequency deviation, as a percentage of 60 Hz, for which the machine will move by an amount equal to its full rating. So:

- if \( R_{pu} = 0.05 \), then the steady-state frequency deviation for which the machine will move by an amount equal to its full rating is \( 0.05 * 60 = 3 \text{hz} \).
- if \( R_{pu} = 0.0417 \), then the steady-state frequency deviation for which the machine will move by an amount equal to its full rating is \( 0.0417 * 60 = 2.502 \text{hz} \).

**(b)** Specify \( \Delta P_C \) to bring \( \Delta \omega \) back to zero (i.e., back to the steady-state frequency \( \omega = \omega_0 \)).
Solution:

Recalling eq. (*):

\[
\Delta \hat{\omega} = \frac{10\Delta \hat{P}_C - 10(s + 1)\Delta \hat{P}_L}{10s^2 + 11s + (1 + 10/R)}
\] (*)

Now we have that

\[
\Delta \hat{P}_C = \frac{\Delta P_C}{s}
\]

and \(\Delta P_L=0\). In this case, eq. (*) becomes:

\[
\Delta \hat{\omega} = \frac{10\Delta P_C / s}{10s^2 + 11s + (1 + 10/R)}
\]

Applying the final value theorem again:

\[
\Delta \omega = \lim_{t \to \infty} \Delta \omega(t)
\]

\[
= \lim_{s \to 0} s\Delta \hat{\omega} = \lim_{s \to 0} s \frac{10\Delta P_C / s}{10s^2 + 11s + (1 + 10/R)}
\]

\[
= \lim_{s \to 0} \frac{10\Delta P_C}{10s^2 + 11s + (1 + 10/R)} = \frac{10\Delta P_C}{1 + 10/R}
\]

that is,

\[
\Delta \omega = \frac{10\Delta P_C}{1 + 10/R}
\]

Solving for \(\Delta P_C\), we get:

\[
\Delta P_C = \frac{\Delta \omega(1 + 10/R)}{10}
\]
Having already computed $1/R=0.4$ in part (a), and with $\Delta \omega = -0.2$, we have

$$\Delta P_C = \frac{-0.2(1+10/2.5)}{10} = -0.1$$

which indicates that for this load increase of 0.1 which results (from primary speed control) in a frequency deviation of -0.2 rad/sec, we need to adjust the speed-changer motor to increase plant output by 0.1 pu in order to correct the steady-state frequency deviation back to 0.

The change to the speed-changer motor would be accomplished by the supplementary control.

### 11.3

**Using the hint provided in the class notes:**

**Hint on Problem 11.3:** At the bottom of page 390, the text says: “The reader is invited to check that with $K_{Pi} = 1/\widetilde{D}_i$ and $T_{Pi} = M_i / \widetilde{D}_i$, Figure 11.10 represents (11.22) in block diagram form. In Figure 11.10 we have $\Delta \omega_i$ as an output and $\Delta P_{Mi}$ as an input and can close the power control loop by introducing the turbine-governor block diagram shown in Figure 11.4.”

This will result in the following block diagram.

![Block Diagram](image)

**Solution:**

$$K_{Pi} = 1/\widetilde{D}_i = 1/10^{-2} = 100$$
From the above figure, using the block diagram relations, we can derive the following equation:

\[ \Delta \hat{\omega} = \frac{K_p}{1 + sT_p} \cdot \frac{1}{1 + \left[ \frac{K_p}{1 + sT_p} \right] \left[ \frac{1}{1 + sT_G} \right] \left[ \frac{1}{1 + sT_T} \right] R} \left( \frac{-1}{s} \right) \]

Putting in known values:

\[ \Delta \hat{\omega} = \frac{100}{1 + 20s} \cdot \frac{1}{1 + \left[ \frac{100}{1 + 20s} \right] \left[ \frac{1}{1 + s} \right] \left( .4 \right)} \left( \frac{-1}{s} \right) \]

To simplify multiply the top and bottom by: \([(1+20s)(1+s)]/20. To get:

\[ \Delta \hat{\omega} = \frac{-5(s + 1)}{s\left( s^2 + \frac{21}{20}s + \frac{41}{20} \right)} \]

