EE 457 Homework #2 Solution

1. Consider the two-bus system shown in Fig. 11. The two generators and transformers are assumed of equal rating – 300 MVA – which is the 3-phase base power for all pu unit data given in what follows.

- Line has series reactance of 0.20 pu
- Pre-fault bus voltage magnitudes are both 1.0 pu.
- The generators are sharing the total real power load equally.
- Assume that the prefault bus voltage at bus 1 is the reference (i.e., has 0 degree phase angle).
- The transformers both have leakage reactance of 0.12 pu.
- Both generators have subtransient reactance of 0.1 pu.
  a. For the pre-fault conditions, compute the pu real power consumed by each load, the pu real power delivered by each generator, the power angle δ, and the pu reactive power delivered by each generator.
  b. Compute the prefault currents into each load.
  c. Compute the fault current for a symmetric three-phase fault occurs on bus 1, with fault impedance Z_f=0.
  d. Compare the fault current computed in (c) with the pre-fault load currents computed in (b).

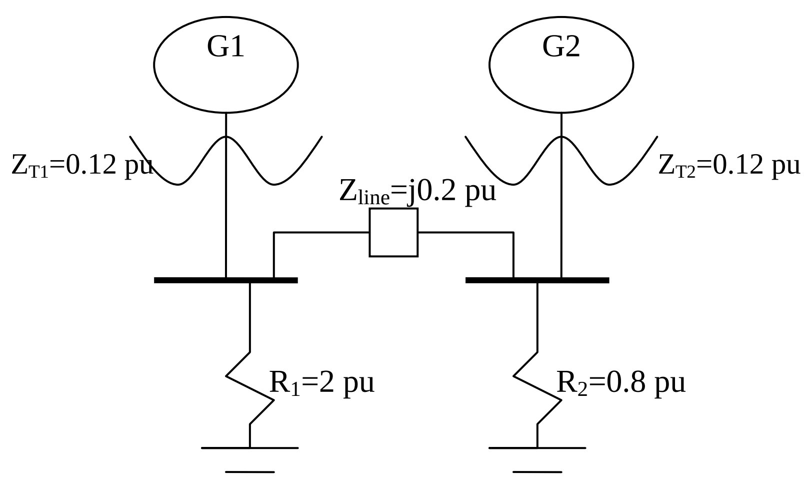


Fig. 11

Solution:

A).

$$P = \frac{|V|^2}{R}$$

$$P_{LoadR1} = \frac{|1|^2}{2} = .5\,pu$$

$$P_{LoadR2} = \frac{|1|^2}{.8} = 1.25\,pu$$

$$P_{G1} = P_{G2} = \frac{1.25 + .5}{2} = .875\,pu$$

The line will therefore carry 1.25 -.875 = .375pu from bus 1 → 2.

$$P_{12} = \frac{|V_1||V_2|}{X}\sin(\delta) = \frac{1}{.2}\sin(\delta) = .375\,pu$$

$$\delta = \sin^{-1}(.375 * .2) = 4.301°$$

$$Q_{12} = Q_1 = -Q_2 = \frac{|V_1|^2}{X} - \frac{|V_1||V_2|}{X}\cos(\delta) = \frac{1}{.2} - \frac{1}{.2}\cos(4.301°) = .014082\,pu$$

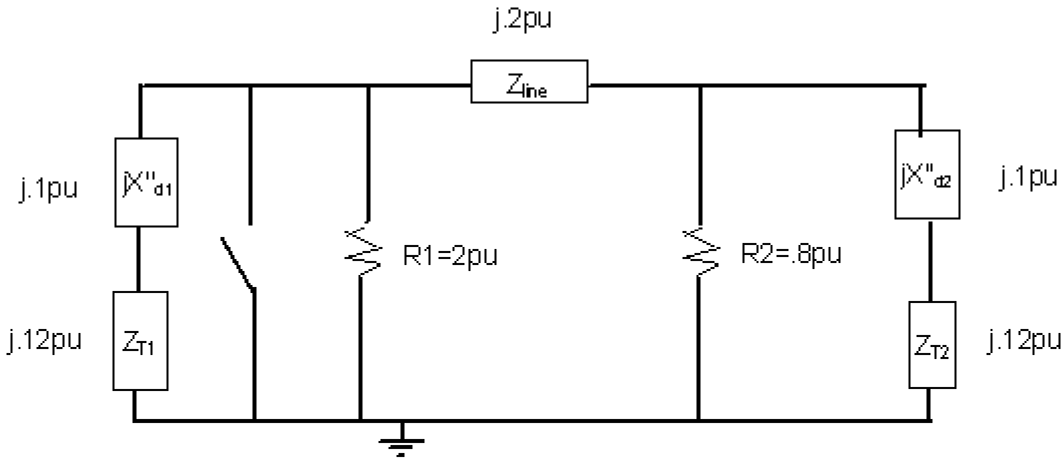
B). Pre-Fault Currents through each load

$$I_{L1} = \frac{V_1}{R_1} = \frac{1}{2} = .5\angle 0°\,pu$$

$$I_{L2} = \frac{V_2}{R_2} = \frac{1\angle -4.301°}{.8} = 1.25\angle -4.301°\,pu$$

