

# EDC2

## 1.0 Introduction

In the previous set of note (EDC1), we developed the EDC problem with losses included and provided the corresponding KKT conditions which lead to its solution procedure. However, in the example that was given, we neglected losses. Now we want to repeat that example, except this time we will include losses. Comparison between the example of these notes and the example of the previous notes will provide insight into the effect of including losses on (a) the solution procedure and (b) the solution itself. In addition, we would like to gain from this effort some intuition in regards to what the penalty factors actually mean.

## 2.0 Problem statement, solution procedure

Recall that the EDC problem with losses is

$$\text{Min } C_T = \sum_{i=1}^m C_i(P_{Gi})$$

Subject to

$$\sum_{i=1}^m P_{Gi} - P_L(P_{G2}, \dots, P_{Gm}) = P_D$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad \forall \quad i = 1, \dots, m$$

The first step to solving this problem is to apply the KKT conditions ignoring the inequality constraints:

$$\frac{\partial F}{\partial P_{Gi}} = \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} - \lambda \left[ 1 - \frac{\partial P_L(P_{G2}, \dots, P_{Gm})}{\partial P_{Gi}} \right] = 0, \quad \forall i = 1, \dots, m \quad (1)$$

$$\frac{\partial F}{\partial \lambda} = \sum_{i=1}^m P_{Gi} - P_L(P_{G2}, \dots, P_{Gm}) - P_D = 0 \quad (2)$$

We found that eq. (1) can be expressed by

$$\lambda = L_i \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} \quad \forall i = 1, \dots, m \quad (3)$$

The basic lambda-iteration solution procedure is the same as the case without losses.

This means that, once we compute the penalty factors, we can proceed as in Fig. 1.

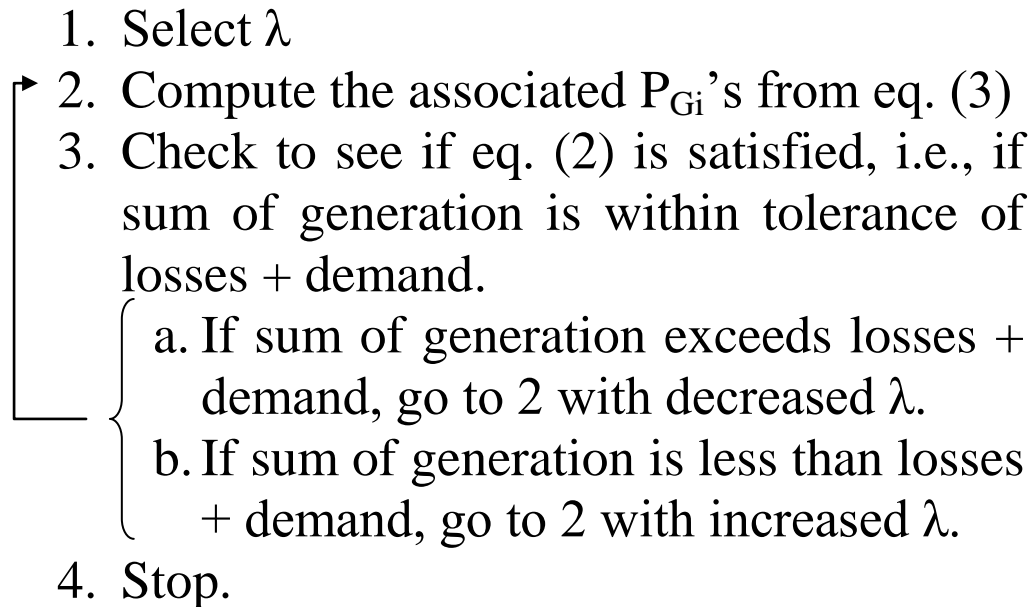
1. Select  $\lambda$
  2. Compute the associated  $P_{Gi}$ 's from eq. (3)
  3. Check to see if eq. (2) is satisfied, i.e., if sum of generation is within tolerance of losses + demand.
    - a. If sum of generation exceeds losses + demand, go to 2 with decreased  $\lambda$ .
    - b. If sum of generation is less than losses + demand, go to 2 with increased  $\lambda$ .
  4. Stop.
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Fig. 1: Algorithm A

However, there is one problem with the Algorithm A:

The penalty factors must be computed to implement the above solution procedure, but for a given loading condition, they depend on the dispatch, which is the solution. Thus, what we need to get the solution depends on having the solution!