Now simplify the denominator so we can perform partial fraction expansion on it. This one is a bit ugly, on homework an easy approach is to set the denominator inside the parentheses equal to zero and solve on your calculator to solve for the roots.

\[ \Delta \hat{\omega} = \frac{-5(s + 1)}{s(s + .525 + j1.332)(s + .525 - j1.332)} \]

After partial fraction expansion we get:
\[ \Delta \hat{\omega} = \frac{A}{s} + \frac{K}{(s + 0.525 + j1.332)} + \frac{K^*}{(s + 0.525 - j1.332)} \]

Solve for A and K using whatever method you desire (i.e. Heavyside’s “Cover Up” Method)

\[ A = -2.439, \quad K = 1.852 \angle -48.863^\circ \]

Finally, to convert to the time-domain we take the Laplace transform which yields:

\[ \Delta \omega(t) = -2.439 + 3.705e^{-525t} \cos(1.332t - 48.863^\circ) \]

### 11.4

First we redraw the figure from 11.3 adding in the tie line. I also slightly changed where \( \Delta \omega \) is defined (note: looks different, but really is same as 11.3) for simplification purposes.
Δδ = \frac{100}{s + 10\left(\frac{100}{1 + 20s}\right)} = \frac{100}{20s^2 + s + 1000}

With the dashed area reduced now to a single “block” the problem can now be solved in almost the exact same fashion as 11.3

Δ\hat{ω} = -\frac{100}{1 + \left[\frac{100}{20s^2 + s + 1000}\right] \left[\frac{1}{(1 + s)}\right]} \cdot (.4)(s)

To simplify multiply the top and bottom by: \([(20s^2+s+1000)(1+s)]/20. \ To get:

Δ\hat{ω} = -\frac{5(s + 1)}{s^3 + 1.05s^2 + 52.05s + 50}

Factor the bottom into 3 terms:

Δ\hat{ω} = \frac{5(s + 1)}{(s + .962)(s + .044 + j7.21)(s + .044 - j7.21)}

After partial fraction expansion we get:

Δ\hat{ω} = \frac{A}{s + .962} + \frac{K}{(s + .044 + j7.21)} + \frac{K^*}{(s + .044 - j7.21)}

Once again use your favorite method to solve for A and K

A = .0036, \ K = -.347 \angle 90.29^\circ

Finally, to convert to the time-domain we take the Laplace transform which yields:
\[ \Delta \omega(t) = -0.036e^{-962t} - 0.694e^{-0.044t} \cos(7.21t - 90.29^\circ) \]

As compared with 11.3
- Amplitude is approximately the same
- Frequency is larger
- Damping is smaller
- s.s. error = 0

11.5
Following Example 11.2, we have:

\[ \frac{\Delta \omega}{\omega_o} = \frac{\Delta f}{f_o} = \frac{0.015}{60} = -0.05\Delta P_A \]

Where \( \Delta P_A \) is in per unit

Solving for \( \Delta P_A \) we get:

\[ \Delta P_A = \frac{0.015}{60 \times 0.05} = 0.005 \text{ pu} \Rightarrow 0.005 \times 250 \text{ MW} = 1.25 \text{ MW} \]

Similarly:

\[ \Delta P_B = \frac{0.015}{60 \times 0.05} = 0.005 \text{ pu} \Rightarrow 0.005 \times 400 \text{ MW} = 2.0 \text{ MW} \]

11.6

\[ \Delta \omega = \frac{-\Delta P_{L1}}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2} = \frac{-0.2}{\frac{1}{0.01} + \frac{1}{0.02} + 0.8 + 1} = -0.00131752 \text{ pu} \]

\[ f_{new} = 60 - 0.00131752(60) = 59.92 \text{ Hz} \]

\[ \Delta P_{tie} = \Delta \omega \left( \frac{1}{R_2} + D_2 \right) = -0.00131752 \left( \frac{1}{0.02} + 1 \right) = -0.0671935 \text{ pu} \]

\[ \Delta P_{tie} = -0.0671935 \times 500 = -33.59 \text{ MW} \]