C). A Fault Occurs at Bus 1

Find the Thevenin Equivalent using the steps described in the class notes.



$$Z_{thev} = [[[(jX''_{d2} + Z_{t2}) \| R2] + Z_{line}] \| R1] \| (jX''_{d1} + Z_{t1})$$

$$Z_{thev} = (.017 + j.1412)\,pu$$

$$I''_f = \frac{V_f}{Z_f + Z_{thev}} = \frac{1}{0 + .17 + j.1412} = 7.02\angle -83.15^\circ\, pu$$

D). There are a couple of things one notices when comparing the prefault current to the fault current:

1. The prefault current phasors are predominantly in phase with the voltage.
2. The short-circuit give rise to current changes having their predominant components lagging the voltage by approximately 90 degrees.
3. The magnitude of the fault current far exceeds the prefault.

2. The one-line diagram of a three-bus power system is shown in Fig. 12. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in pu on a common 100 MVA base. Determine the fault current, the bus voltages, and the line currents *during the fault* when a balanced three-phase fault with fault impedance Z_f=j0.16 pu occurs on bus 1. Assume that all pre-fault bus voltages are 1.0 pu.

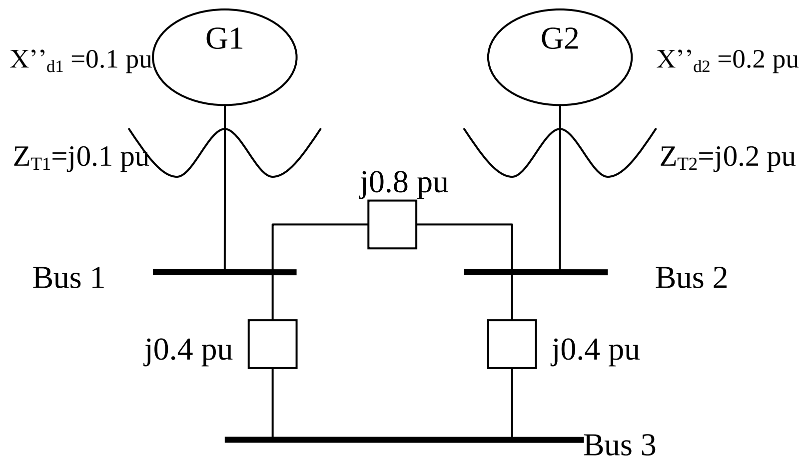
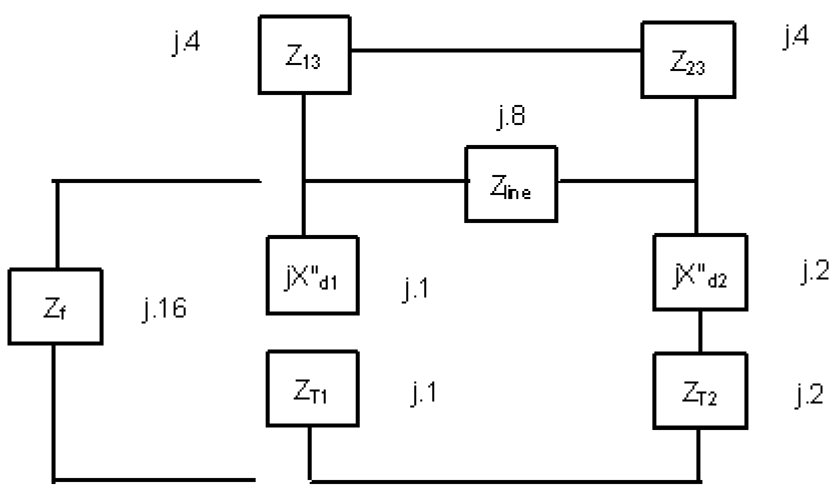


Fig. 12

Solution:

The first step is to find the Thevenin Equivalent Impedance. To do so we first redraw the circuit.