One thing we can do here is to add another loop to Algorithm A. This loop will update

the penalty factors after each iteration of Algorithm A, and then repeat Algorithm A. The new algorithm is shown in Fig. 2.

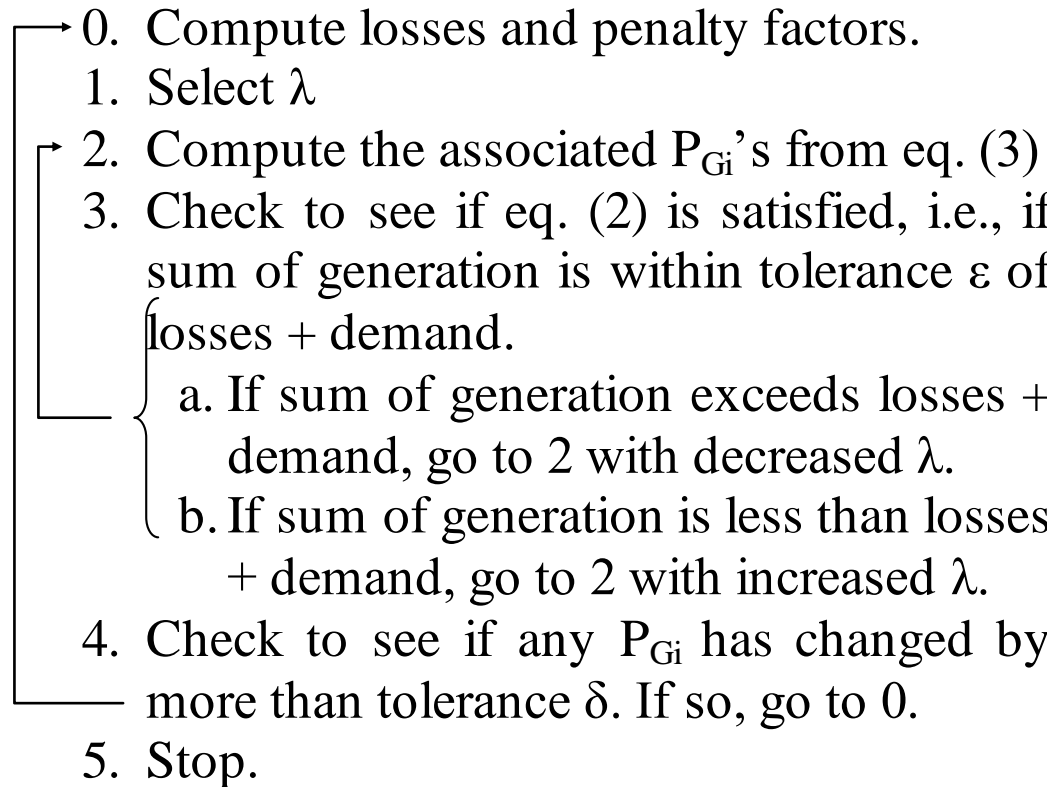


Fig. 2: Algorithm B

One last comment.... In Algorithm B, step 0 requires computation of losses and penalty factors. This step requires the  $P_{Gi}$ 's. Therefore, the first step of the algorithm requires that we guess the solution. One can simply use engineering judgment in making this guess, but a better way is to use an EDC calculation without losses.

### 3.0 Example

We again turn to examples 11.8 and 11.9 in the text where the fuel-cost curves are given as:

$$C_1(P_{G1}) = 900 + 45P_{G1} + 0.01P_{G1}^2 \quad (4)$$

$$C_1(P_{G2}) = 2500 + 43P_{G2} + 0.003P_{G2}^2 \quad (5)$$

As in the previous notes, we will first solve the problem with a load of 600 MW rather than 700 MW. Generator limits are given as

$$50MW \leq P_{G1} \leq 200MW \quad (6)$$

$$50MW \leq P_{G1} \leq 600MW \quad (7)$$

In addition, we now specify a loss function:

$$P_L(P_{G2}) = 0.0002(P_{G2} - 525)^2 \quad (8)$$

derived based on the system of Fig. 3.

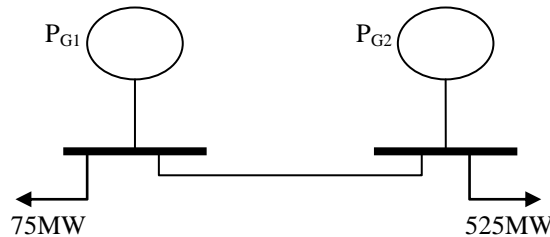


Fig. 3

One observes that the loss function is 0 if  $P_{G2}=525$  MW. This is consistent with Fig. 3 since with  $P_{G2}=525$  MW, the 525 MW load at the right-hand-side bus causes zero flow in the line, and therefore zero losses. But if  $P_{G2}$  is greater (or lesser) than 525 MW, there will be flow on the line from (to) the right-hand-bus, and therefore some losses will exist.

The penalty factors are given by:

$$L_i = \frac{1}{\left[1 - \frac{\partial P_L(P_{G2}, \dots, P_{Gm})}{\partial P_{Gi}}\right]} \quad (9)$$

Applying eq. (9) to eq. (8), we have:

$$L_1 = 1$$

$$L_2 = \frac{1}{\left[1 - \frac{\partial}{\partial P_{G2}} 0.0002(P_{G2} - 525)^2\right]} = \frac{1}{[1 - 0.0004(P_{G2} - 525)]}$$

We observe that the unit 1 penalty factor is 1, but the unit 2 penalty factor will change depending on the dispatch.

Let's apply our Algorithm B. To do so, we need a starting solution. We will use the solution from EDC without losses, which was obtained in the previous notes. For the 600 MW case, which is our interest here, we got  $P_{G1}=61.64$  MW,  $P_{G2}=538.46$  MW,  $\lambda=46.23$ .

Step 0: Compute losses and penalty factors:

$$P_L(P_{G2}) = 0.0002(P_{G2} - 525)^2$$

$$= 0.0002(538.46 - 525)^2 = 0.03623MW$$

$$L_1 = 1$$

$$L_2 = \frac{1}{[1 - 0.0004(538.46 - 525)]} = 1.0054$$

Step 1: We will use  $\lambda=46.23$  which comes from the EDC solution without losses.

Step 2: To obtain the  $P_{Gi}$ 's, we need eq. (3):

$$\lambda = (1)(45 + 0.02P_{G1})$$

$$\lambda = (1.0054)(43 + 0.006P_{G2})$$

Solving each of these for  $P_{Gi}$ , we obtain:

$$P_{G1} = \frac{\lambda - 45}{0.02}$$

$$P_{G2} = \frac{\lambda / 1.0054 - 43}{0.006}$$

Using our guess for  $\lambda$  of 46.23, we get:

$$P_{G1} = \frac{46.23 - 45}{0.02} = 61.5$$

$$P_{G2} = \frac{46.23 / 1.0054 - 43}{0.006} = 496.94$$

Step 3: So our power balance equation is

$$P_{G1} + P_{G2} = 61.5 + 496.94 = 558.4$$

$$P_D + P_L = 600 + 0.03623 = 600.03623$$

Clearly we are too low, so let's increase  $\lambda$ .

Step 1: Try  $\lambda = 46.4$ .

Step 2:

$$P_{G1} = \frac{46.4 - 45}{0.02} = 70$$

$$P_{G2} = \frac{46.4 / 1.0054 - 43}{0.006} = 525.13$$



Step 3: So our power balance equation is

$$P_{G1}+P_{G2}=70+525.13=595.13$$

$$P_D+P_L=600+0.03623=600.03623$$

So we are still a little too low so we need to increase  $\lambda$ .