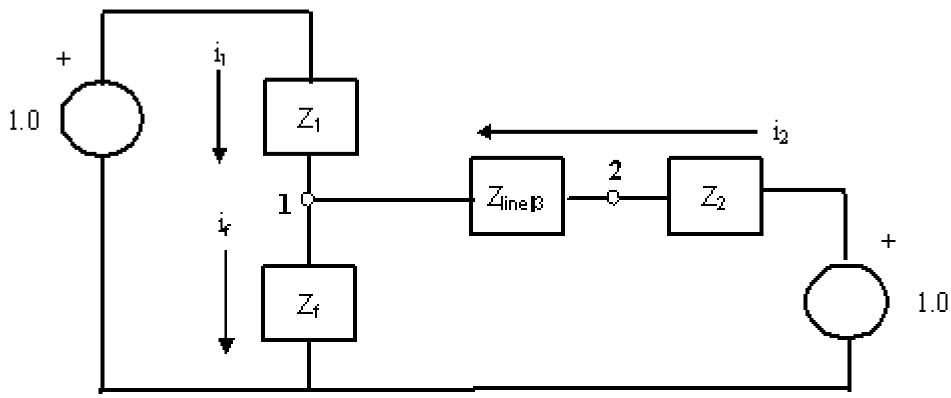


$$Z_{thev} = [[(Z_{13} + Z_{23}) \| Z_{line}] + jX''_{d2} + Z_{T2}] \| (jX''_{d1} + Z_{T1})$$

$$Z_{thev} = j.16 pu$$

$$I''_f = \frac{V_f}{Z_f + Z_{thev}} = \frac{1}{j.16 + j.16} = 3.125\angle -90°$$

Next we'll find the branch current. To start we simplify and redraw the circuit to first find only the current through each generator.



$$Z_1 = j.1 + j.1 = j.2$$

$$Z_2 = j.2 + j.2 = j.4$$

$$Z_{line\|3} = (j.4 + j.4) \| j.8 = j.4$$

Now that we know the fault current we can solve for the current through Gen1 and Gen2 using KVL.

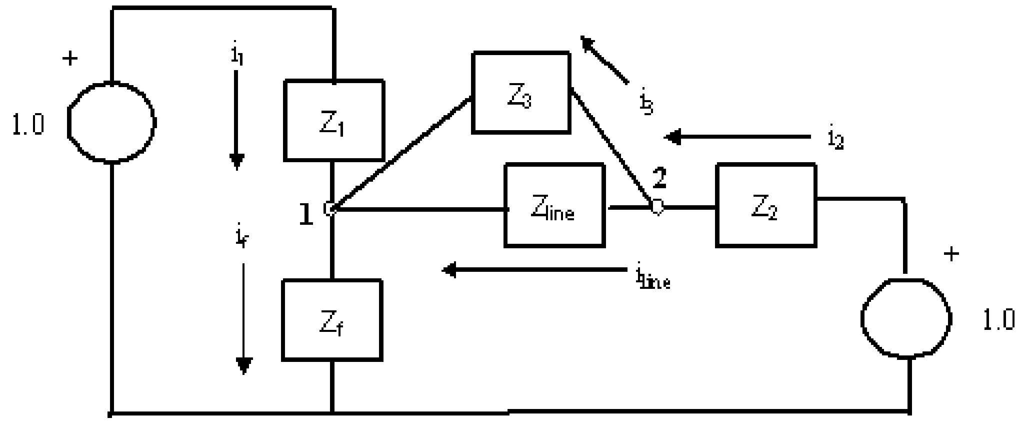
$$1.0 - i_1(j.2) - i_f(j.16) = 0$$

$$1.0 - i_2(j.4 + j.4) - i_f(j.16) = 0$$

$$i_1 = 2.5\angle -90° pu$$

$$i_2 = .625\angle -90° pu$$

We can again redraw the circuit to find the current through the line and the Bus 3 loop



Since Z_3 (j.4+j.4) is equal to Z_{line} we know the i_2 will simply split between them. Therefore:

$$i_3 = i_{line} = .3125\angle -90^\circ pu$$

We can now compute the bus voltages

$$V_1 = i_1 Z_1 = (2.5\angle -90^\circ)(j.2) = .5\angle 0^\circ pu$$

Use node voltage to find the other voltages.

$$\frac{V_2 - 1}{j.4} + \frac{V_2 - V_1}{j.8} + \frac{V_2 - V_3}{j.4} = 0$$

$$\frac{V_3 - V_2}{j.4} + \frac{V_3 - V_1}{j.4} = 0$$

2 Equations, 2 Unknowns….

$$V_2 = .75\angle 0^\circ pu$$

$$V_3 = .625\angle 0^\circ pu$$