Step 1: We can use linear interpolation to obtain a new guess for  $\lambda$  according to:

$$\frac{46.4 - 46.23}{595.13 - 558.4} = \frac{\lambda - 46.4}{600.03623 - 595.13} \Rightarrow \lambda = 46.4227$$

Step 2:

$$P_{G1} = \frac{46.4227 - 45}{0.02} = 71.135$$

$$P_{G2} = \frac{46.4227 / 1.0054 - 43}{0.006} = 528.89$$

Step 3: So our power balance equation is

$$P_{G1}+P_{G2}=71.135+528.89=600.025$$

$$P_D+P_L=600+0.03623=600.03623$$

So we are almost perfect. We could do another iteration at this point, but it will be better to recompute losses and penalty factors.

Step 0: Compute losses and penalty factors:

$$P_L(P_{G2}) = 0.0002(P_{G2} - 525)^2$$
$$= 0.0002(528.89 - 525)^2 = 0.0030MW$$

$$L_1 = 1$$

$$L_2 = \frac{1}{[1 - 0.0004(528.89 - 525)]} = 1.0061$$

Step 1: Use the  $\lambda=46.4227$  from the last iteration.

Step 2:

$$P_{G1} = \frac{46.4227 - 45}{0.02} = 71.135$$

$$P_{G2} = \frac{46.4227/1.0061 - 43}{0.006} = 523.5397$$

Step 3: So our power balance equation is

$$P_{G1} + P_{G2} = 71.135 + 523.5397 = 594.6747$$

$$P_D + P_L = 600 + 0.003 = 600.003$$

So need to increase  $\lambda$ .

Step 1: Try 46.46.

Step 2:

$$P_{G1} = \frac{46.46 - 45}{0.02} = 73.0$$

$$P_{G2} = \frac{46.46/1.0061 - 43}{0.006} = 529.7$$

Step 3: So our power balance equation is

$$P_{G1} + P_{G2} = 73 + 529.7 = 602.7$$

$$P_D + P_L = 600 + 0.003 = 600.003$$

So need to decrease  $\lambda$ .

Step 1: We can use linear interpolation to obtain a new guess for  $\lambda$  according to:

$$\frac{46.46 - 46.4}{602.7 - 594.6747} = \frac{46.46 - \lambda}{602.7 - 600.003} \Rightarrow \lambda = 46.4398$$

Step 2:

$$P_{G1} = \frac{46.4398 - 45}{0.02} = 71.99$$

$$P_{G2} = \frac{46.4398/1.0061 - 43}{0.006} = 526.3725$$

Step 3: So our power balance equation is

$$P_{G1} + P_{G2} = 71.99 + 526.3725 = 598.3625$$

$$P_D + P_L = 600 + 0.003 = 600.003$$

So need to increase  $\lambda$ .

Step 1: We can use linear interpolation to obtain a new guess for  $\lambda$  according to:

$$\frac{46.46 - 46.4398}{602.7 - 598.3625} = \frac{\lambda - 46.4398}{600.003 - 598.3625} \Rightarrow \lambda = 46.4474$$

Step 2:

$$P_{G1} = \frac{46.4474 - 45}{0.02} = 72.37$$

$$P_{G2} = \frac{46.4474/1.0061 - 43}{0.006} = 527.6314$$

Step 3: So our power balance equation is

$$P_{G1} + P_{G2} = 72.37 + 527.6314 = 600.0014$$

$$P_D + P_L = 600 + 0.003 = 600.003$$

This is probably close enough. Solution is

$$P_{G1} = 72.37 \text{ MW}, P_{G2} = 527.63 \text{ MW}, \lambda = 46.23.$$

Recall the solution without losses was

$$P_{G1} = 61.64 \text{ MW}, P_{G2} = 538.46 \text{ MW}, \lambda = 46.23.$$

But losses are only 0.003 MW!

Why are solutions so different when the losses are almost 0?

Answer: Because in the solution with losses, the minimization of cost-rate required not only using the most economic units but also that we minimize losses so we would not need to supply as much demand. Therefore, the solution attempted to find the right tradeoff between the economics of the units, and it attempted to move  $P_{G2}$  close to 525 MW.